Theoretical aspects

of relativistic spectral features from accretion disc spots

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in collaboration with:

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Objects and models

- Active galactic nuclei
- Stellar-mass black holes

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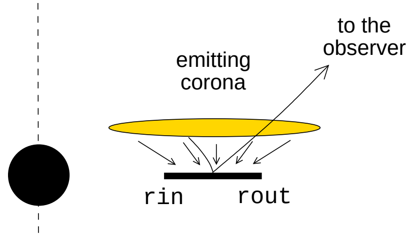
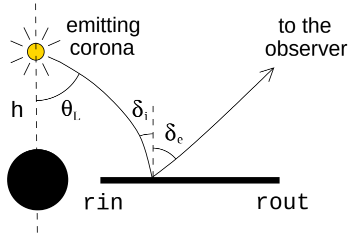
- Stellar-mass black holes
- Central black hole
- Accretion disc
  - geometrically thin, planar, non-self-gravitating
  - time-dependent, non-axisymmetric
- A spectral feature (reflection on the disc surface)





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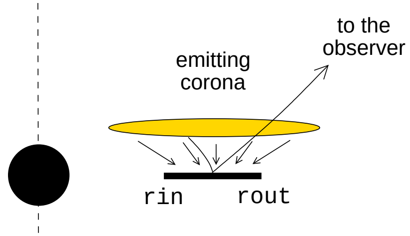
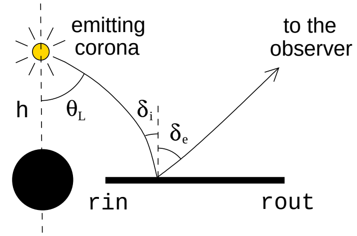
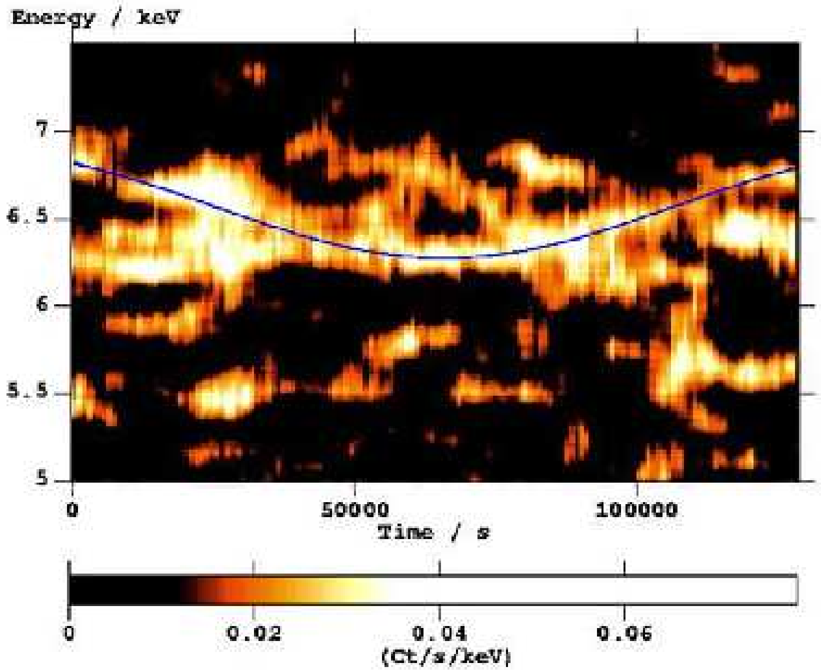


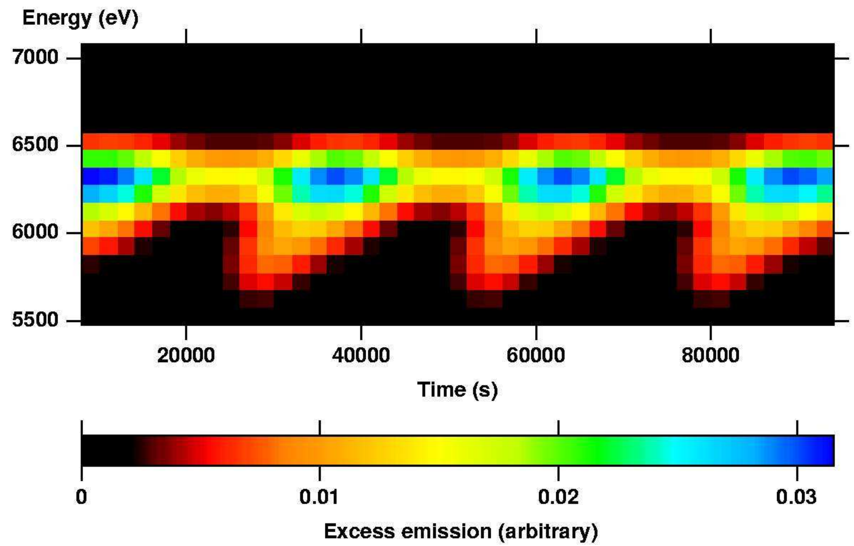


- Accretion disc
  - geometrically thin, planar, non-self-gravitating
  - time-dependent, non-axisymmetric
- A spectral feature produced by reflection
- GR effects taken into account
- Link to a spectrum-fitting procedure (XSPEC)

Motivation: spots?



Turner et al. (2006), A&A 445, 59
(Seyfert galaxy Mrk 766 from XMM-Newton)



Iwasawa et al. (2004), MNRAS 355, 1073
(Seyfert galaxy NGC 3516 from XMM-Newton)

Motivation: lensing?

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2101 CONSTITUTION AVENUE

WASHINGTON, D.C.

Prof. Albert Einstein **Sept. 16, 1936**

Institute for Advanced Study

Dear Prof. Einstein:

Last spring an apparently sincere laymen in science, Rudi Mandl, came into our offices here in the building of the National Academy of Sciences and dicussed a proposed test for the relativity theory based on observations during eclipses of stars.

We supplied Mr. Mandl with a small sum of money to enable him to visit you at Princeton and discuss it with you...

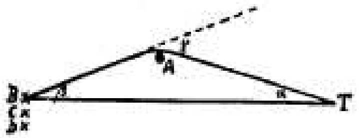
Motivation: lensing?

Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson*.

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geq \gamma \geq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1\overset{''}{.}7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant), T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B, von T aus gesehen, sei α, und der Winkel zwischen A und T, von B aus gesehen, sei β. Es ist dann

$$\gamma = \alpha + \beta\,.$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0. Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A, analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D, so würden die Spiegelbilder umgekehrt gelegen sein wie in einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B, wenn von A aus gerechnet wird (D wäre am nächsten zu A).



Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

Petrograd, 1924 Jan. 28. *O. Chwolson.*

Antwort auf eine Bemerkung von *W. Anderson*.

Daß ein Elektronengas einer Substanz mit negativem Brechungsvermögen optisch äquivalent sein müßte, kann bei dem heutigen Stand unserer Kenntnisse nicht zweifelhaft sein, da dasselbe einer Substanz von verschwindend kleiner Eigenfrequenz äquivalent ist.

Aus der Bewegungsgleichung

$$\varepsilon X = \mu\, d^2x/dt^2$$

eines Elektrons von der elektrischen Masse ε und der ponderabeln Masse μ folgt nämlich für einen sinusartig pendelnden Prozeß von der Frequenz ν die Gleichung

$$\varepsilon X = -(2\pi\nu)^2 \mu x\,.$$

Berücksichtigt man, daß εx das »Moment« eines schwingenden Elektrons ist, so erhält man für die Polarisation $p = n\varepsilon x$ eines Elektronengases mit n Elektronen pro Volumeinheit

$$p = -\varepsilon^2 n / [\mu (2\pi\nu)^2] \cdot X\,.$$

Hieraus folgt, daß die scheinbare Dielektrizitätskonstante

$$D = 1 + 4\pi p/X = 1 - \varepsilon^2 n/(\pi\mu\nu^2)$$

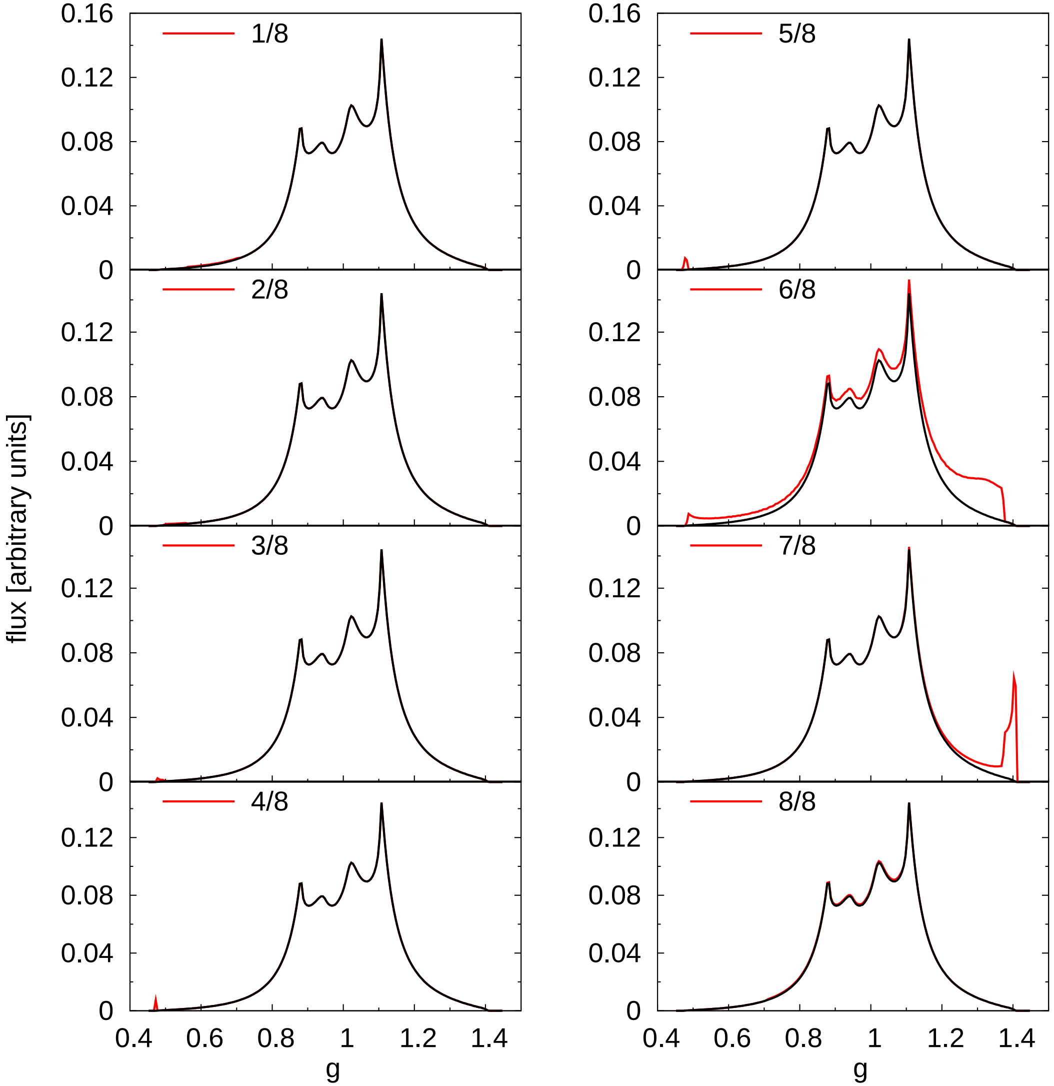
ist. $\sqrt{D}$ ist in diesem Falle der Brechungsexponent, also jedenfalls kleiner als 1. Es erübrigt sich bei dieser Sachlage, auf das Quantitative einzugehen.

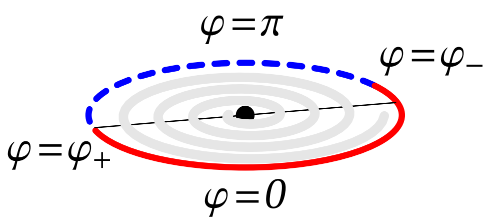
Es sei noch bemerkt, daß ein Vergleich des Elektronengases mit einem Metall unstatthaft ist, weil die bei der elementaren Theorie der Metalle zugrundegelegte »Reibungskraft« bei freien Elektronen fehlt; das Verhalten der letzteren ist allein durch die Einwirkung des elektrischen Feldes und durch die Trägheit bedingt.

Berlin, 1924 April 15. *A. Einstein.*

O. Chwolson (1924), “Über eine mögliche Form fiktiver Doppelsterne”, Astronomische Nachrichten, 221, 329

Motivation: interplay of both?





(J. Svoboda, Thesis)

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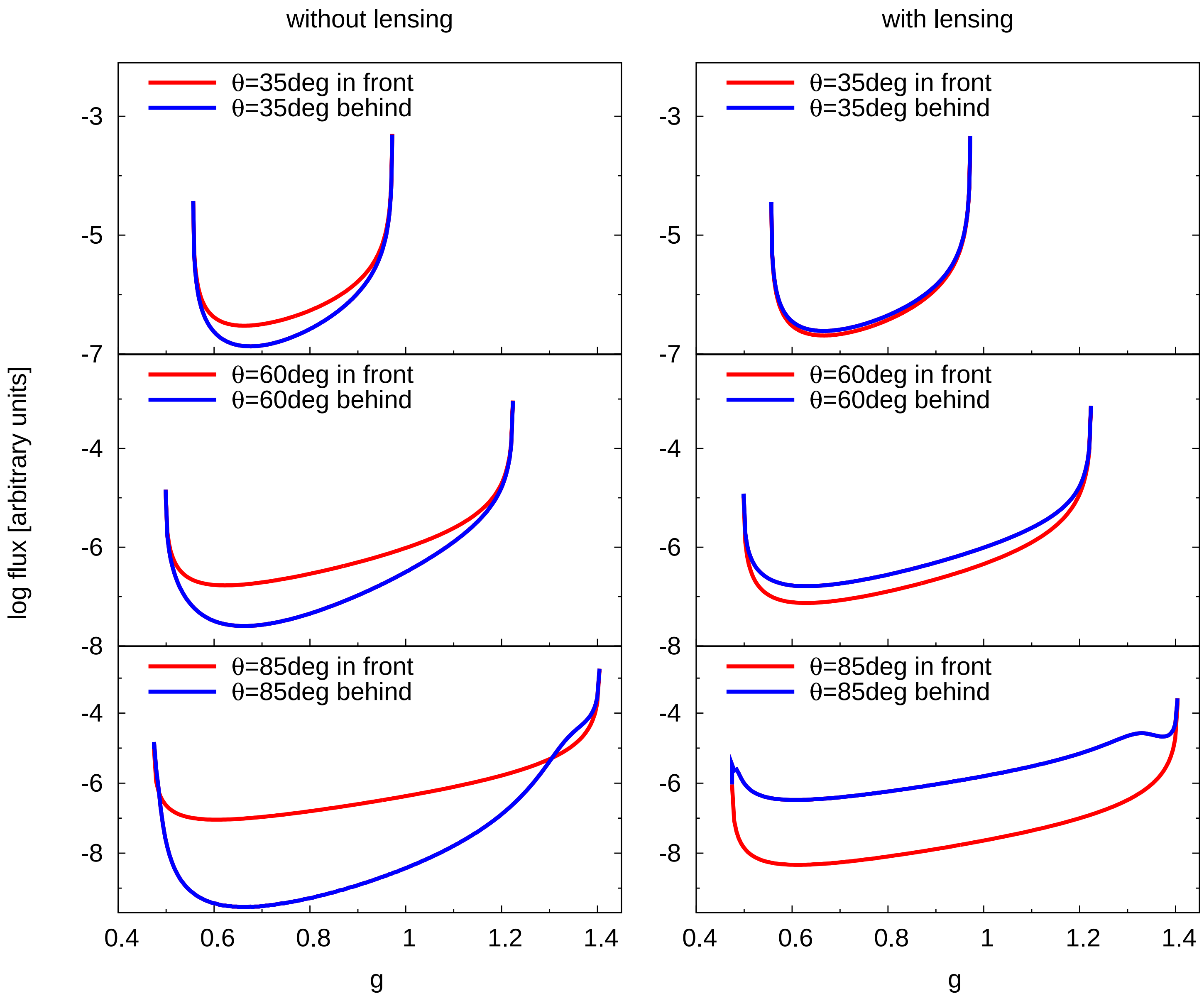

High-frequency elmg. waves

Basic equations – vacuum case: $F^{\mu\nu}{}_{;\nu} = 0, \qquad {}^{\star}F^{\mu\nu}{}_{;\nu} = 0.$

$$E^{\alpha} = F^{\alpha\beta}u_{\beta},\ {}^{\star}F_{\mu\nu} \equiv \tfrac{1}{2}\varepsilon_{\mu\nu}{}^{\rho\sigma}F_{\rho\sigma}$$

An electromagnetic wave is an approximate test-field solution of the Maxwell equations:

$$F_{\alpha\beta} = \Re e \left[u_{\alpha\beta}\, e^{\Im\psi(x)} \right].$$

A fixed background geometry is asssumed (BH metric).

- Phase $\psi(x)$ ... rapidly varying function
- Amplitude $u_{\alpha\beta}$ ... slowly varying function
- Wave vector $k_{\alpha} \equiv \psi_{,\alpha}$ ... paralel transport, null geodesics

$$k_{\alpha;\beta}\, k^{\beta} = 0, \quad k_{\alpha}k^{\alpha} = 0.$$

Polarization tensor

Propagation law in empty space:

$$DF_{\alpha\beta} - 2\theta F_{\alpha\beta} = 0$$

$\theta \equiv \frac{1}{2} k^{\alpha}{}_{;\alpha}$ (expansion of the null congruence), $D \equiv u^{\alpha}\nabla_{\alpha}$.

- Polarization tensor ... $J_{\alpha\beta\gamma\delta} \equiv \frac{1}{2}\langle F_{\alpha\beta} F_{\gamma\delta}\rangle$
- In an observer rest-frame ... $J_{\alpha\beta} \equiv J_{\alpha\beta\gamma\delta}\, u^{\gamma}\, u^{\delta} = \langle E_{\alpha} \bar{E}_{\beta}\rangle$
- Stokes parameters ... $S_{A} \equiv \frac{1}{2}(k_{\alpha}u^{\alpha})^2 F_{A} \quad (A = 0, \dots, 3)$

F_{A} ... constructed by projecting the polarization tensor,

$J_{\alpha\beta}u^{\beta} = 0, \quad J_{\alpha\beta}k^{\beta} = 0, \quad \omega = u_{\alpha}k^{\alpha}$.

References: [1] Sir George Stokes (1852), Trans. Cambridge Phil. Soc., 9, 399

[2] Chandrasekhar (1950), *Radiative Transfer* (Oxford: Clarendon)

[3] Cocke & Holm (1972), Nature, 240, 161

[4] Jauch & Rohrlich (1955), *The Theory of Photons and Electrons* (Reading: Wesley)

Stokes parameters

"On the composition and resolution of streams of polarized light from different sources"

$$S_0 \equiv J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(1)} + e^{\alpha}_{(2)}e^{\beta}_{(2)}\right) = \langle |E_{(1)}|^2 + |E_{(2)}|^2 \rangle$$

$$S_1 \equiv J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(1)} - e^{\alpha}_{(2)}e^{\beta}_{(2)}\right) = \langle |E_{(1)}|^2 - |E_{(2)}|^2 \rangle$$

$$S_2 \equiv J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(2)} + e^{\alpha}_{(2)}e^{\beta}_{(1)}\right) = \langle E_{(1)}\bar{E}_{(2)} + E_{(2)}\bar{E}_{(1)} \rangle$$

$$S_3 \equiv \Im J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(2)} - e^{\alpha}_{(2)}e^{\beta}_{(1)}\right) = \Im\langle E_{(1)}\bar{E}_{(2)} - E_{(2)}\bar{E}_{(1)} \rangle$$

S_1, S_2, and S_3 determine the polarization state.

References: [5] Anile (1989), *Relativistic fluids and magneto-fluids* (Cambridge)
[6] Madore (1974), Comm. Math. Phys., 38, 103
[7] Bičák & Hadrava (1975), A&A, 44, 389
[8] Breuer & Ehlers (1980), Proc. Roy. Soc. Lond. A, 370, 389
[9] Broderick & Blandford (2003), MNRAS, 342, 1280

Propagation law

Normalized Stokes parameters:

$$s_1 = S_1/S_0, \quad s_2 = S_2/S_0, \quad s_3 = S_3/S_0.$$

Degree of polarization:

$$\Pi_l = \sqrt{s_1^2 + s_2^2}, \quad \Pi_c = |s_3|, \quad \Pi = \sqrt{\Pi_l^2 + \Pi_c^2}.$$

Propagation through an arbitrary (empty) space-time:

$$F_{A,\,\mathrm{em}}\, dS_{\mathrm{em}} = F_{A,\,\mathrm{em}}\, dS_{\mathrm{obs}},$$

$$1 + z = \frac{(k_\alpha u_\alpha)_{\mathrm{em}}}{(k_\alpha u_\alpha)_{\mathrm{obs}}}, \quad S_A = \frac{k_A}{(1+z)^2 dS}.$$

Five transfer functions

- The energy shift (gravitational and Doppler)
  - emitted photons are coming from places with high gravity
  - photons are emitted from rapidly moving matter

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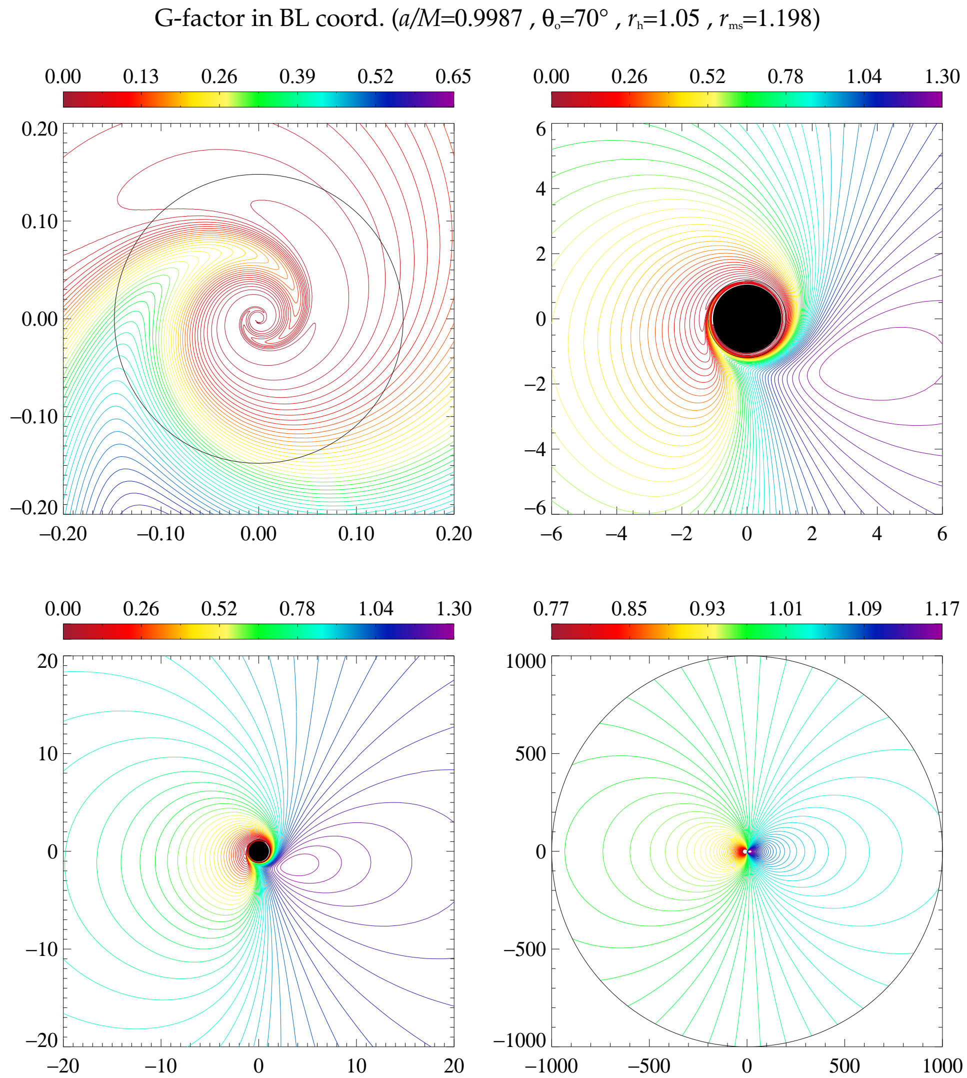
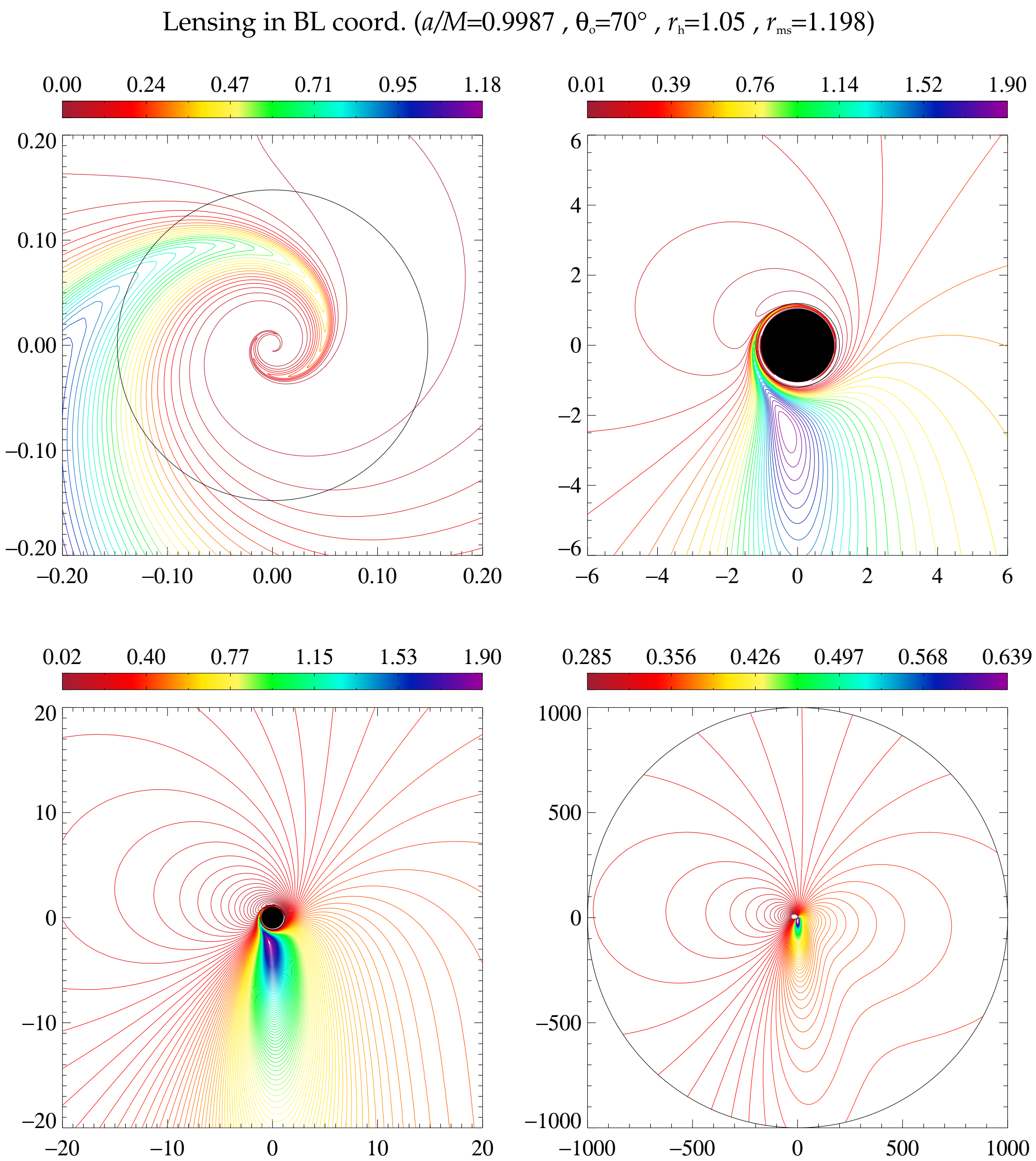
- The light-time effect
  - mutual time delays of photons at detector
- The change of polarization angle
  - Polarization vector is parallel transported through gravitational field

The shift of photon energy, z



Dovčiak et al. (2004), ApJSS, 153, 205

Lensing effect, $S_{\rm em}/S_{\rm obs}$



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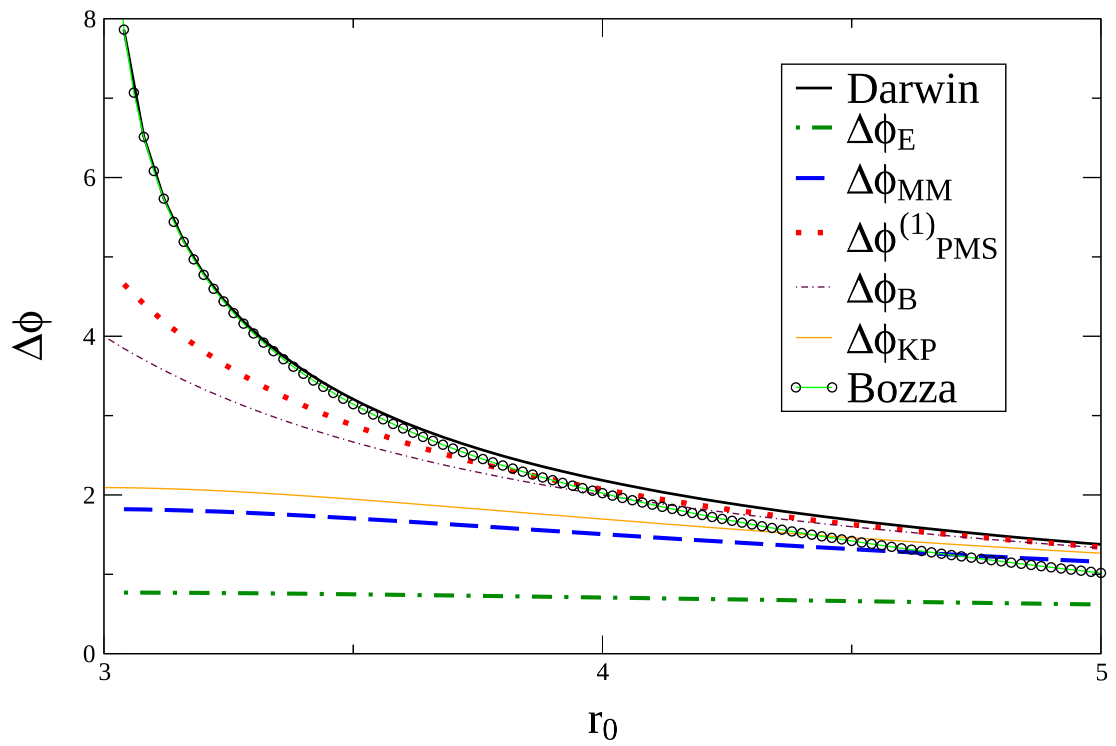


Darwin (1959)
Einstein (1911)
Mutka & Mähönen (2002)
Amore & Diaz (2006)
Beloborodov (2002)
Keeton & Peters (2005)
Bozza (2003)

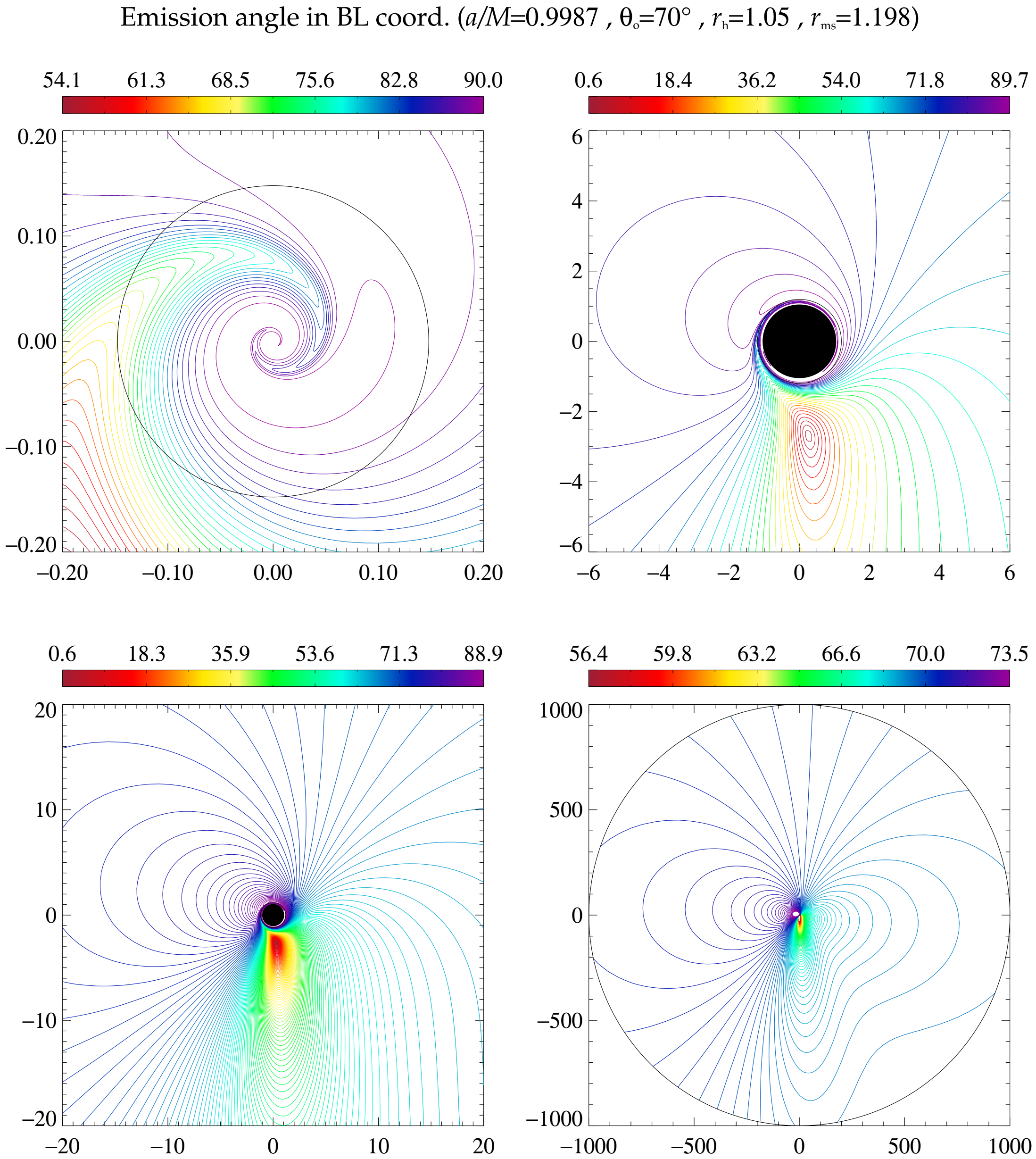
Light deflection and gravitational lensing: exact formula and analytical approximations.

In Schwarzschild metric:

$$\delta\phi_D = 4\sqrt{r_0/(GM\Upsilon)}\left[F\left(\tfrac{\pi}{2},\kappa\right) - F\left(\varphi,\kappa\right)\right]$$
$$\approx \frac{4GM}{r_0} + \frac{7.78097G^2M^2}{r_0^2} + \frac{17.1047G^3M^3}{r_0^3} + \mathcal{O}\left[(GM/r_0)^4\right].$$

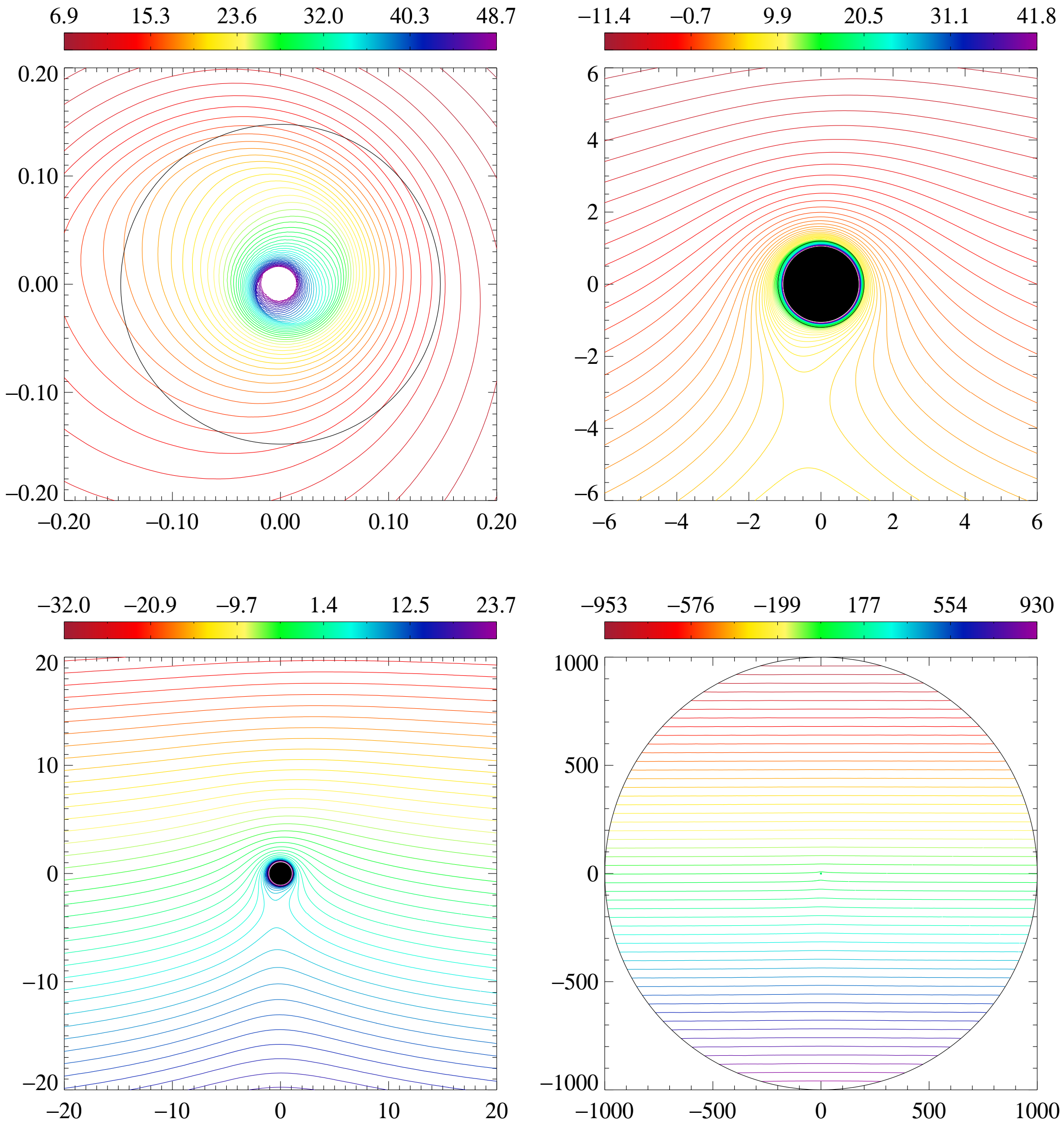
Amore & Diaz, Phys. Rev. D73 (2006) 083004

Emission angle, $\cos\delta_{em}$

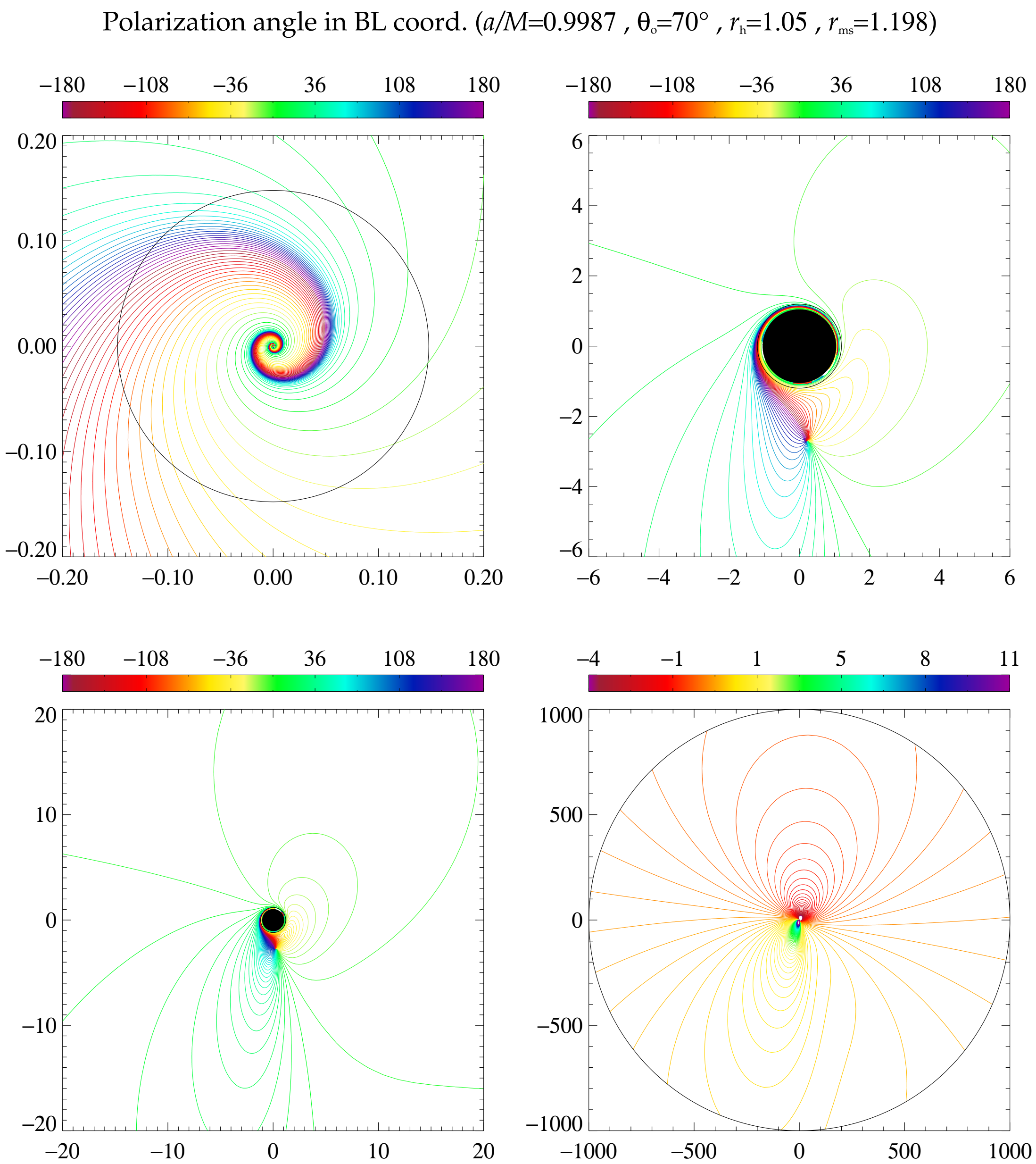


Light-time effect, δt

Time delay in BL coord. (a/M=0.9987 , θ_o=70° , r_h=1.05 , r_{ms}=1.198)



Polarization angle, $\cos\psi$



Wave fronts in a BH spacetime

Schwarzschild metric,

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

Eikonal equation,

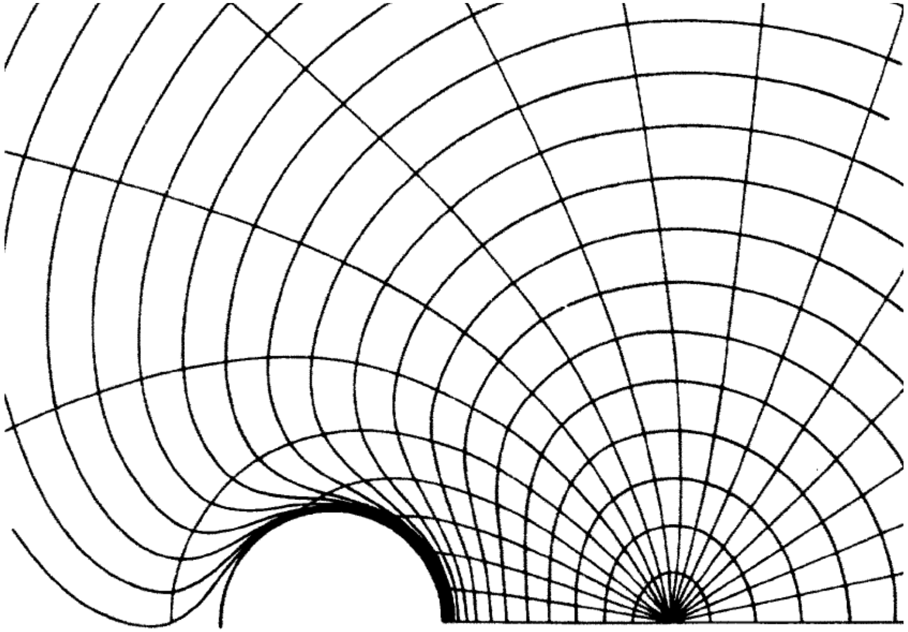
$$-\left(1 - \frac{2M}{r}\right) (\psi_{,r})^2 + \left(1 - \frac{2M}{r}\right)^{-1} (\psi_{,t})^2 - r^{-2}(\psi_{,\phi})^2 = 0.$$

Solved by separation of variables, $\psi(t, r, \phi) \equiv R(r) + \alpha\phi - \omega t$,

$$\left(1 - \frac{2M}{r}\right) (R')^2 = \left(1 - \frac{2M}{r}\right)^{-1} \omega^2 - r^{-2}\alpha^2.$$

Wave front: $\boxed{\psi(t_0 + n\,\delta t, r, \phi) = \psi(t_0, r_0, 0).}$

Wave fronts in a BH spacetime



Wave fronts do not depend on polarization (in geometrical optics approximation).

The analogy:

light propagation in a vacuum curved spacetime versus material media in a flat spacetime.

The effective permeability: $\mu = \sqrt{1 - 2M/r}$.

Mashoon (1973); Hanni (1977); . . .

Wave fronts in a BH spacetime

Kerr metric,

$$ds^2 = -\frac{\Delta}{\Sigma}\Big(dt - a\sin^2\theta\, d\phi\Big)^2 + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma\, d\theta^2$$

$$+\frac{\sin^2\theta}{\Sigma}\Big[a\, dt - (r^2 + a^2)\, d\phi\Big]^2.$$

The separation of variables and solution for the eikonal equation (Carter),

$$\psi = R(r) + T(\theta) + \alpha\phi - \omega t.$$

Wave fronts exhibit the frame dragging effect.

Waves in a refractive medium

Basic equations – current-free:

$$I^{\mu\nu}{}_{;\nu} = 0, \qquad {}^{\star}F^{\mu\nu}{}_{;\nu} = 0.$$

$$I_{\mu\nu} = \mu^{-1} F_{\mu\nu}, \quad I_{\mu\nu} \equiv \tfrac{1}{2} \chi^{\rho\sigma}_{\mu\nu}(N)\, F_{\rho\sigma}$$

Wave solution:

$$F^{\alpha\beta} = \Re e \left[e^{\Im \omega \phi} \sum_{n=0}^{\infty} \omega^{-n} F^{\alpha\beta}_{(n)} \right], \qquad I^{\alpha\beta} = \Re e \left[e^{\Im \omega \phi} \sum_{n=0}^{\infty} \omega^{-n} I^{\alpha\beta}_{(n)} \right]$$

- Zeroth order: $F^{\mu\nu}_{(0)} k_\nu = 0, \quad I^{\mu\nu}_{(0)} l_\nu = 0,$
- Rays: $k^\nu = \hat{\mu}^{-1} \left[l^\nu - (N^2 - 1)\, \Omega u^\nu \right]$,
- $l^\mu l_\mu = (N^2 - 1)\Omega \neq 0, \quad \Omega = l^\mu u_\mu,$

$$\left(|\psi|^2 k^\alpha \right)_{;\alpha} = 0, \qquad \psi_\alpha u^\alpha = 0.$$

Waves in dispersive plasmas

Basic equations:

$$T^{\mu\nu}{}_{;\nu} = -F^{\mu\alpha} j_\alpha, \quad F^{\mu\nu}{}_{;\nu} = 4\pi\mu_0\, j^\mu, \quad {}^\star F^{\mu\nu}{}_{;\nu} = 0.$$

$$j^\mu_{\text{tot}} = -4\pi e \left(\underline{nu^\mu} - n_i u_i^\mu\right), \quad \left(nu^\mu\right)_{;\mu} = 0$$

Linearization + two scales: $F^{\mu\nu} \rightarrow \hat{F}^{\mu\nu}(X^\alpha, \psi), \quad n \rightarrow \hat{n} \ldots$

slow ... $X^\alpha \equiv \epsilon x^\alpha$, fast ... $\psi \equiv \epsilon^{-1}\Theta(\epsilon x^\alpha)$, wavefront ... $l_\mu \equiv \psi_{,\mu}$

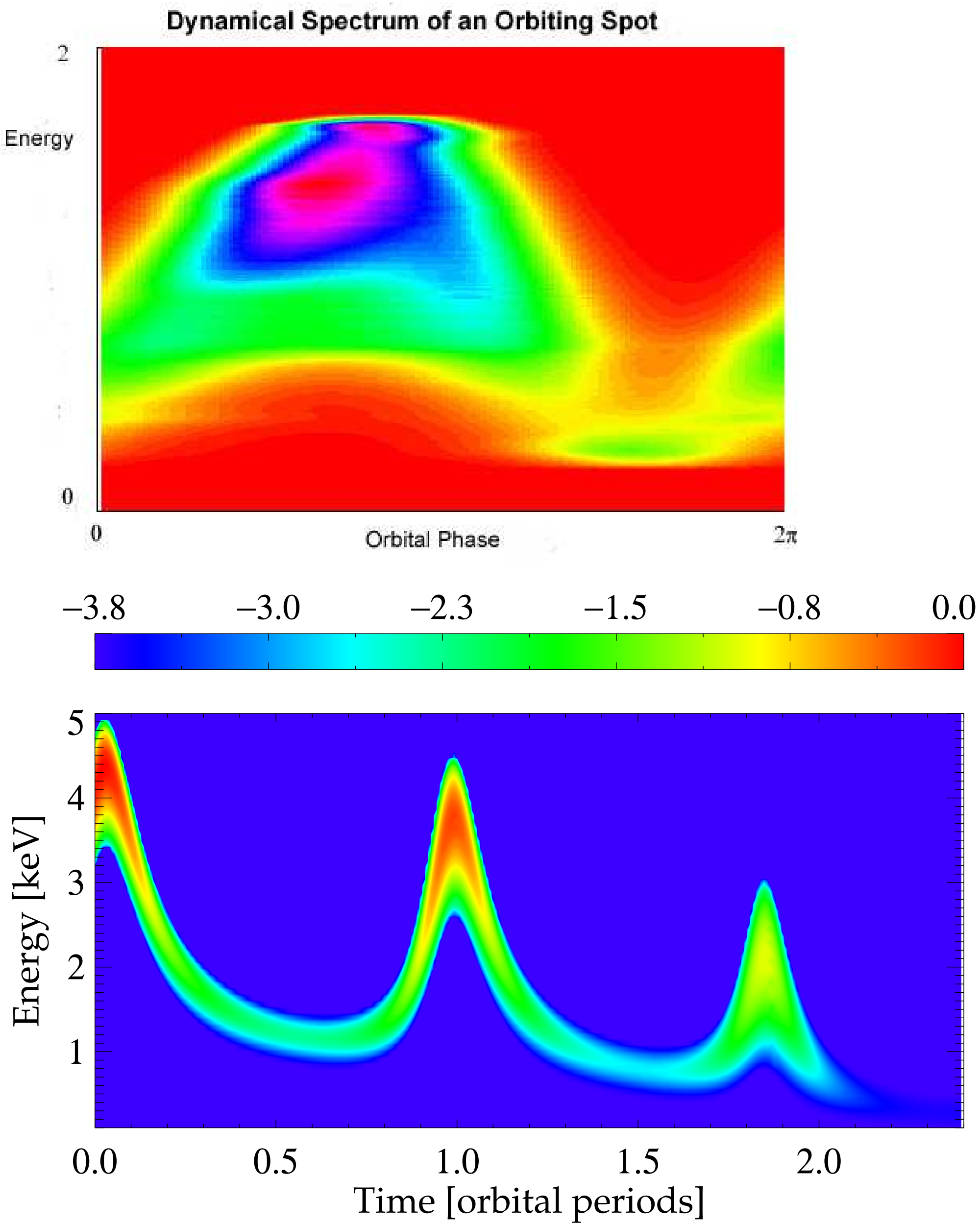
Wave solution:

$$\hat{F}^{\alpha\beta} = e^{\Im\psi} \sum_{n=0}^{\infty} \epsilon^n \hat{F}^{\alpha\beta}_{(n)}$$

(Anile & Pantano 1977)

- Background fields vary on slow scale ... $g_{\mu\nu} \equiv g_{\mu\nu}(X^\alpha)$
- Local frequency of the wave ... $\Omega \equiv u_\mu l^\mu$
- Dispersion relation ... $l_\mu l^\mu = -4\pi\mu_0 e^2 n m^{-1} \equiv -\Omega_p^2$

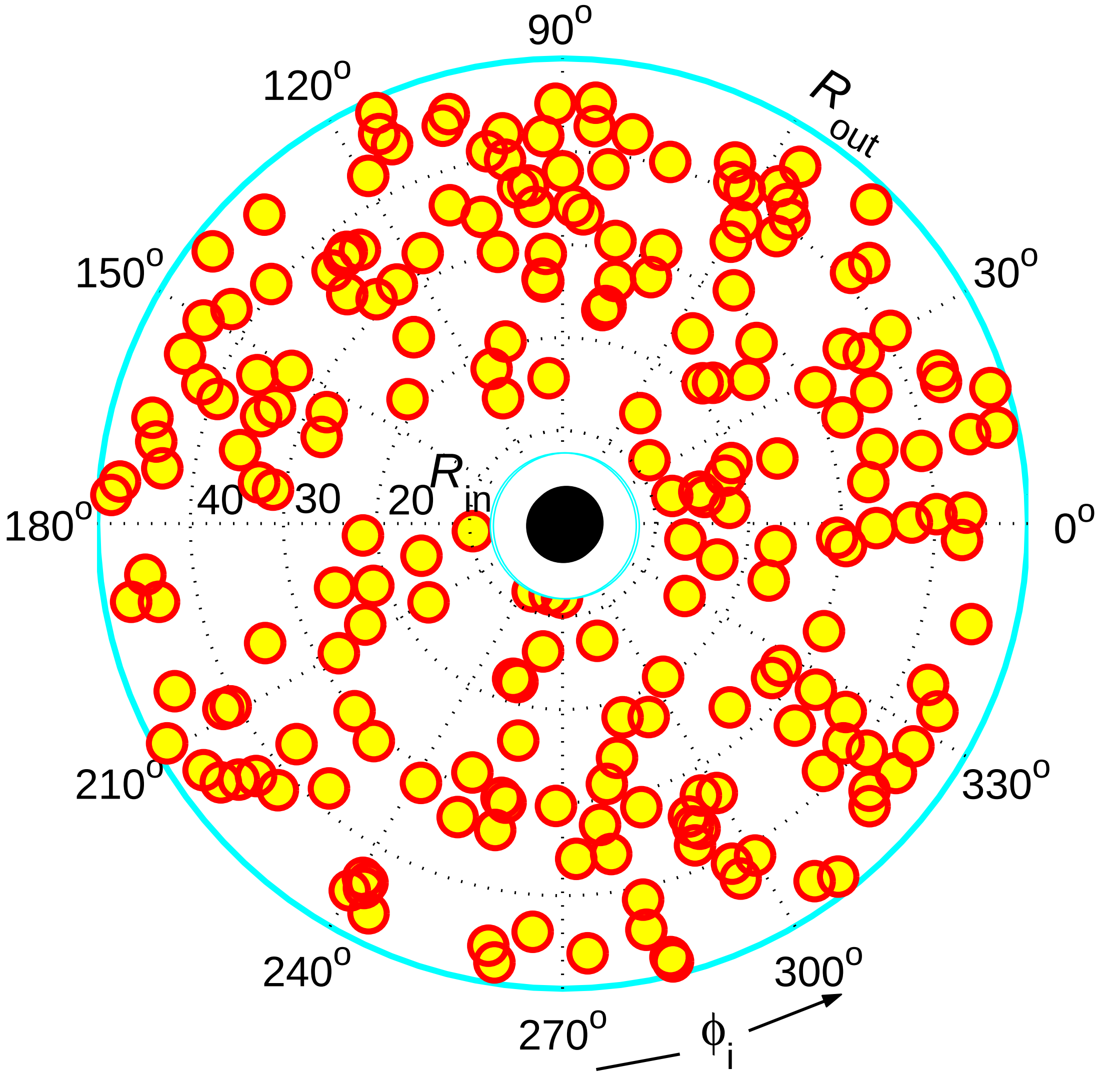
Example: an orbing spot



- Fabian, Iwasawa, Reynolds, Young, (2000), “Broad Iron Lines in Active Galactic Nuclei”, PASP, 112, 1145
- Reynolds & Nowak (2003), “X-rays from active galactic nuclei: relativistically broadened emission lines”, Phys. Rep., 337, 389

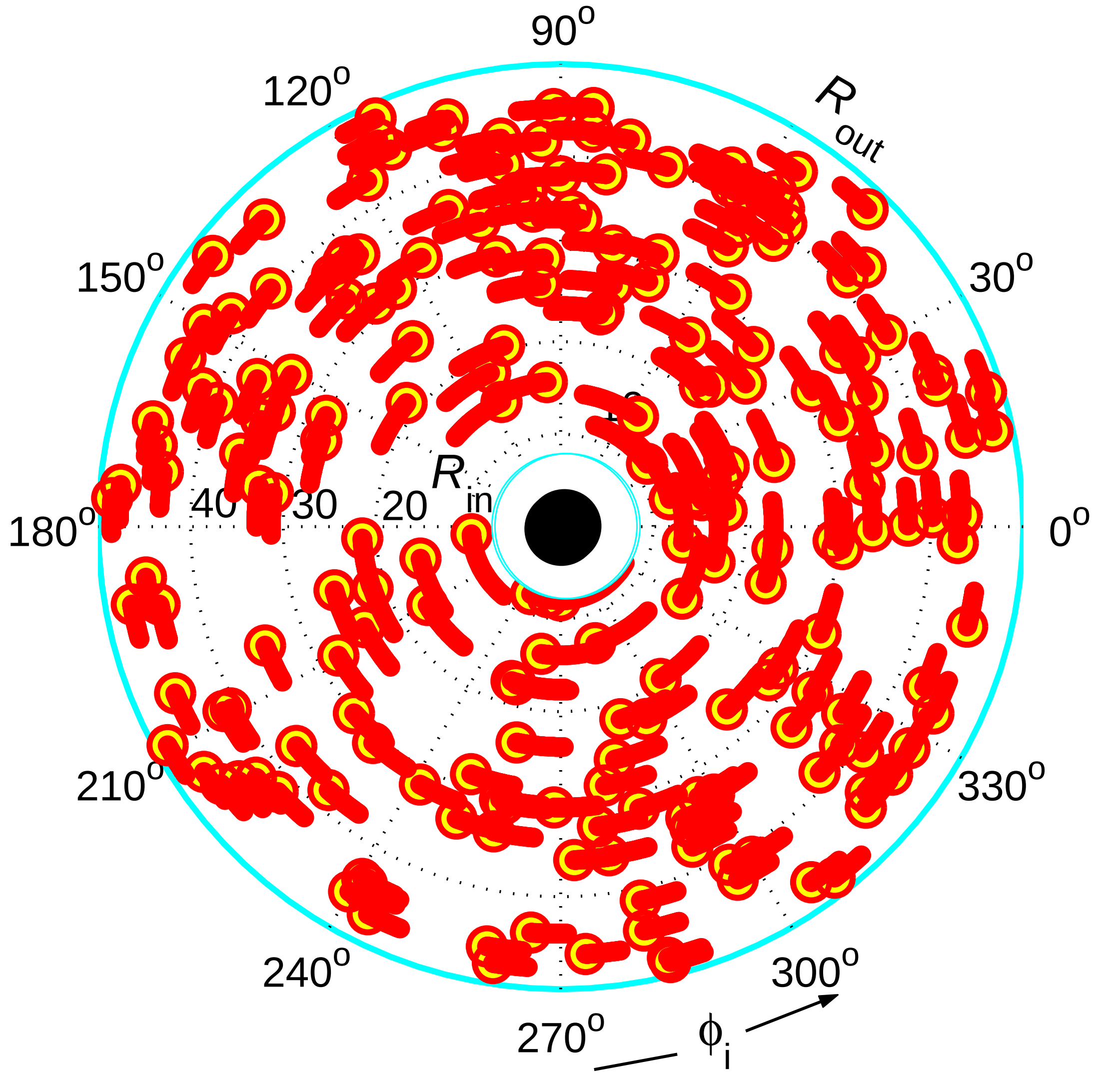
(see a poster by Dovčiak et al.)

Flare/spot model



Spectra from individual flares/spots – talk by René Goosmann

Flare/spot model



Czerny et al. (2004), A&A, 420, 1

Flare/spot model, $F_{\mathrm{var}}(E)$

Assumption: L_{i} scales with the flare distance from the center,

$$L_{\mathrm{i}}(R_{\mathrm{i}}) \propto \left(\frac{R_{\mathrm{i}}}{R_{\mathrm{in}}}\right)^{-\beta_{\mathrm{rad}}}$$

with the incident radiation flux being a power-law of R_{i}.

Fractional variability amplitude F_{var} (Vaughan et al. 2003):

$$F_{\mathrm{var}}^2 \equiv \left(\frac{\sigma^2}{L_{\mathrm{X}}^2 T_{\mathrm{obs}}}\right)^2 = \frac{1}{N_{\mathrm{mean}}} \frac{\int_{R_{\mathrm{in}}}^{R_{\mathrm{out}}} p(R_{\mathrm{i}}) L_{\mathrm{i}}^2 \,\mathrm{d}R_{\mathrm{i}}}{\left[\int_{R_{\mathrm{in}}}^{R_{\mathrm{out}}} p(R_{\mathrm{i}}) L_{\mathrm{i}} \,\mathrm{d}R_{\mathrm{i}}\right]^2}.$$

In our case,

$$F_{\mathrm{var}}^2 = \frac{(\beta_{\mathrm{rad}}-2)^2(1-\zeta^{2\beta_{\mathrm{rad}}-2})(1-\zeta^2)}{2\zeta^2(2\beta_{\mathrm{rad}}-2)(1-\zeta^{\beta_{\mathrm{rad}}-2})^2}\frac{1}{N_{\mathrm{mean}}}.$$

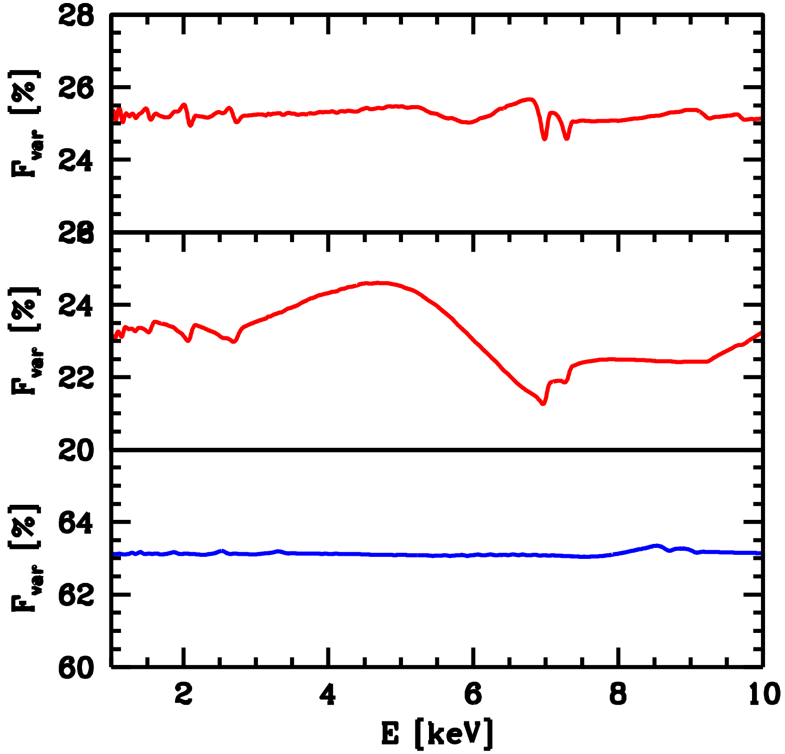
...more complex than the simple $1/N_{\mathrm{mean}}$.

Flare/spot model, $F_{\rm var}(E)$

...variability is enhanced if flare luminosity scales with the flare radius $\propto$ to the disc flux ($\beta_{\rm rad} = 3$, $R_{\rm out} \gg R_{\rm in}$):

$$F_{\rm var}^2 \simeq \frac{1}{8}\frac{R_{\rm out}^2}{R_{\rm in}^2}\,N_{\rm mean}^{-1}.$$

$F_{\rm var}(E)$ for three models:



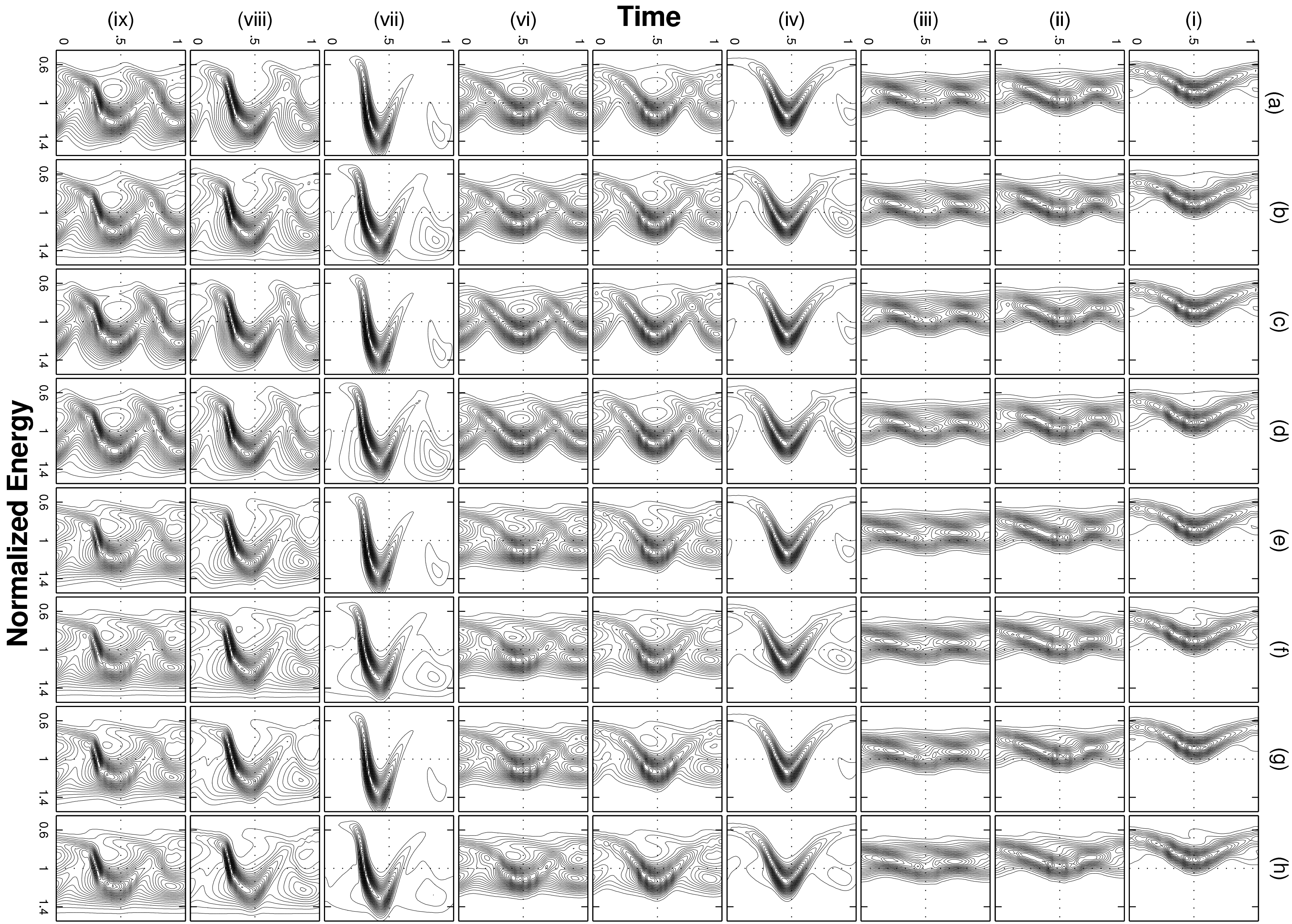
$\longleftarrow$ $R_{\rm in} = 6$, $N = 30$, $i = 30^\circ$ (Schwarzschild)

$\longleftarrow$ $R_{\rm in} = 1.2$, $N = 100$, $i = 30^\circ$ (Kerr)

$\longleftarrow$ $R_{\rm in} = 1.2$, $N = 100$, $i = 70^\circ$ (Kerr)

Obs. duration $\simeq 10^5$ s (Czerny et al. 2004).

Example: a spiral wave ?



Summary

- GR enhances variability at large inclination angles, provided the disc extends close to BH $\rightarrow$ factor of 3 difference between Seyfert 1 vs. 2 type galaxies for a Kerr BH; factor of 1.4 for a Schwarzschild BH.
- The predicted intra-day variability level of Seyfert galaxies is well explained by the flare/spot model if $N_{\mathrm{mean}} \sim 30$–$100$ for $a = 0$, and factor of 10 more for $a = 1 \longrightarrow$ larger number of spots than the usual expectations.
- The energy dependence of F_{var} is typically weaker in the model than in data (Markowitz et al. 2003).

Discrepancies can possibly be resolved by relaxing simplifying assumptions of the model.