ANALYSIS OF AN ASTROMETRIC FIZEAU INTERFEROMETER FOR GAIA

Sacha Loiseau, Stuart Shaklan

Observatoire de Paris, F-92195 Meudon, France
and
Jet Propulsion Laboratory, California Institute of Technology,
4800 Oak Grove Drive, Pasadena, California 91109, USA

ABSTRACT

GAIA's concept tends to bring together solutions chosen for the Hipparcos astrometry satellite and interferometric techniques. Like Hipparcos, GAIA is a continuously scanning instrument for which the integration time on any observed object is limited by the field of view of the detector. If a final astrometric accuracy of 10 microarcsec is aimed at, a field of view of 1° in diameter is needed. We present a design for the proposed 2.6-m baseline Fizeau interferometer with two 40 cm apertures, and overall dimensions compatible with the size of the Ariane 5 payload shroud. It has a 0.9° diffraction limited field of view. The response of the optical system to small perturbations on each optical element is given in terms of fringe visibility, which is shown to depend only on sub-aperture spot separation. The robustness of the design to thermal, mechanical and manufacturing errors is discussed. The unavoidable distortion present in wide field optical systems is analyzed in terms of displacement of the interference fringes. This work is intended as a step towards an end-to-end modelling tool.

Key words: Optical Design, Interferometry, Tolerances, Fringe Visibility.

1. INTRODUCTION

GAIA is an extremely daring concept. Although some technical solutions can be directly adapted from Hipparcos, most of them are technological challenges requiring innovative solutions or breakthroughs in various areas, such as detectors, metrology and stable structures.

The main body of the instrument, the interferometer itself, is to be designed with several constraints. For our application, namely wide field astrometry, the Fizeau configuration seems more appropriate because it offers a significantly wider field of view than a Michelson with the same entrance pupil configuration.

The strict performances requirements on such an instrument lead to severe stability constraints on the optical and mechanical structure. For instance, achieving an astrometric accuracy of 10 microarcsec (μas) using a 3 metre baseline requires a measurement accuracy in fringe position of 15 picometers at a wavelength of 550 nm. Moreover, the optical path difference between the two interfering beams must be stabilized so as to remain smaller than a fraction of the coherence length $\lambda^2/\Delta\lambda$. In the case of a Fizeau interferometer, the Airy disks formed by the two apertures must add coherently on the focal plane, imposing stability constraints on the optical train.

A study of the interferometer tolerances to mechanical perturbations is performed to define the robustness of the system in terms of fringe contrast. It is shown that as the interferometer is mechanically perturbed (e.g. tilt or displacement of any optic), the fringe contrast drops mainly as a function of spot separation in the image plane, with aberrations playing only a secondary role. The main aberration present in our system, namely distortion, is analyzed both for the geometrical spot and the diffraction image. The variations of distortion with the motion of the optics is studied as well.

2. AN ALL REFLECTIVE COMPACT THREE-MIRROR TELESCOPE

Like Hipparcos, GAIA is a continuously rotating instrument for which the calibration parameters of the instrument are part of the astrometric data, solved for during the global data reduction. This leads to an intrinsic limitation of the integration time on any observed object: its upper bound is roughly $(\Omega T)/4\pi$ where Ω is the solid angle subtended by the field of view, and T the total duration of the mission. Since the latter is not expandable, the field of view of the instrument (of each of the three identical interferometers) should be maximized. Simple calculations on astrometric precision for photon noise limited observations show that a field of view of 0.9–1° in diameter is needed to reach the desired astrometric precision. The Fizeau type interferometer traditionally offers a wider field of view than the Michelson type, hence our choice of optical configuration. The natural first step to the design of a Fizeau interferometer is the design of the corresponding mono-pupil telescope on which a two-aperture mask will later be placed, set by the need for a wide field of view and by its necessary compactness: it had to comply with the dimensions of the Ariane 5 payload shroud which is a 4.4-m circular envelope. It seemed logical to consider a three-mirror telescope, capable of being compact with a sufficiently long focal length. A reasonable size of a few decimeters for the focal plane sets the value of the latter: with a field of view of $\sim 1°$, a value between 10 and 15 m seems adequate.

2.1. A Three-Mirror Design

Adding a third mirror to the system introduces additional degrees of freedom compared to a two-mirror design, making it possible to obtain an aplanatic and anastigmatic configuration (Korsh 1977).

Our design constraints were to use three conic mirrors, to achieve a diffraction-limited field of view of 0.9°, maintain a total length no greater than 3.5 m, and to constrain the effective focal length to 11.55 m. We optimized for the best composite focus over the whole 0.9° field of view, letting the conic constants, separations, and focal lengths of the mirrors vary. In the final configuration, the three mirrors composing the telescope are hyperboloids with reasonable conic constants. The specifications of the mirrors are shown in Table 1.

Surface	z	D [m]	r	K
primary mirror	3.52	3.00	11.57	-1.41
secondary mirror	0.02	1.25	5.36	-3.58
tertiary mirror	3.52	1.12	10.02	-5.09
focal plane	0.00	0.18	∞	—

Table 1: Parameters of the three-mirror telescope from which the interferometer is derived. z = coordinate of the vertex of the surface, D = outer diameter of surface, r = radius of curvature, K = conic constant. Light enters in the $+z$ direction. All mirrors are concave towards $+z$ (negative curvature). The focal surface is flat.

The tertiary and the primary share a common vertex but have different curvatures. The aperture stop is located on the primary mirror. The focal plane is located 2 cm behind the secondary. Its proximity could facilitate thermal control, passive or active. The overall length of the telescope is 3.5 m and its width 3 m, thus fitting in the Ariane 5 envelope. The longest possible separations between the mirrors inside the 3.5 m envelope makes it possible to have relatively slow beams, better for aberrations. The final focal ratio of the system is 3.85, which, if we consider a 2.6-m baseline Fizeau interferometer, is not sufficient to reach the Nyquist sampling limit of the fringes, making direct fringe detection difficult to achieve. It is however interesting to study the behavior of the fringes in the focal plane, even if a pupil plane detection is chosen eventually. The optical layout is shown in Fig. 1.

2.2. Image Characteristics

At the working wavelength, 550 nm, the diameter of the first Airy ring of the diffraction pattern of the telescope is 5.17 μm. The rms sizes of the geometrical spots given by Code V (ORA) are 2.5 μm on axis, 4.3 μm at 0.32° (corresponding to $0.45/\sqrt{2}$) and 5.5 μm at 0.45°, making it almost perfectly diffraction limited over a FOV of 0.9°. The Strehl ratio (ignoring the central obscuration) at full field is 0.875. The spot diagram, displayed in Fig. 2, shows that the system has low aberrations. Because of the wide field of view, there is quite a large amount of distortion in the system. A careful study of the distortion present in the interferometer is described later. The aberrations of the corresponding Fizeau interferometer, as described in the following section, will only be a fraction of that.

3. A DESIGN FOR A FIZEAU INTERFEROMETER

The ray sampling densities of most optical design programs are inadequate to characterize the performance of a Fizeau interferometer. We desire a large number of rays across two widely separated and relatively small subapertures, while at the same time demanding diffraction analysis using both subapertures simultaneously for interferometry. We used the Controlled Optics Modeling Package (Redding et. al. 1992), which offers a large number of rays (up to 2048 × 2048) and the possibility of implementing small sub-apertures by means of obscurations on the primary mirror. Furthermore, COMP has a built-in full diffraction propagation feature, making it possible to obtain interference patterns on the focal plane. A layout of the Fizeau interferometer is shown in Fig. 3. Each of the three original mirrors could be reduced to two small sub-apertures. For the primary, these apertures have a diameter of 40 cm.

3.1. Interference Fringes with a Fizeau Interferometer

The purpose of building an interferometer is to measure the phase and amplitude of the interference fringes resulting when a star is observed. The phase gives an extremely sensitive measure of the star's position relative to other stars (whose phases are being simultaneously measured), while the modulation amplitude is related to the signal-to-noise ratio of the measurement. In an ideal instrument, each subaperture forms perfectly overlapping, unaberrated Airy patterns, and the amplitude is maximized. In reality, however, the large field of view and limited degrees of freedom have led to a design that introduces small aberrations and image separation. Perturbations such as subaperture tilts, displacements, and defocus will further reduce the fringe modulation. In this section, we present the formalism for analyzing the fringe modulation. The next section then utilizes these results and presents a tolerance analysis of the design.

We begin by assuming that the incoming light is monochromatic and that it emanates from a distant point source. The assumption of an unresolved source is reasonable since the vast majority of stars observed by GAIA are unresolved with a 3 m baseline.

The incoming plane wavefront passing through the two subapertures P_1 and P_2 is defined by

$$P_n = a\, e^{-i\vec{k}\cdot\vec{r}} P(\vec{r} + (-1)^n \vec{B}/2) e^{-i\phi_n(\vec{r})} \qquad (1)$$

where $\vec{B}$ is the baseline vector, ϕ_n are the aberrations of the wavefront, P is the subaperture pupil i.e. $P = 1$ for $|\vec{r}| < d/2$, $P = 0$ if not, and d is the subaperture diameter. The wavefront is moving in direction $\vec{k}$ when it is incident on the subapertures.

The optical system performs a Fourier Transform of the wavefront, resulting in an image plane field given by $\tilde{P_1} + \tilde{P_2}$, where tilde over a symbol indicates the Fourier Transform of its corresponding function in the pupil plane. The observed intensity is proportional to the square of the field, so that one has:

$$I = |\tilde{P_1}|^2 + |\tilde{P_2}|^2 + 2\mathrm{Re}(\tilde{P_1}\tilde{P_2^*}) \qquad (2)$$

The signal modulation is proportional to the amplitude of the cross term.

The formalism giving the expression of the intensity on the focal plane in aberration-free and aberrated cases is described in the paper by Loiseau & Shaklan (1995). In the following section however, we give the details of the estimation of the fringe visibility.

3.1.1. Estimation of the Fringe Visibility

It is convenient to analyze the fringes by taking the Fourier transform of their intensity distribution. Squaring gives the spectral density (or power spectrum) of the image, which presents non-overlapping low- and high-frequency terms. Roddier & Léna (1984) have shown that the ratio of high to low frequency energies gives a good estimate of $\frac{V^2}{2}$. Their equations include effects of resolving the source with the interferometer but based on a perfect system with no aberrations. We show, in Section 3.4., that this ratio is proportional to a ratio of convolution integrals of intensities displaced by perturbations applied to the sytem. We applied this method to estimate the visibility of the fringes given by COMP. The intensity curve corresponding to the diffraction pattern given by COMP is displayed in Fig. 4. The method then consisted of taking the 2-D Fourier transform of the fringes, squaring its modulus and calculating the ratio of appropriate terms. This spectral density is displayed in Fig. 5.

The fringe contrast for various field angles is given in Table 2. The contrast of these monochromatic fringes is high at all field angles in both viewing direction, even up to 0.8° in the field (contrast = 0.98 and 0.88), confirming the good performances of the design.

X, Y field angles [deg]	0	0.32	0.45
visibility (Y)	0.995	0.989	0.984
visibility (X)	0.995	0.986	0.974

Table 2: Variations of fringe contrast with field angles in both directions (along baseline axis and perpendicularly).

3.1.2. Fringe Visibility Reduction with Finite Optical Bandwidth

With a finite optical bandwidth $\Delta\lambda$, the loss of visibility is slightly increased. Indeed, assuming equal power at all frequencies, the reduction factor due to chromatic effects that should be applied to the theoretical contrast can be expressed as:

$$|\Gamma_{\Delta\lambda}| = \frac{\sin\left(\pi\frac{\Delta l \Delta\lambda}{\lambda_0^2}\right)}{\pi\frac{\Delta l \Delta\lambda}{\lambda_0^2}} \qquad (3)$$

where Δl is the optical path difference between the two interfering beams. If a bandwidth equal to $\Delta\lambda = \lambda/10$ is assumed, an optical path difference of the same value would lead to a negligible reduction of the fringe contrast, but an optical path difference of λ (still a tenth of the coherence length) gives a correction factor of 0.986. For a bandwidth $\Delta\lambda = \lambda/4$, the coherence length is 4λ and an optical path difference of λ reduces the fringe visibility by a factor 0.900. The impact of polychromaticity becomes more important at large field angles.

Our program makes it possible to co-add interference patterns obtained for different wavelengths. Table 3 gives the fringe contrast obtained with bandwidths of 60 nm and 100 nm, for the unperturbed system, at three field angles. As predicted, depending on the field angle, the reduction factor on the fringe visibility ranges from 0.8 to 0.9 for a bandwidth equal to $\Delta\lambda \sim \lambda/9$ (60 nm), even at full field for a slightly perturbed system. For a wider bandwidth, $\Delta\lambda \sim \lambda/5$ (100 nm), the fringe contrast is reduced by the same order of magnitude for the unperturbed system. After perturbation of the system, the consequences of such a bandwidth are, of course, a little bit more dramatic and shown in Section 3.3.2..

Field angle [deg]	0	0.32	0.45
$\Delta\lambda \sim \lambda/5$	0.822	0.764	0.781
$\Delta\lambda \sim \lambda/9$	0.887	0.812	0.823

Table 3: Variations of the fringe contrast with field angles for polychromatic fringes with two different optical bandwidths, for an unperturbed system.

3.2. Distortion

Distortion is the only aberration which does not affect image quality but only its position. With an astrometric interferometer, we are interested in measuring very accurately the phase of the interference fringes. If they are displaced by unpredicted distortion, significant errors can occur. To avoid that, the first thing is building a distortion free or at least a low distortion instrument. The second is to carefully analyze the present distortion so as to come up with a precise model of the aberrations over the whole field of view which would eventually allow an accurate measurement of the desired quantities, after correction of the aberration effects.

Our design was optimized for a wide field of view (0.9°) and is thus subject to distortion, since it scales as the cube of the field angle. Using COMP, its ray trace and diffraction features, we analyzed the distortion of the system for both the geometrical spot and the true diffracted image.

In a perfect system with no distortion, the centroid of the geometrical spot follows the variations of $f \times \tan\theta$ where f is the paraxial focal length and θ the field angle. In a real system, the centroid's position is displaced by a third order term. We first plotted the difference between the actual centroid position and $f \times \tan\theta$ versus field angle. The relevant curve is the one for which f is a least-squares fit of the system [centroid position versus $\tan\theta$]. These two curves are displayed in Fig. 6.

The second and most interesting step was to compute the position of the fringe pattern (the position of the central fringe for instance) and trace its variations across the whole field of view. Again, this could be done using COMP's full diffraction propagation. The variations of the fringe postion with respect to field angle are plotted on Fig. 7 where it can be seen that there is $\sim$ 40 mas of distortion at the edge of the field (0.45°). This corresponds to one fringe of displacement, an effect that would severely blur fringes if they were intergrated across the focal plane.

3.2.1. Variations of Distortion with Perturbations of the Optics

As the optical elements of our sytem are perturbed, we compute the position of the central fringe, as explained previously. It seems that the amount of distortion is quite insensitive to these perturbations, at least for the order of magnitude we considered. For instance, Fig. 8 shows the same curve as Fig. 7 after tilting one of the primary sub-apertures by 4 μrad. It can be seen that the magnification of the fringe position is slightly different than before. The amount of distortion at the edge of the field is only 10% greater than before. Severe perturbations on one of the tertiary sub-apertures for instance, lead to similar results thus showing that the system does not seem to be subject to large variations of distortion within a wide range of perturbations.

3.3. Tolerances of the Design

One of the main purposes of this work is to try to define tolerances of a Fizeau interferometer to thermal and mechanical perturbations. In this section, the system is perturbed in various ways and the fringe visibility is estimated for each case.

3.3.1. Theoretical Structure and Thermal Variations

To estimate the expected displacements due to these variations, we considered a structure made of low expansion material, supporting the different elements of the interferometer. The way we imagined it is displayed in Fig. 9. A translation of the central rod, joining the virtual primary vertex to the virtual secondary vertex, will create a rotation of the orthogonal rods which join the subapertures on each surface (primary, secondary, tertiary), if the contact point between them has three degrees of freedom. To estimate the values of these translations and rotations, we considered a Coefficient of Thermal Expansion (CTE) of $10^{-8}\,K^{-1}$, which seems quite reasonable today. A temperature variation of 10 K for instance, will expand the 3.5-m central rod by $10^{-8} \times 10 \times 3.5 = 0.35\,\mu$m. The resulting rotation for the 1.5-m lever joining the two primary apertures would be $0.35 \times 10^{-6}/1.5 = 0.23$ micro-radian. This value is rather pessimistic considering the performances of active thermal control. It could be divided by a factor as large as 1000, depending also on the value of the CTE which range is $5.10^{-8}\,K^{-1} - 5.10^{-9}\,K^{-1}$. A value of 10 K for temperature variation gives us a first idea.

3.3.2. Perturbations and Resulting Contrasts

In the case of a multi-pupil instrument, such as an interferometer, it is essential to look at perturbations of individual optical elements of the system, i.e., the segments composing the 'virtual' mirrors of the instrument as opposed to a global tip/tilt or translation of the whole telescope, or of one of its filled apertures, if any. Driving COMP and SCOMP (Subroutine COMP) by a fortran program, we propagated rays and diffraction through both apertures separately, making it possible to perturb each optical element independently. The amplitudes of the propagated electric fields are then added and squared

tilts (Y) [10^{-6} rad]	0.1	0.3	0.6	0.7
visibility	0.975	0.887	0.633	0.532
tilts (Y) [10^{-6} rad]	-0.1	-0.3	-0.6	-0.7
visibility	0.999	0.965	0.790	0.698
tilts (X) [10^{-6} rad]	0.1	0.3	0.6	0.7
visibility	0.987	0.922	0.689	0.589
translation (Z) [μm]	-1.0	-0.5	0.5	1.0
visibility	0.992	0.994	0.996	0.997
translation (Y)[μm]	1.0			
visibility	0.979			

Table 4: Variations of the fringe visibility with perturbations applied to one of the two primary apertures, for on axis observations.

field angle [deg]	0	0.32	0.45	0.8
visibility	0.887	0.863	0.845	0.839

Table 5: Variations of the fringe visibility with field angle for a primary sub- aperture tilted by 0.3 micro-radian.

to obtain the diffraction pattern on the common focal plane. The fringes are then treated and analyzed as described previously. The term perturbations includes tips, tilts and translations in every possible direction. As shown in Fig. 9, the optical axis is the Z axis and the axis supporting the baseline of the interferometer is Y. A tilt in Y means that the element rotates around the X axis. When a tilt in Y is applied, the aperture really rotates around the virtual primary vertex, therefore, a translation is applied to compensate for the displacement of the vertex of the aperture. This translation is not mentioned in what follows. A field angle in Y means that the Y axis rotated around the X axis: the observer always looks orthogonally to the Y axis.

Results concerning the perturbations applied to one of the two primary sub-apertures are shown in Table 4. These results are for on-axis observations. The slightly different values between positive and negative Y axis tilts are due to the fact that initially, the centroids of the images formed by each aperture do not perfectly overlap, inducing a non-symmetrical situation. A negative tilt improves the initial configuration and thus enhances the fringe contrast. The latter remains greater than 0.88 for perturbations of amplitude up to 0.3 micro-radian, which is rather satisfying.

For off-axis obervations after perturbation, the results are displayed in Table 5: a 0.3 micro-radian tilt leads to a contrast of 0.85 at 0.32° in the field. Since the system has a wide almost diffraction limited field of view, one can expect the contrast to be quite constant over the whole field after any kind of perturbations: the variation is less than 5% of the on-axis value of the visibility.

Since each of the three mirrors could be divided into two much smaller sub-apertures, it is interesting to apply the same kind of perturbations to the secondary and tertiary sub-apertures. Since these mirrors are closer from the optical axis than the primary apertures, they will suffer from a larger tilt for the same expansion of the central rod of our theoretical structure. An expansion of 0.35μm led to a tilt of 0.23 micro-radian for the primary. It would tilt a secondary sub-aperture by 0.65 micro-radian and

field angle [deg]	0	0.32	0.45
visibility	0.907	0.884	0.868

Table 6: Variations of the fringe visibility with field angle for a tilt of a secondary sub-aperture by 0.65 micro-radian.

tilt [10^{-6} rad]	0	0.1	0.3	0.6
visibility	0.995	0.990	0.976	0.944
tilt 0.875 μrad				
Field angle [deg]	0	0.32	0.45	
Visibility	0.900	0.878	0.862	
tilt 1 μrad				
Field angle [deg]	0	0.32	0.45	
Visibility	0.876	0.851	0.834	

Table 7: Variations of the fringe visibility with the tilt of one of the tertiary sub-apertures for on axis observations, and for three field angles for a tilt of 0.875 micro-radian and 1 micro-radian.

a tertiary one by 0.875 micro-radian. Fringe contrasts obtained after perturbations, at three field angles, are shown in Table 6 and 7 for secondary and tertiary mirror respectively.

For nominal values corresponding to a 10 K temperature variation, the contrast stays above 0.83 for a single perturbation. A perturbation applied to two elements of the system has much more dramatic consequences on the fringe visibility. for instance, tilting a primary and a tertiary aperture by the nominal values given above (0.3 and 0.875 micro-radian respectively) leads to a contrast of 0.66, probably insufficient in terms of signal-to-noise ratio.

As for fringe contrast variations with a finite bandwidth for a perturbed system, the previous results should be on average multiplied by a factor around 0.85. For instance, for a bandwidth of 60 nm, after tilting a primary sub-aperture by 0.3 μrad, the contrast at the edge of the field is 0.71, to be compared to 0.85 for quasi-monochromatic fringes. It is therefore recommended to keep a quite narrow bandwidth so as to maintain high fringe visibility.

However, the temperature variations we considered in this section seem pessimistic. Sub-Kelvin thermal stabilization appears realistic for future missions. The tolerances we derived could then be loosened by a significant factor, as large as 100. The constraint on the bandwidth could then also be relaxed, which is a good point for the number of detected photons.

3.4. A Relationship Between Fringe Visibility and Centroid Separation

In an optical system such as a Fizeau interferometer, the contrast of the fringes obtained in the focal plane depends on two main parameters: the optical aberrations of the system and the degree of superposition of the images formed by each aperture. As a matter of fact, each of the two apertures of the interferometer produces a diffraction pattern on the focal plane. Interference fringes are the result of the coherent superposition of these two images.

For the unperturbed system, Fig. 10 shows the geometrical spot diagram of the interferometer: the centroids of the two spots are separated by $\sim 2\mu$m, i.e., about a twentieth of the Airy disk diameter.

The quality of the fringes depends directly on how well the images overlap. This can be quantified by the measure of the linear separation between the centroids of the spots given by each aperture. In a system with low aberrations, or in a low-aberration regime, the visibility of the fringes is only a function of this separation. To empirically prove this univocal relationship, we plotted the fringe visibility as a function of the separation of the centroids, whose positions are calculated for each configuration of the optical elements, for each family of perturbations. The curves show an excellent agreement. Moreover, it could be fitted using another way to express the contrast V as a function of spot centroid separation.

Let's consider again the Fourier Transform (denoted here by F for clarity) of intensity I:

$$\begin{aligned} \tilde{I} &= F(|\tilde{P_1}|^2) + F(|\tilde{P_2}|^2) + F(\tilde{P_1}\tilde{P_2}^* + \tilde{P_1}^*\tilde{P_2}) \quad (4) \\ &= P_1 \star P_1^* + P_2 \star P_2^* + P_1 \star P_2^* + P_1^* \star P_2 \end{aligned}$$

where $\star$ denotes the convolution of two distributions. This equation contains a low frequency term (the sum of the auto-correlations) and high-frequency terms centered at $\pm\vec{B}$. The terms do not overlap, enabling us, by squaring this expression, to obtain:

$$|\tilde{I}|^2 = |P_1 \star P_1^* + P_2 \star P_2^*|^2 + |P_1 \star P_2^*|^2 + |P_1^* \star P_2|^2 \quad (5)$$

We showed previously that the ratio of the high to low frequency energies gave a good estimation of $\frac{V^2}{2}$.

$$C = \frac{V^2}{2} = \frac{\int\int |P_1 \star P_2^*|^2 + |P_1^* \star P_2|^2 d^2 f}{\int\int |P_1 \star P_1^* + P_2 \star P_2^*|^2 d^2 f} \quad (6)$$

For unaberrated wavefronts ($\arg P_1 = \arg P_2$), the modulation of the fringes obtained with an unresolved source is equal to 1/2 and the contrast to unity. Using the convolution theorem, it is straightforward to show that the numerator of Eq. 6 is equal to the overlap integral of the images formed by each subaperture in the absence of interference which is the cross-correlation of two Airy patterns evaluated at the separation $\alpha_1 - \alpha_2$. Likewise, the denominator can be expressed in terms of overlap integrals. For the case of displaced images, one finally gets:

$$\begin{aligned} C &\propto \frac{2\int I(\rho-\alpha_1)I(\rho-\alpha_2)d\rho}{2\int I(\rho)^2 d\rho + 2\int I(\rho-\alpha_1)I(\rho-\alpha_2)d\rho} \quad (7) \\ &= \frac{2\int I(\rho)I(\rho+\alpha_1-\alpha_2)d\rho}{2\int I(\rho)^2 d\rho + 2\int I(\rho)I(\rho+\alpha_1-\alpha_2)d\rho} \end{aligned}$$

The distribution we obtained for various perturbations matches the curve given by Eq. 7, as demonstrated in Fig. 11. Using this interpolating function, we can obtain values accurate to within 2% of the true value.

The importance of this derived relationship resides in the possibility to have a quick access to the fringe visibility, i.e., to the signal-to-noise ratio, for any kind of perturbation applied to the optical system, by measuring the resulting displacement of the centroid of the spot produced by the perturbed part of the system. Moreover, by measuring the separation of the centroids in units of Airy disk diameter, the derived function can be generalized to Fizeau interferometers.

In this section, we have shown that the aberrations of the system only play a secondary role on the loss of visibility, compared to the relative tilts of the entrance subapertures. The main tolerances on these perturbations are, considering that a visibility of $\sim$0.75 is acceptable, $\sim 0.5\,\mu$rad for a tilt of a primary subaperture, $\sim 0.8\,\mu$rad for a tilt of a secondary subaperture and $\geq 1\,\mu$rad for a tilt of a tertiary subaperture.

Tolerances on translations of the optics are less stringent, allowing motions of several microns along both the optical axis and the perpendicular ones. It is also possible to define a quality criterion in terms of the separation of the light centroids. If a visibility of 0.75 or higher is needed, our results show that the separation between the centroids of the images of each sub-aperture must remain smaller than 32% of the diameter of the corresponding theoretical Airy spot.

4. CONCLUSION

A design for an all-reflective compact three-mirror telescope has been presented. This telescope has a diffraction limited field of view of $\sim 0.9^\circ$ and seems rather easy to manufacture. Mirrors have reasonable eccentricities and radii of curvatures but would naturally require a precise manufacturing (especially on conic constants specifications) so as to obtain an accurate compensation of the aberrations. The complete telescope fits into a 3.5-m circular envelope and is thus appropriate for Ariane 5. This design is used to model a Fizeau interferometer with 2.6-m baseline and 40 cm apertures. We derived a numerical function describing the variations of the fringe visibility with the motion of the centroids of the images formed by each aperture of the interferometer.

In the future, this relationship will enable us to quickly describe the response of the instrument to perturbations in terms of fringe visibility. We can define a quality criterion for the fringes in terms of separation of the centroids. We showed that when this separation remains smaller than 30% of the diameter of the theoretical unaberrated Airy pattern, the quasi-monochromatic visibility stays above 0.8. For polychromatic fringes ($\Delta\lambda = 60$ nm), this value drops to 0.7. The instrument maintains good fringe visibility for perturbations corresponding to a temperature variation of 10 K, a pessimistic value, over the whole structure made of low expansion material, and of the order of a few tenths of micro-radians for tips and tilts. It is likely that a far more precise thermal control would be achieved, thus substantially relaxing the tolerances derived here.

5. Acknowledgements

We wish to thank Lennart Lindegren and Michael Perryman for providing the subject of this study and the information we needed to perform it. Thanks also to Erik Høg for useful comments. The research described was partially performed at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. This work was partially completed while the first author was at the Jet Propulsion Laboratory. This research was also supported by Matra Marconi Space.

REFERENCES

Code V is a registred product of Optical Research Associates, Pasadena, California.

Korsh, D., 1977, Anastigmatic Three-Mirror Telescope, Applied Optics, 16, No. 8, 2074.

Lindegren, L., Perryman, M. 1994, The GAIA mission, Technical report

Loiseau, S., Shaklan, S., 1995, Optical design and tolerancing of a Fizeau interferometer, 1995, submitted to Applied Optics.

Redding, D., Needels, L., Wallace, K.,Yu, J., Levine, M., 1992, Controlled Optics Modeling Package User Manual, Version 1.0, JPL document D-9816.

Roddier, F., Léna, P., 1984, Journal of Optics, 15, No. 4, 171.

Figure 1: *Cross section of the three-mirror design. The primary and the tertiary mirror share the same vertex. The overall length of the telescope is 3.5 m for a primary diameter of 3 m. The final focal ratio is 3.85.*

Figure 2: *Geometrical spot diagram. The rms spot sizes are 2.5, 4.3 and 5.5 μm for the three field angles 0°, 0.32°, and 0.45°.*

Figure 3: *Optical layout of the Fizeau interferometer. The Ariane 5 4.4-m envelope is shown as well.*

Figure 4: *Fringes integrated along the direction perpendicular to the direction of the baseline, for on-axis observation. The x-axis scale is arbitrary.*

Figure 5: *Spectral density of the image.*

Figure 6: *Variations of the difference between the true centroid position and $f \times \tan\theta$ with respect to field angle.*

Figure 7: *Variations of the fringe position with respect to field angle. The amount of distortion at the edge of the field (0.45°) is ∼40 mas.*

Figure 8: *Variations of the fringe position with respect to field angle with a primary sub-aperture tilted by 2 μrad.*

Figure 9: *Theoretical structure made of low expansion material and supporting the optical system. It could be composed of a central rod and several secondary rods supporting the sub-apertures on the three surfaces. Further explanations on this structure are given in Section 3.3.1.*

Figure 10: *Spot diagram given by COMP for the interferometer. The geometrical spots overlap on the focal plane to within a small fraction of the Airy disk diameter. The overall size of the spot is 2 μm. Units are centimeters.*

Figure 11: *Fitting function for the visibility vs. centroid separation relationship. The curved is derived from Eq. 7.*