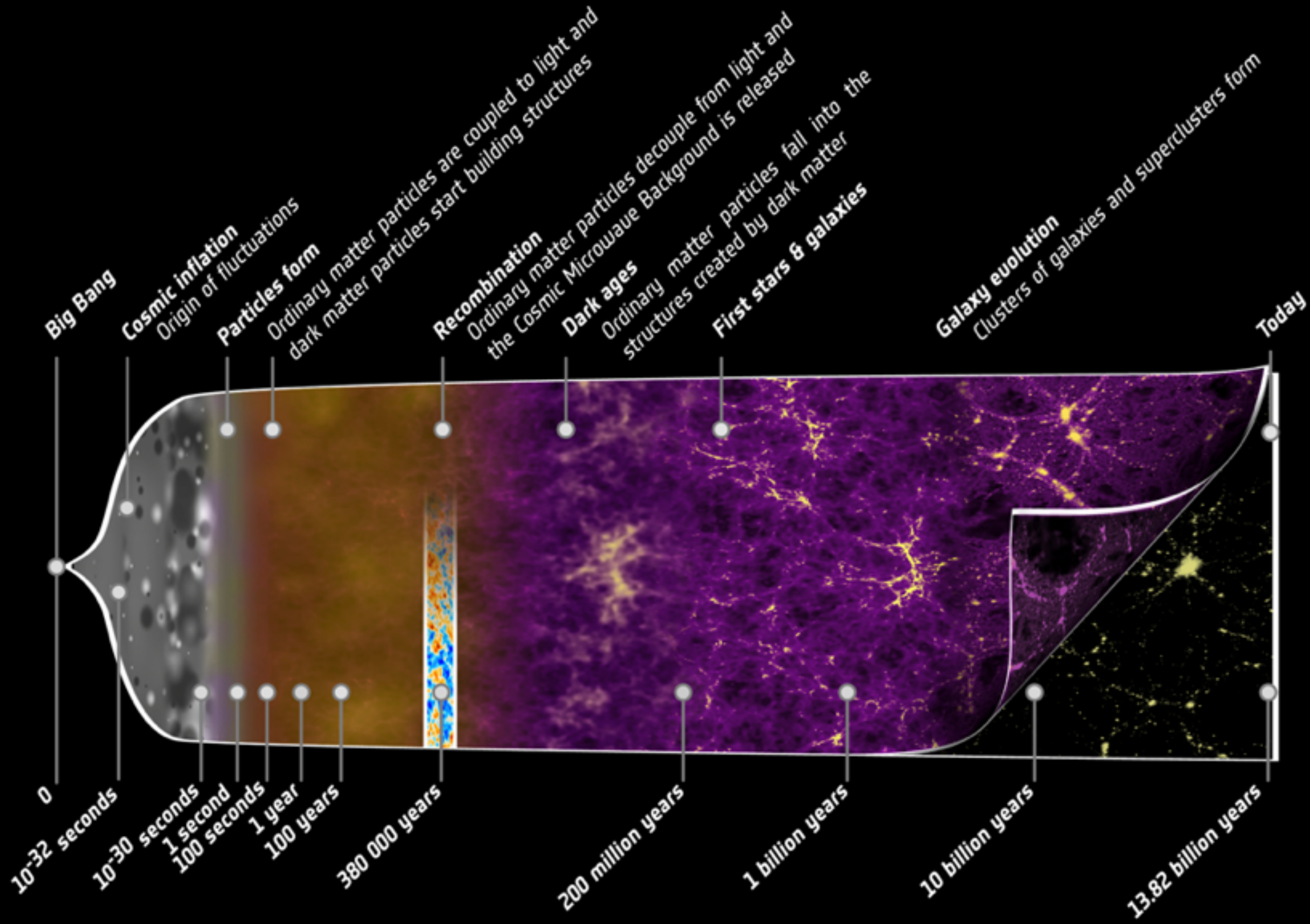
Inflation



December 2014

Inflationary dynamics

Almost exponential expansion
Only small departures from
Cosmological Constant because
Inflation has to end

$$p = w\rho = (-1+\epsilon)\rho$$

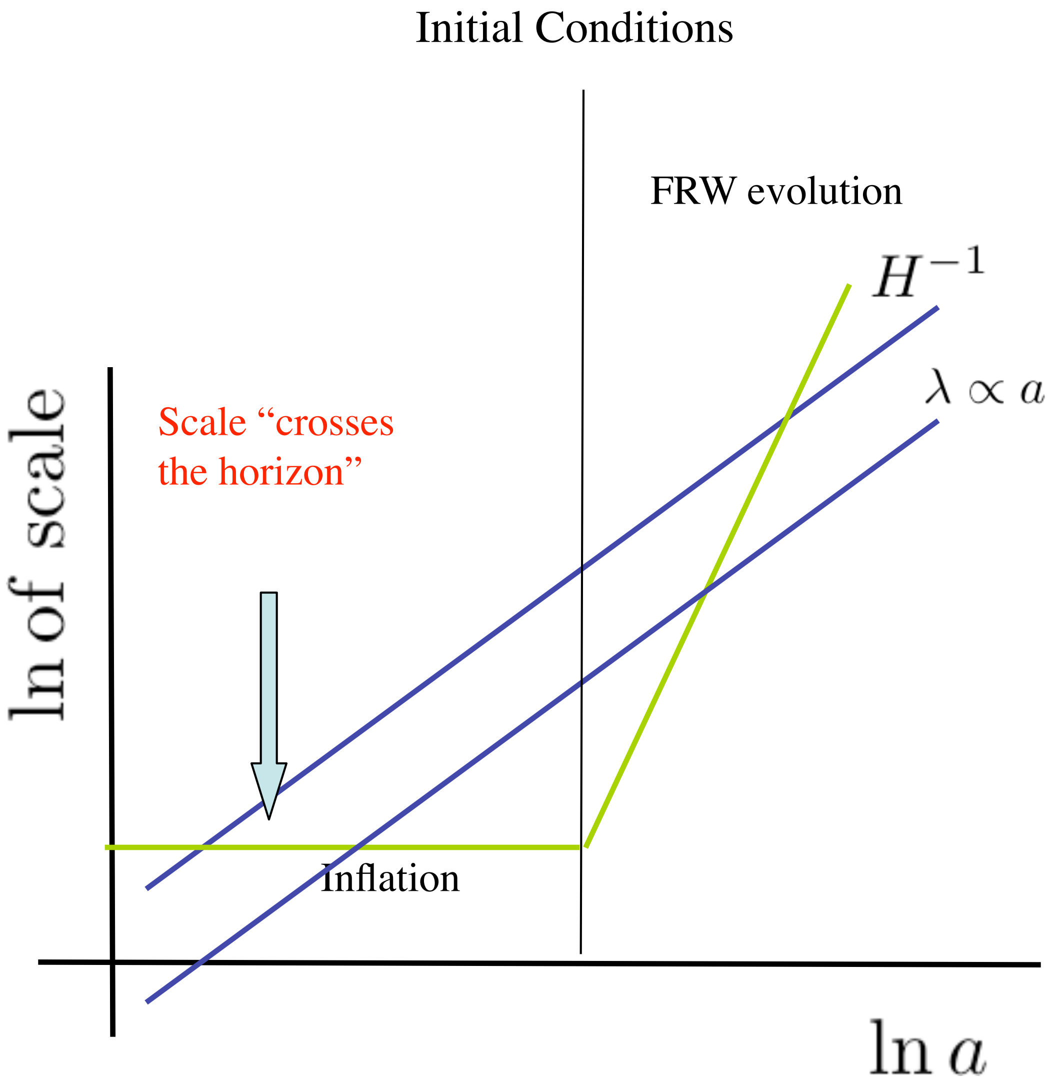
$$\epsilon = -\frac{\dot{H}}{H^2}$$

During this period the Universe must have expanded by roughly 60 enfolds

$$N = \ln(a_{\rm final}/a_\star) \approx 60$$

The origin of fluctuations

The clock fluctuations are "frozen" at horizon crossing (frequency of order H). *We are probing the theory at an energy H which is roughly constant in time*. What observe is the fluctuations in the expansion of one region relative to the other due to the clock fluctuations.



Amplitude of scalar and tensor fluctuations as a function of scale

$$\Delta_{\rm s}^2(k) \equiv \Delta_{\mathcal{R}}^2(k) = \frac{1}{8\pi^2}\frac{H^2}{M_{\rm pl}^2}\frac{1}{\varepsilon}\bigg|_{k=aH} ,$$

$$\Delta_{\rm t}^2(k) \equiv 2\Delta_h^2(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\rm pl}^2}\bigg|_{k=aH} ,$$

Tensor to scalar ratio

$$r \equiv \frac{\Delta_{\rm t}^2}{\Delta_{\rm s}^2} = 16\,\varepsilon_\star \, . \qquad \epsilon = -\frac{\dot H}{H^2}$$

Planck 2013

$$n_s - 1 \approx -0.04 \neq 0$$

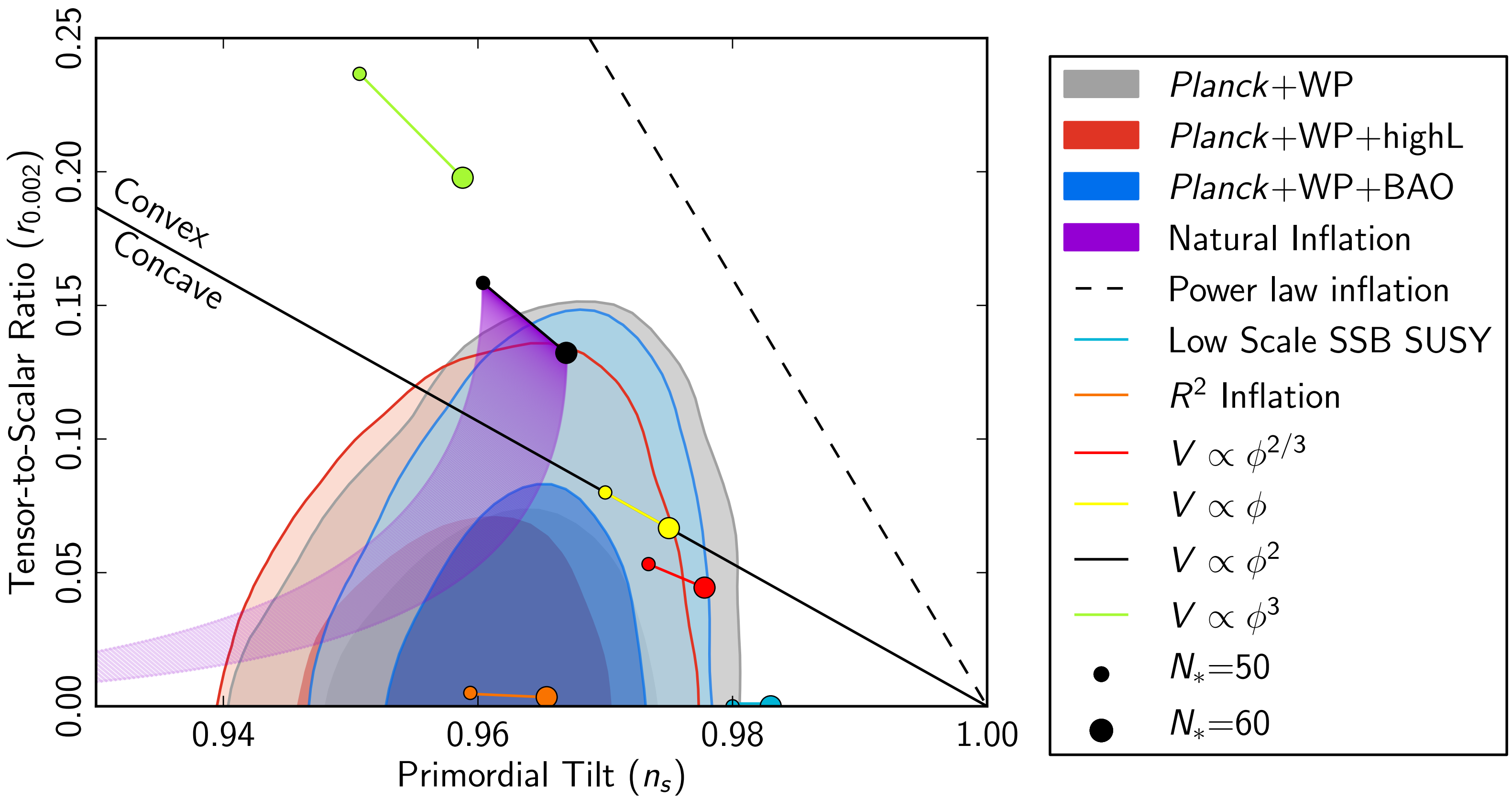


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

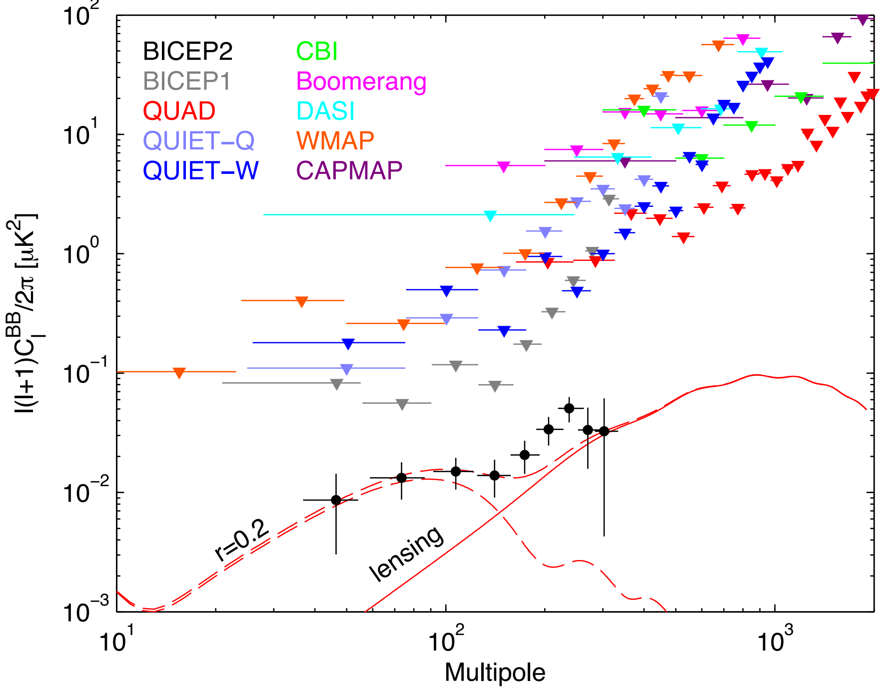
$$\boxed{V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\,\mathrm{GeV}}\,.$$

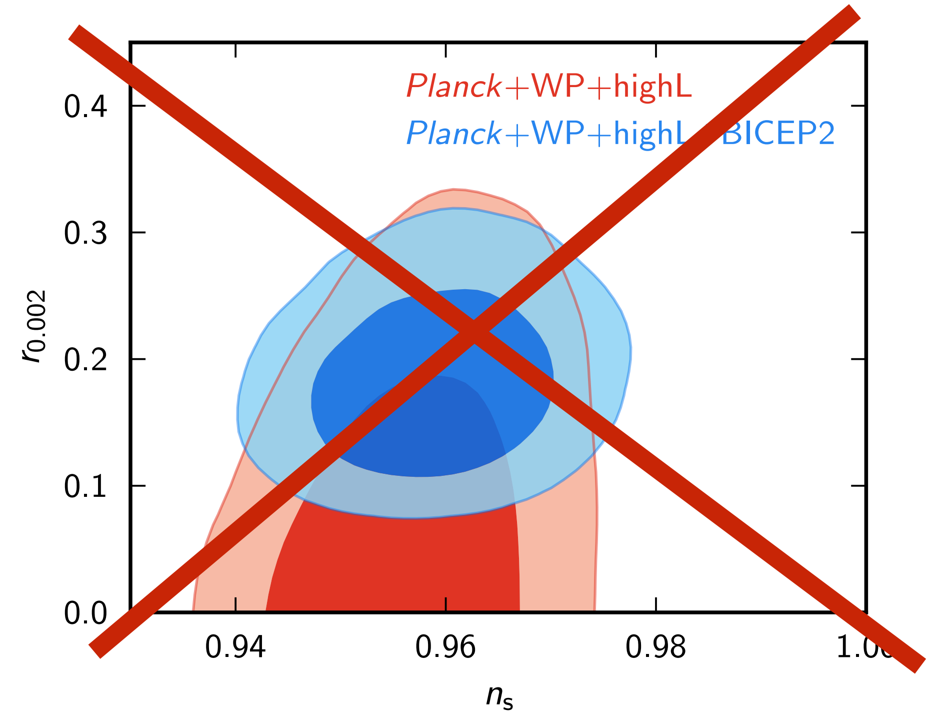
Observable gravity waves imply inflation happened around the GUT scale.

$$\boxed{\Delta\phi \sim \left(\frac{r}{0.002}\right)^{1/2} \left(\frac{N_\star}{60}\right) \; M_{pl}}$$

Observable gravity waves imply super-Planckian field excursions.

BICEP 2





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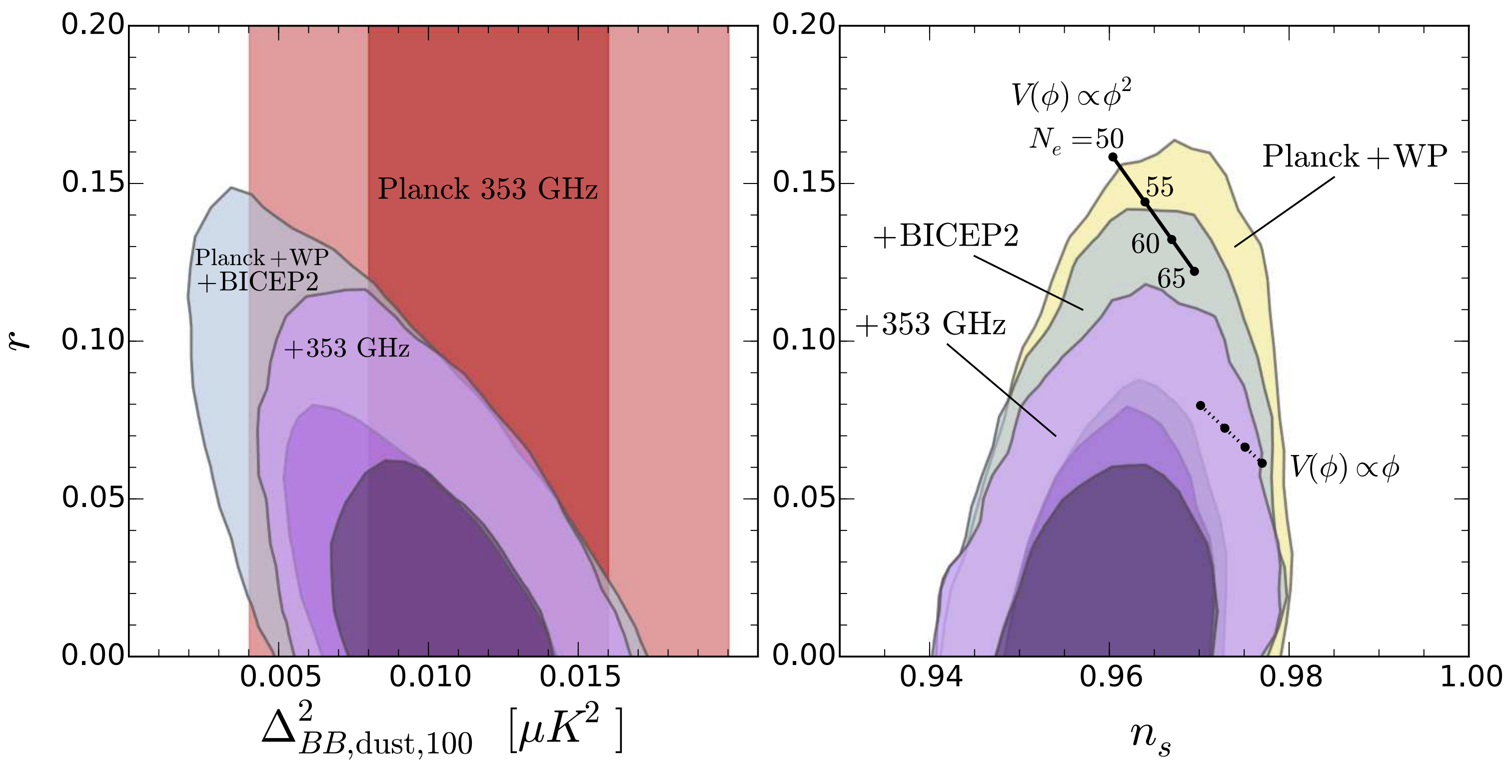


$$\Delta\phi \sim \left(\frac{r}{0.002}\right)^{1/2} \left(\frac{N_\star}{60}\right) M_{pl}$$

<u>Observable gravity waves imply super-Planckian field excursions.</u>
<u>Far away from the minimum.</u>

Potential clue: The size of the tilt

$$\epsilon = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN}$$

$$\Delta_s \propto \frac{H^2}{M_{pl}^2 \epsilon} \propto k^{(ns-1)}$$

$$a(t) \approx e^{Ht} = e^N \rightarrow dN = Hdt$$

$$(n_s - 1) = \frac{d\ln\Delta}{dlnk} = -2\epsilon - \frac{d\ln\epsilon}{dN}$$

Derivatives with respect to scale translate to derivatives with respect to time or N.

What if the tilt is related to the end of inflation?

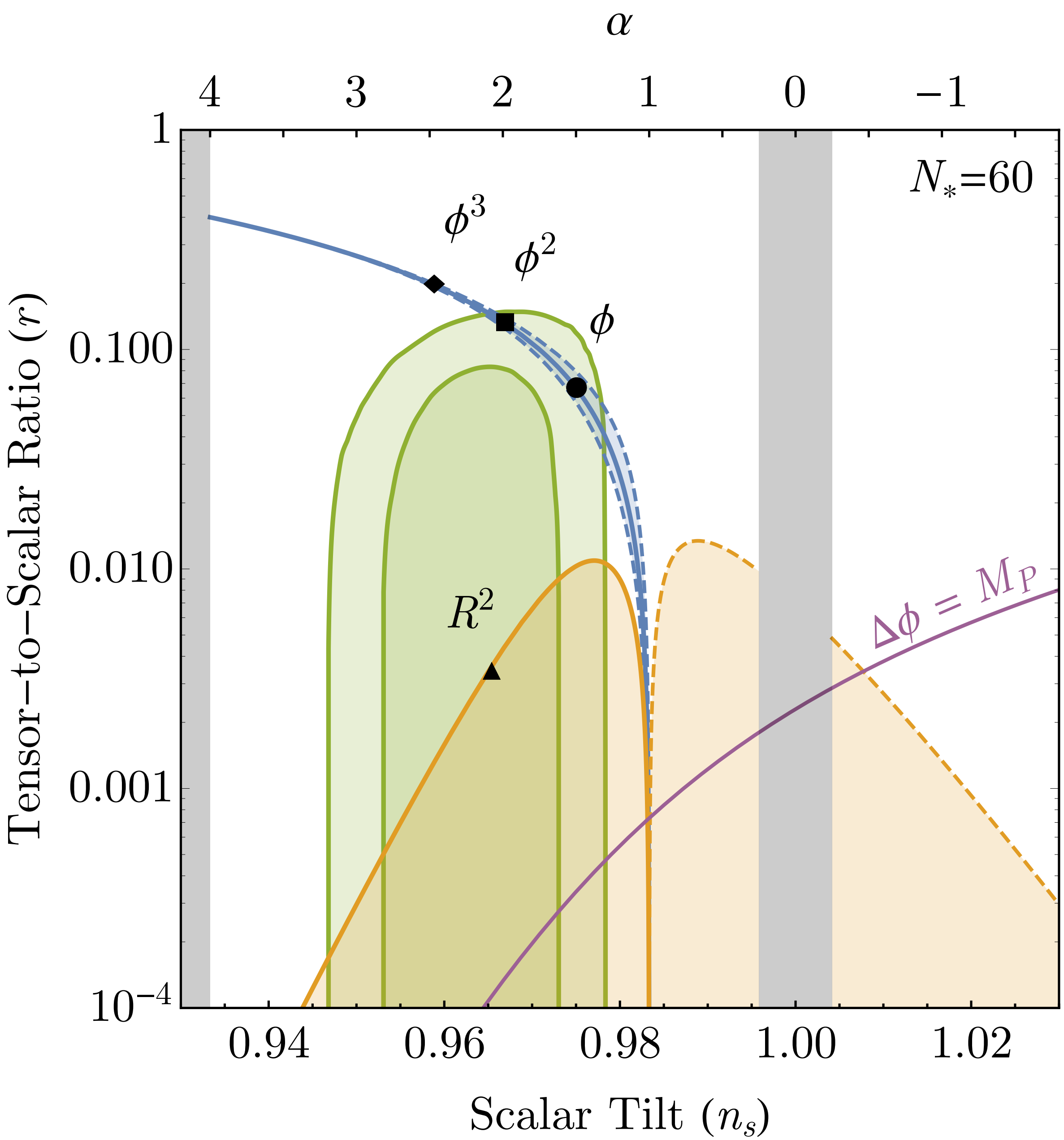
$$n_s - 1 = -\frac{\alpha}{N}$$

Not necessarily the case, for example:

$$V = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda(\psi^2 - M^2)^2 + \lambda'\phi^2\psi^2 \qquad n_s - 1 \approx 2\eta = \frac{8m^2 M_P^2}{\lambda M^4}$$

the tilt is determined by parameters and is independent of N.

Consequences of $n_s - 1 = -\dfrac{\alpha}{N}$



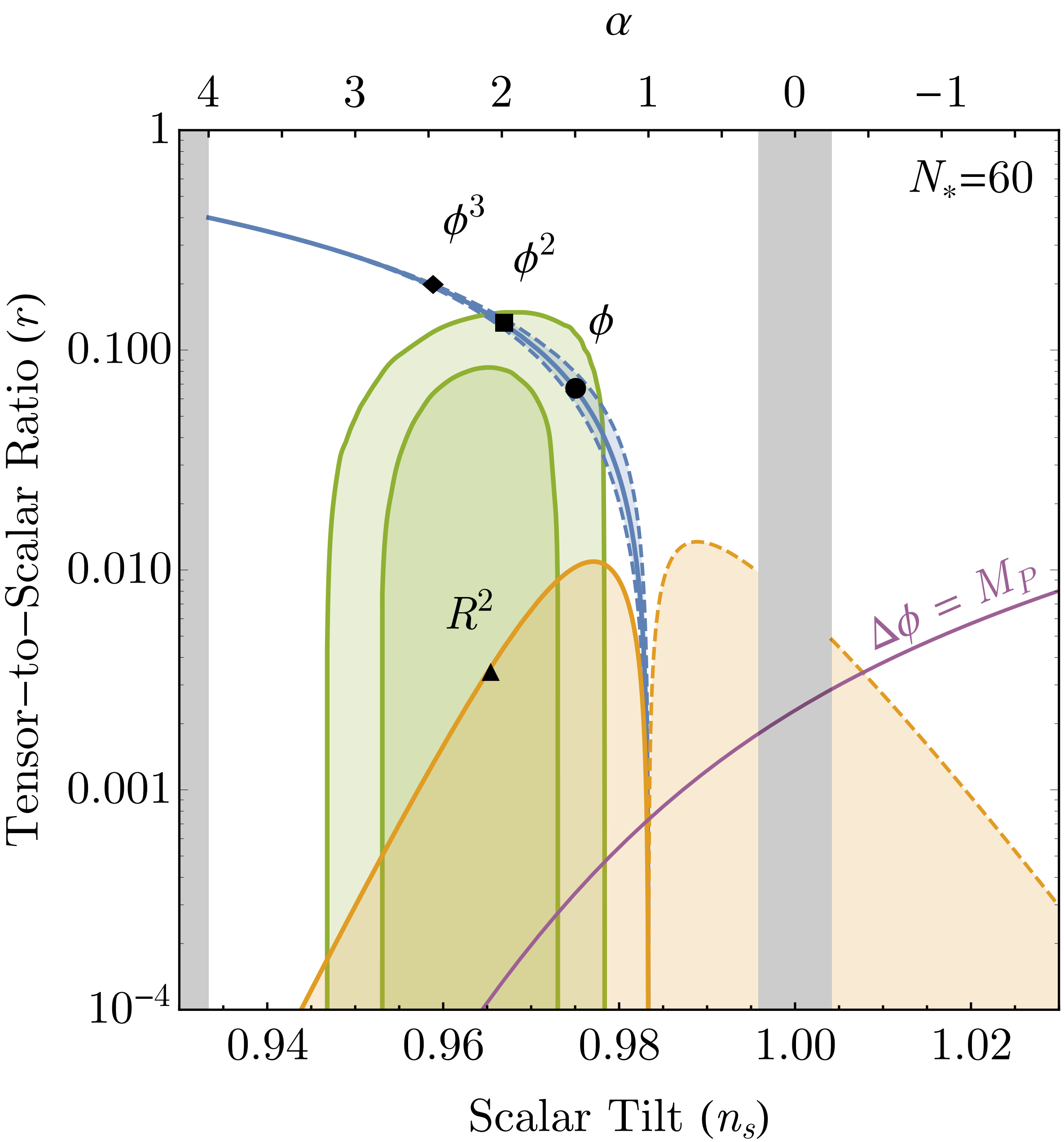
$$n_s - 1 = -2\epsilon + \frac{\mathrm{d}\log\epsilon}{\mathrm{d}N} = -\frac{\alpha}{N}$$

$$\epsilon(N) = \frac{1}{2(\alpha-1)^{-1}N + AN^{\alpha}}$$

$$r = 16\epsilon$$

The Lyth Bound

$$\frac{d\phi}{dN} = \sqrt{2\epsilon(N)}M_{pl} = \sqrt{\frac{r(N)}{16}}M_{pl}$$



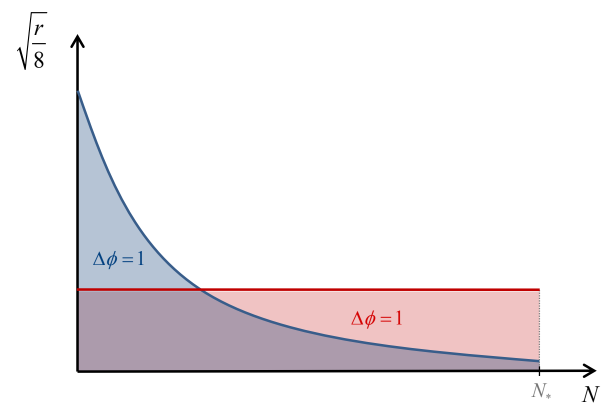


FIG. 1. *Two curves indicating $\sqrt{r(N)/8}$. The central idea is that both have identical areas and lead to $\Delta\phi = 1$. The flat curve depicts the Lyth bound, while the tilted curve indicates the improvement when taking the spectral index into account.*

1408.6839
Garcia-Bellido et al.

The fossil we already have

$$n_s - 1 \approx -0.04 \neq 0$$

No fluctuations in composition

Departures from Gaussianity

$$\zeta = \zeta_g(1 + \frac{3}{5} f_{NL}\zeta_g) = \Phi_g + f_{NL}\Phi_g^2$$

$$\frac{\mathrm{Non-Gaussian}}{\mathrm{Gaussian}} < 10^{-3} - 10^{-4}$$

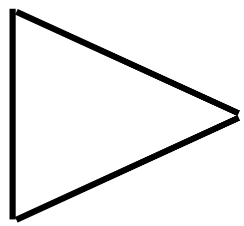
Planck 2013

	Independent KSW	**ISW-lensing subtracted** KSW
SMICA		
Local	9.8 ± 5.8	**2.7 ± 5.8**
Equilateral	−37 ± 75	**−42 ± 75**
Orthogonal	−46 ± 39	**−25 ± 39**

Three point-function in single field slow roll inflation

$$ds^2 = -dt^2 + a^2(t)e^{2\zeta(x,t)}dx^2$$

$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle$

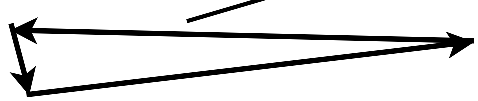


$$F_{\rm stand}(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \frac{1}{8}(2\pi)^4 P_{\mathcal{R}}^2 \cdot \frac{1}{\prod k_i^3}\left[(3\epsilon - 2\eta)\sum_i k_i^3 + \epsilon\sum_{i\neq j} k_i k_j^2 + 8\epsilon\frac{\sum_{i>j} k_i^2 k_j^2}{k_t}\right]$$

suppressed in the squeezed limit

Local piece

$k_3 \ll k_2, k_1$



Is the small scale power modulated in a way that correlates with the long wavelength modes ?

$$F_{\rm stand} \rightarrow (n_s - 1)P(k_{\rm short})P(k_{\rm long})$$

<u>The small scale power is independent of the amplitude of the long mode.</u>

$$x_F = e^{\zeta}x \quad k_F = e^{-\zeta}k$$

This is a consequence of the "sequential hiding" of modes and attractor solution.

Only background that gives scale invariant Gaussian perturbations for the adiabatic mode is de Sitter.

Thus in other scenarios you are forced to get the fluctuations in the curvature from a conversion from another field.

$$ds^2 = -dt^2 + a^2(t)e^{2\zeta(x,t)}dx^2$$

Modes are all super-horizon during conversion.
They are all observable at the same time as they are all changing the equation of state at the same time. There is no suppression in the squeezed limit.

Contrast with the adiabatic inflationary fluctuations

$$ds^2 = -dt^2 + a^2(t)e^{2\zeta(x,t)}dx^2$$

Curvature $\qquad k_3 \ll k_2, k_1$

$$\frac{\dot{\zeta}}{H} \qquad \frac{\nabla^2\zeta}{a^2H^2} \longrightarrow \left(\frac{k_3}{k_1}\right)^2$$

Locally observable effect are very small. Signal suppressed by $\left(\frac{k_3}{k_1}\right)^2$

Actual result $\qquad f_{NL}^{equil} \sim \epsilon$

We are actually seeing time delay fluctuations

Local non-Gaussianity is zero because of attractor nature of inflation. Can only be true for clock.

Equilateral part is really tiny because these are time delay fluctuations, actual change in the space-time is down by epsilon.

Large self-interactions

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{8}\frac{(\partial\phi)^4}{M^4} \longrightarrow f_{NL}^{eq} \sim \frac{\dot{\phi}^2}{M^4}$$

In non-Gaussianities are large, then field is not slowly rolling in the background solution.

The theory of the perturbations cannot be extrapolated to the energy scale relevant for the background solution. (This is always the case for small sound speeds)

Example: DBI

$$\mathcal{L} = -M^4\sqrt{1 - \frac{(\partial\phi)^2}{M^4}}$$

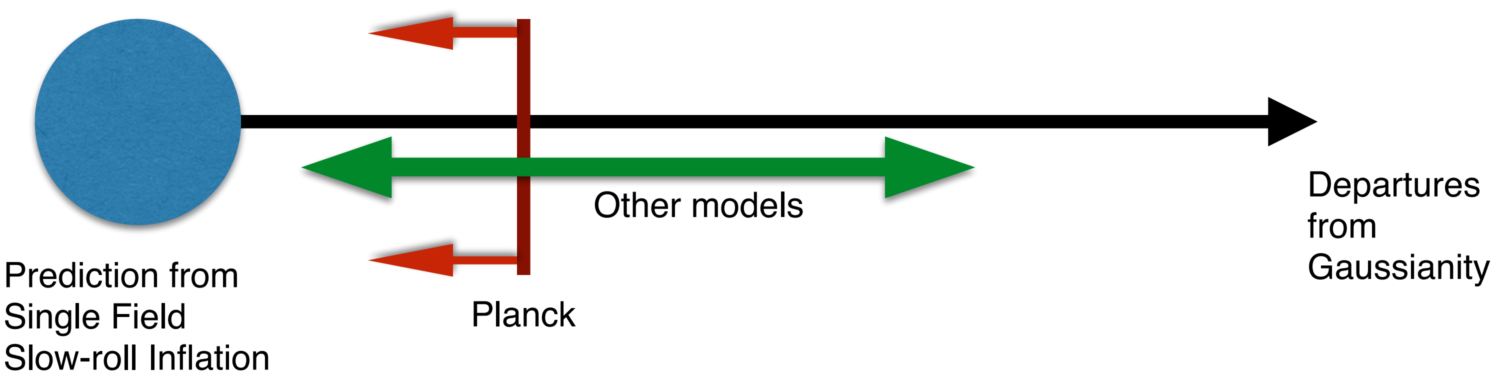
Planck 2013

	Independent KSW	ISW-lensing subtracted KSW
SMICA		
Local	9.8 ± 5.8	**2.7 ± 5.8**
Equilateral	-37 ± 75	**-42 ± 75**
Orthogonal	-46 ± 39	**-25 ± 39**

Target

$$f_{NL}^{local} \leq 1$$ Single field Inflation

$$f_{NL}^{eq} \leq 1$$ Slow-Roll single field Inflation



Summary

If ns -1 is proportional to 1/N there is a ``forbidden region" in the ns-r plane.

There are clear targets for non-G where even an upper limit becomes very informative.