DIRECT DETECTION OF GRAVITY WAVES THROUGH HIGH PRECISION ASTROMETRY

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ABSTRACT

It is generally accepted that a first ever direct detection of gravity waves would herald a new era in astronomy and in fundamental physics. Ever since the early sixties, increasingly larger human and material resources are being invested in the detection effort. Unfortunately, the gravity wave effects one has had to exploit so far are extraordinarily small and are usually very many orders of magnitude smaller than the noise involved. The detectors that are presently at the most advanced stage of development hope to register extremely rare, instantaneous longitudinal shifts that are expected to be orders of magnitude smaller than one Fermi.

However, it was recently shown that gravity waves can manifest themselves through much larger effects than previously envisaged. One of these new effects is the periodic, apparent shift in a star's angular position due to a foreground gravity wave source. The comparative largeness of this effect stems from its being proportional not to the inverse of the gravity wave source's distance to the Earth, but to the inverse of its distance to the star's line of sight. In certain optimal but not unrealistic cases, the amplitude of this effect can reach the critical level of one micro-arcsecond, thus raising the prospect that the long-awaited first direct detection of gravity waves could be achieved by a high-precision astrometry space mission such as GAIA.

Key words: gravity waves, interferometric astrometry, GAIA

1. ASTROMETRIC EFFECTS

Perhaps the earliest understood physical effect of gravity waves, is their modulating of proper distances (Einstein 1916, Einstein 1918, Weyl 1922, Eddington 1924.) The first bar detectors (Weber 1960), as well as recent detection projects such as LIGO (Vogt 1991) and VIRGO (Bradaschia et al. 1990), are based on that effect.

The experimental challenge facing such detection efforts is daunting. The expected distance modulations have about the same magnitude as the gravity-wave's amplitude, which is typically smaller than 10^{-22} in the vicinity of the Earth. Thus, these experiments involve detecting shifts much smaller than one Fermi in distances of the order of a kilometer.

Not long ago, it was proposed to explore an approach to gravity-wave detection that is based on accelerations of null, rather than timelike geodesics (Fakir 1991).

The simplest illustration of this idea is the shifting of apparent stellar positions due to an intervening gravitational pulse (Fakir 1992). Suppose a supernova flash hits the Earth, coming from the northern celestial hemisphere. This is an indication that a gravitational pulse has also just whipped passed the Earth, and is now interposed between us and all the southern celestial hemisphere. It was calculated that the angular positions of southern stars would then experience apparent shifts of the order of the pulse amplitude h:

$$|\delta\alpha| \approx \frac{1}{2} h \sin\alpha \quad , \tag{1}$$

where α is the angle of incidence of the light rays with respect to the gravitational wave front. In the case of 'pulses with memory' (Kovacs & Thorne 1978, Smarr 1979, Braginski & Grishchuk 1985, Grishchuk & Polnarev 1989) such shifts can be quasi-permanent.

Quantitatively, this version of the effect does nothing to improve the prospects of gravity-wave detection. The angular shifts resulting from Eq. (1) should be smaller than 10^{-17} arcsec, while the precision that seems achievable today in this context through interferometric astrometry and very-long-baseline interferometry is about 10^{-6} arcsec.

However, this effect presents a feature that distinguishes it qualitatively from most others: in Eq. (1), h is not the amplitude of the waves when they meet the Earth. Instead, h is the amplitude of the waves when they meet the stellar photons, which only much later reach the Earth. This amounts to a prospect of *remote probing of gravity waves.*

Since $h < h_{\rm Earth}$ in the case above, this feature can only worsen the observational situation in this particular illustration. However, this same feature can dramatically improve the detectability of the effect, if the configuration defined by the gravity-wave source, the Earth and the light source is, in a sense, inverted (Fakir 1994a,b.) In the new configurations, as we shall see below, probing the waves at a distance could mean probing them in regions of space where h is not smaller, but much larger than $h_{\rm Earth}$.

In the previous illustration, the Earth was placed between the gravity-wave source and the light source. Consider, now, a situation where it is the gravity-wave source that is placed between the Earth and the light source. Then the photons, during their journey towards the Earth, would have encountered gravity-wave crests with heights ranging from $h_{\rm light\ source}$ to $h(b)$ to $h_{\rm Earth}$, where b is the distance of closest approach between the photons and the gravity-wave source, the 'impact parameter'.

The hope, of course, is that the photon will 'remember' the highest amplitude of gravity waves it sees on its way to the Earth. If so, the analogue of Eq. (1) for the new configuration would exhibit $h(b)$ on the right-hand side, rather than the much smaller $h_{\rm Earth}$ or $h_{\rm light\ source}$. The whole scheme would then amount to remote probing of *strong* gravity-wave sites.

A priori, there are several reasons to fear that this scheme would not work. The physics of the photons' encounter with gravity waves is more involved in this latter case of spherical wave fronts than in the former plane-wave case.

For example, one could question whether the deflections acquired by a photon during the 'ingoing' phase (approaching the gravity-wave source) are not cancelled by deflections during the outgoing phase. Fortunately, the calculation shows that this is not the case.

One could also wonder if there would be deflections at all during the outgoing phase: the gravity-wave crests travel at the same speed as the photon itself. Now, the photon is only sensitive to *variations* in h, and it would see no such variations if it travels along with the gravity waves. Nevertheless, in most actual situations, photons and gravity-wave fronts travel *at an angle*. Hence, in the outgoing phase also, photons may see changes in h and experience deflections.

Let the gravity-wave mode of interest be described by

$$h = \frac{H}{r}\exp\{i\Omega(r - t + t_{\rm ph})\} \ , \tag{2}$$

where $t_{\rm ph}$ determines the wave's phase. H is a constant that encodes the intrinsic strength of the source. Working in a spherical transverse-traceless gauge, projecting the problem onto a plane containing the Earth and the light and gravity-wave sources, and considering the optimal alignment case where $b\Omega$ is of order 1, one finds (Fakir 1994a)

$$|\delta\phi|_{\rm optimal} \approx \frac{3}{4}\pi\Omega H = \frac{3}{2}\pi^2|h(r=\Lambda)| \ , \tag{3}$$

where Λ is one gravitational wavelength. The angle ϕ is close to 0, $\pi/2$ and π at the light source, the gravity-wave source and the Earth, respectively.

Let us generalize this result to arbitrary values of the impact parameter b. We can infer from Eq. (16) of the above reference that

$$\delta\phi \approx \frac{H}{b}e^{i\Omega t_{\rm ph}}\int_0^\pi d\phi \exp\left\{i\Omega b\frac{1+\cos\phi}{\sin\phi}\right\}$$

$$\times\left[\sin\phi - \frac{3}{2}\sin^3\phi + i\Omega b\left(\frac{\sin^2\phi}{2} - 1 - \cos\phi\right)\right] \tag{4}$$

(This was obtained by comparing the two ends of the trajectory: $\phi \approx 0$ and $\phi \approx \pi$. One can show that $\delta\phi = b[u_1(\phi\approx 0) + u_1(\phi \approx \pi)]$, where u_1 is the fluctuation of $1/r$.)

Eq. (4) can be rewritten as

$$\delta\phi \approx \frac{H}{b}e^{i\Omega t_{\rm ph}}\int_0^\infty \frac{4xe^{ib\Omega x}}{(1+x^2)^2}$$

$$\times\left[1 - \frac{6x^2}{(1+x^2)^2} - \frac{ibx^3}{1+x^2}\right]dx \tag{5}$$

which integrates nicely to the analytical formula

$$\delta\phi \approx \frac{1}{2}H\Omega e^{i\Omega t_{\rm ph}}\left[(b\Omega+1)e^{b\Omega}E_1(b\Omega)\right.$$

$$\left. + (b\Omega - 1)e^{-b\Omega}E_1(-b\Omega)\right] \tag{6}$$

$E_1(z)$ is the exponential integral function

$$E_1(z) = \int_1^\infty \frac{e^{-zt}}{t}dt \ , \quad Re(z) > 0 \ , \tag{7}$$

extended analytically to the entire complexe plane except $z = 0$. It is straightforward to verify that Eq. (5) integrates to Eq. (3) in the limit $b\Omega << 1$.

Thus, the gravity-wave-induced deflection is equal to the wave amplitude at only one gravitational wavelength from the source, times a factor that decreases slightly faster than $1/b\Omega$.

Besides the future prospects of achieving angular resolutions of the order of 10^{-7} arcsec for radio sources by space-based interferometry, there has been considerable progress, recently, towards reaching a very high angular precision for optical sources as well (Perryman et al. 1992, Høg 1993 and several papers in these proceedings.) Also, the increase in angular resolution power has been accompanied by a considerable improvement in photometric sensitivities, potentially revealing a number of new stellar systems that could be relevant to this study.

There are several actual astronomical configurations to which this approach can be applied. The candidates fall into two classes. In the first, the gravity-wave source and the light source are aligned with the Earth by pure chance. They are two unrelated, far apart celestial objects. Because of the large number of binary stars in the Galaxy, also because of their relatively large gravity-wave amplitude and wavelength, a lucky alignment of a binary star with some more distant light source would be the typical candidate in this class. Neutron stars are too scarce, are too weak gravity wave emitters, and the most interesting have too short wavelengths to qualify for astrometric detection.

Numerically, candidates in this first category could produce optimal shifts of about 10^{-6} arcsec, which falls within the precision attainable by a space-based astrometric project such as GAIA. Such shifts could be produced, for instance, by a very fast binary source with $H \sim 5$ cm and an orbital period of about an hour (i.e. the gravity-wave period is $2\pi/\Omega \sim 30$ minutes.) For more details about how GAIA could serve as a gravity wave detector using this effect, see Makarov 1995. It is argued there that it could be possible to detect gravity waves through the effect described above within an astrometric mission like GAIA, by scanning the sky systematically for all possible galactic gravity wave sources, including the (in principle) very numerous invisible neutron-star binaries.

Alternatively, and perhaps as a first detection attempt that would be in keeping with the present outline of the GAIA project, one could first select a not too distant gravity wave source, typically a fast binary system that is within 100 to 1000 parsecs from the Earth. One then has to find a background star that lies within a few arcseconds of the binary. The proper motion of the binary increases the likelihood of such alignments over a few years period. There is a number of promising sources in the galaxy to be investigated for such alignments with background stars, including cataclysmic variables and massive X-ray binaries (Fakir & Quist 1995.) Because the gravitational signal would be (1) periodic and (2) known to a very high accuracy, it is possible to use techniques such as data folding and other versions of filter matching to reduce the noise to the expected level of the gravitational signal.

A second class of *a priori* candidates is formed by cases where the gravity-wave source and the light source are locked into tight gravitationally bound systems. Common examples of this in the galaxy are stars (as light sources) and binaries (as gravity-wave sources) locked into multiple-star systems or even globular clusters. Of particular observational importance is the case of a binary formed by a neutron star (as the gravity-wave source) and some companion star (as the light source.)

Comparison of typical gravitational wavelengths and typical separations shows that the alignment requirement, for this category, is satisfied naturally. Unfortunately, because of the proximity between light source and deflector, the observationally relevant apparent angular shift of the stellar image is much smaller than the deflection angle. Eventually, however, it was shown that another effect could be exploited in the detection of gravity waves from some of the most interesting members of this class of candidates, namely those which comprise a pulsar (Fakir 1994a.)

Take, for instance, a system like the well studied binary pulsar PSR B1913+16 (Taylor 1992.) It turns out that this system and alike could be very promising sites for direct gravity-wave detection. (This is, of course, besides the indirect evidence for the existence of gravity waves already provided by the observed secular slow-down of this binary pulsar.)

Consider first, as the gravity-wave source, the dark neutron star that revolves around the actual 17 Hz pulsar. Once every $7^{\mathrm{h}}45^{\mathrm{m}}$, the two stars come to within only one light-second (about half a solar radius) of each-other. This is, at most, of the same order of magnitude as the darker companion's gravitational wavelength.

In principle, there are two more sources of gravity waves that could be affecting the apparent position of that same light source. One is the pulsar itself. Being a neutron star that rotates 17 times per second, it should be emitting gravity waves at a frequency of 34 Hz. However, (1) the angle between the electromagnetic and the gravitational directions of propagation is very small in this case, (2) here there is no incoming, only an outgoing phase. As mentioned above, the combination of these two facts means that the light from the pulsar is unlikely to be deflected by the pulsar's own gravity waves.

The other additional source of waves is the binary system as a whole. (These are the waves for which there is already indirect observational evidence.) Here also, the shortness of the incoming phase and the smallness of the angle between the electromagnetic and the gravitational directions of propagation are a concern. More importantly, there are more considerations that have to be taken into account, before one can make predictions in this case. The photons, here, originate from the gravity-wave source itself, and traverse the near-zone ($r < \Lambda$) before reaching the radiation zone, where our calculations are valid. Such cases necessitate a separate study, where, in particular, dynamical Newtonian contributions to the deflection would have to be included.

Numerically, the deflections produced in this case may well reach the 10^{-6} arcsec (Fakir 1994a.) This could be the case if, for instance, the companion is a 10 Hz neutron star (i.e. the gravitational frequency is $\Omega/2\pi = 20\ \mathrm{sec}^{-1}$), radiating perhaps through the Chandrasekhar-Friedman-Schutz mechanism (Chandrasekhar 1970, Friedman & Schutz 1978), with a strength $H = 10^{-6}$m. However, as we mentioned earlier, these deflections do not translate into significant apparent angular shifts in this case. Nevertheless, the consideration of pulsars as sources of the deflected light lead to another gravity-wave detection prospect, which we now summarize for completeness.

Following the above study, the next logical step is to try to exploit the exceptional properties of pulsars, especially the high stability of their period. This was achieved through the exploitation of an effect that has little to do with light deflection, namely the gravity-wave-induced *modulation of time delays*, in the very same astronomical configurations discussed above (Fakir 1994b.)

The possibility of gravity-wave detection through the modulation of pulsar frequencies by *plane* waves has already been extensively explored (Detweiler 1979, Romani & Taylor 1983, Hellings & Downs 1983, Stinebring et al. 1990.) The experimental effort in this field has made it possible to detect fractional frequency modulations as faint as 10^{-15}. Thus, stringent upper limits could be imposed on the cosmological and the galactic gravity-wave backgrounds. Recently, we also learned that, in the wake of this effort, the contribution of individual binary stars was also considered in one instance (Sazhin 1978.) Unfortunately, this initial investigation was not followed up by the consideration of more promising candidates, such as systems consisting of gravitationally bound light and gravity-wave sources.

In complete analogy with our discussion of the gravity-wave-induced light deflection effect, the hope here is that, (1) it can be shown rigorously that the crossing of a zone of spherical gravity waves does result in a net frequency modulation; (2) the strongest gravity-wave amplitudes encountered along the trajectory do contribute to the net modulation.

The same worries we had initially for the working of the light-deflection effect (see above), can be expressed here. Once again, the calculation shows these worries not to be founded (Fakir 1994b.) It was shown that spherical gravity waves can induce time-delay fluctuations $\delta(\Delta t)$ that vary at a rate

$$\frac{d}{dt_{\mathrm{ph}}}\delta(\Delta t) \approx \frac{1}{2}\Omega H e^{i\Omega t_{\mathrm{ph}}}$$

$$\times \int_{\phi_{\text{initial}}}^{\phi_{\text{final}}} d\phi \sin\phi \exp\left\{ ib\Omega \frac{1+\cos\phi}{\sin\phi} \right\} \quad (8)$$

(Δt is the total time it takes a photon to travel from the light source to the Earth, via the gravity-wave source.)

Here also, the problem has a compact analytical solution. A change of variables can put Eq. (8) in the form

$$\left| \frac{d}{dt_{\text{ph}}} \delta(\Delta t) \right| \approx 2\Omega H \left| \int_0^\infty \frac{x}{(1+x^2)^2} e^{ib\Omega x} dx \right| \quad (9)$$

This integrates to

$$\left| \frac{d}{dt_{\text{ph}}} \delta(\Delta t) \right| \approx H\Omega \left| \frac{b\Omega}{2} \left(e^{b\Omega} E_1(b\Omega) \right. \right.$$

$$\left. \left. - e^{-b\Omega} E_1(-b\Omega) \right) - 1 \right| \quad (10)$$

Hence, numerically, this second effect behaves just like the first one (Eq. (6)), at least in orders of magnitude: for optimal alignments, it is as high as the waves' amplitude only one gravitational wavelength away from the source. For larger impact parameters, the effect decreases roughly like $1/b\Omega$.

To use the same numerical illustration as for the previous effect, a binary star with $H \sim 5$ cm and a gravity-wave period (half the orbital period) $T \sim 2\pi/\Omega \sim 30$ minutes, would produce fractional frequency modulations of about 5×10^{-13}. A neutron star with $H \sim 10^{-6}$m and $T = 2\pi/\Omega \sim 0.05$ sec, would yield frequency modulations as strong as 10^{-12}.

Retrospectively, this latter approach to gravity-wave detection has exploited a perhaps curious observational fact. For several cases of gravity-wave sources that are members of gravitationally bound stellar systems, the stellar separations can be as small as only one gravitational wavelength or so. Thus, in a dense globular cluster, the average stellar separation is of the same order of magnitude as the gravitational wavelength of a typical binary star. For a binary system, one member of which is a neutron star, the orbital size can be comparable to that neutron-star's gravitational wavelength. Hence, there exists many astronomical sites where light sources are constantly moving close to, or even within gravity-wave near-zones.

To summarize, what happens in this detection scheme in the most common case (which is the one relevant to high precision astrometry) is that the photons from the background star travel huge distances virtually unperturbed, then cross regions of strong gravity waves where their direction of propagation is shifted, then travel on towards the Earth where they eventually deliver the record of their encounter with strong gravity waves. This amounts to a possibility of probing regions of *strong* gravity waves at a distance, thus avoiding the extraordinary smallness that plagues most other gravity wave effects.

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