# The impact of the slow solution in the winds of massive stars

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Collaborators:
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Ignacio Araya, Universidad de Valparaíso

# History

Late ‘60: first UV spectral observations

**ROCKET OBSERVATIONS OF MASS LOSS FROM HOT STARS***

DONALD C. MORTON

*Princeton University Observatory, Princeton, N.J., U.S.A.*

**Abstract.** Rocket observations have shown that the far-ultraviolet resonance lines have P-Cygni profiles in the spectra of many hot stars, including of and Wolf-Rayet stars and OB supergiants. Velocity shifts as high as $-3000$ km $sec^{-1}$ have been measured for the short-wavelength edges of some of the lines. Estimates of the rates of mass loss range from $10^{-8}$ to $10^{-6}$ $M_\odot$ $year^{-1}$.

# History

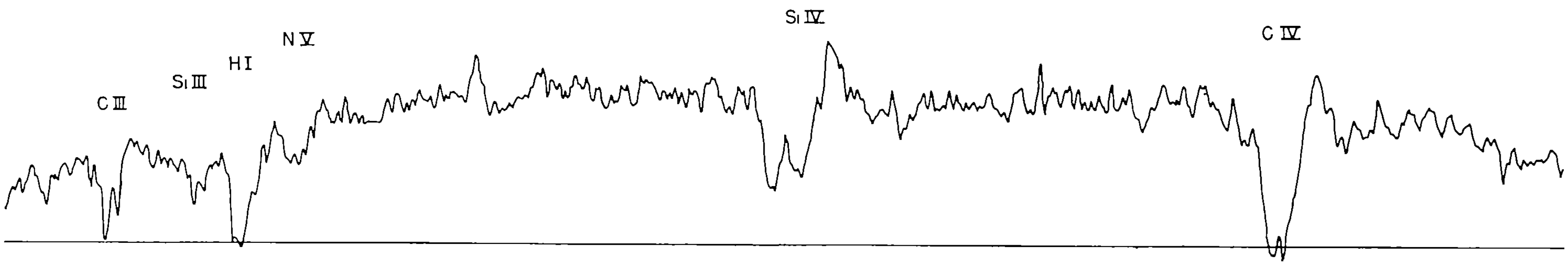


Fig. 1. Densitometer tracing, on an intensity scale, of the far-ultraviolet spectrum of ζ Orionis, photographed by Princeton on September 10, 1966. The distribution of intensity with wavelength includes the unknown response of the spectrograph. Wavelengths increase towards the right from 1140 to 1630 Å. The H I line is interstellar, but all the other identified absorption features are circumstellar with large Doppler shifts to shorter wavelengths.



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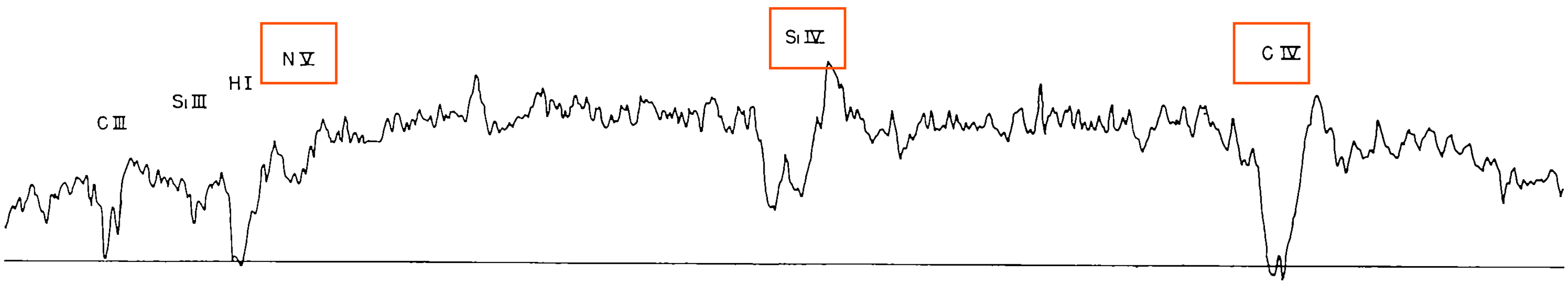


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# Theory

Only known Theory:

Parker's Model for the Solar Wind (Parker, E.N.: 1960, ApJ 132, 821)

For O stars => Teff = $10^7$ K

But at this Temperature C IV - N V - Si IV Don´t Exist

Destroyed by collisional ionization



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# Radiation Driven Winds

**Lucy & Solomon** (1970, ApJ, 159, 870): Wind driven by resonance lines
Obtained only mass loss rates of about 1/100th of the observed values

**Castor, Abbott & Klein** (1975, ApJ, 195, 157)
Wind driven by an ensemble of lines (scattering)
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## The Standard Model (m-CAK)

# 1D - Hydrodynamics

Assumptions:
Stationary - Low viscosity - Spherical symmetry - No Mag. Fields.

From Mass and Momentum Conservation laws:



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$$4\pi r^2 \rho v = \dot{M},$$

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# Line Force: Very Simplified Version

Contribution by **one** line i at frequency $\nu_i$

$$g^i_{rad} = \frac{photon\ momentum\ absorbed\ by\ line\ i}{time\ mass}$$



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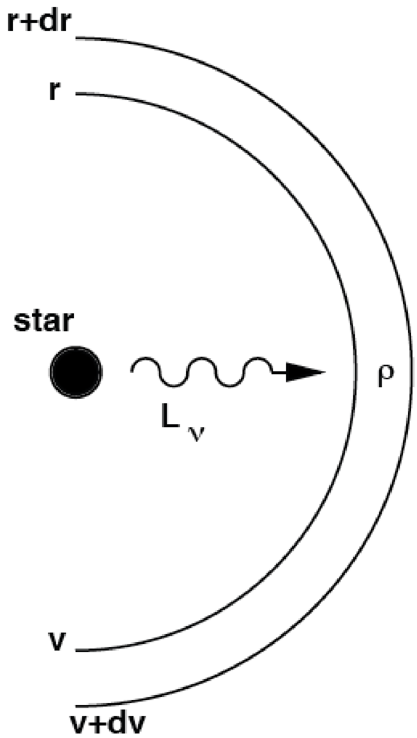




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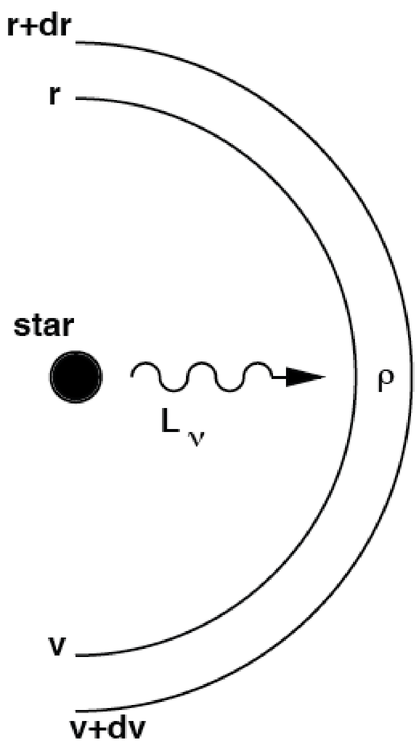


$$dm = 4\pi r^2 \rho dr$$




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# Line Force: Very Simplified Version

Contribution by **one** line i at frequency $\nu_i$

$$\frac{L}{c}\frac{L_{\nu_i}(1-e^{-\tau_i})d\nu^{\mathrm{WIDTH}}}{L} = \frac{L}{c^2}\frac{\nu_i L_{\nu_i}}{L}(1-e^{-\tau_i})dv.$$



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total photon
momentum rate
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fraction absorbed by one spectral line in an outer shell of thickness dr

# Line Force: Very Simplified Version

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optical thickness

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total photon momentum rate provided by the star

fraction absorbed by one spectral line in an outer shell of thickness dr

$$d\nu^{\mathrm{WIDTH}} = \nu_i \frac{dv}{c}$$

dv/dr

$$dm = 4\pi r^2 \rho dr$$



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# Line Force: Super Simplified Version

Contribution by **one** line i at frequency $\nu_i$

Dependence on the Velocity gradient

$$g_{rad}^{Th}(r) = \frac{n_e \sigma_e L}{c 4\pi r^2 \rho}$$

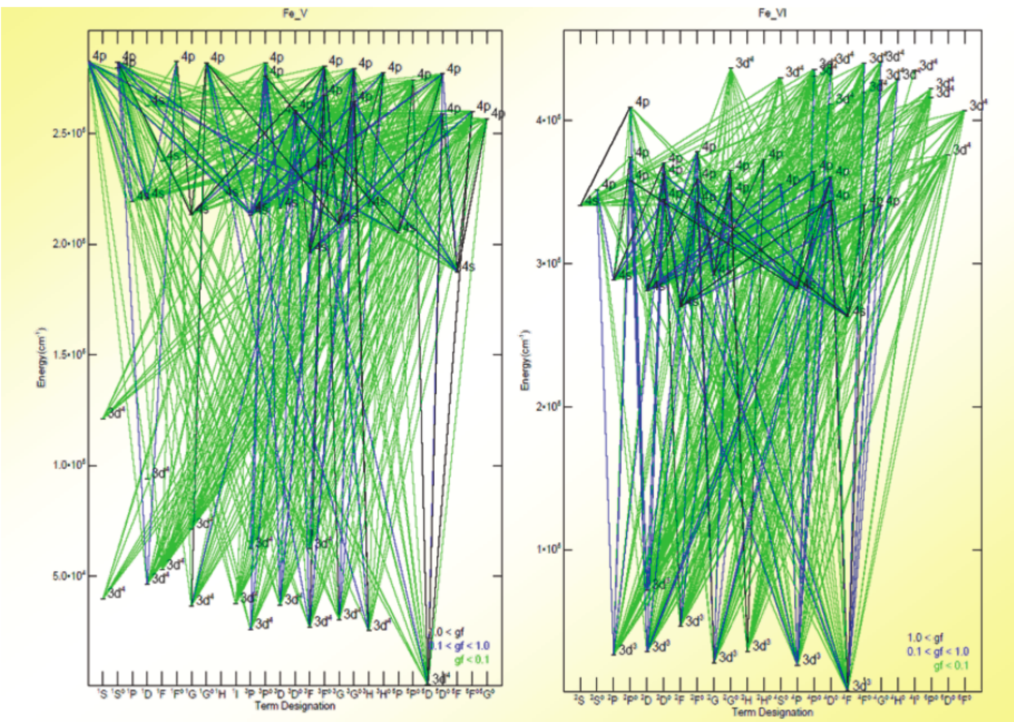
$$g_{rad}^{i} = g_{rad}^{Th} \, \frac{1}{c} \, \frac{\nu_i L_{\nu_i}}{L} \, \left(1 - e^{-\tau_i}\right) \, \frac{1}{n_e \sigma_e} \, \frac{dv}{dr}$$

FORCE MULTIPLIER

# Line Force

Contribution from an ensemble of lines

Currently: 4.2 Mega lines, 150 ionization stages (H –Zn),

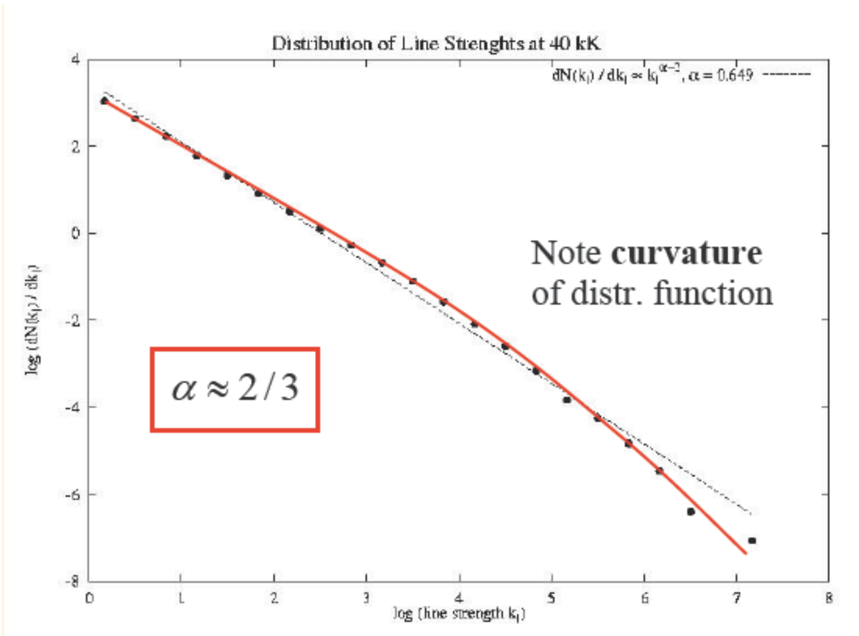



| | I | II | III | IV | V | VI | VII | VIII |
|---|---|---|---|---|---|---|---|---|
| 1 | H_I | | | | | | | |
| 2 | He_I | He_II | | | | | | |
| 6 | C_I | C_II | C_III | C_IV | C_V | | | |
| 7 | N_I | N_II | N_III | N_IV | N_V | N_VI | | |
| 8 | O_I | O_II | O_III | O_IV | O_V | O_VI | | |
| 9 | F_I | F_II | F_III | F_IV | F_V | F_VI | | |
| 10 | Ne_I | Ne_II | Ne_III | Ne_IV | Ne_V | Ne_VI | | |
| 11 | Na_I | Na_II | Na_III | Na_IV | Na_V | Na_VI | | |
| 12 | Mg_I | Mg_II | Mg_III | Mg_IV | Mg_V | Mg_VI | | |
| 13 | Al_I | Al_II | Al_III | Al_IV | Al_V | Al_VI | | |
| 14 | Si_I | Si_II | Si_III | Si_IV | Si_V | Si_VI | | |
| 15 | P_I | P_II | P_III | P_IV | P_V | P_VI | | |
| 16 | S_I | S_II | S_III | S_IV | S_V | S_VI | S_VII | |
| 17 | Cl_I | Cl_II | Cl_III | Cl_IV | Cl_V | Cl_VI | | |
| 18 | Ar_I | Ar_II | Ar_III | Ar_IV | Ar_V | Ar_VI | Ar_VII | ArVIII |
| 19 | K_I | K_II | K_III | K_IV | K_V | K_VI | | |
| 20 | Ca_I | Ca_II | Ca_III | Ca_IV | Ca_V | Ca_VI | | |
| 22 | Ti_I | Ti_II | Ti_III | Ti_IV | Ti_V | | | |
| 23 | V_I | V_II | V_III | V_IV | V_V | | | |
| 24 | Cr_I | Cr_II | Cr_III | Cr_IV | Cr_V | Cr_VI | | |
| 25 | Mn_I | Mn_II | Mn_III | Mn_IV | Mn_V | Mn_VI | | |
| 26 | Fe_I | Fe_II | Fe_III | Fe_IV | Fe_V | Fe_VI | Fe_VII | FeVIII |
| 27 | Co_I | Co_II | Co_III | Co_IV | Co_V | Co_VI | Co_VII | |
| 28 | Ni_I | Ni_II | Ni_III | Ni_IV | Ni_V | Ni_VI | Ni_VII | NiVIII |
| 29 | Cu_I | Cu_II | Cu_III | Cu_IV | Cu_V | Cu_VI | | |
| 30 | Zn_I | Zn_II | Zn_III | | | | | |

atomic data status:

| excellent | good | poor | bad |
|---|---|---|---|

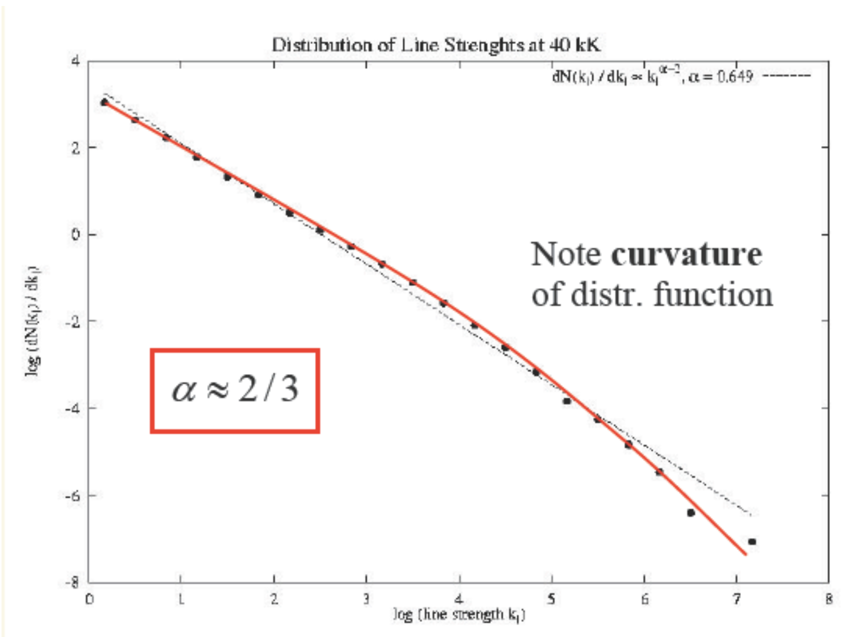
# Line Force




Logarithmic plot of line-strength distribution function for an O-type wind at 40,000 K and corresponding power-law fit
(Puls et al. 2000, A&AS 141)

# Line Force

$$n(k,\nu)\,d\nu\,dk \propto k^{\alpha-2}\,dk$$

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Mass-loss

Ensemble of lines

$$g^{\text{line}} = \frac{C}{r^2}\left(r^2 v \frac{dv}{dr}\right)^{\alpha}$$



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# CAK & m-CAK Models

Castor, Abbott & Klein (1975), Abbott (1982), Friend & Abbott (1986), Pauldrach et al. (1986)



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$$g^{\text{line}} = \frac{C}{r^2} CF\left(r, v, \frac{dv}{dr}\right) \left(r^2 v \frac{dv}{dr}\right)^{\alpha} \left[\frac{n_E}{W(r)}\right]^{\delta}$$

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Eigenvalue
(Mass-loss)

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Changes in Ionization



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Correction Factor

Changes in Ionization



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# Radiation Driven Wind Hydrodynamics



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Mass Conservation $\longrightarrow$ $4\pi r^2 \rho v = \dot{M}$



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Mass Conservation → $4\pi r^2 \rho v = \dot{M}$

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Inertia

Gas Pressure

Gravity

Line Force



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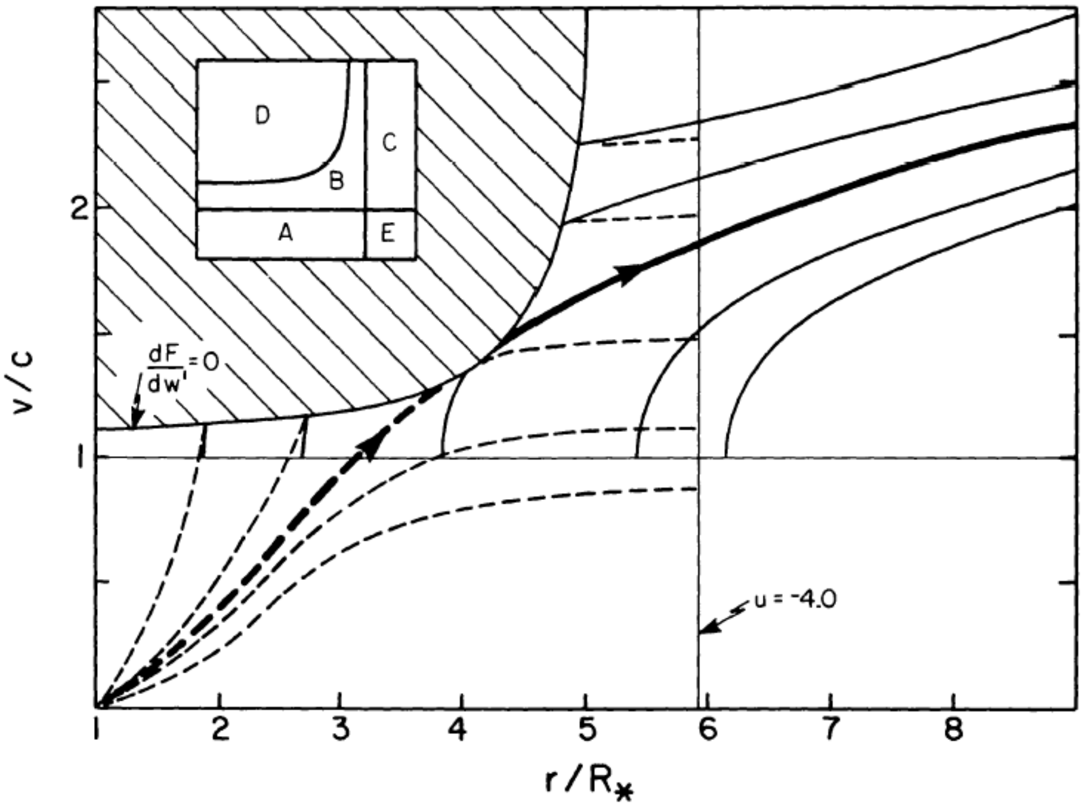
# First Topological Analysis

## Non-Rotating Solution Schema



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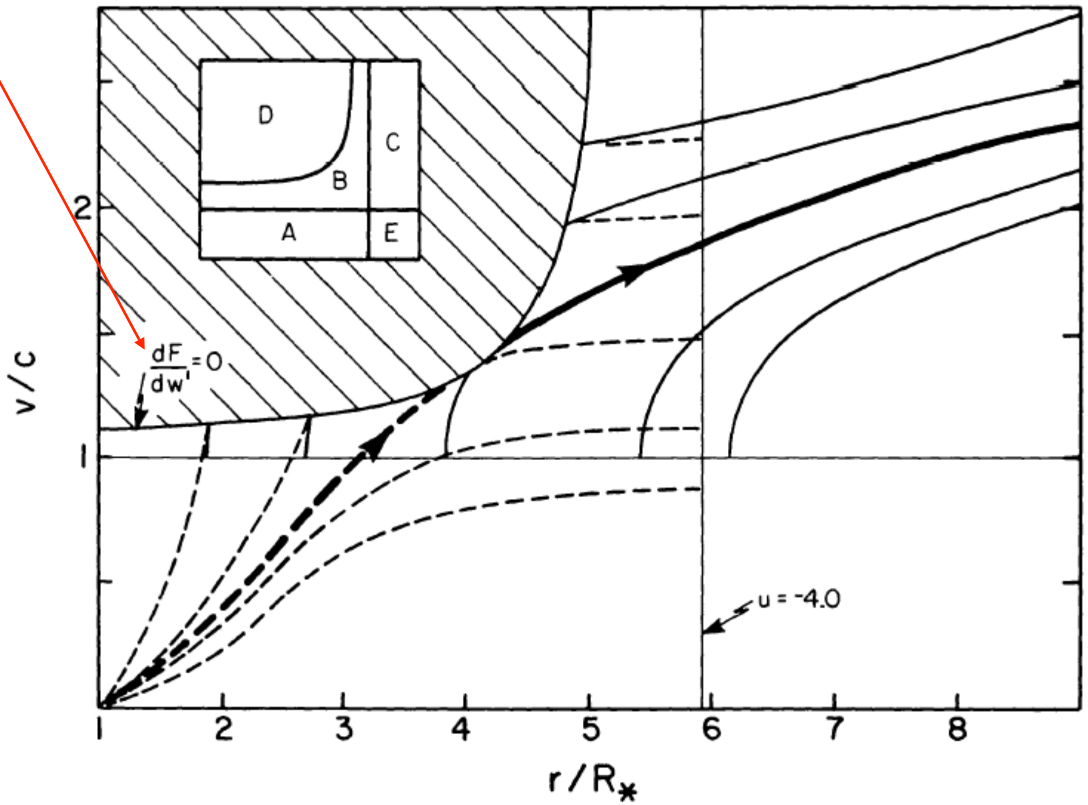
# First Topological Analysis

## Non-Rotating Solution Schema

### Singularity Condition

$$\frac{\partial}{\partial w'} F(u, w, w') = 0$$



# First Topological Analysis

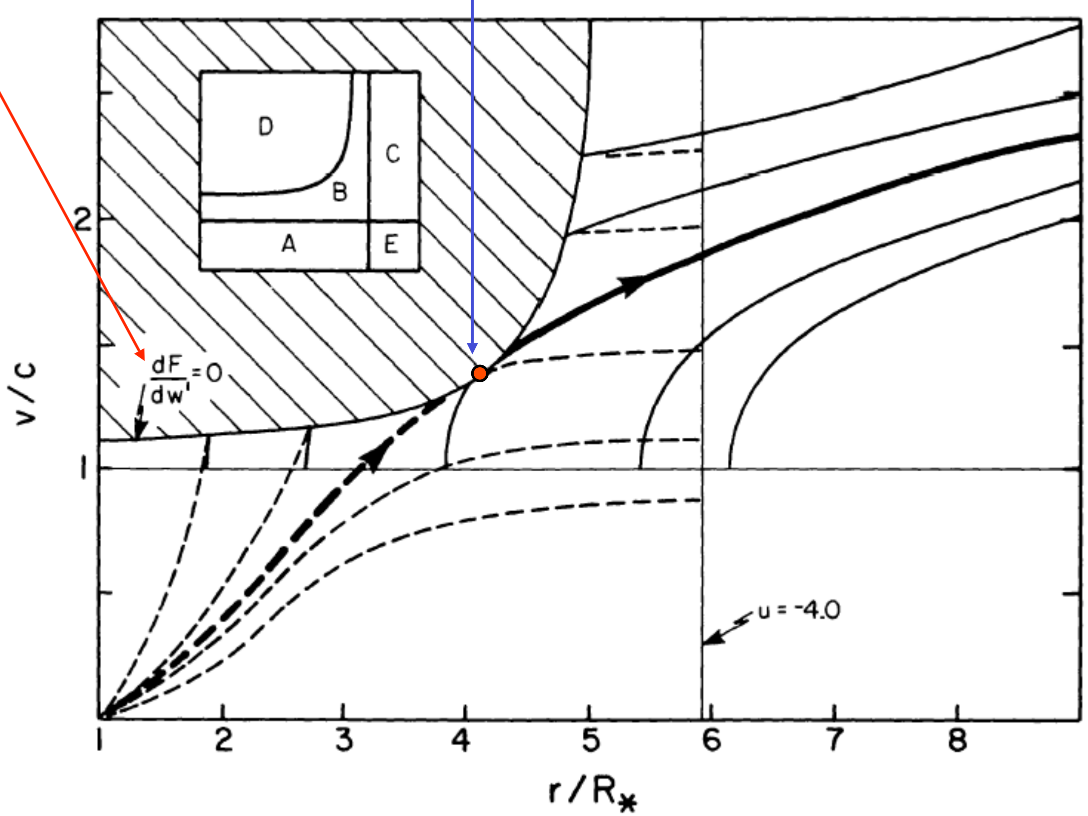
## Non-Rotating Solution Schema

**Singularity Condition**

$$\frac{\partial}{\partial w'} F(u, w, w') = 0$$

**Regularity Condition**

$$\frac{d}{du} F(u, w, w') = \frac{\partial F}{\partial u} + \frac{\partial F}{\partial w} w' = 0.$$



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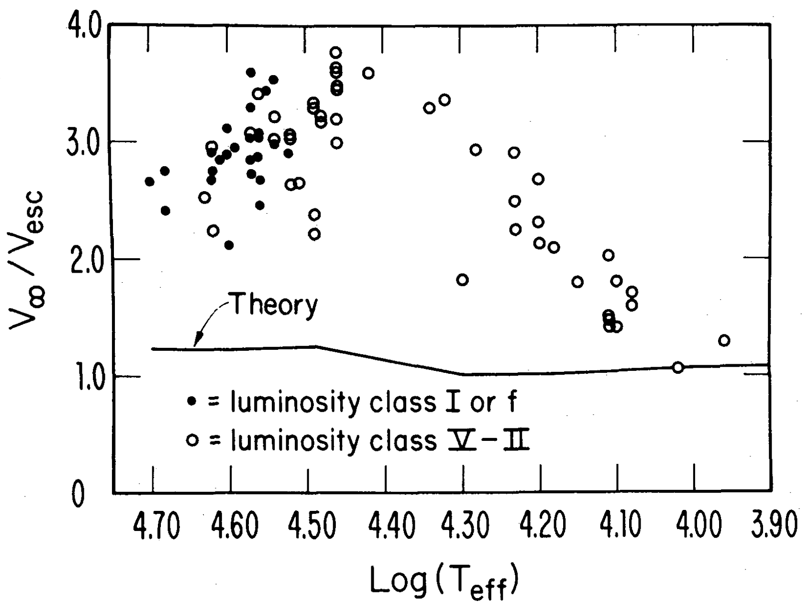
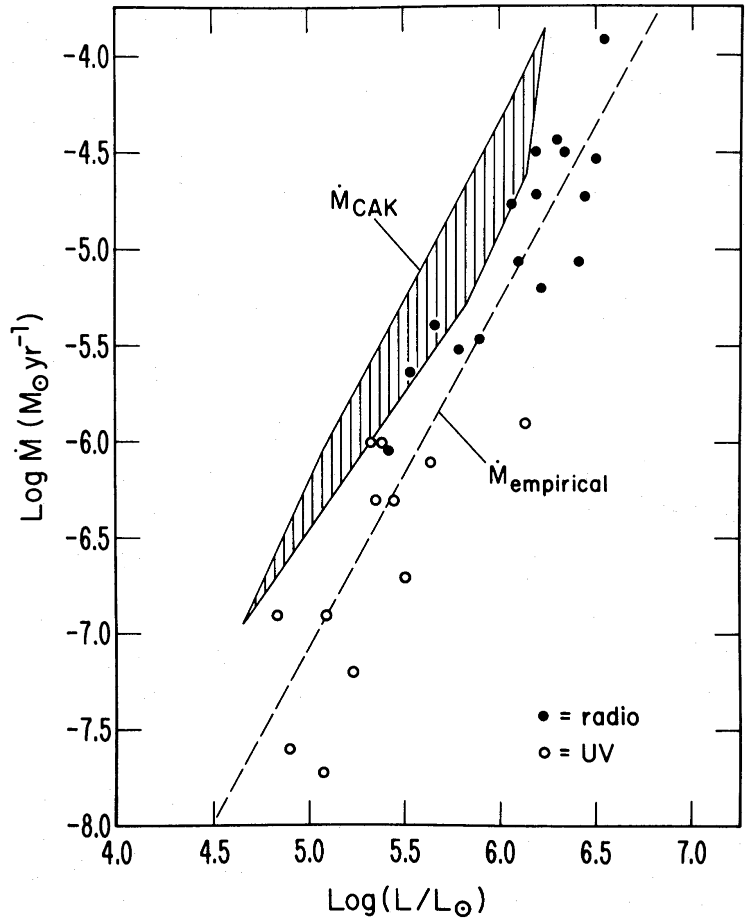
# CAK Model



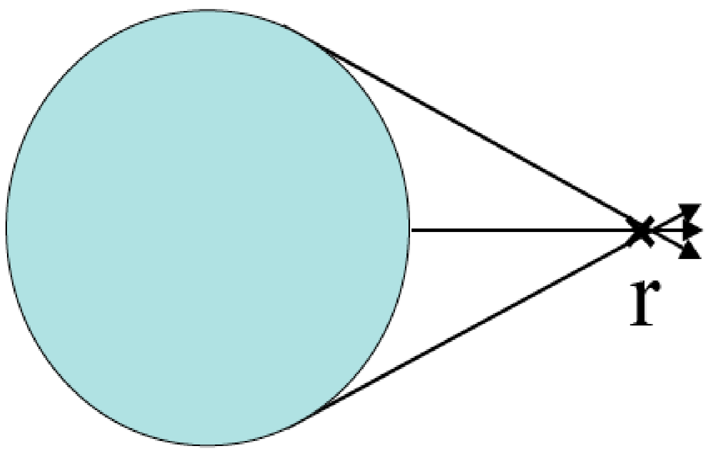

## CAK Theory:

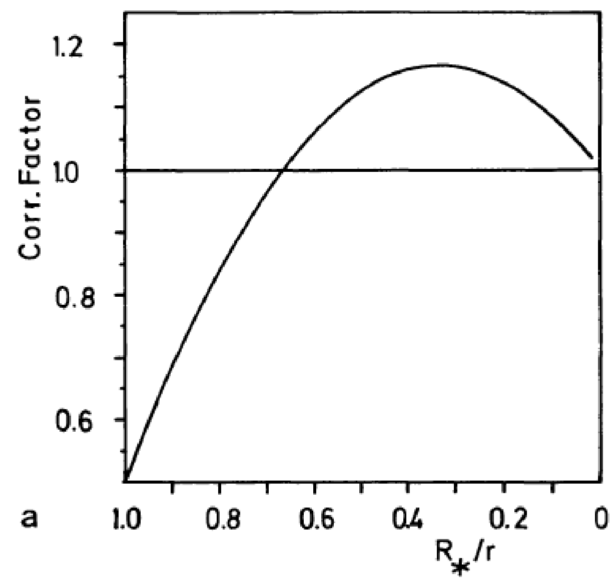
- Qualitatively acceptable agreement with the observational values
- Theory weak assumption:

Point Star approximation

# m-CAK Model

modified-CAK Theory:
Finite Disk Correction Factor




Friend & Abbott
ApJ, 311,701,1986

Pauldrach, Puls & Kudritzki
A&A, 164,86, 1986



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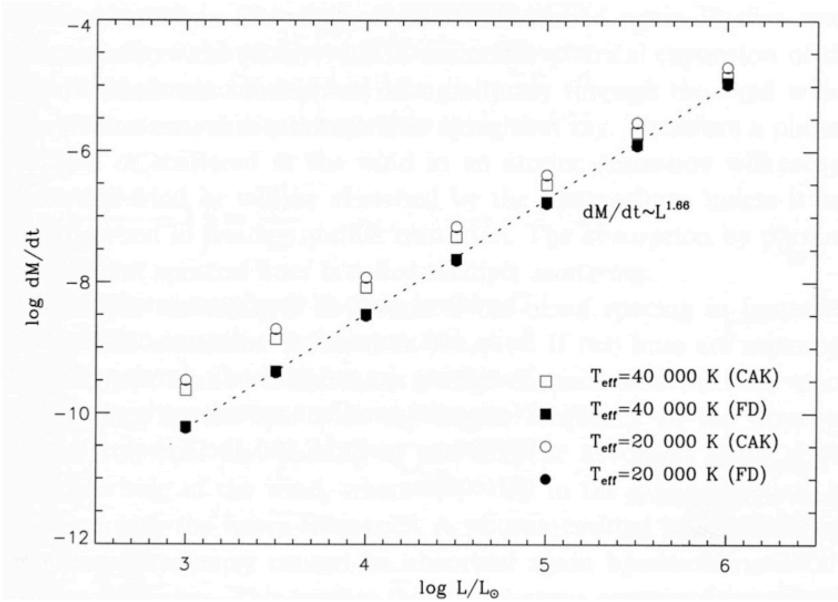
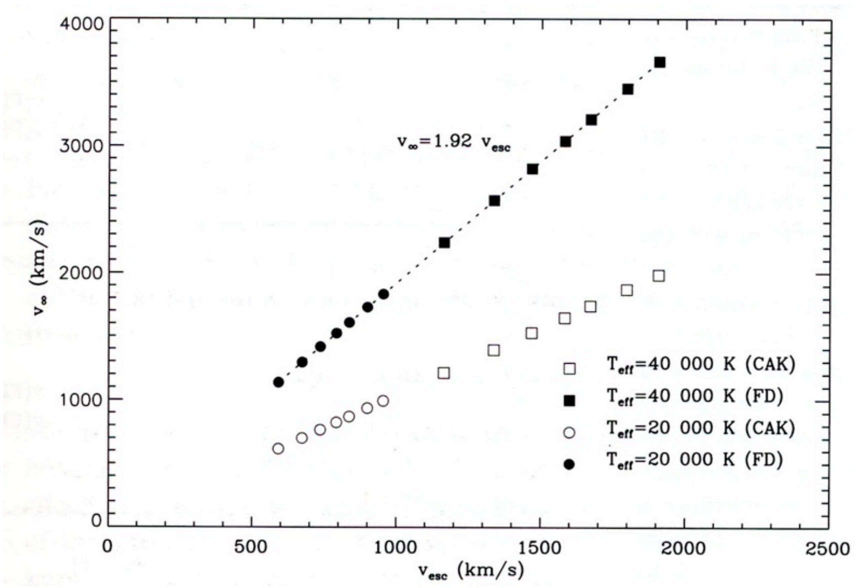
# m-CAK Model

m-CAK: better agreement with observations



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# The effect of Rotation in 1D models

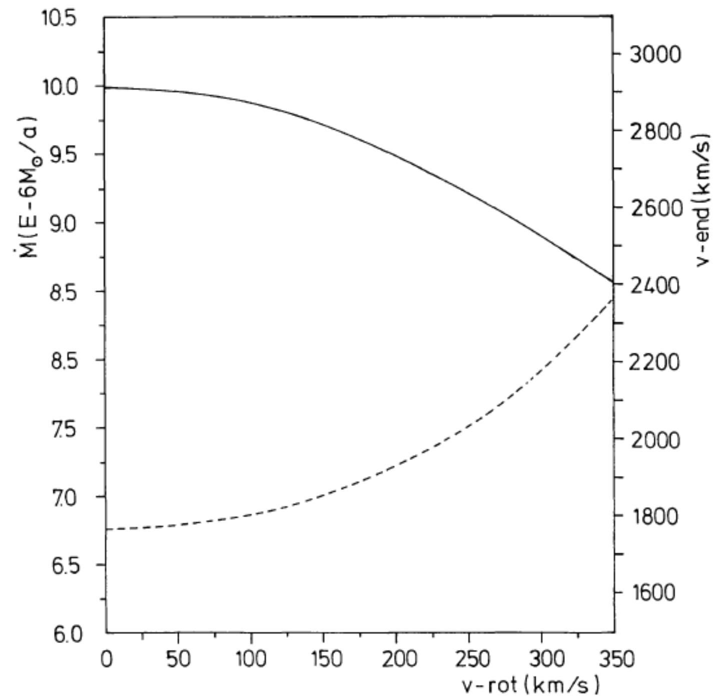
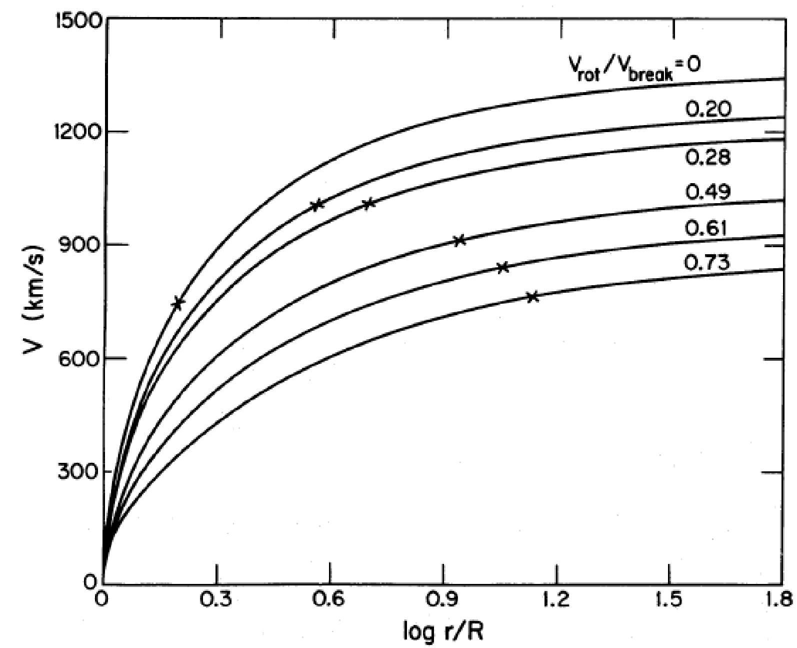



**Fig. 4.** The dependence of $\dot{M}$ (dashed) and $v_\infty$ (fully drawn) on $v_{rot}$ for the 05f-star

Pauldrach et al.
A&A, 164,86, 1986

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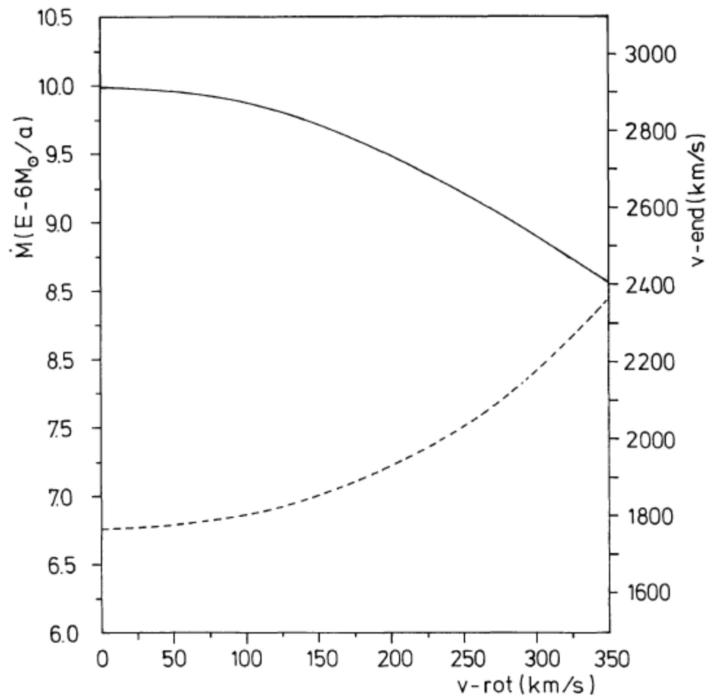


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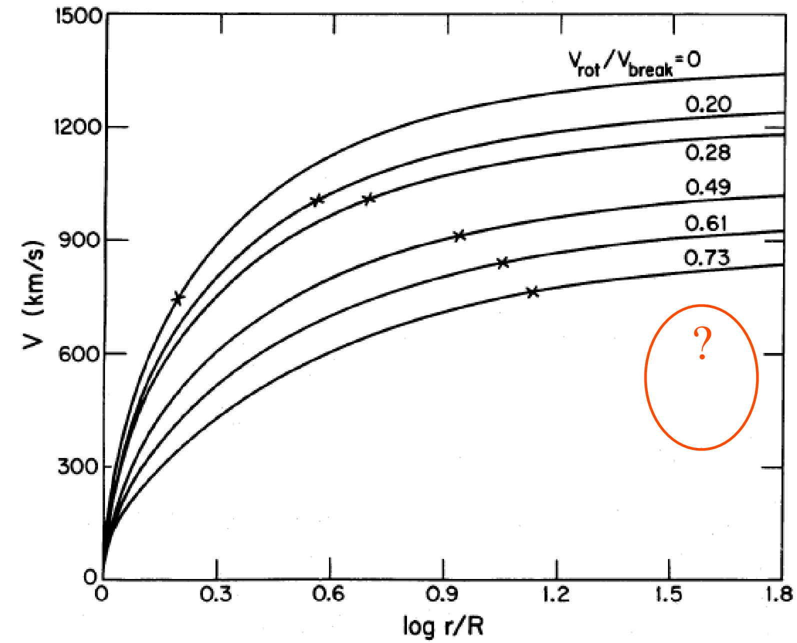


Friend & Abbott
ApJ, 311,701,1986

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# The effect of Rotation in 1D models

Friend & Abbott
ApJ, 311,701,1986

rotational velocities were used, the mass-loss rate might become very large. We were unable to find solutions for larger rotational velocities, mainly because of numerical difficulties involving the finite disk factor when the effective escape speed falls below some critical value. In a study of Be star winds, Poe and Friend (1986) have pushed the rotational velocity closer to the breakup value, and they find that the mass-loss rate does



506 *F. X. de Araújo, J. A. de Freitas Pacheco and D. Petrini*

$\chi = 0.7$ respectively. We have encountered severe numerical difficulties for models with $\chi \geq 0.8$. When we used $\alpha = 0.56$ we managed to obtain the solutions $v(r)$ until a certain radius $r \leq 5R$, but for the $\alpha = 0.4$ model we could not find the localization of the critical point. Somewhat analogous problems

# Radiation Driven Wind Hydrodynamics with Rotation: Revisited

Mass Conservation ⟶ $4\pi r^2 \rho v = \dot{M}$



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Inertia

Gas Pressure

Gravity

Centrifugal force

Line Force



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Changing of variables

Equation of Motion



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## Changing of variables

## Equation of Motion

$$u = \frac{-R_*}{r},$$
$$w = \frac{v}{a},$$
$$w' = \frac{dw}{du},$$



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$$u = \frac{-R_*}{r},$$
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## Equation of Motion

$$F(u, w, w') = 0$$

$$F(u,w,w') \equiv \left(1 - \frac{1}{w^2}\right) w \frac{dw}{du} + A + \frac{2}{u} + a_{\text{rot}}^2 u - C' C F g(u) (w)^{-\delta} \left( w \frac{dw}{du} \right)^{\alpha} = 0$$



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## Changing of variables

$$u = \frac{-R_*}{r}, \quad w = \frac{v}{a}, \quad w' = \frac{dw}{du},$$

## Equation of Motion

$$F(u, w, w') = 0$$

$$F(u,w,w') \equiv \left(1 - \frac{1}{w^2}\right) w \frac{dw}{du} + A + \frac{2}{u} + a_{\mathrm{rot}}^2 u - C' C F g(u) (w)^{-\delta} \left( w \frac{dw}{du} \right)^{\alpha} = 0$$

$$A = \frac{GM(1-\Gamma)}{a^2 R_*} = \frac{v_{\mathrm{esc}}^2}{2a^2},$$

$$C' = C \left( \frac{\dot{M} D}{2\pi} \frac{10^{-11}}{a R_*^2} \right)^{\delta} (a^2 R_*)^{(\alpha - 1)},$$

$$g(u) = \left( \frac{u^2}{1 - \sqrt{1 - u^2}} \right)^{\delta},$$

$$a_{\mathrm{rot}} = \frac{v_{\mathrm{rot}}}{a},$$



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## Changing of variables

$$u = \frac{-R_*}{r},$$
$$w = \frac{v}{a},$$
$$w' = \frac{dw}{du},$$

## Equation of Motion

$$F(u, w, w') = 0$$

$$F(u, w, w') \equiv \left(1 - \frac{1}{w^2}\right) w \frac{dw}{du} + A + \frac{2}{u} + a_{\mathrm{rot}}^2 u - C' C F g(u)(w)^{-\delta} \left(w \frac{dw}{du}\right)^{\alpha} = 0$$

$$A = \frac{GM(1-\Gamma)}{a^2 R_*} = \frac{v_{\mathrm{esc}}^2}{2a^2},$$

$$C' = C \left(\frac{\dot{M} D}{2\pi} \frac{10^{-11}}{a R_*^2}\right)^{\delta} (a^2 R_*)^{(\alpha-1)},$$

$$g(u) = \left(\frac{u^2}{1 - \sqrt{1 - u^2}}\right)^{\delta},$$

$$a_{\mathrm{rot}} = \frac{v_{\mathrm{rot}}}{a},$$

# with the influence of the ROTATION…

$$u = \frac{-R_*}{r},$$
$$w = \frac{v}{a},$$
$$w' = \frac{dw}{du},$$



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# with the influence of the ROTATION…

$$u = \frac{-R_*}{r}, \quad w = \frac{v}{a}, \quad w' = \frac{dw}{du},$$

$$\longrightarrow \quad Y = w\, w', \quad Z = w/w'$$



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# with the influence of the ROTATION…

$$u = \frac{-R_*}{r}, \quad w = \frac{v}{a}, \quad w' = \frac{dw}{du}, \quad \longrightarrow \quad Y = w\,w', \quad Z = w/w'$$

$$\left(1 - \frac{1}{YZ}\right) Y \quad + A + 2/u + a_{rot}^2 u \quad - C' f_1(u,Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

$$\left(1 - \frac{1}{YZ}\right) Y \quad - C' f_2(u,Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

$$\left(1 + \frac{1}{YZ}\right) Y \quad -2Z/u^2 + a_{rot}^2 Z \quad - C' f_3(u,Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$



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## with the influence of the ROTATION…

$$\left.\begin{aligned} u &= \frac{-R_*}{r}, \\ w &= \frac{v}{a}, \\ w' &= \frac{dw}{du}, \end{aligned}\right\} \longrightarrow \begin{aligned} Y &= w\, w' \\ Z &= w/w' \end{aligned}$$

$$\left(1 - \frac{1}{YZ}\right) Y \quad + A + 2/u + a_{rot}^2 u \quad - C' f_1(u,Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

$$\left(1 - \frac{1}{YZ}\right) Y \quad - C' f_2(u,Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

$$\left(1 + \frac{1}{YZ}\right) Y \quad -2Z/u^2 + a_{rot}^2 Z \quad - C' f_3(u,Z)\, g(u)\, Z^{-\delta/2}\, Y^{\alpha-\delta/2} = 0$$

At the Singular Point: $u_s, Y_s, Z_s, C'$



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# Without any Approximation!



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# Without any Approximation!

$$Y = \frac{1}{Z} + \left(\frac{f_2}{f_1 - f_2}\right)\left(A + \frac{2}{u} + a_{rot}^2 u\right)$$

$$C'(\dot{M}) = \frac{1}{g f_2}\left(1 - \frac{1}{YZ}\right) Z^{\delta/2}\, Y^{1-\alpha+\delta/2}$$



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# Without any Approximation!

$$Y = \frac{1}{Z} + \left( \frac{f_2}{f_1 - f_2} \right) \left( A + \frac{2}{u} + a_{rot}^2 u \right)$$

$$C'(\dot{M}) = \frac{1}{g f_2} \left( 1 - \frac{1}{Y Z} \right) \; Z^{\delta/2} \; Y^{1-\alpha+\delta/2}$$

and

$$R(u, Z) \equiv -\frac{2}{Z} + \frac{2Z}{u^2} - a_{rot}^2 Z + f_{123}(u, Z) \left( A + \frac{2}{u} + a_{rot}^2 u \right)$$

# $R(u\,,\,Z) = 0$ ← singular point location



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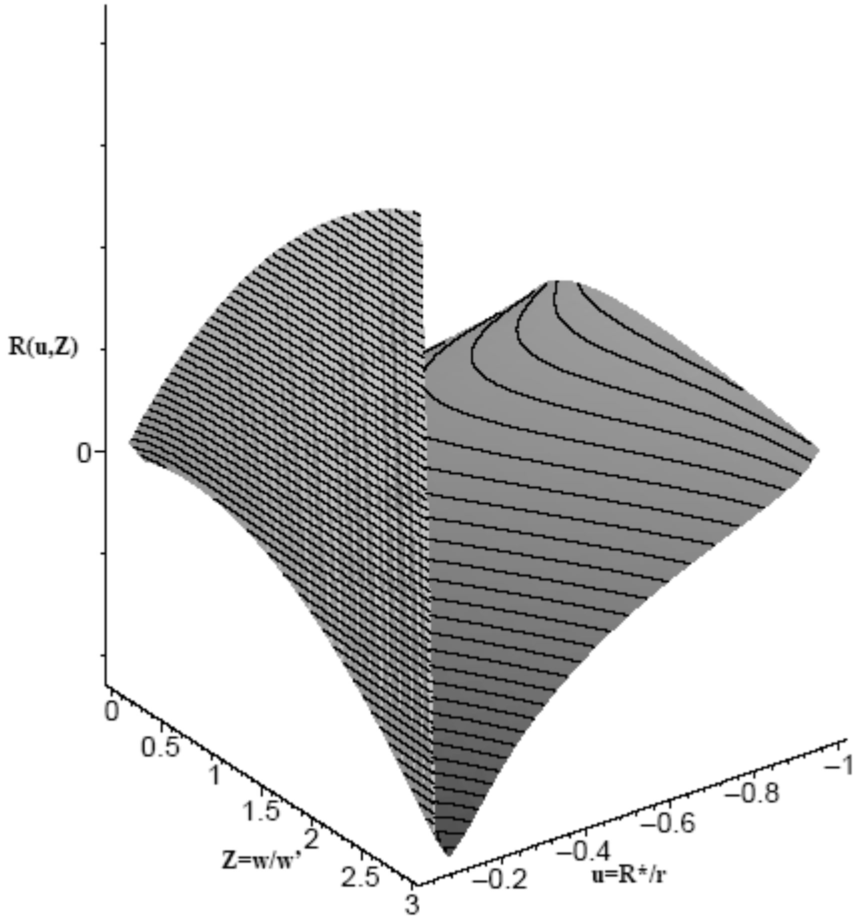
# $R(u\ ,\ Z) = 0$ ← singular point location

$$v_{\rm rot}/v_{\rm bkup} = 0.5$$

# R(u , Z) = 0 ← singular point location




$$v_{\rm rot}/v_{\rm bkup} = 0.5$$



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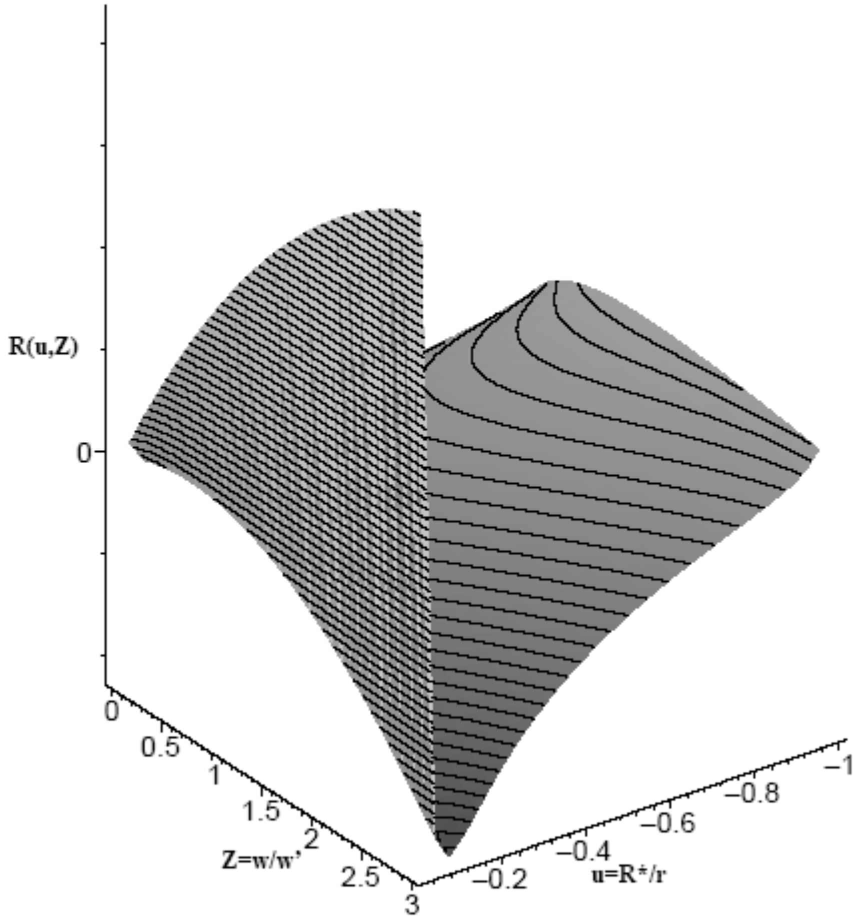
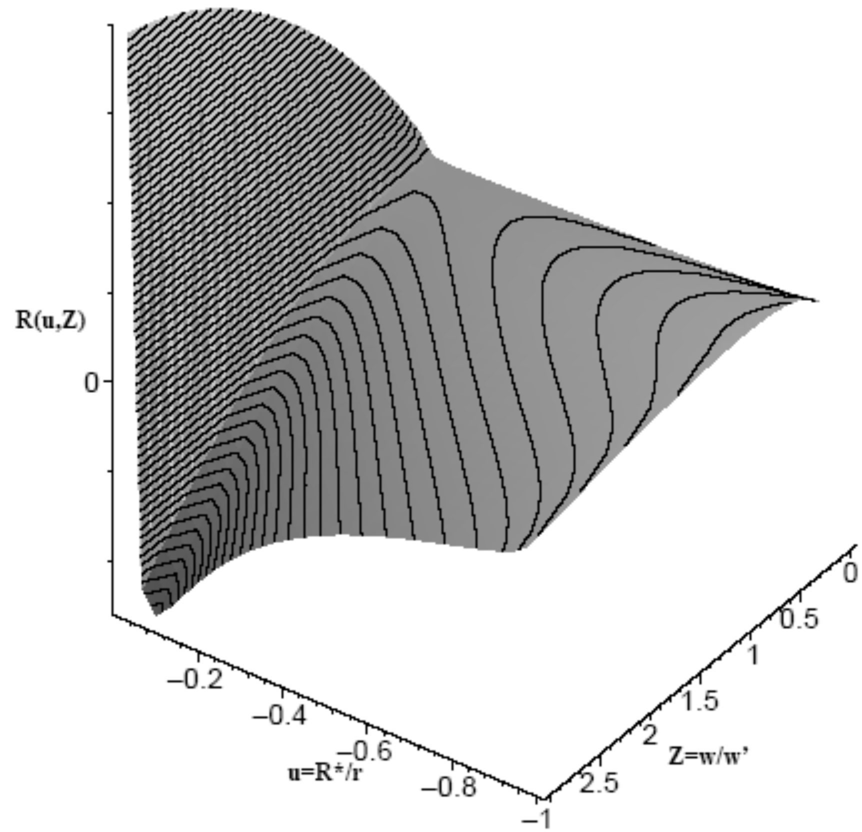
# R(u , Z) = 0 ← singular point location




$$v_{\mathrm{rot}}/v_{\mathrm{bkup}} = 0.5$$



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$v_{\rm rot}/v_{\rm bkup} = 0.5$



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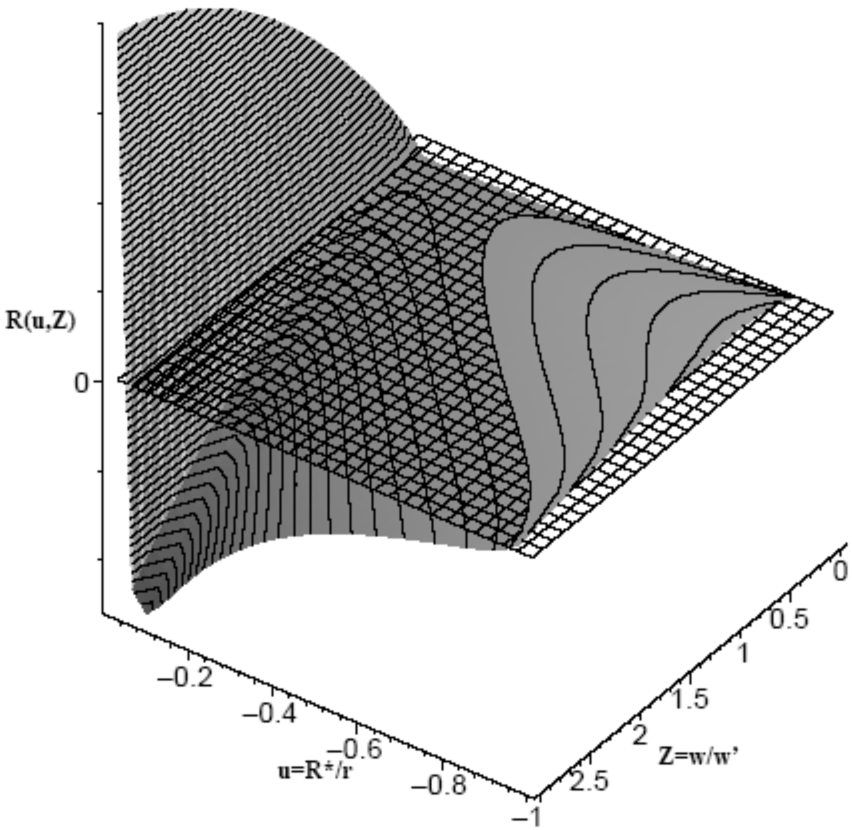
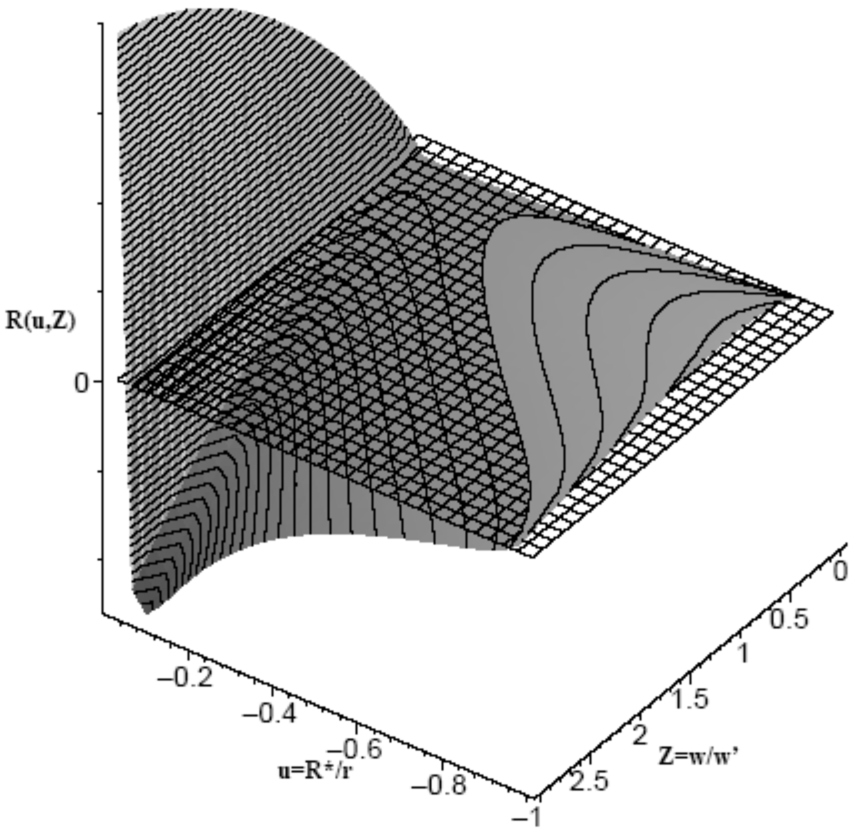
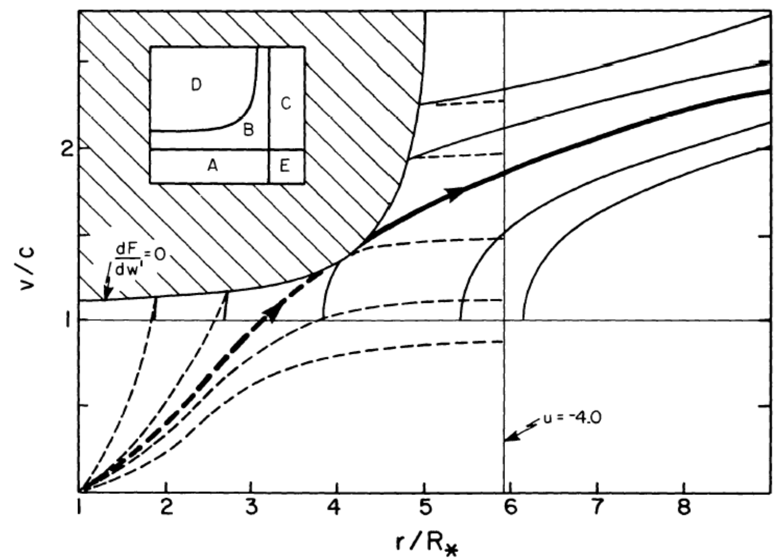
$$v_{\rm rot}/v_{\rm bkup} = 0.5$$



$$v_{\rm rot}/v_{\rm bkup} = 0.5$$



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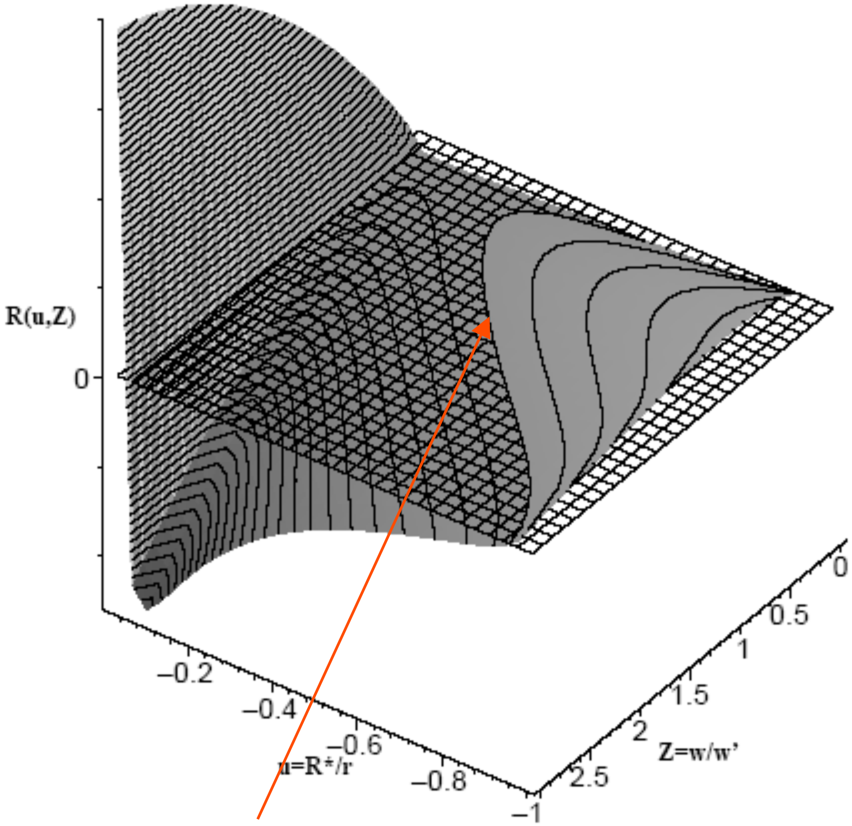
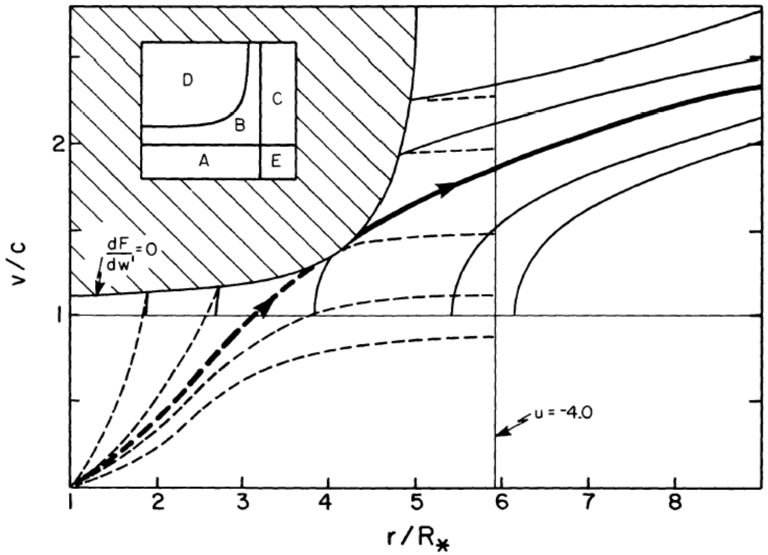
$v_{\rm rot}/v_{\rm bkup} = 0.5$




CAK family
of singular points



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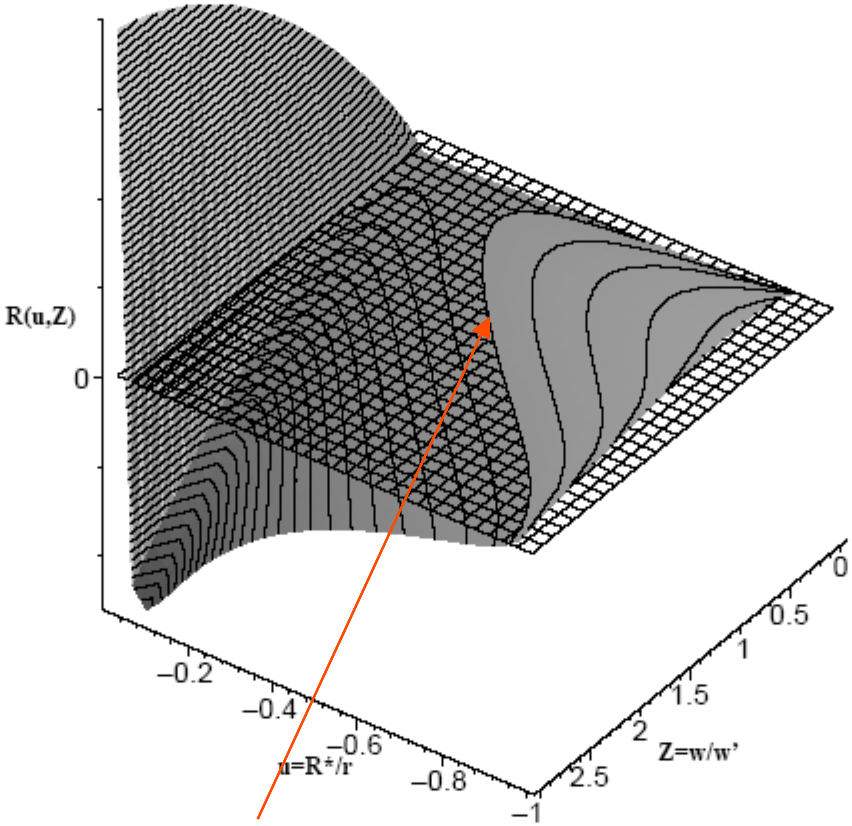
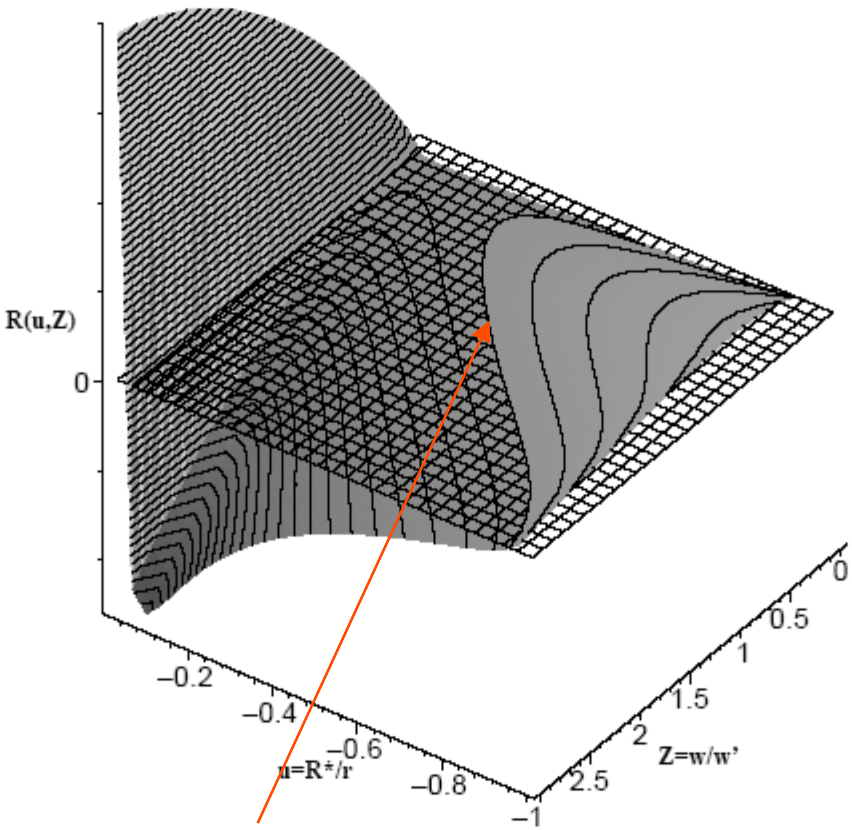
$$v_{\mathrm{rot}}/v_{\mathrm{bkup}} = 0.5$$




CAK family
of singular points

$v_{\rm rot}/v_{\rm bkup} = 0.5$

$v_{\rm rot}/v_{\rm bkup} = 0.9$




CAK family
of singular points



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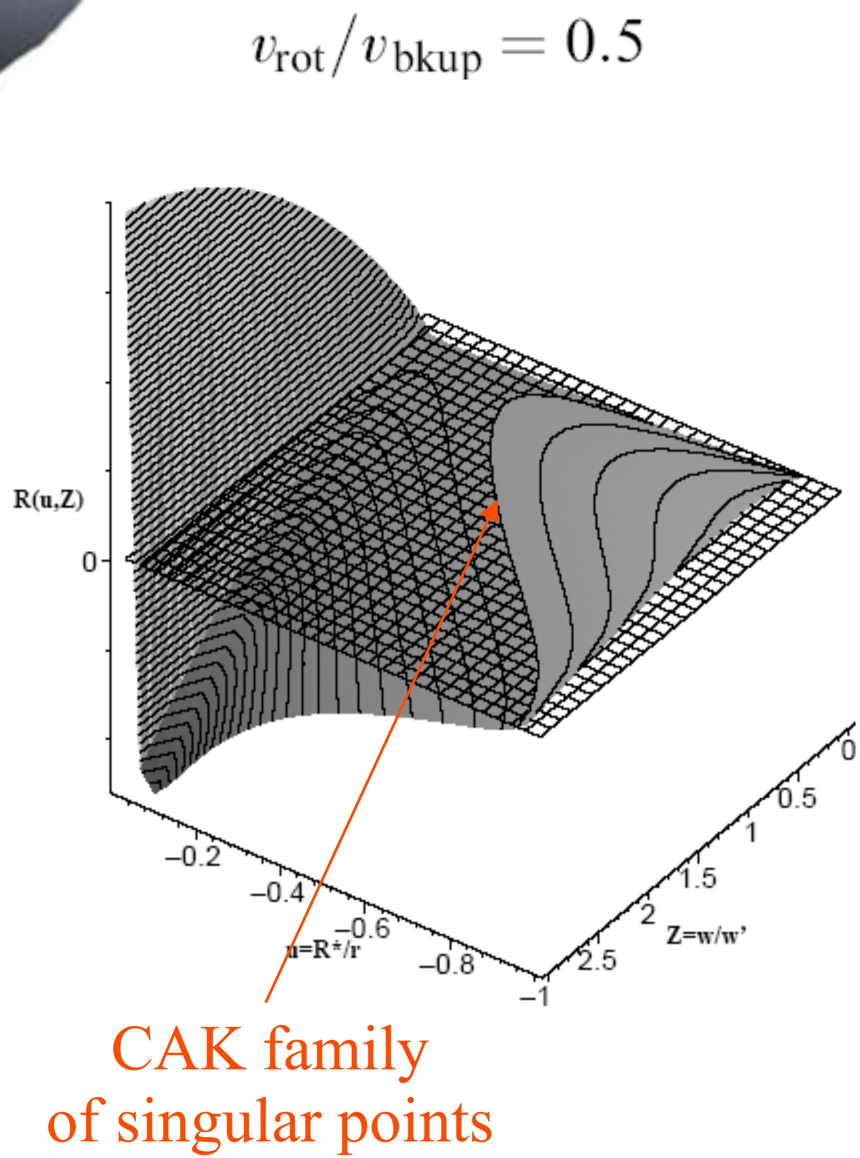
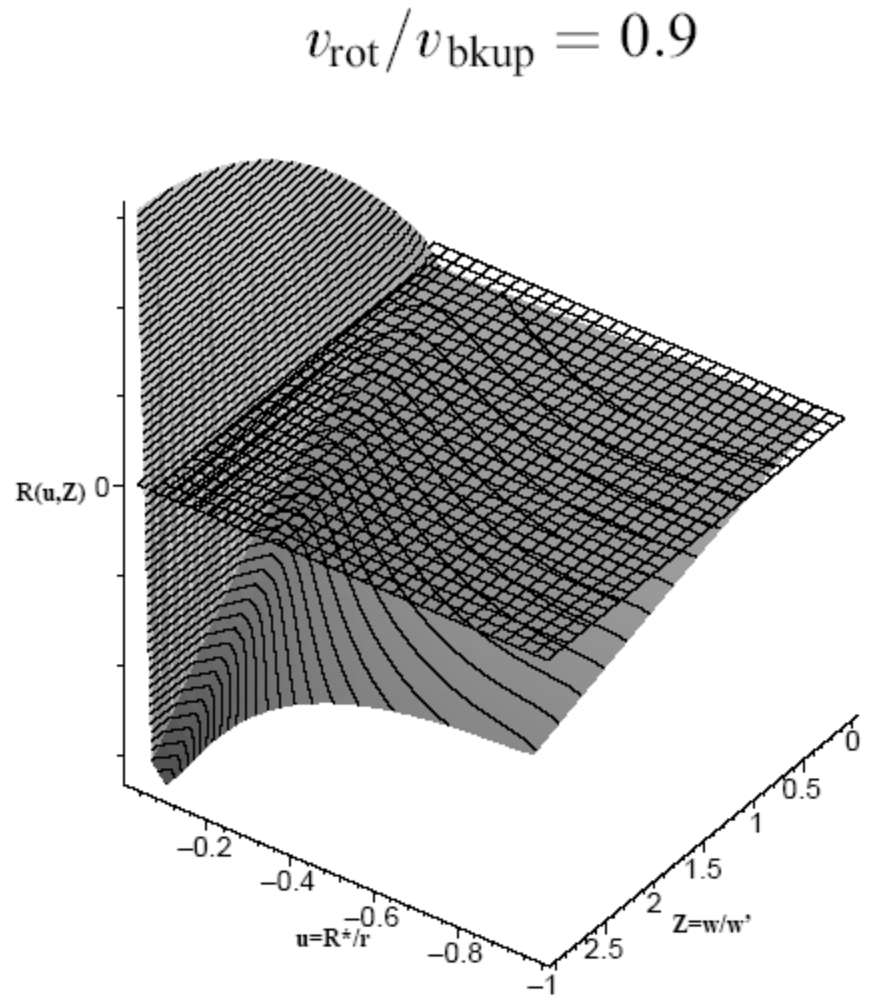


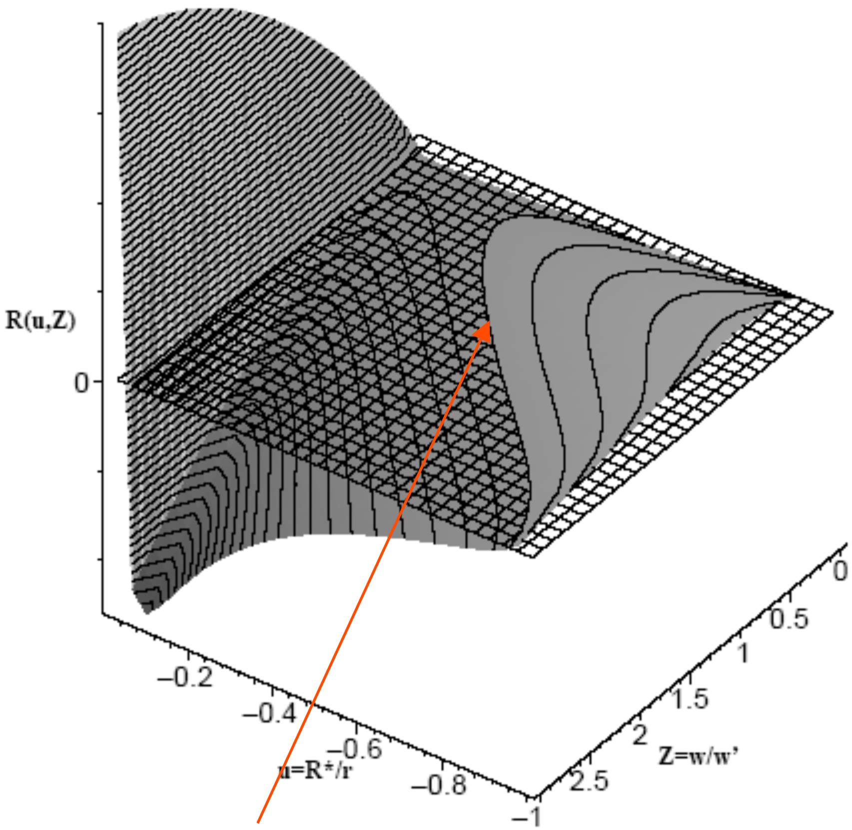
$v_{\rm rot}/v_{\rm bkup} = 0.5$
R(u,Z)
0
-0.2
-0.4
-0.6
u=R*/r
-0.8
-1
0
0.5
1
1.5
2
2.5
Z=w/w'
CAK family
of singular points

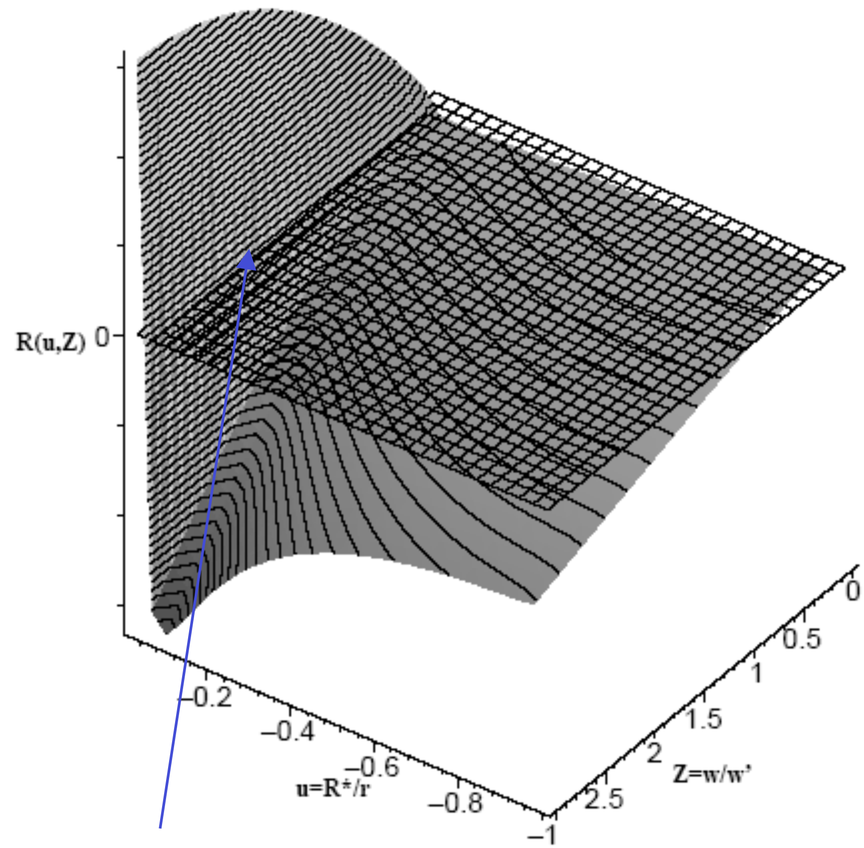
$v_{\rm rot}/v_{\rm bkup} = 0.9$
R(u,Z)
0
-0.2
-0.4
-0.6
u=R*/r
-0.8
-1
0
0.5
1
1.5
2
2.5
Z=w/w'

$$v_{\rm rot}/v_{\rm bkup} = 0.5$$

$$v_{\rm rot}/v_{\rm bkup} = 0.9$$

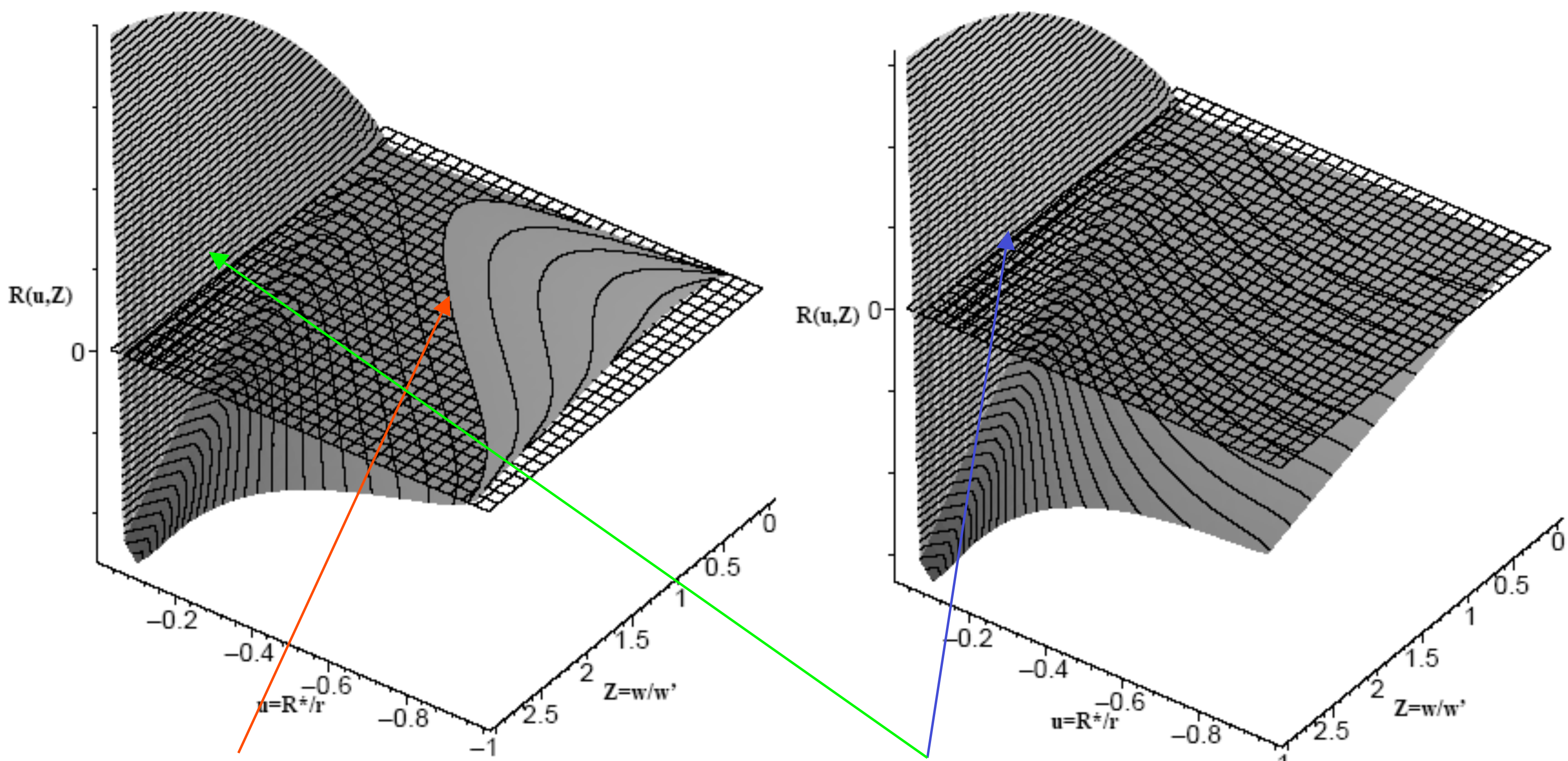



CAK family
of singular points




NEW family
of singular points

$v_{rot}/v_{bkup} = 0.5$ $v_{rot}/v_{bkup} = 0.9$




CAK family
of singular points

NEW family
of singular points



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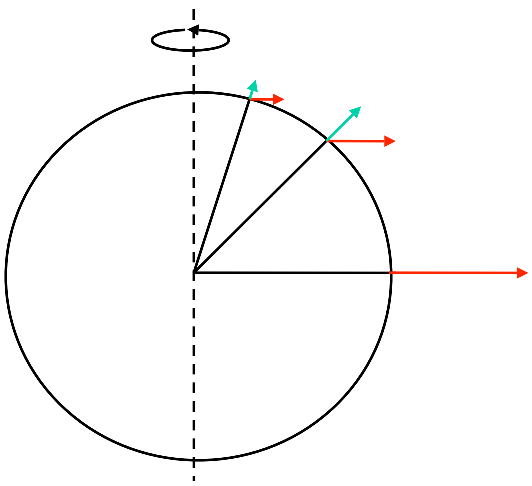
Departamento Física y Astronomía

# Applications of the Slow solution

# B[e] Supergiants





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# m-CAK

- Bistability line force parameters ($T_{eff}$=25,000K):
  One set for polar latitudes and
  other set for equatorial latitudes

- Fast and Slow solution (m-CAK)

- Rotation parameter $\Omega$

# B[e] Supergiant Wind

## Stellar Parameters

$$T_{\rm eff} = 25\,000\ {\rm K}$$
$$M/M_\odot = 17.5$$
$$L/L_\odot = 10^5$$

form Pelupessy et al. (2000)

## Line-Force Parameters

| $T$ [K] | $\alpha$ | $k$ | $\delta$ |
|---|---|---|---|
| 30 000 | 0.65 | 0.06 | 0 |
| 17 500 | 0.45 | 0.57 | 0 |

form Pelupessy et al. (2000)

**Table 17.** Escape velocities, effective gravity and rotational velocities derived from $v_\infty/v_{\rm esc} = 1.3$ and stellar parameters given in Tab. 14. $M_{\rm ZAMS}$ values are from Zickgraf et al. (1986).

| Star | $M_{\rm ZAMS}$ | $M_{\rm B[e]}$ | $v_{\rm esc}[{\rm km\,s^{-1}}]$ | $\log g_{\rm eff}$ | $\Gamma$ | $\Gamma_{\rm rad}$ | $\Gamma_{\rm rot}$ | $v_{\rm rot}[{\rm km\,s^{-1}}]$ | $v_{\rm crit}[{\rm km\,s^{-1}}]$ | $\Omega$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Hen S22 | 52 | 35 | 60 | 0.72 | 0.98 | 0.56 | 0.42 | 240 | 318 | 0.75 |
| R 82 | 30 | 20 | 55 | 0.65 | 0.98 | 0.32 | 0.66 | 224 | 283 | 0.79 |
| R 50 | 45 | 30 | 40 | 0.15 | 0.99 | 0.40 | 0.59 | 204 | 277 | 0.74 |

form Zickgraf (1998)

# B[e] Supergiant m-CAK Wind

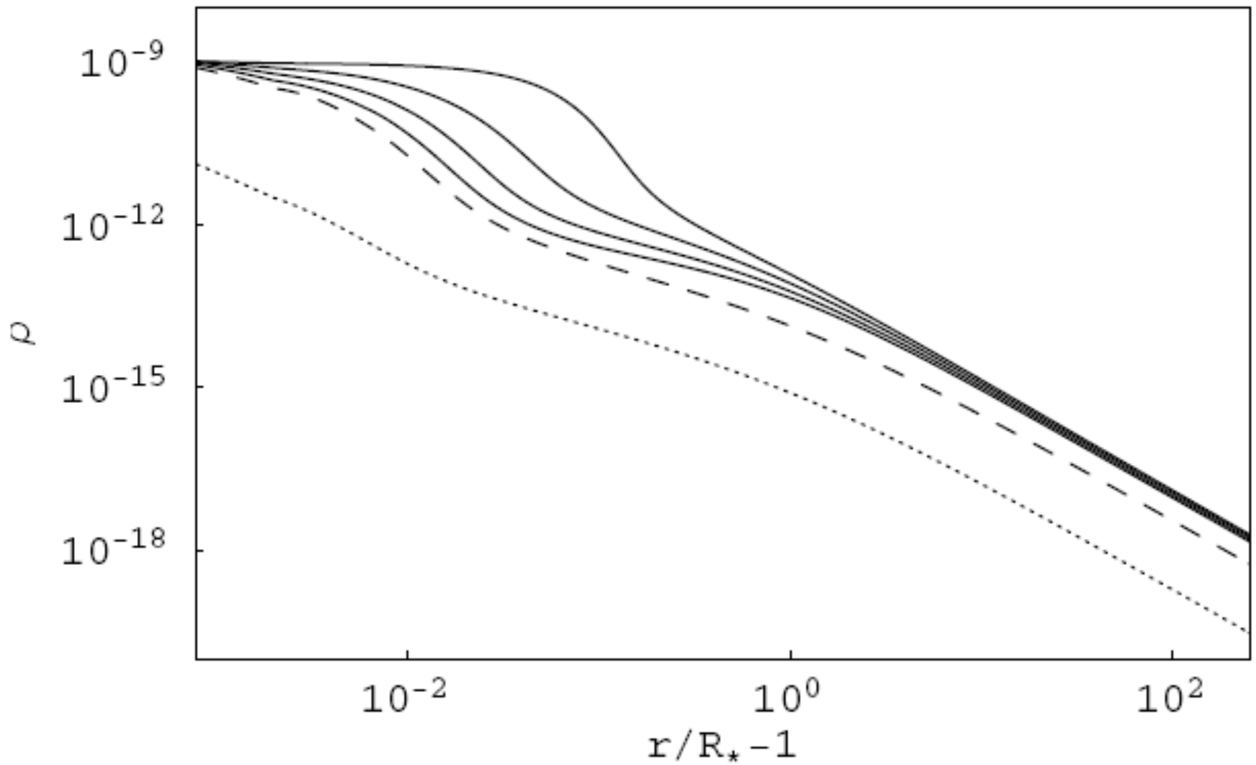


**Fig. 3.** m-CAK model: density (in g cm$^{-3}$) versus $r/R_* - 1$. Polar density is in dotted-line; equatorial density for $\Omega = 0.6$ (fast solution) is in dashed-line and equatorial densities for $\Omega = 0.7, 0.8, 0.9, 0.99$ are in continuous-line, the higher is $\Omega$, the higher is the density.



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# B[e] Supergiant m-CAK Wind

## density contrast

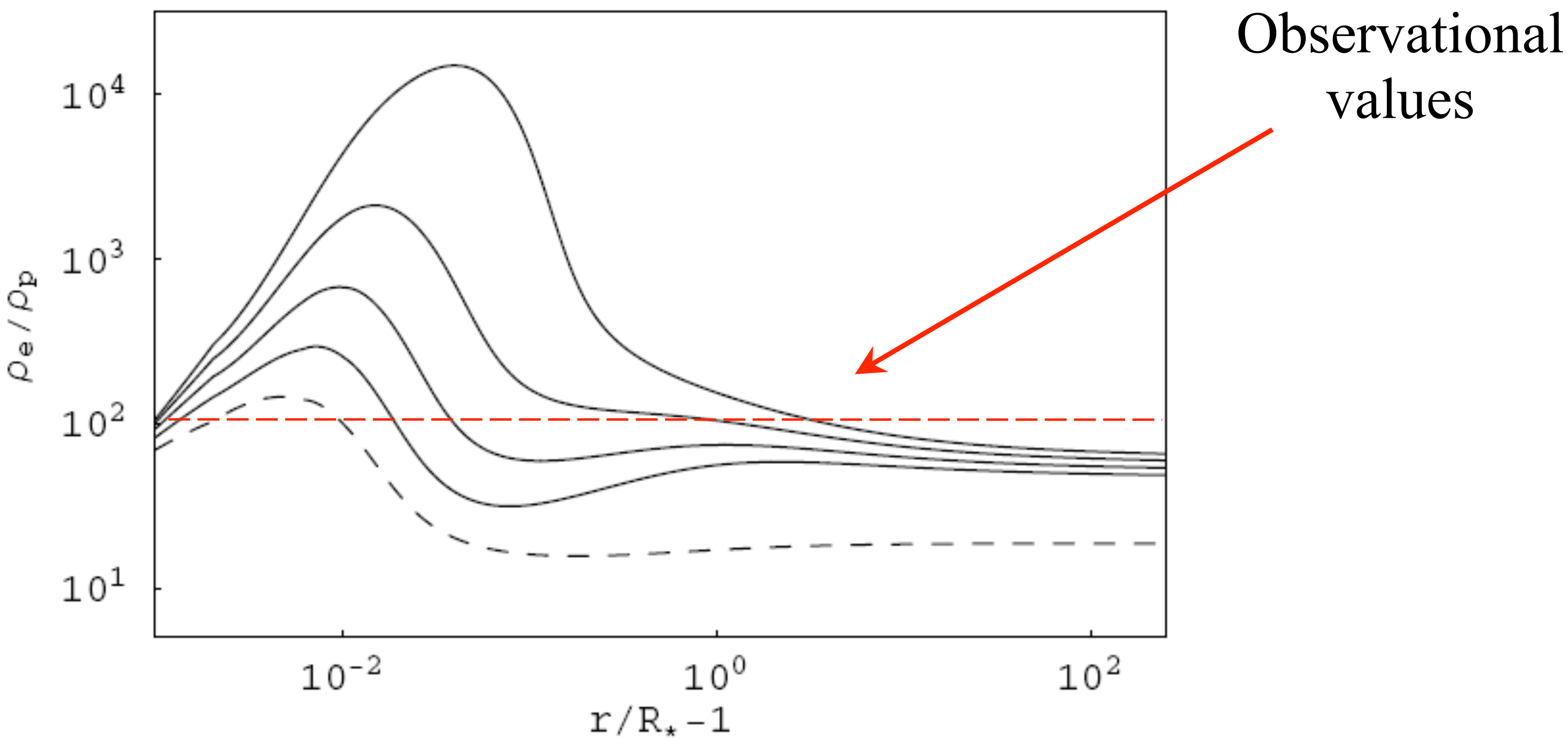


**Fig. 4.** m-CAK model: density contrast versus $r/R_* - 1$, dashed-line is for $\Omega = 0.6$ and continuous-line are for $\Omega = 0.7, 0.8, 0.9, 0.99$. The higher is $\Omega$, the higher is the density contrast.




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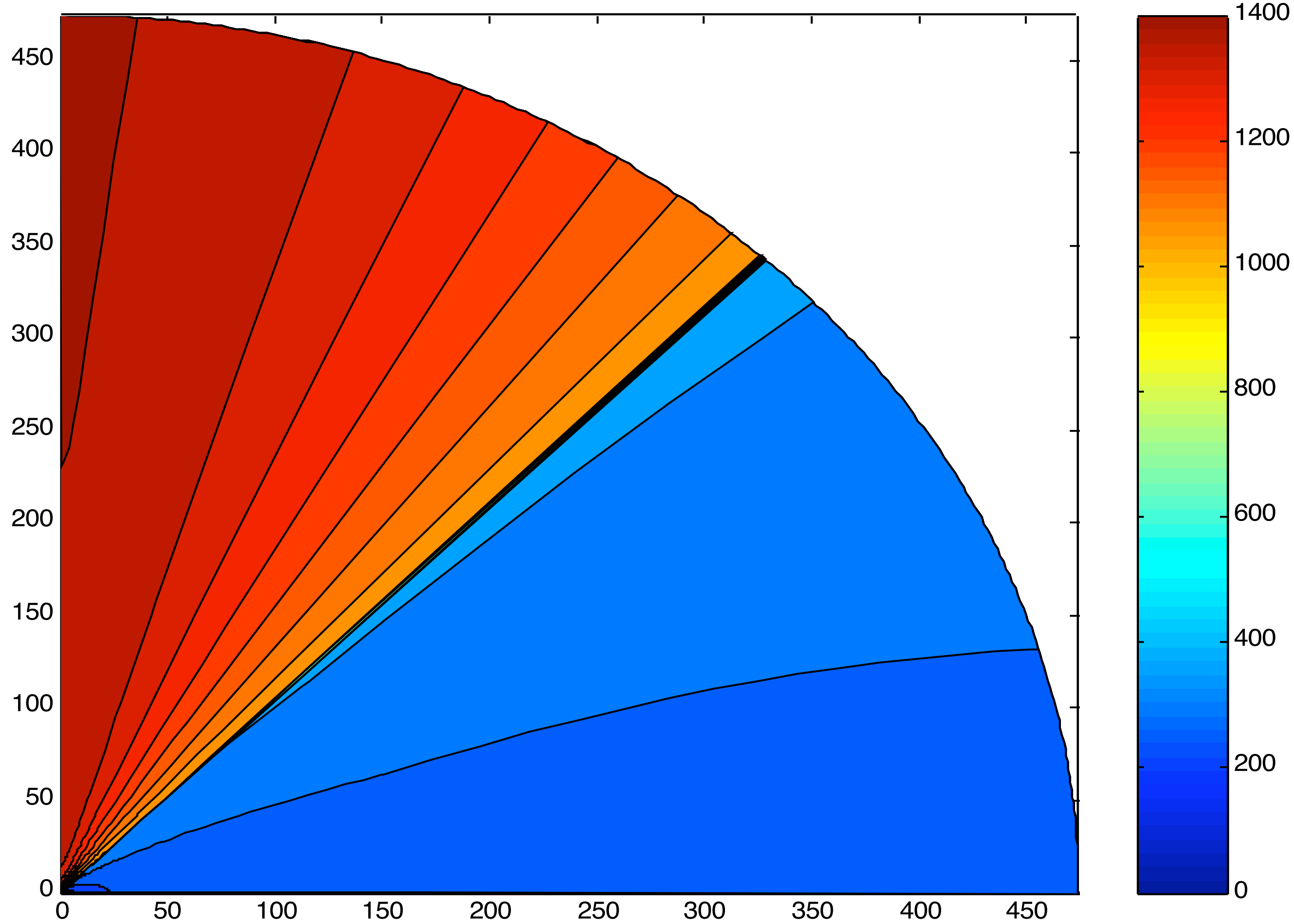
# B[e] Supergiant m-CAK Wind

## "2D" Wind



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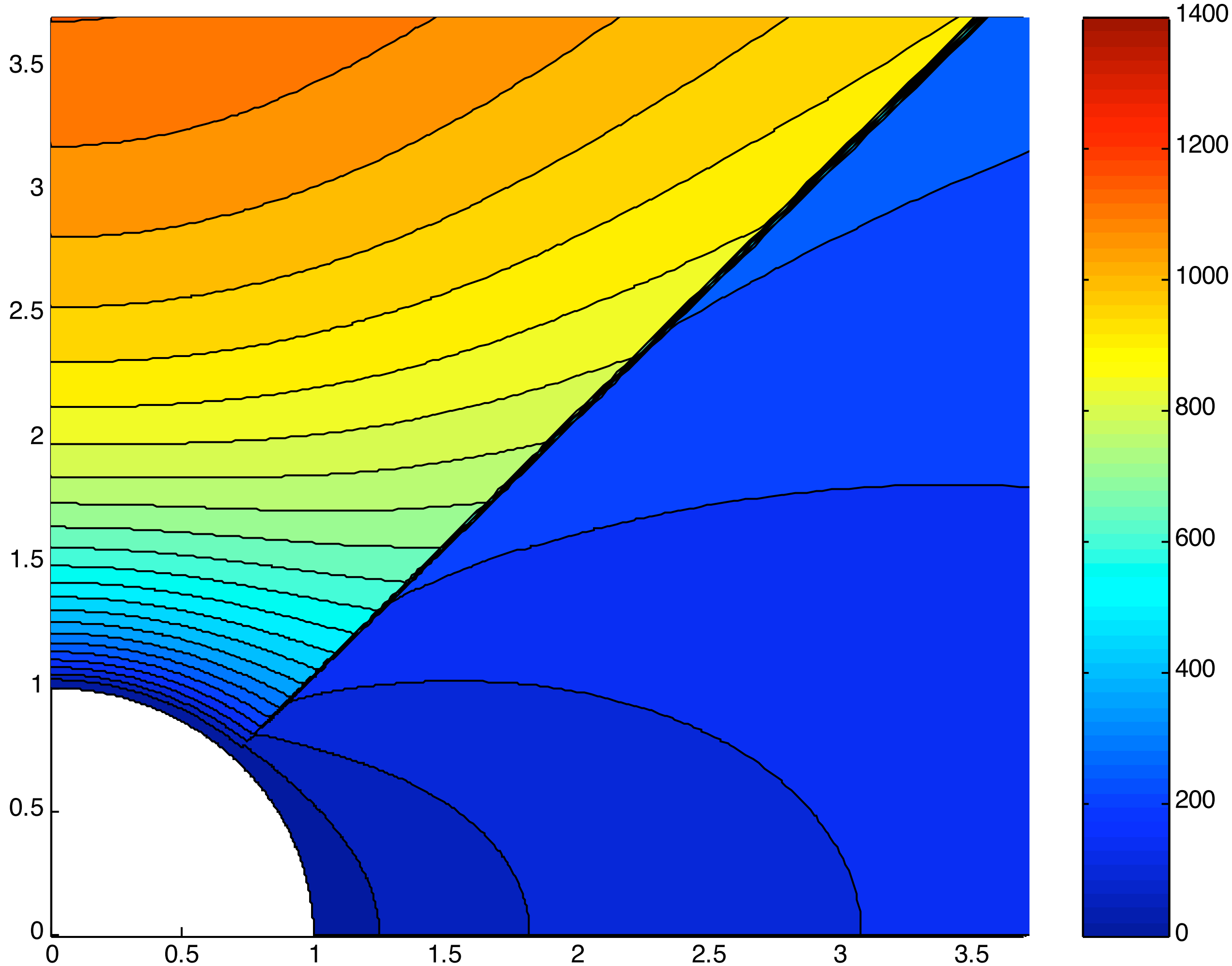
# B[e] Supergiant m-CAK Wind

## "2D" Wind



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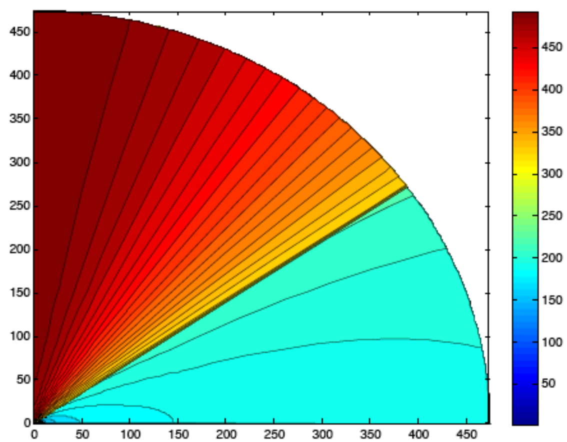
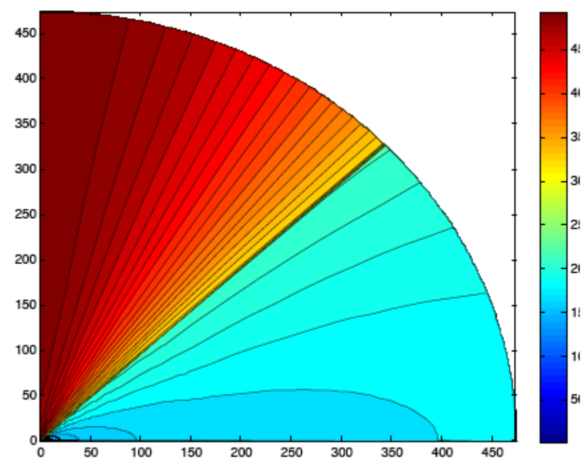
# B Supergiant m-CAK Wind

## Disk Aperture Angle
## HD 206165

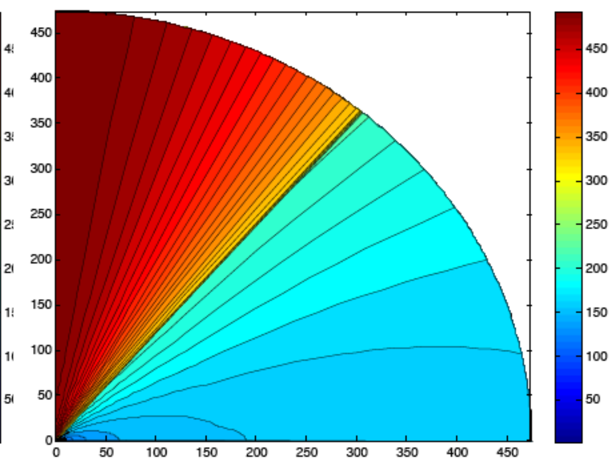
$\Omega = 0.75$



$\Omega = 0.85$



$\Omega = 0.95$





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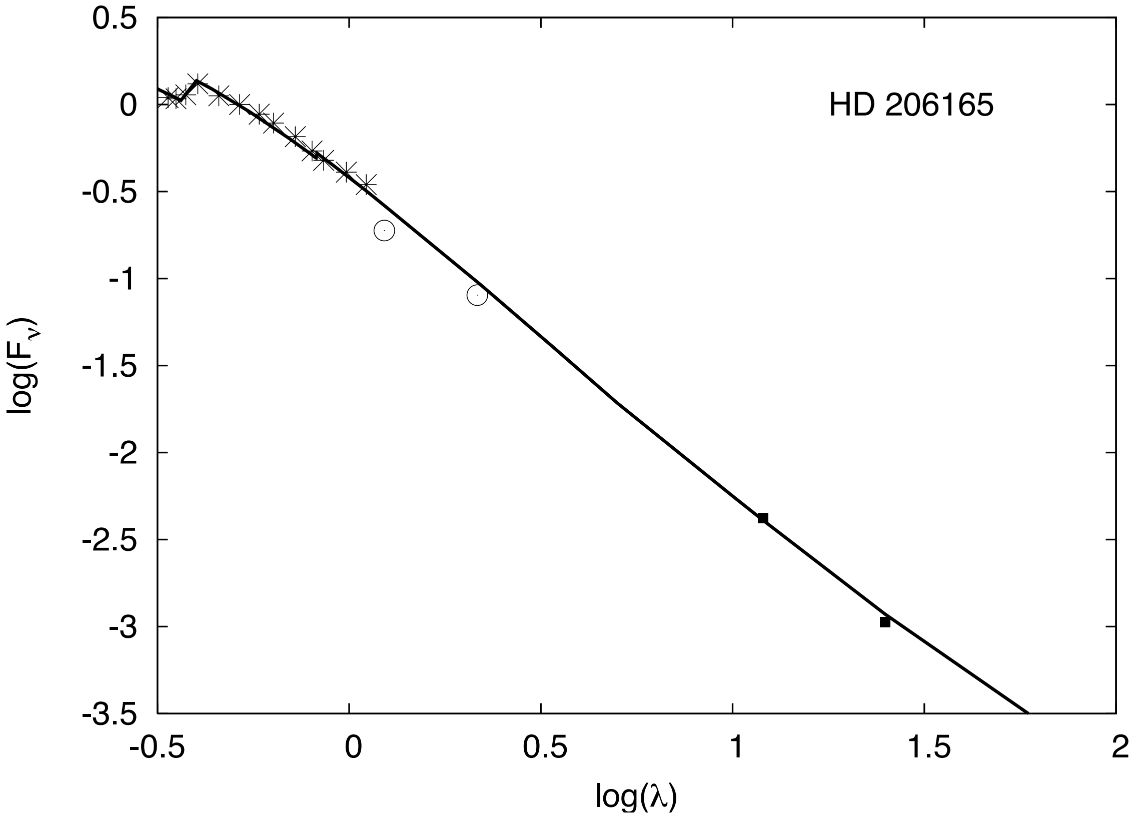
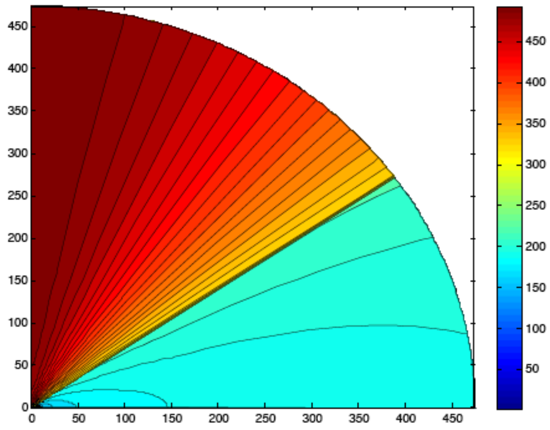
# B Supergiant m-CAK Wind

Continuum Spectra HD 206165 (A. Torres et al. 2010, sub.)




$\Omega = 0.75$





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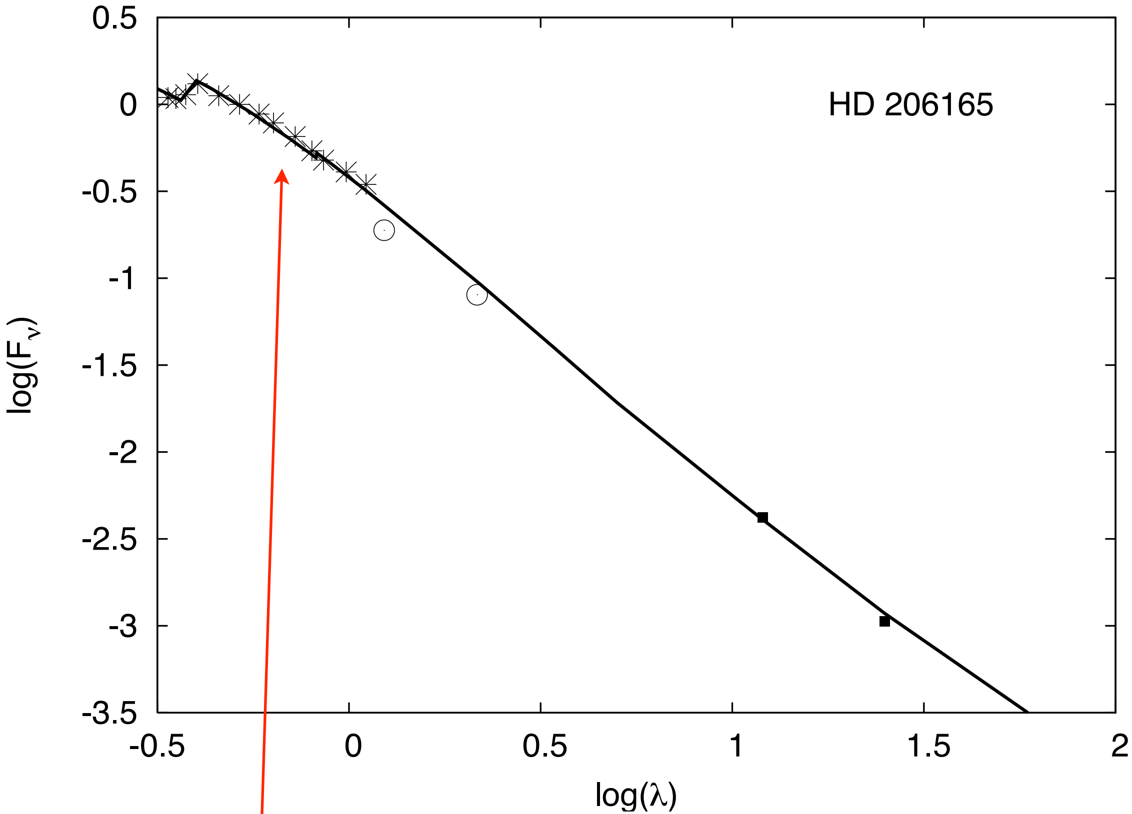
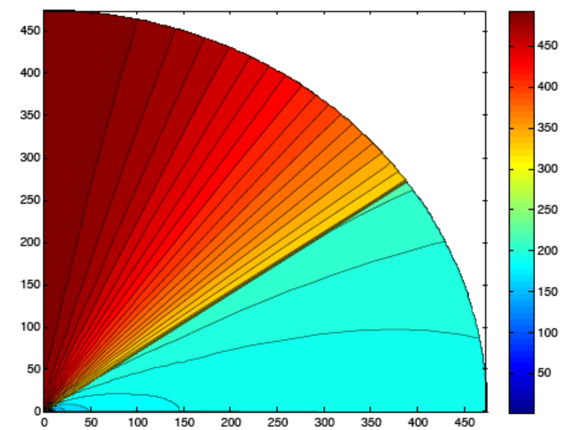
# B Supergiant m-CAK Wind

## Continuum Spectra HD 206165 (A. Torres et al. 2010, sub.)



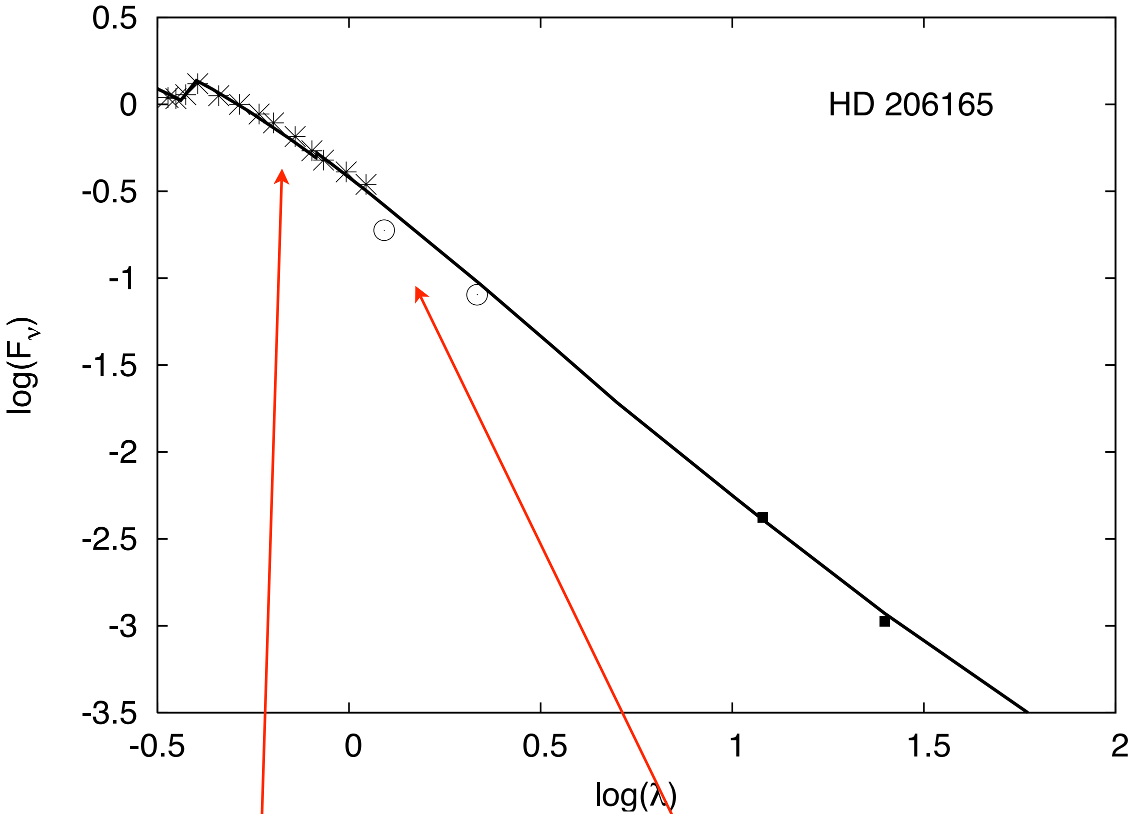

$\Omega = 0.75$



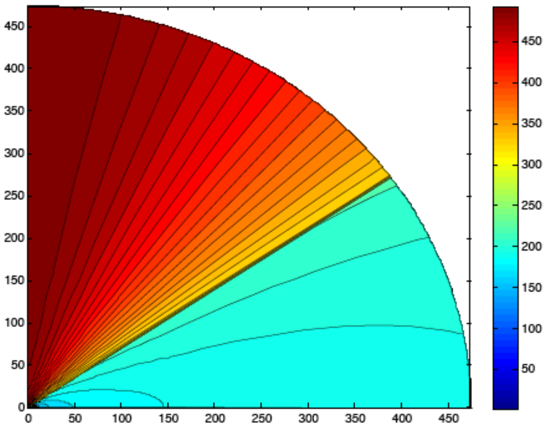

13 colors photometry

# B Supergiant m-CAK Wind

Continuum Spectra  HD 206165 (A. Torres et al. 2010, sub.)




$\Omega = 0.75$



13 colors photometry

2-mass



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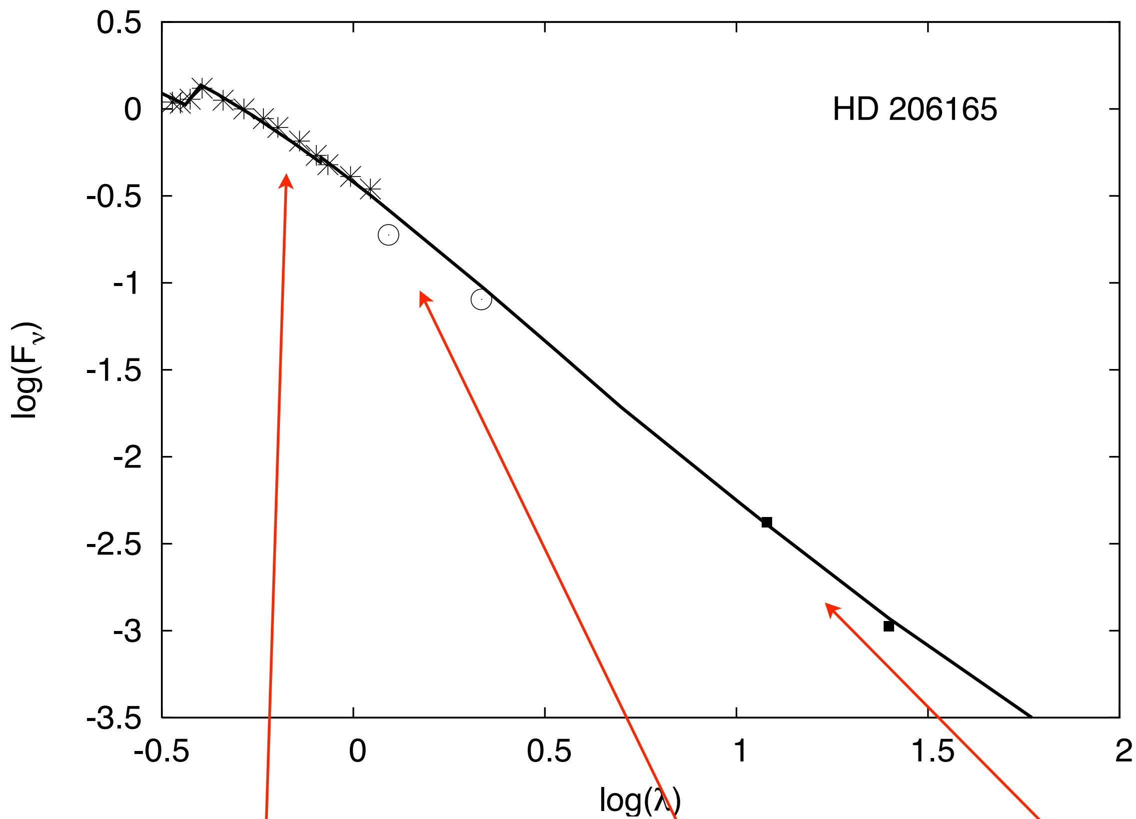
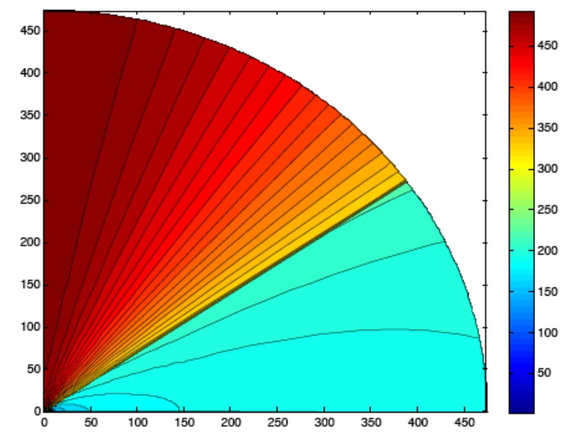
# B Supergiant m-CAK Wind

Continuum Spectra HD 206165 (A. Torres et al. 2010, sub.)




$\Omega = 0.75$



13 colors photometry

2-mass

IRAS Fluxes



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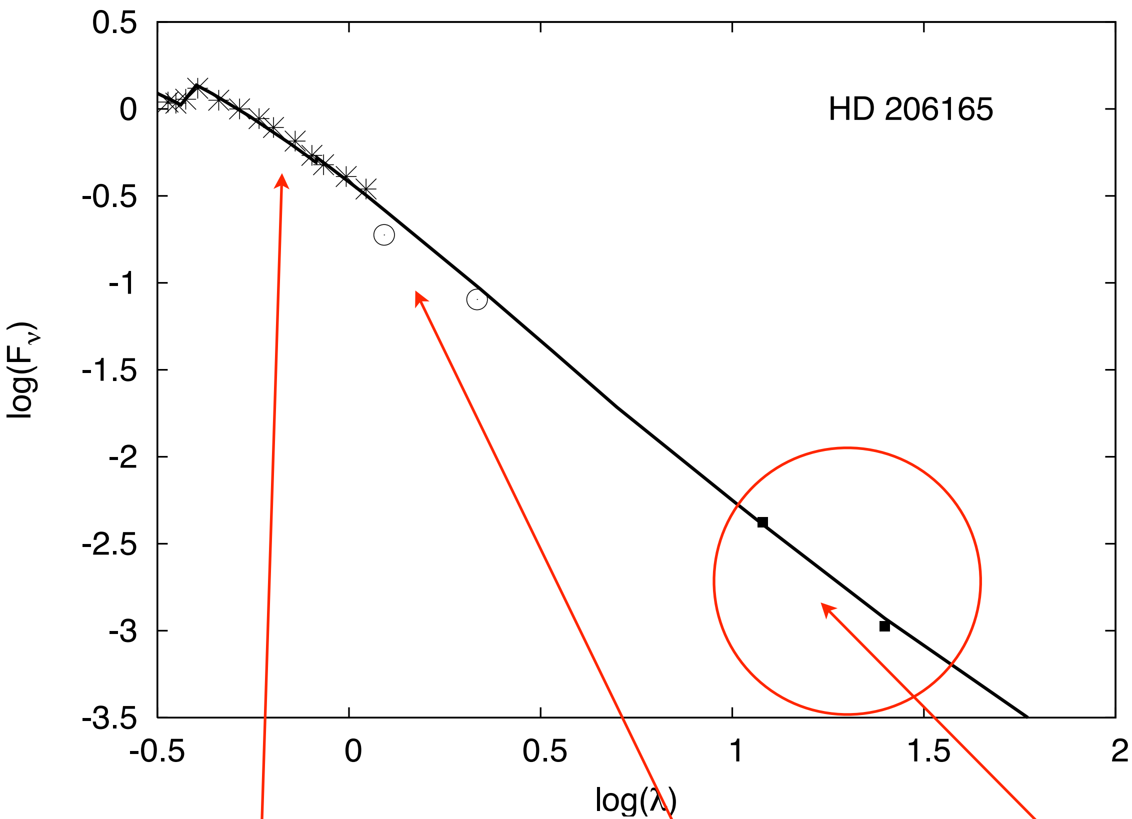
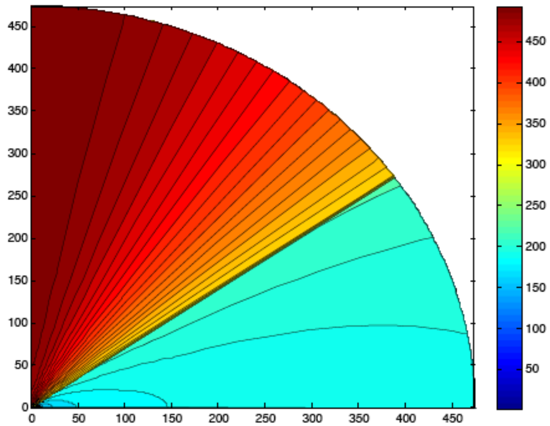
# B Supergiant m-CAK Wind

Continuum Spectra HD 206165 (A. Torres et al. 2010, sub.)




$\Omega = 0.75$



13 colors photometry

2-mass

IRAS Fluxes



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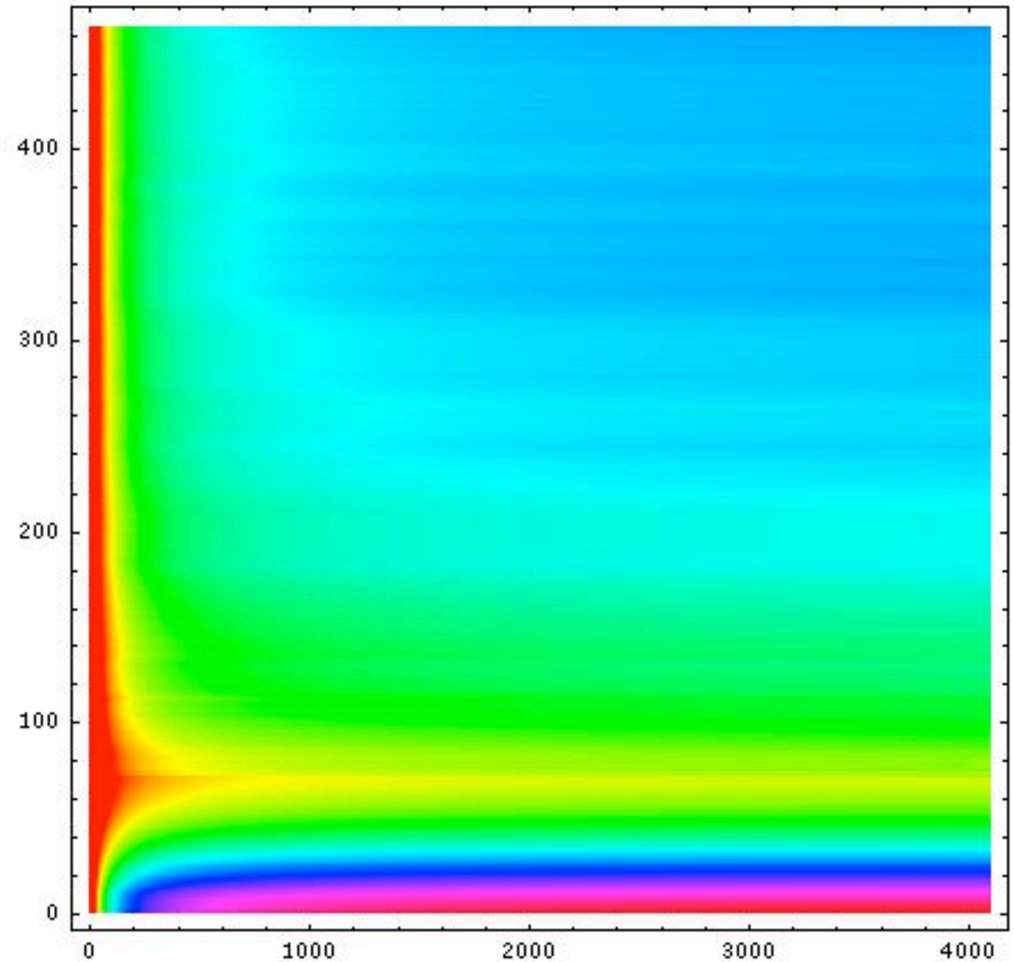
# Time dependent Hydrodynamic

# Time dependent Hydrodynamic

ZEUS-3D
CAK model



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# Time dependent Hydrodynamic

## ZEUS-3D
## m-CAK model

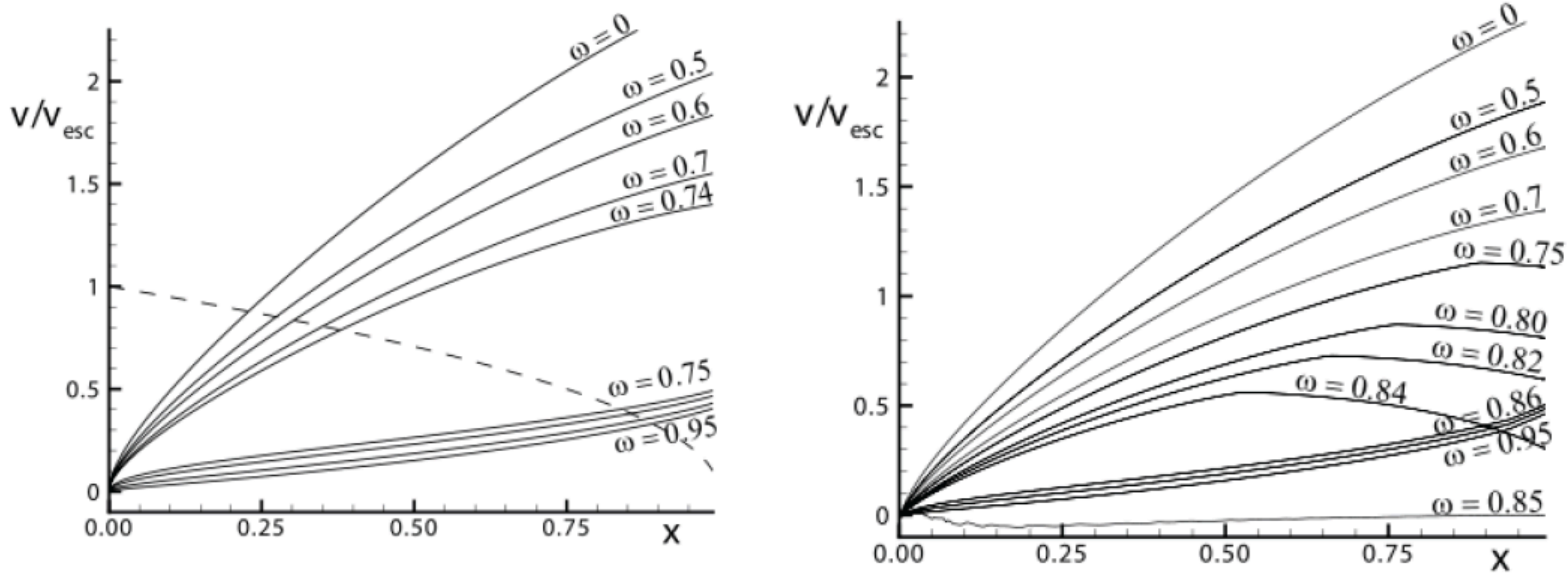


Fig. 36. Condiciones iniciales propuestas por Madura, Owocki & Feldmeier (2007)

# Conclusions

Slow wind solution may solve some of the problems from massive stars hydrodynamic:

- Winds from B[e] Supergiants (outflowing disk)
- Time dependent Calculations (bifurcation, oscillation, clumping?)
- 2D-calculations
- Observations (constrains to theory)



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# F I N



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