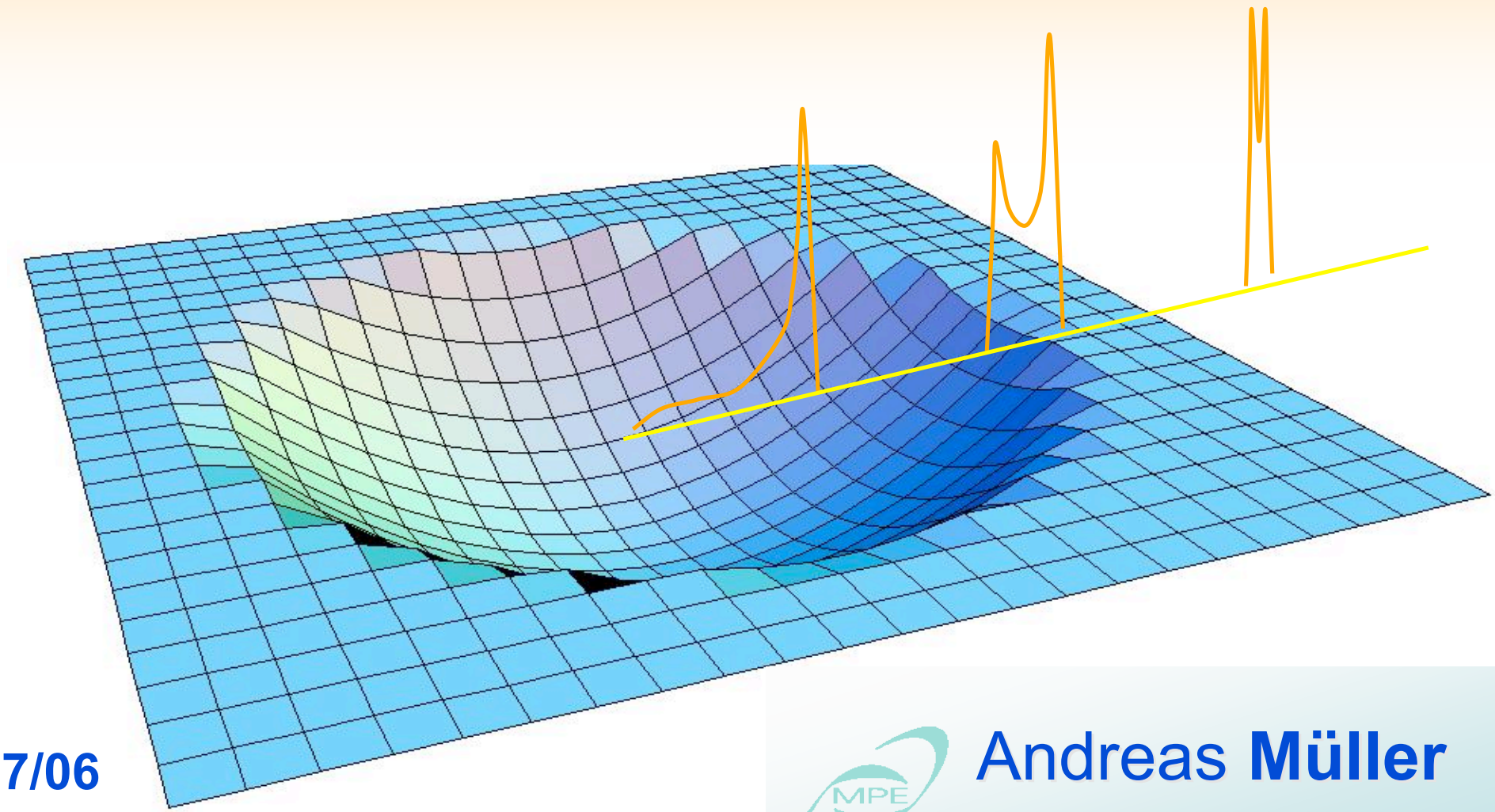
The Onset of **General Relativity:**

Gravitationally Redshifted Emission Lines



06/27/06

ESAC/XMM Workshop, Madrid

Andreas **Müller**

MPE Garching

Paper entitled

On the Signatures of Gravitational Redshift: The Onset of Relativistic Emission Lines

accepted for A&A

(coming soon on astro-ph)

Co-author:

Margrethe Wold (ESO Garching)

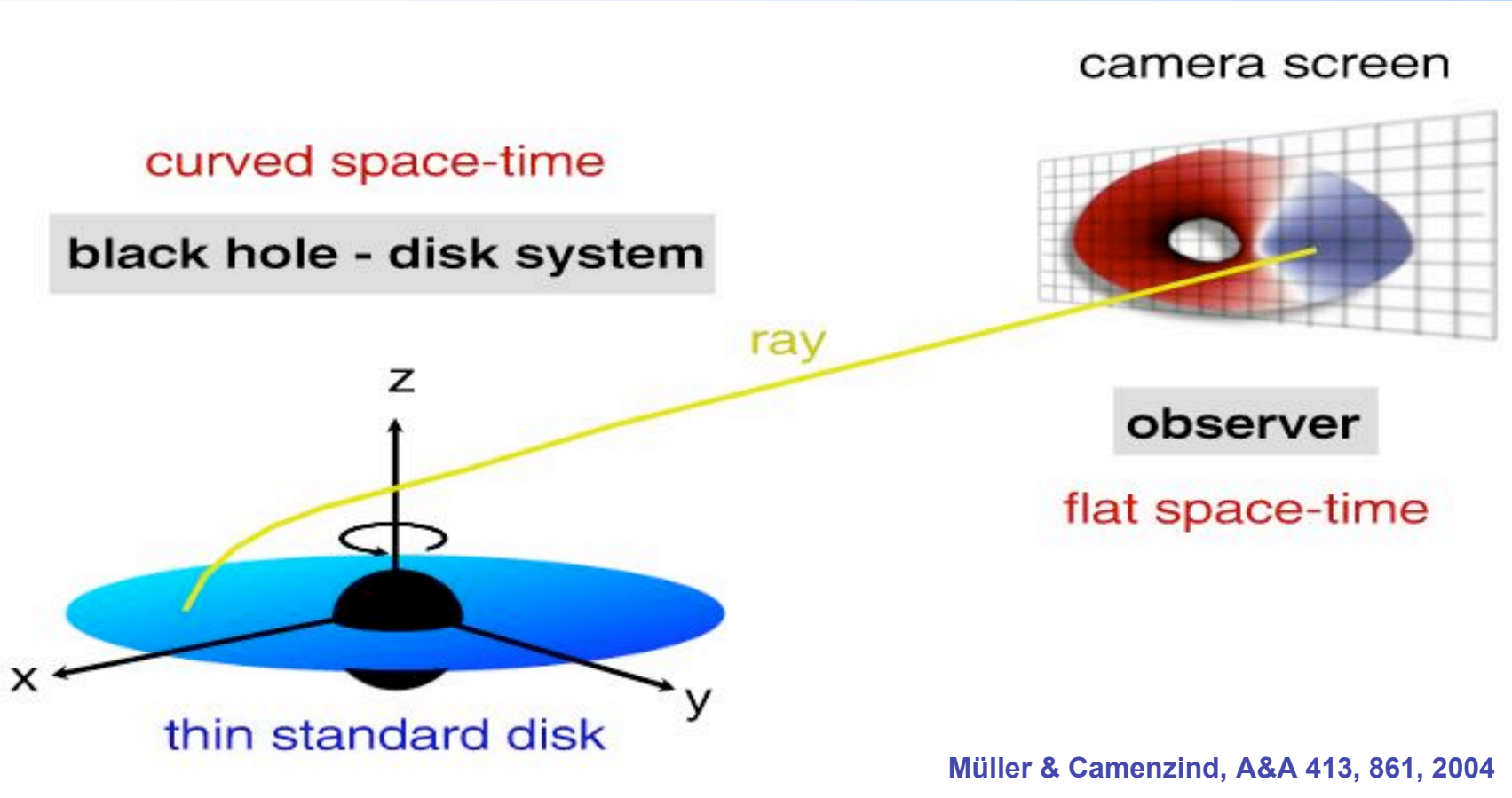
Outline

- **Method:** Kerr Ray Tracing
- **Model:** Equatorial Keplerian Rings
- **Tool:** Line Characterisations

- **Result:** Radial decay of Gravitational Redshift
- **Result:** Two modes of Gravitational Redshift
- **Result:** {M, i} measurements
- **Result:** Probing BH spin

- **Outlook:** Multi-wavelength Challenge

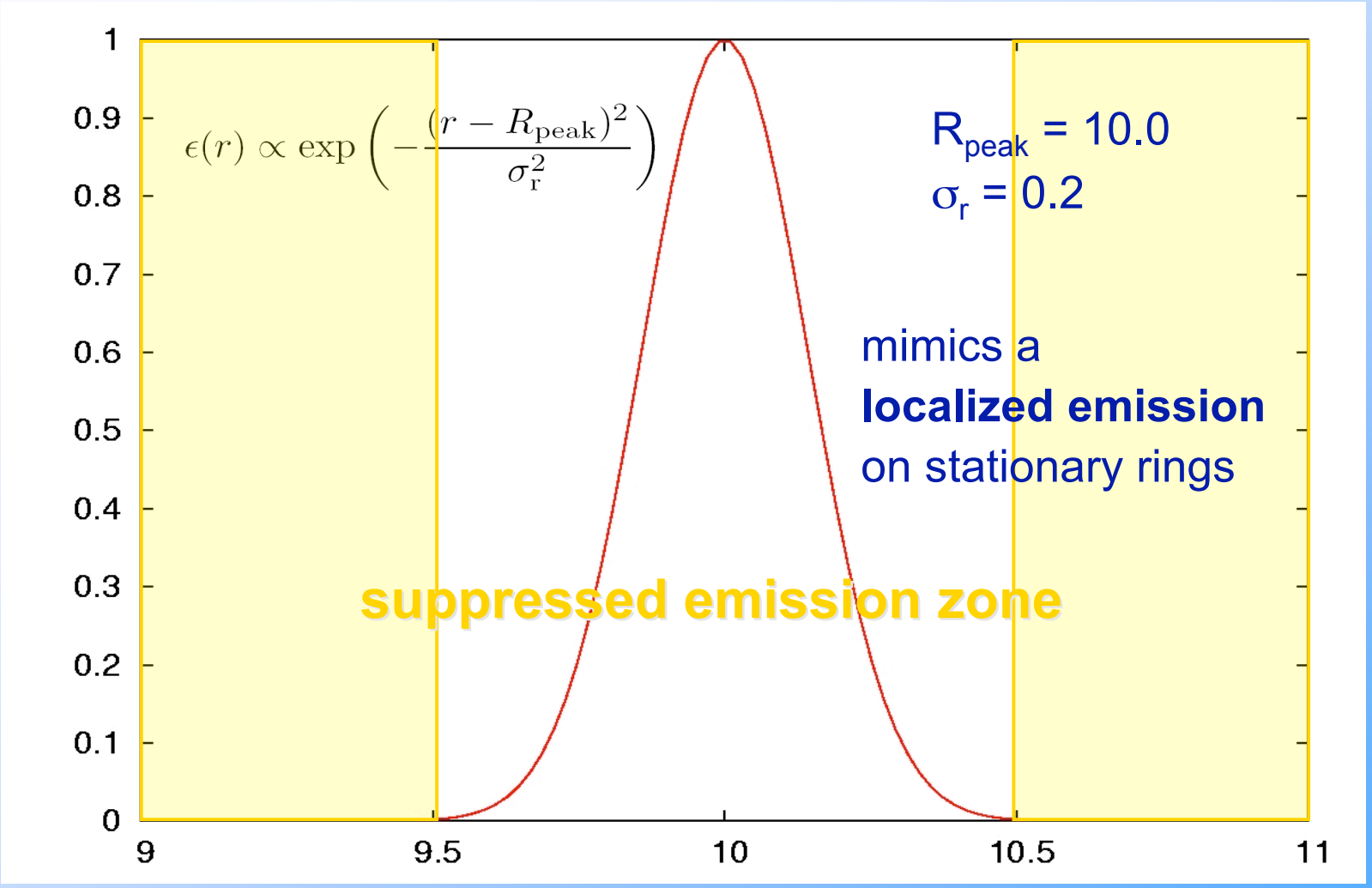
Method: Kerr Ray Tracing



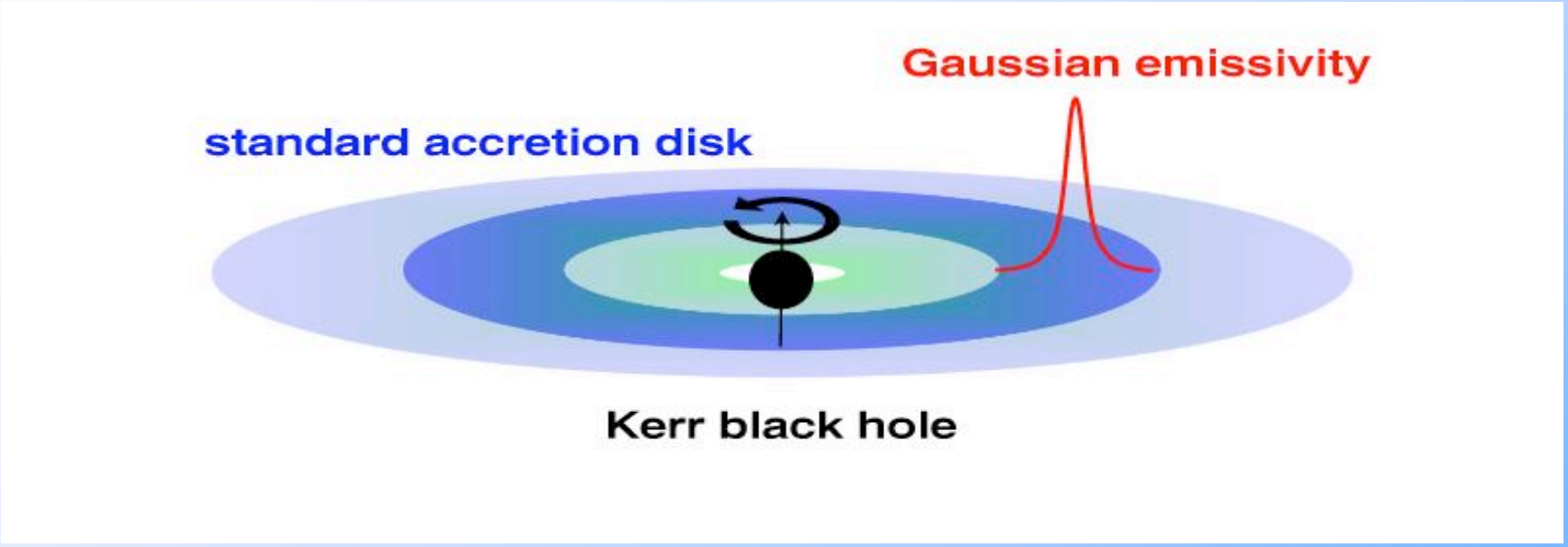
Müller & Camenzind, A&A 413, 861, 2004

Solve null geodesics equations in Kerr geometry with specified velocity field of emitter
Solver: Kerr Black Hole Ray Tracer (KBHRT)

Gaussian emissivity

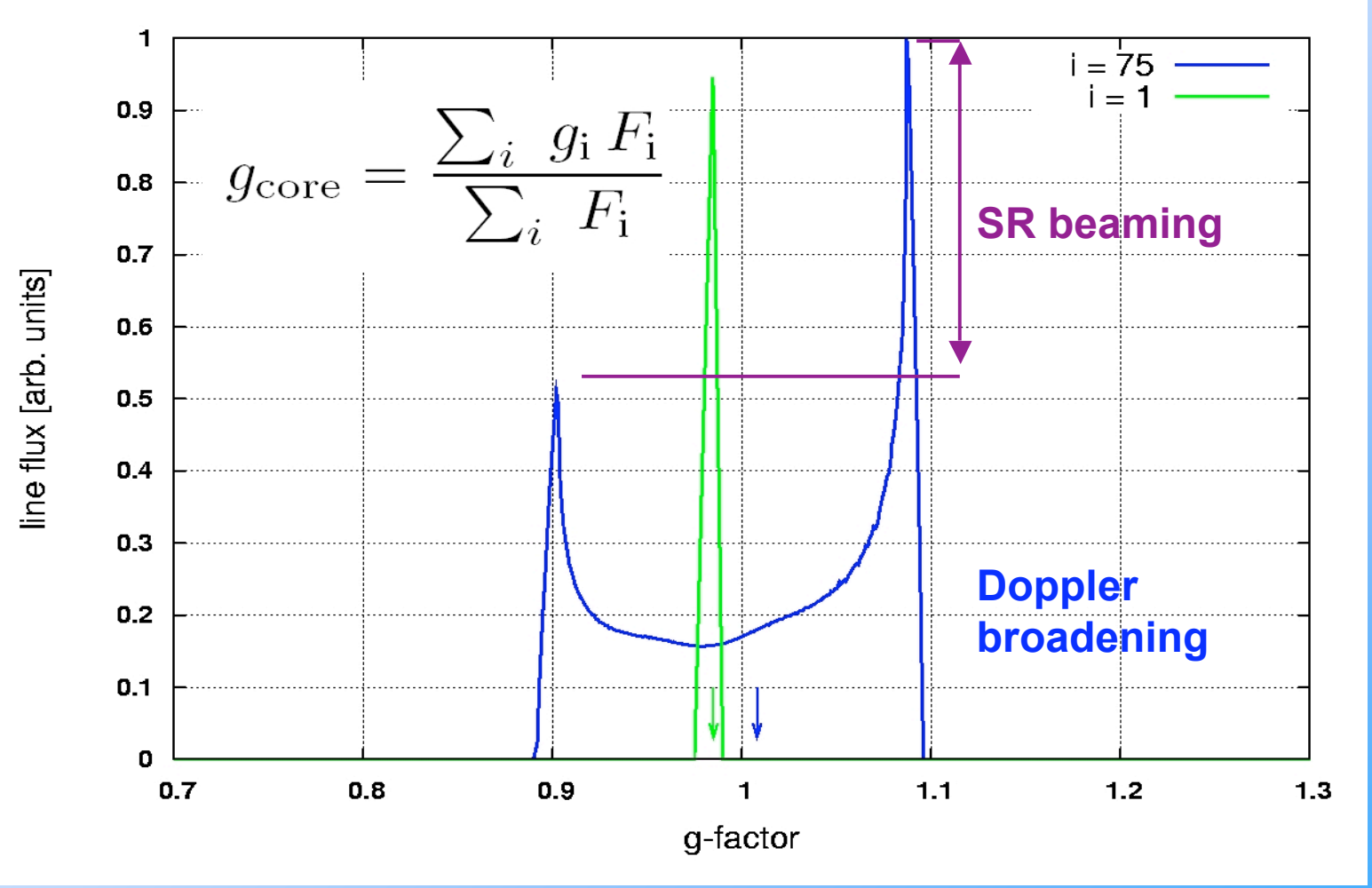


Model: equatorial Keplerian rings



- assume δ-function for line emission in rest frame
- shift Gaussian emissivity over disk to study radial dependence of ring emission
- i ~ 0° face-on orientation for gravitational redshift analysis

Example Line profiles: 1° vs. 75°



R_{peak} = 100.0 r_g
a/M = 0.998

Keplerian rotating ring

Gaussian emissivity

g_{core} = 0.985

g_{core} = 1.008

$g = \nu_{obs} / \nu_{em}$

Schwarzschild geometry

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + r^2 \sin^2\theta \, d\Phi^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \, d\theta^2$$

$= -\alpha^2$

$\alpha = d\tau/dt = (-g_{tt})^{1/2}$ **lapse function, „Schwarzschild factor“**

- redshift:

$$z(r) = \frac{\Delta\lambda}{\lambda} = \frac{1 - \alpha(r)}{\alpha(r)} = \left(1 - \frac{2GM}{rc^2}\right)^{-1/2} - 1$$

$$z(r) \approx \frac{GM}{rc^2} = r_g/r \text{ for } r \gg R_S$$

asymptotically linear decay

Kerr geometry *(Boyer-Lindquist form)*

$$ds^2 = -\alpha^2 dt^2 + \tilde{\omega}^2 (d\phi - \omega dt)^2 + \rho^2 / \Delta \; dr^2 + \rho^2 d\theta^2$$

with functions

$$\begin{aligned}
\alpha &= \frac{\rho\sqrt{\Delta}}{\Sigma}, \\
\Delta &= r^2 - 2Mr + a^2, \\
\rho^2 &= r^2 + a^2 \cos^2\theta, \\
\Sigma^2 &= (r^2 + a^2)^2 - a^2 \Delta \sin^2\theta, \\
\omega &= \frac{2aMr}{\Sigma^2}, \\
\tilde{\omega} &= \frac{\Sigma}{\rho} \sin\theta.
\end{aligned}$$

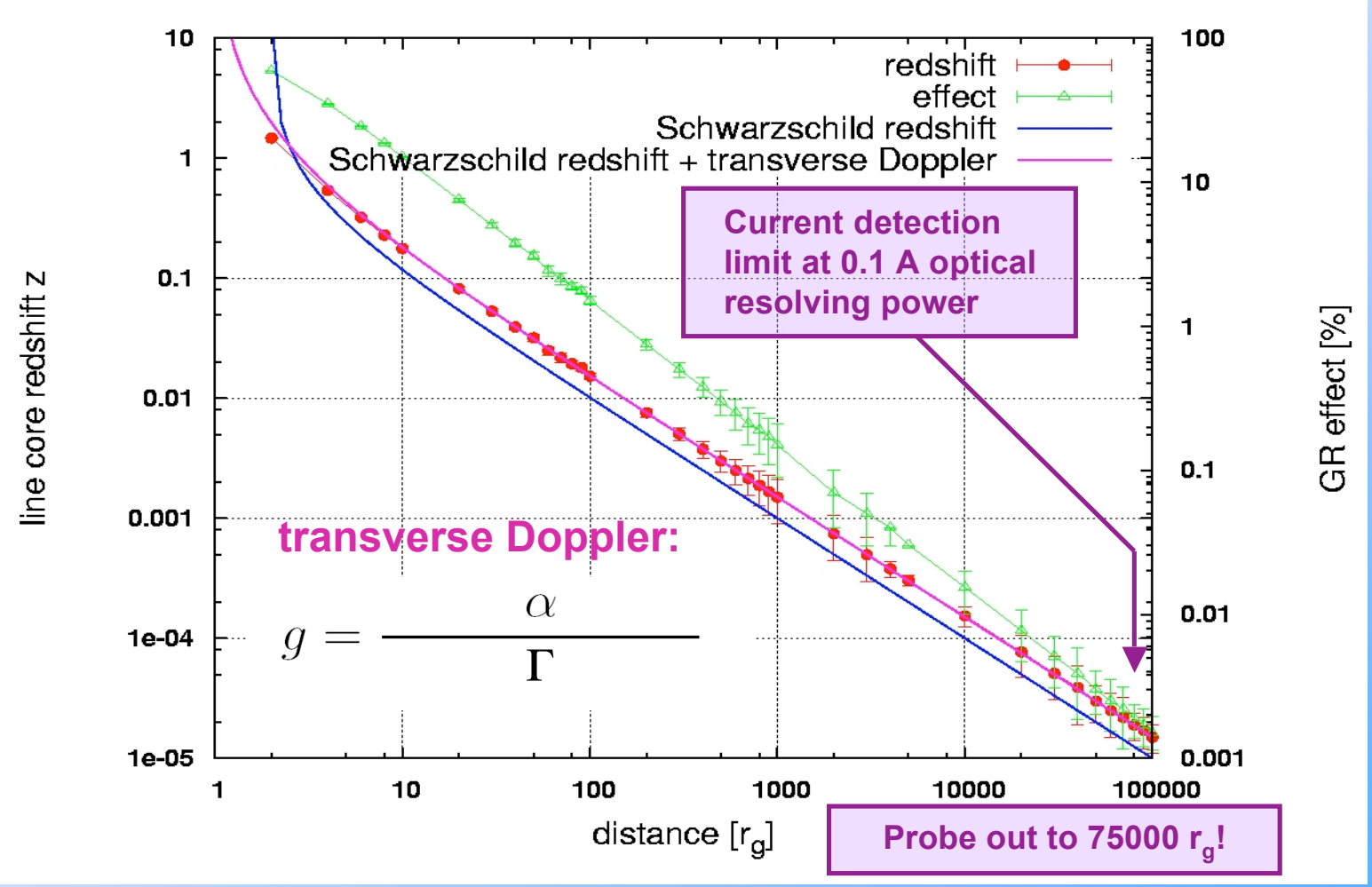
BH mass *M*,
BH spin *a*

- redshift: ($\theta = \pi/2$)

$$\alpha = \sqrt{\frac{r(r^2 - 2Mr + a^2)}{r^3 + a^2(r + 2M)}}$$

asymptotically linear decay

Line core redshift gradient



simulations:

i = 1°,

Keplerian rings,

Gaussian emissivity

$z_{core} = 1/g_{core} - 1$

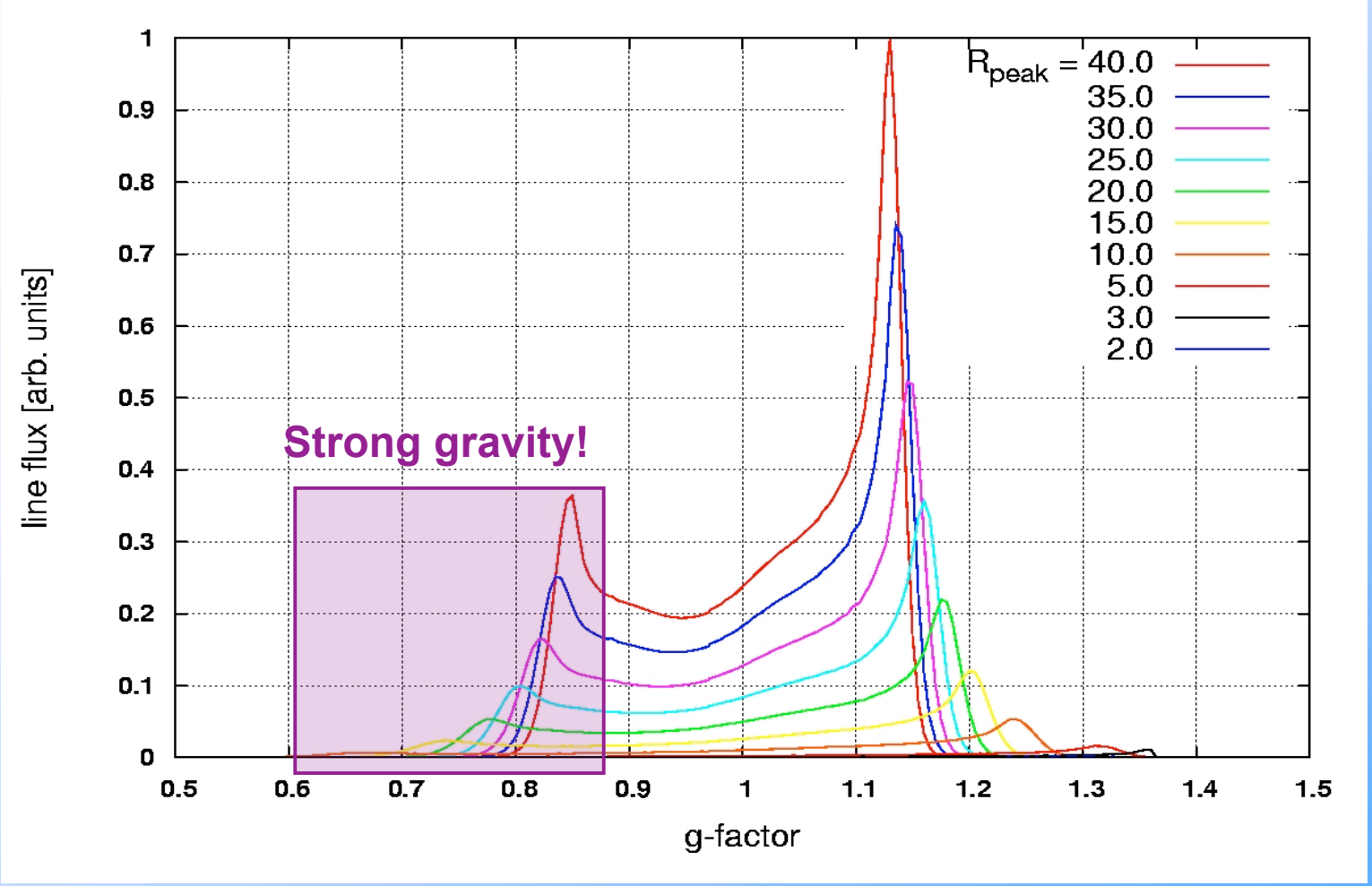
$(1 - g_{core}) \times 100$

Schwarzschild:

$$\alpha\,|_{a=0} = \sqrt{1 - \frac{2M}{r}}$$

$z = 1/\alpha - 1$

Line decay from 40 to 3 r_g



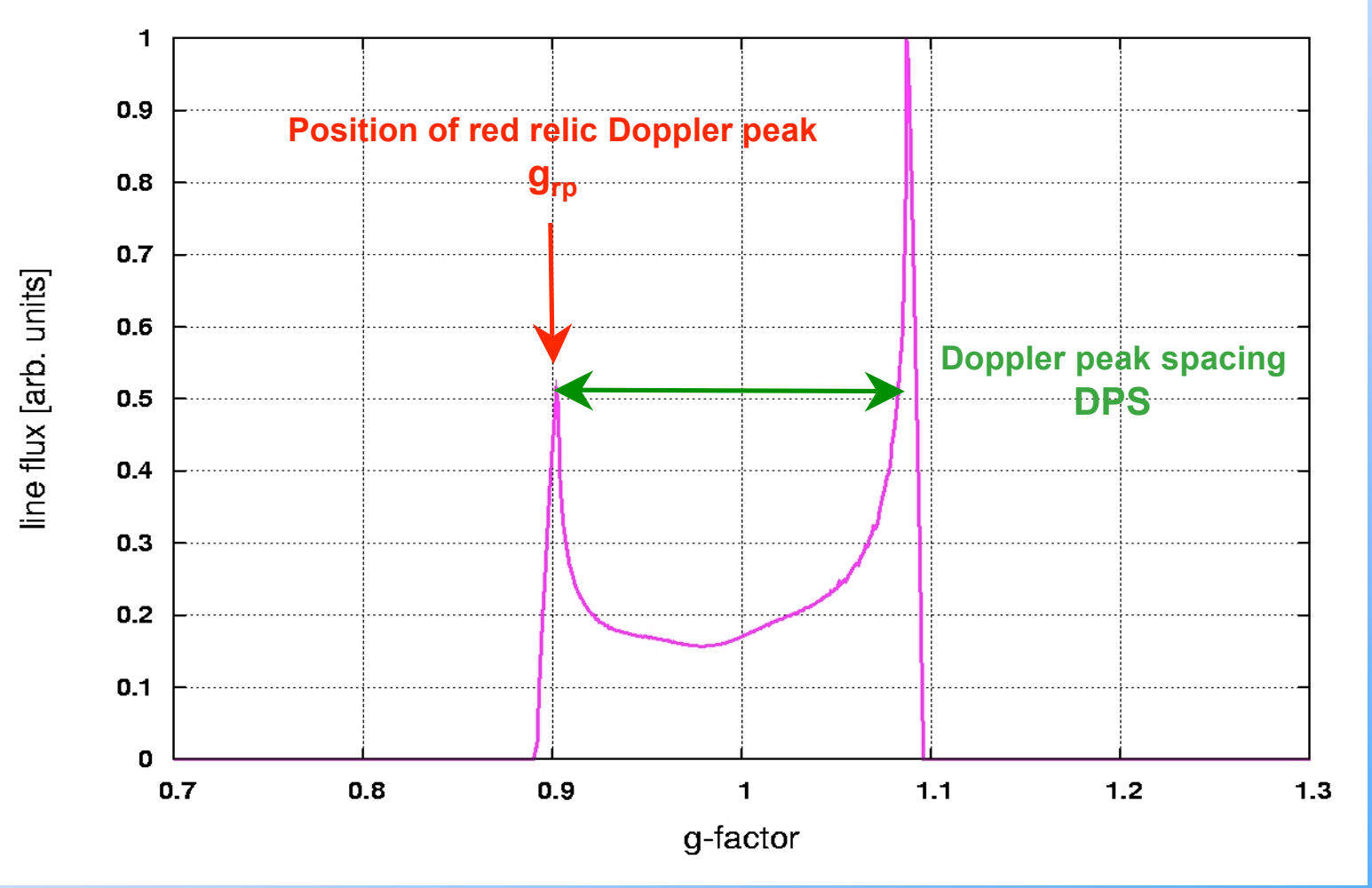
i = 75°
a/M = 0.998

Keplerian rotating rings,

Gaussian emissivity: ~ 1 r_g narrow

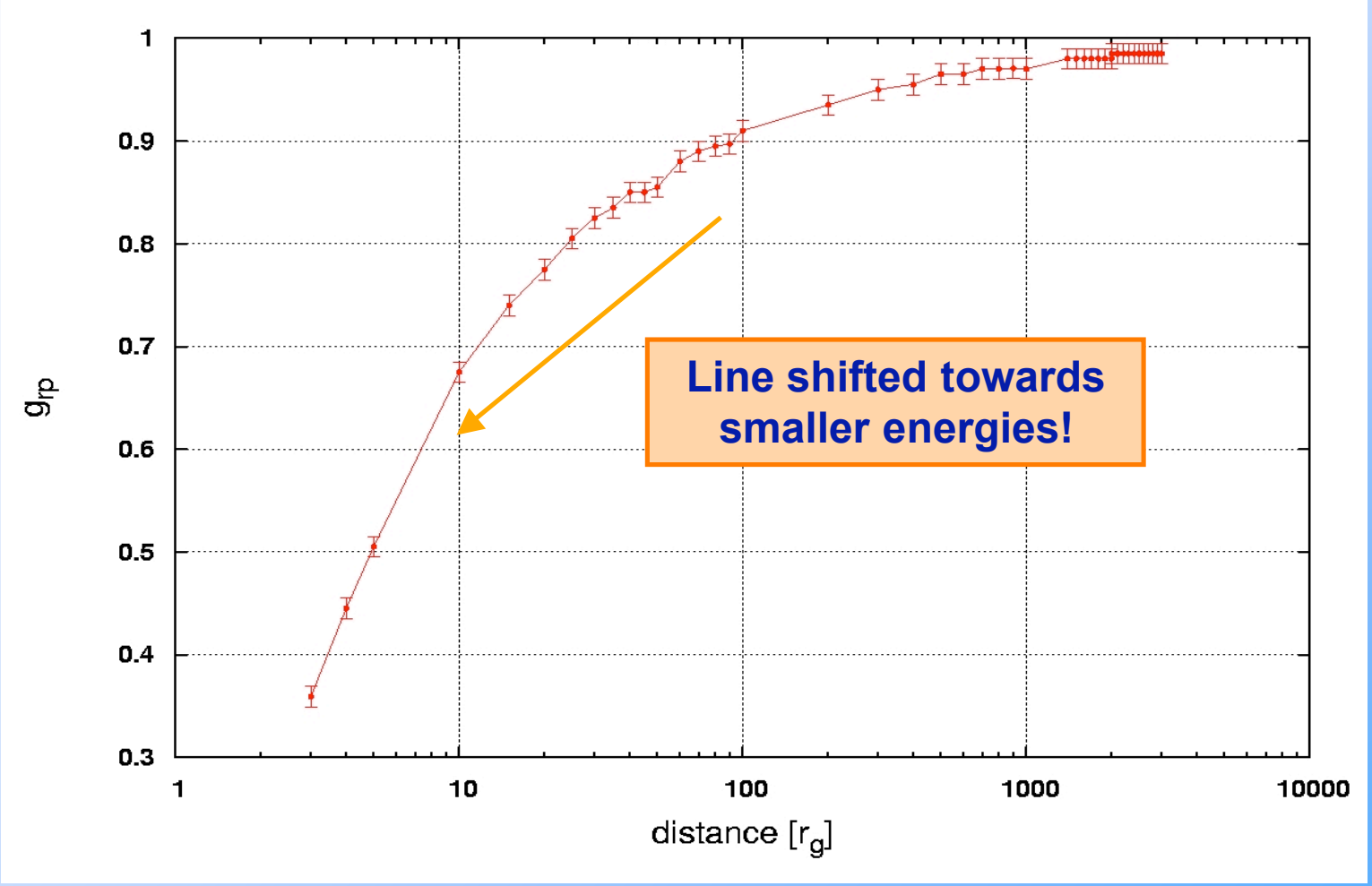
$g = \nu_{obs} / \nu_{em}$

Two other Line Criteria



Müller & Camenzind, A&A 413, 861, 2004

Red line wing evolution



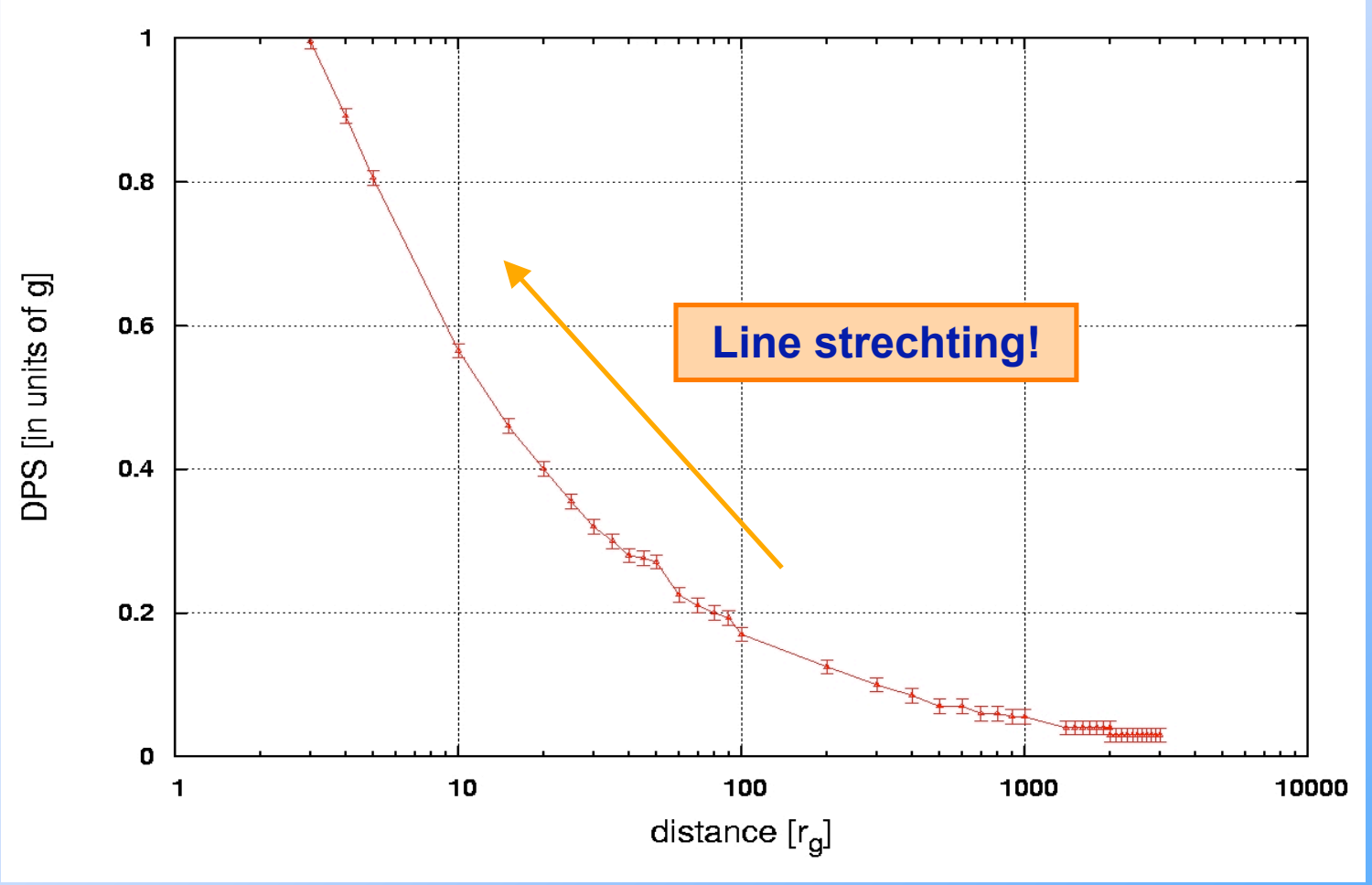
i = 75°
a/M = 0.998

Keplerian rotating rings,

Gaussian emissivity: ~ 1 r_g narrow

$g = \nu_{obs} / \nu_{em}$

A Line torn apart



i = 75°
a/M = 0.998

Keplerian rotating rings,

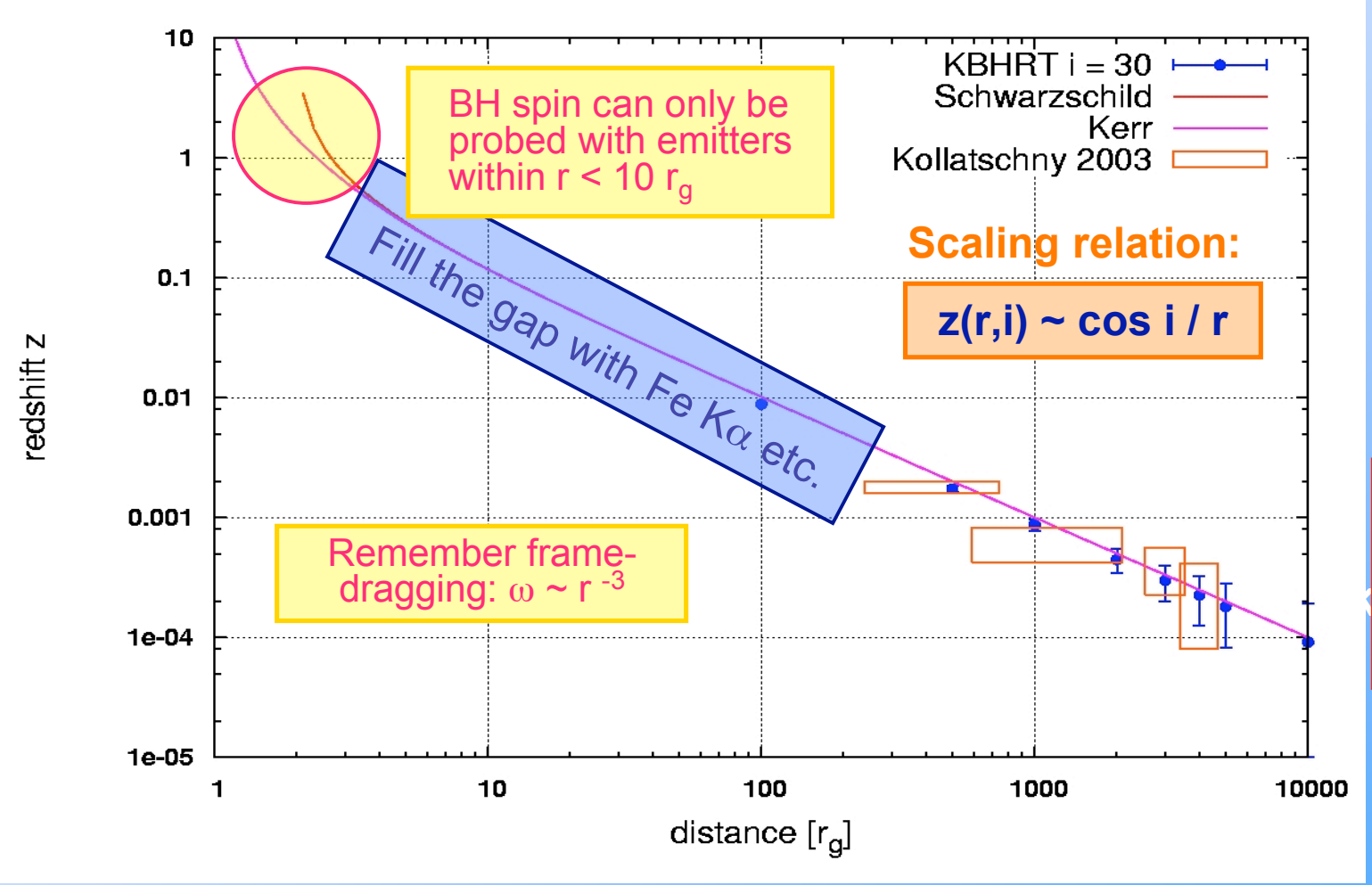
Gaussian emissivity: ~ 1 r_g narrow

$g = \nu_{obs} / \nu_{em}$

Note on Generality

- $g = E_{obs}/E_{rest}$ ⇨ use *any emission line* specified by E_{rest}
  - X-rays e.g. Fe Kα
  - optical lines e.g. Hα, Hβ, HeI, HeII
  - etc.
- study can be applied *to any black hole* of arbitrary mass
  - SMBHs in AGN and non-active galaxies
  - stellar-mass black holes
  - intermediate-mass black holes

Probing BH spin and Multi-wavelength challenge



Schwarzschild:

$$\alpha\,|_{a=0} = \sqrt{1 - \frac{2M}{r}}$$

Kerr:

$$\alpha = \sqrt{\frac{r(r^2 - 2Mr + a^2)}{r^3 + a^2(r + 2M)}}$$

Optical BLR line data:

NLS1 Mrk 110

Kollatschny, A&A 412 L61, 2003

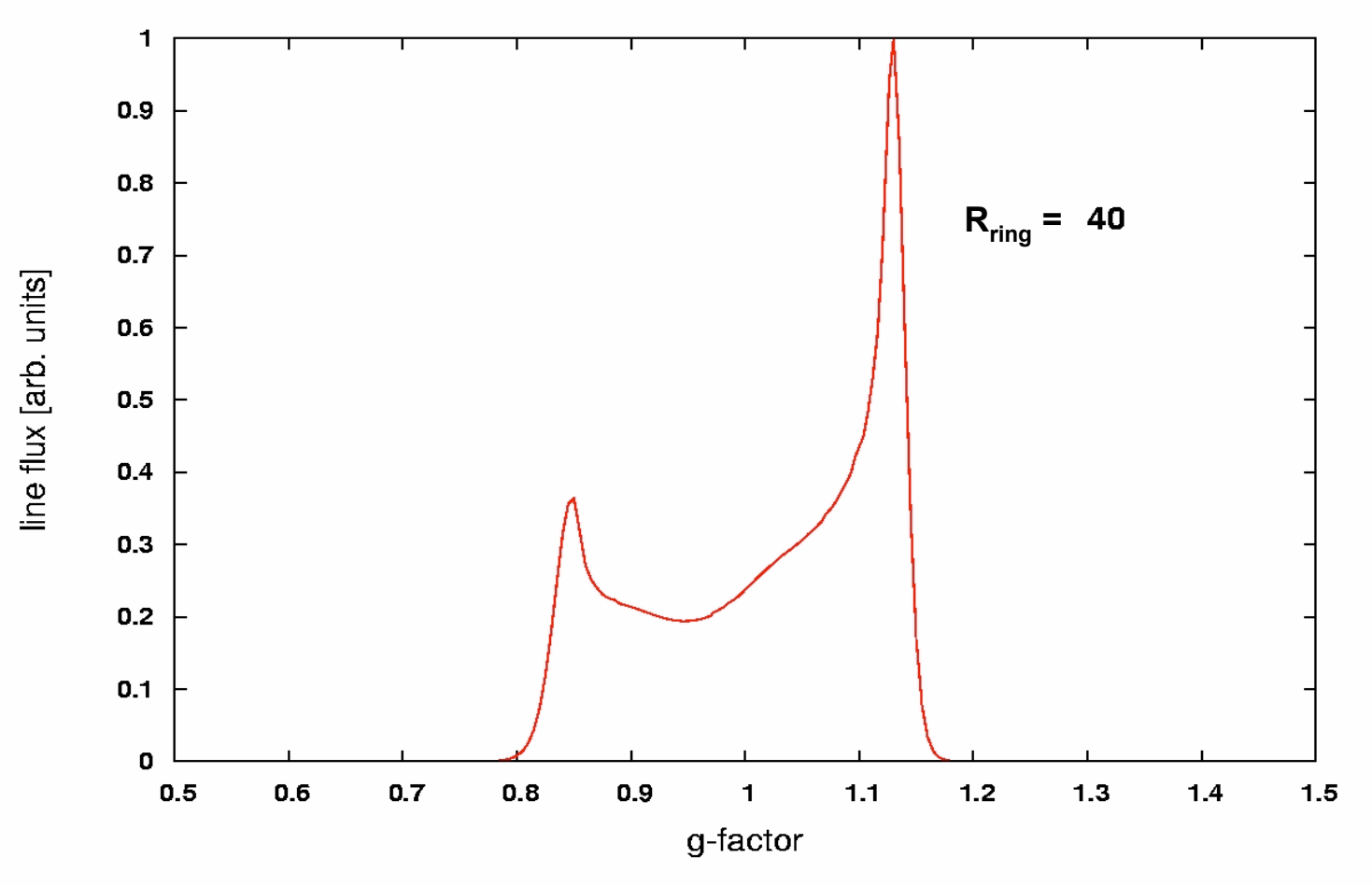
M_{BH} = 1.4 x 10^8 $M_\odot$

assumed

Conclusions

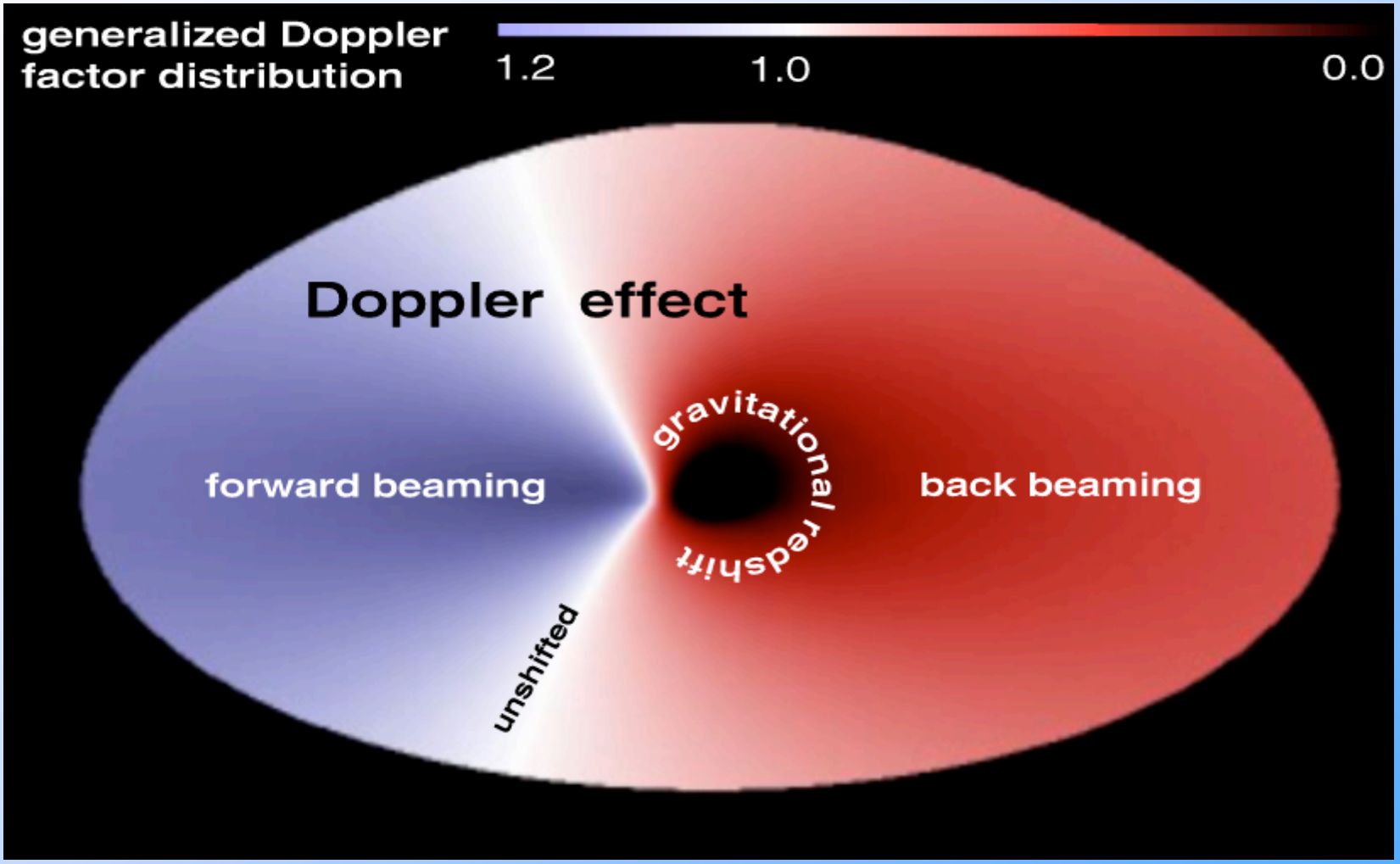
- **g_{core} suitable to analyse emission lines and quantify gravitational redshift**
- **Two modes:** *shift* **and** *distortion*
- **Redshift far-field scaling: z ~ 1/Distance**
- **Redshift can be** *probed out to 75000 r_g* **with current optical resolution (VLT)**
- **Distortion starts at a few tens r_g**
- **Close emitters, r < 10 r_g, can** *probe BH spin*
- **Multi-wavelength observations of** *any redshifted feature* **provide tool for robust** *{M,i} measurements*

Thank you!


line flux [arb. units]
g-factor
R_{ring} = 40
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5

Additional Slides:

Step 1: Rendering Image



$g = \nu_{obs} / \nu_{em}$

$g = (1+z)^{-1}$

Step 2: Compute Spectral Flux

$$F^{obs} = \int_{image} d\Xi \; I_\nu^{obs}$$

- **spectral flux integral**

$$I_\nu^{obs} = g^3 \, \hat{I}_\nu^{em}$$

- use conservation of phase-space current *(Misner 1973)*

$$\hat{F}_\nu^{em} = \pi \hat{I}_\nu^{em} = \epsilon(r)\delta\left(\nu_{em} - \nu_0\right)$$

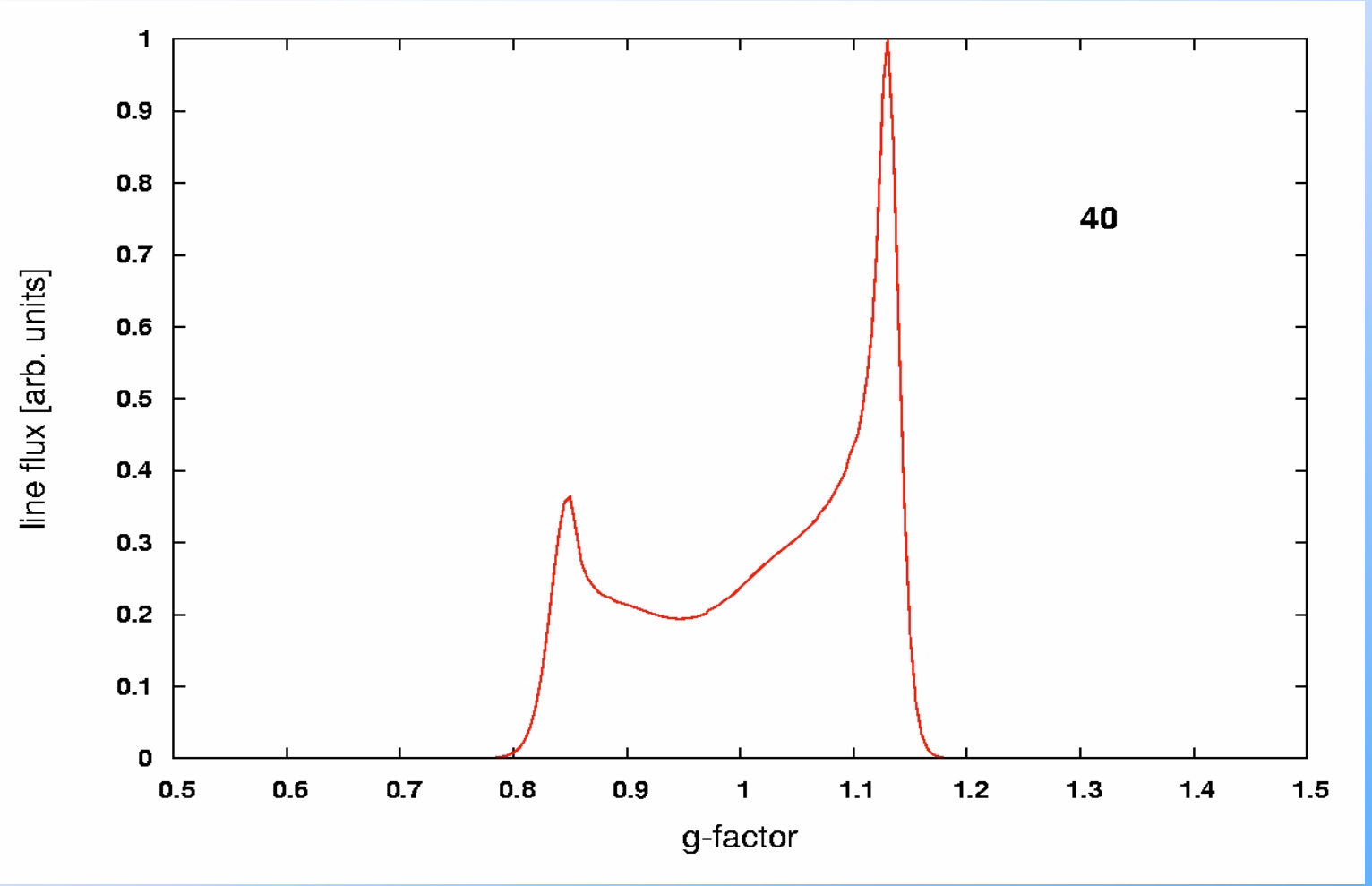
- assume line shape in rest frame: **δ-distribution**
- specify radial emissivity profile

$$F_{obs}(E_{obs}) = \int_{image} \epsilon(r) g^4 \delta(E_{obs} - gE_0) d\Xi$$

- evaluate tuple {g, ΔΩ, r} on each pixel and sum over pixels!

GR Doppler factor *g* is a function of metric and emitter velocity field!

Line decay from 40 to 3 r_g



variable R_{peak}

i = 75°
a/M = 0.998

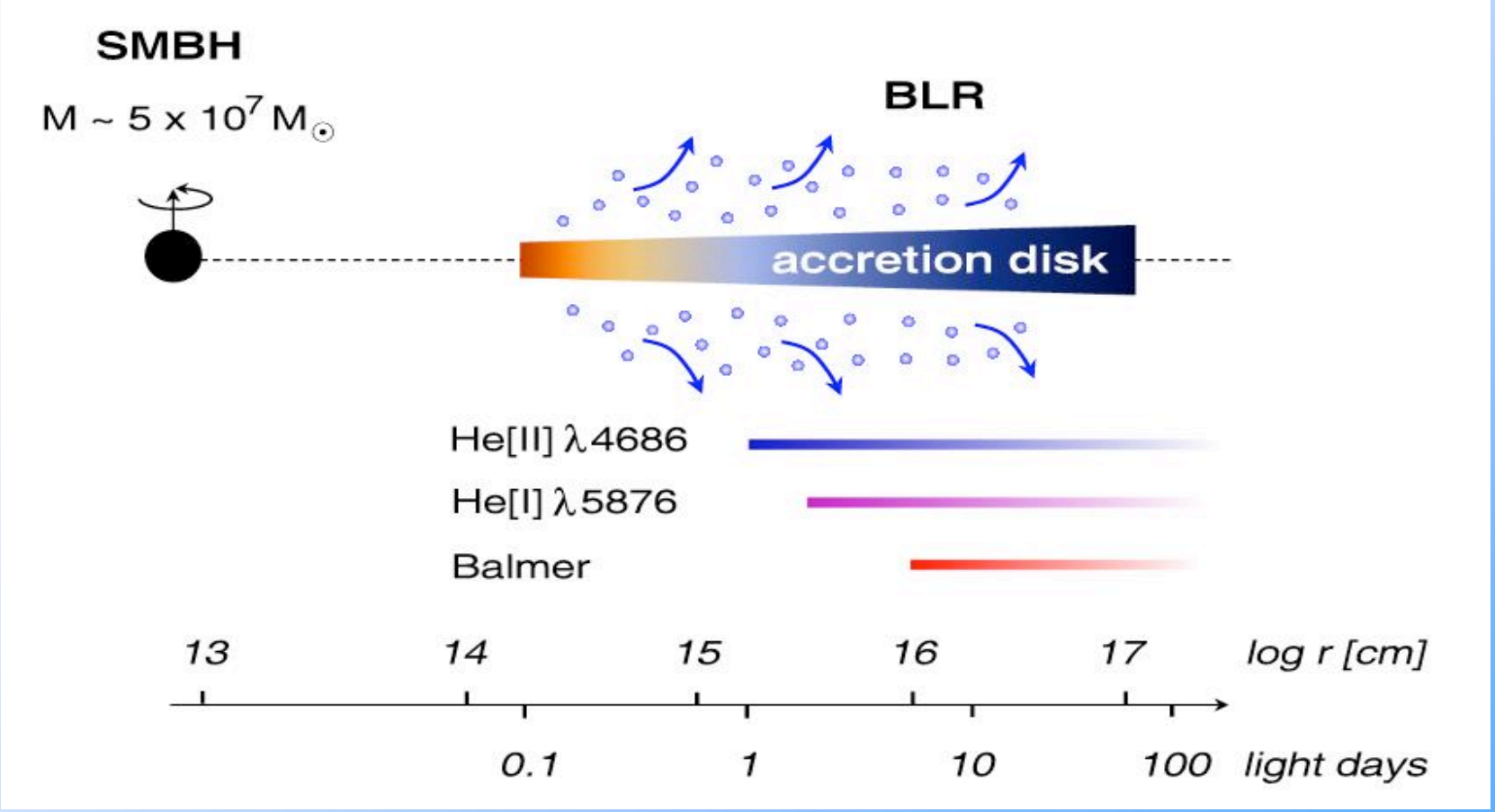
Keplerian rotating rings,

Gaussian emissivity:
~ 1 r_g narrow

$g = \nu_{obs} / \nu_{em}$

Optical Lines from the BLR

BLR structure



favored model

this work:

only Keplerian rotation

line emitting rings in equ. plane

idea from Kollatschny's website

Observational method

- take optical lines Hα, Hβ, HeI, HeII
- rms line width measures line variability
- reverberation mapping: emitter distance calculated from cross-correlating light curves of lines with continuum
- time lag measurements: time interval τ for line responding on continuum
  - ➢ orbital radius R = τc
- differential redshift measurements of rms line centers Δv
  - ➢ redshifts Δz = Δv/c

$$M_{grav} = c^2 G^{-1} R \Delta z$$

Kollatschny, A&A 412, L61, 2003

Optical data

Kollatschny, A&A 412, L61, 2003

Table 1. Rms line widths (*FWHM*) of our strongest emission lines: Hα, Hβ, HeIλ5876, and HeIIλ4686; differential redshift of rms line centers Δv_{cent} (rms); cross-correlation lags τ and central black hole mass estimation M_{grav} derived from gravitational redshift.

$M_{grav} = c^2 G^{-1} R \Delta z$

Line		*FWHM* (rms) [km s^{-1}]	Δv_{cent} (rms) [km s^{-1}]	τ [days]	M_{grav} [10^7 $M_\odot$]
(1)		(2)	(3)	(4)	(5)
HeII	broader ↑	4444 ± 200	541 ± 60	3.9 ± 2	13 ± 3
HeI		2404 ± 100	186 ± 60	10.7 ± 6	12 ± 4
Hβ		1515 ± 100	118 ± 50	24.2 ± 4	17 ± 4
Hα		1315 ± 100	74 ± 50	32.3 ± 5	14 ± 5

$\Delta z = \Delta v / c$

0.0018
0.0006
0.0004
0.0002

$R = \tau c$ [10^{16} cm]

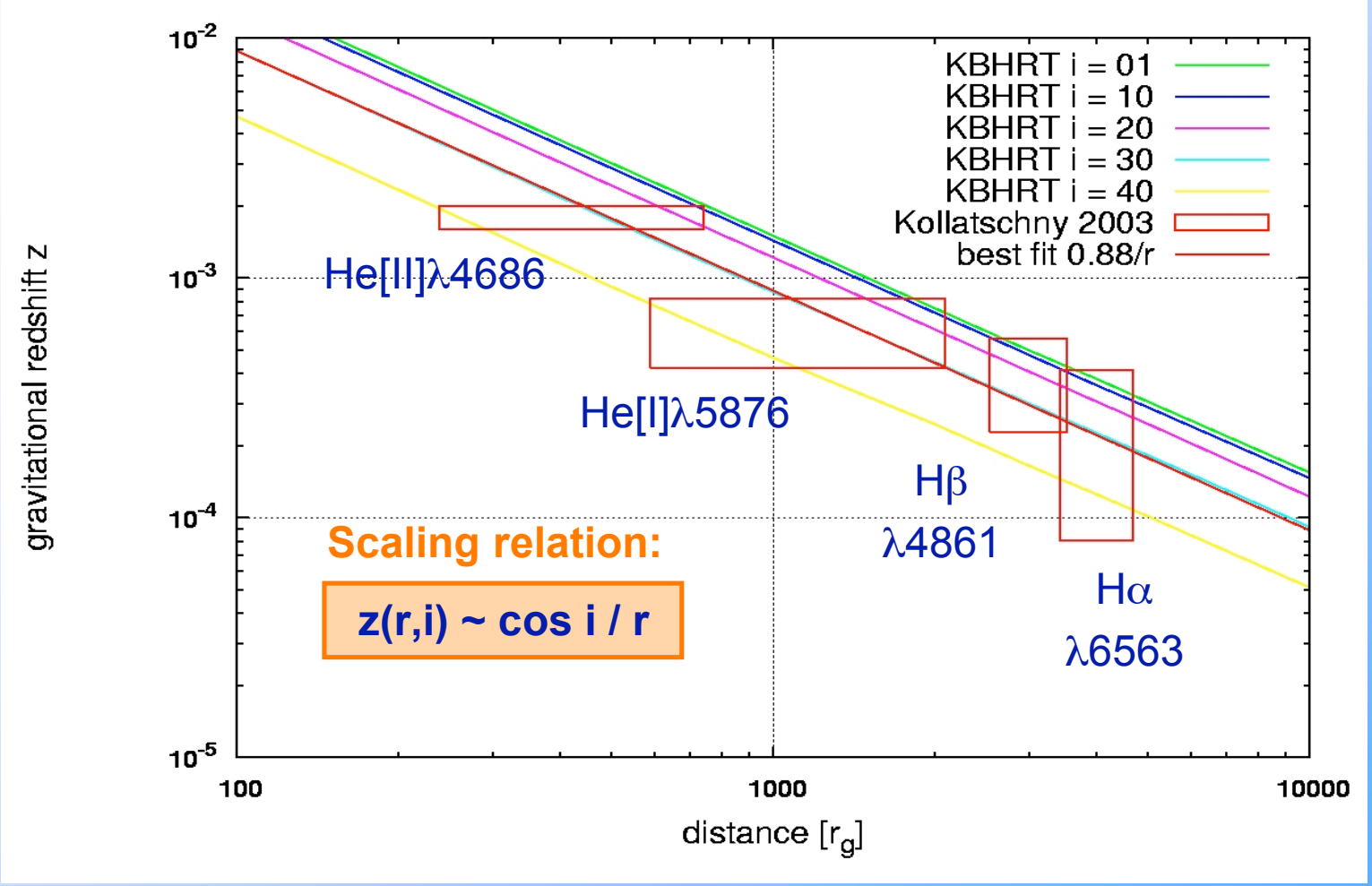
1.0
2.8
6.3
8.4

line-of-sight Doppler widths

mean value: $M_{grav} = 1.4 \times 10^8 M_\odot$

assume & rescale

Mrk 110 - Inclination of inner disk

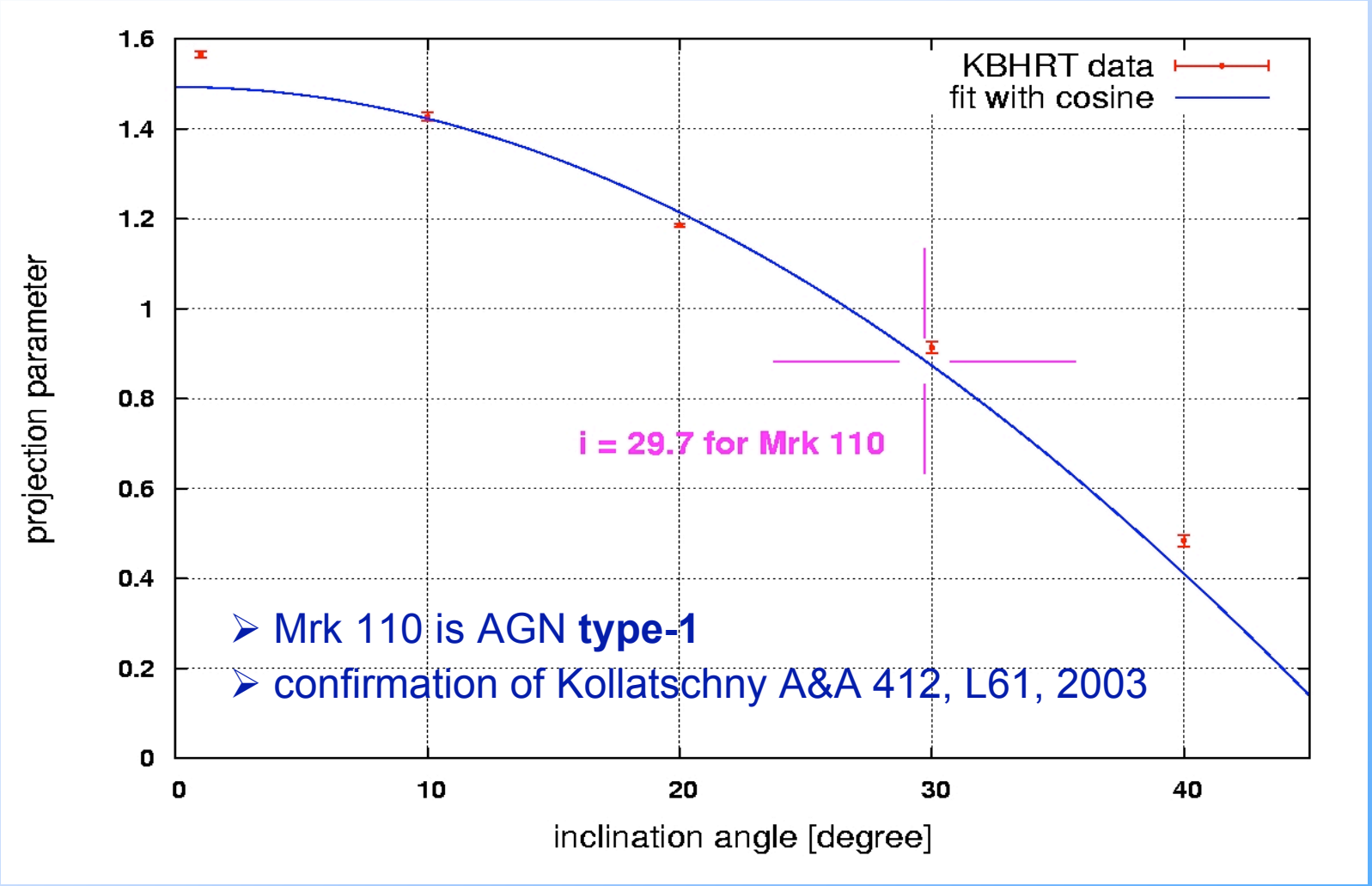


Test different inclinations

assume
M = 1.4 x 10^8 $M_\odot$

Obs. data: Kollatschny, A&A 412, L61, 2003

Mrk 110 - Inclination of inner disk



- Mrk 110 is AGN **type-1**
- confirmation of Kollatschny A&A 412, L61, 2003

Conclusions – Broad Line Region

- Flat Keplerian BLR model with emitting rings allow to fit optical gravitationally redshifted lines
- However: spherical BLR, complex dynamics, opacity effects, disk warping
- Fits with cos i/r in z-r plot deliver {M, i}
  CAUTION: scaling breaks down at small radii
- Inner disk inclination of NLS-1 Mrk 110 fitted to i = 30° by assuming M