

# THE HIPPARCOS CATALOGUE DOUBLE AND MULTIPLE SYSTEMS ANNEX

Lennart Lindegren

Lund Observatory, Box 43, SE-22100 Lund, Sweden

## ABSTRACT

The Hipparcos Catalogue provides astrometric results for 117 955 catalogue entries, including the position at epoch 1991.25, the parallax, and the proper motion components. For most of the entries these five parameters, representing the assumed uniform space motion of a single star, were sufficient to represent the astrometric measurements. Many of the objects are in reality double or multiple stars, and for 17 917 entries additional parameters were introduced to cope with the various manifestations of multiplicity. Details on the additional parameters are contained in the Double and Multiple Systems Annex, divided in five parts according to the type of solution: resolved systems with astrometric data on each component (13 211 entries with a total of 24 588 components), astrometric binaries with curved orbital motion but without a periodic solution (2622 entries), astrometric binaries with orbital solutions (235 entries), astrometric binaries revealed through the variability of one of the components ('variability-induced movers'; 288 entries), and objects where the single-star hypothesis was rejected based on the residuals although no specific physical model of the multiplicity could be determined ('stochastic' solutions; 1561 entries). More than a third of the entries in the Annex were not previously known to be non-single. The construction of the Annex and the characteristics of the different types of solution are described.

Key words: Hipparcos; space astrometry; double stars; multiple stars.

## 1. INTRODUCTION

The presence of double and multiple stars in the Hipparcos Catalogue presented a considerable challenge for the data reductions and for preparing the published data. On one hand, the Hipparcos instrument was not ideally suited for the observation of resolved objects, and their treatment severely complicated the reductions. On the other hand, these systems are by themselves highly interesting objects and it was desirable to obtain precise and reliable parallaxes for them in spite of the observational difficulties. It was also realised early on that Hipparcos would be able to detect many new double and multiple stars, a capability that should of course be maximally exploited.

While most stars are in reality members of double or multiple systems, only a minority of the systems are manifestly non-single as seen by Hipparcos. Thus, 85 per cent of the entries in the Hipparcos Catalogue could in practice be treated as single stars. The remaining 15 per cent, or 17 917 entries out of a total of 117 955 entries with associated astrometry, required special solutions to cope with the various manifestations of multiplicity. The main part of the Hipparcos Catalogue (Perryman 1997) gives the five astrometric parameters—position, parallax and proper motion—for *all* the entries, while additional information on the non-single entries are found in the Double and Multiple Systems Annex. The main catalogue as well as the Annex, in both printed and machine-readable forms, are contained in the 17-volume publication, The Hipparcos and Tycho Catalogues (ESA 1997). Full documentation of the contents of the Annex is given in Volume 1, and details on the construction of the data in Volume 3.

A major difficulty in constructing the Double and Multiple Systems Annex was that no unique model exists for the non-single objects. A range of models using different sets of astrometric and photometric parameters had to be considered, and the choice between them was far from trivial. Given the limited time available for this process, a somewhat schematic classification had to be adopted which in some cases does not give full justice to the observations. Furthermore, the combination of Hipparcos observations with other (existing or future) observations will in many cases allow more detailed or accurate solutions. Provisions have been made to preserve intermediate data which permit at least a partial re-analysis of the double and multiple star data (Section 5).

## 2. THE STANDARD MODEL OF STELLAR MOTION

The 85 per cent entries in the Hipparcos Catalogue treated as effectively single stars were reduced according to the 'standard model of stellar motion'. The basic assumption is that the objects are point sources moving with constant space velocity relative to the solar system barycentre. In this model the apparent motion of a star on the sky is fully described by the standard five astrometric parameters:  $(\alpha, \delta)$  for the barycentric direction at the catalogue epoch J1991.25; the parallax  $\pi$  representing barycentric distance; and the proper motion compo-

nents  $\mu_{\alpha*} = \mu_{\alpha} \cos \delta$  and  $\mu_{\delta}$  representing the rate of change of the barycentric direction. (Only for a handful of nearby high-velocity stars is perspective acceleration significant over the three-year observation interval, making it necessary to assume a non-zero radial velocity in the model.)

In addition to the truly single stars, this model is a good approximation for a number of limiting cases of double or multiple systems: very close binaries (separation  $\varrho \lesssim 2$  mas), where the photocentre practically coincides with the centre of mass; very wide doubles ( $\varrho \gtrsim 30$  arcsec), where the components could be observed and reduced separately as single stars; systems with large magnitude difference ( $\Delta Hp \gtrsim 4$  mag) and astrometric binaries with relatively long periods ( $P \gtrsim 30$  yr), where the motion of the primary or photocentre over the three years of observation was nearly uniform.

An important feature of the standard model is that the astrometric and photometric reductions of these objects could be completely decoupled, permitting a considerable simplification of the logistics of the reductions and of the presentation of the results. This is so because the intensity of the point source simply enters as a scaling factor to the detector signal. In contrast, with two or more components contributing to the detector signal, the phase information (astrometry) of the components is always intertwined with the amplitude information (photometry).

### 3. MANIFESTATIONS OF MULTIPLICITY

The observably non-single objects can be broadly divided into two categories: resolved systems, for which the detector signal could be decomposed into the contributions from distinct components; and unresolved systems, or astrometric binaries, for which only the photocentres could be observed.

#### 3.1. Resolved Systems

The Rayleigh criterion for the angular resolution of the Hipparcos telescope (aperture diameter 0.29 m) is about 0.45 arcsec. Actually, double stars of moderate brightness ratio ( $\Delta Hp \lesssim 2$  mag) gave a measurable broadening of the diffraction peak already at a separation of 0.10–0.13 arcsec. A major reason for this surprisingly high resolution was that the instrument response to a ‘single’ star could be calibrated very accurately, making even small deviations noticeable. One of the most sensitive criteria for detecting such systems was based on the difference between the two magnitude scales of the main detector:  $Hp_{dc}$  calculated from the non-modulated (‘dc’) component of the detector signal; and  $Hp_{ac}$  calculated from the modulated (‘ac’) detector signal. With proper calibration, using unresolved stars, the difference  $Hp_{ac} - Hp_{dc}$  was simply a logarithmic measure of the visibility of the object at the spatial frequency of the modulating grid. A separation of 0.1 arcsec in an equal-magnitude pair could already reduce the visibility by about 3 per cent, causing differences of up to 0.03 mag between the ‘ac’ and ‘dc’ magnitudes

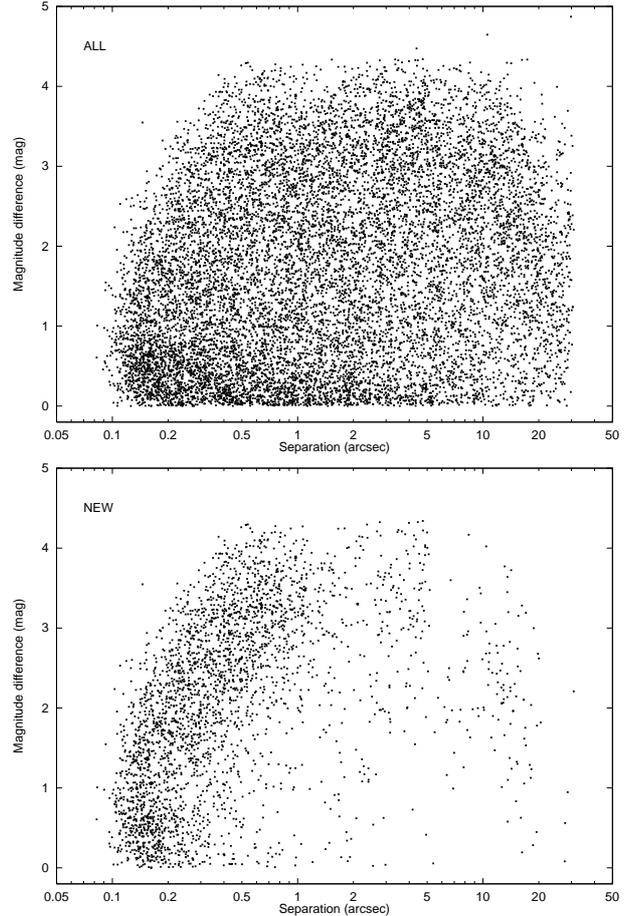


Figure 1. Distribution of separation and magnitude difference for 12 393 component pairs in Part C of the Double and Multiple Systems Annex (top), and for the subset of 2996 pairs discovered with Hipparcos (bottom).

of the object. For small separations the effect is proportional to  $\varrho^2$ , making the detection limit relatively sharp.

Other criteria for detecting resolved systems were based on the asymmetry (or phase shifts) produced by the superposition of the component signals. Some 10 000 Hipparcos entries were known as double or multiple stars already before the mission, and could be used to verify the efficiency of these criteria.

Once a resolved system had been detected, a model consisting of two or more point-source components could be fitted to all the scans across the object. This fitting gave the (relative) intensities of the components as well as their positions, together with a parallax and proper motion usually assumed to be common for the system. Figure 1 shows the distribution of separation and magnitude differences for all resolved component pairs contained in the Double and Multiple Systems Annex. It appears that the Annex provides a fairly complete census of binaries among the Hipparcos stars with  $\Delta Hp < 3.5$  mag and  $\varrho > 0.12$  to 0.3 arcsec (depending on  $\Delta Hp$ ). The boundary at  $\varrho \simeq 30$  arcsec corresponds to the limit where the components could be treated as separate entries in the Hipparcos Catalogue. Most of

the newly discovered pairs (Figure 1, bottom) have separations below 1 arcsec.

### 3.2. Unresolved Systems (Astrometric Binaries)

Unresolved pairs, i.e. with separations below 0.1 to 0.3 arcsec (depending on  $\Delta Hp$ ), were observed as point sources located at the photocentre of the pair. From the shape of the detector signal, these were indistinguishable from single stars. However, orbital motion sometimes caused the path of the photocentre to deviate from the uniform motion expected for a single star. Such systems could then be detected as ‘astrometric binaries’. Similar results could be obtained for systems having a large magnitude difference, or an invisible companion, where the photocentre coincides with the visible component. Thus, stars with brown-dwarf or planetary companions potentially fall in this category.

The way in which an astrometric binary manifests itself in the Hipparcos data depends mainly on two parameters: the apparent size  $a_0$  of the absolute orbit of the photocentre, and the period  $P$  of the orbit in relation to the length of the observation interval (3.3 years).  $a_0$  depends both on the apparent size of the relative orbit and on the mass and intensity ratios: it can be noted that the detection of astrometric binaries is generally unfavourable for pairs with nearly equal magnitudes. However, even when  $a_0$  is significant, detection depends on the non-uniformity of the motion of the photocentre. Three cases can be distinguished (Figure 2):

A. If the observation interval is short compared with the period, then the motion of the photocentre will appear uniform and the standard model of stellar motion can be applied. There is no indication of multiplicity in the Hipparcos data; the object will not be included in the Double and Multiple Systems Annex, nor flagged as suspected double. Nevertheless, the derived proper motion may deviate significantly from the proper motion of the barycentre of the system, and hence from the mean proper motion of the photocentre over a much longer time span, say 50 years. This could result in a discrepancy between the proper motion as given by Hipparcos and by ground-based observations.

B. If the observation interval covers some fraction of the orbital period, the motion of the photocentre may be detectably non-uniform, but the coverage is not sufficient for a full orbit determination. In this case a polynomial representation of the motions in  $\alpha$  and  $\delta$ , as functions of time, may be used. Quadratic or cubic polynomials may describe the motion over the observation interval, but cannot be extrapolated to other epochs. The position and proper motion of such a system are not well-defined concepts (see Section 4.2).

C. If the observation interval is comparable to or longer than the orbital period, it may be possible to fit a complete set of orbital parameters to the photocentric motion. This orbit is superposed on the uniform motion of the barycentre, for which the standard model of stellar motion is applicable.

No definitive limits in  $P$  can be designated, corre-

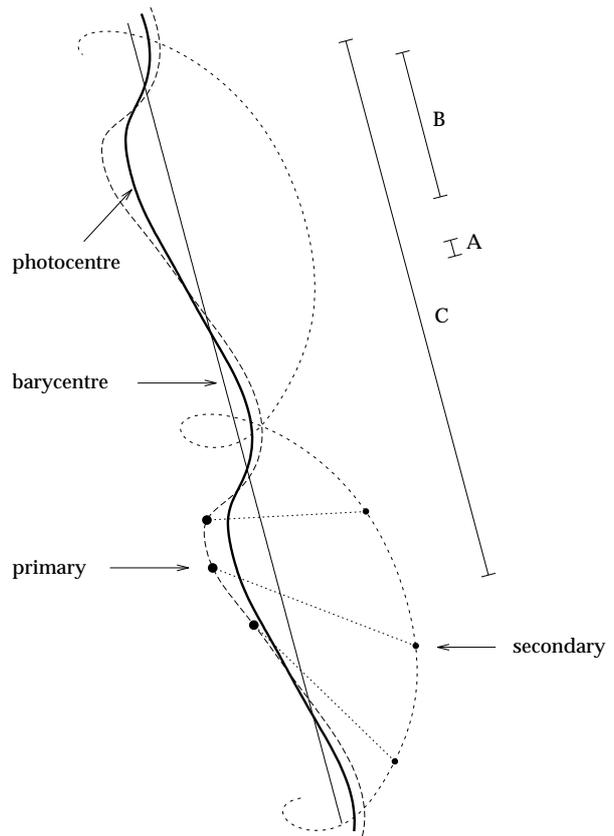


Figure 2. For close binaries, i.e. with separations less than 0.1 to 0.3 arcsec, the point observed by Hipparcos was the photocentre of the components. The orbital motions of the components about the barycentre of such a system generally caused the photocentre to trace a non-linear path on the sky, in the diagram represented by the thick curve. The type of solution permitted by the Hipparcos data depended mainly on the length of the observation interval—about 3.3 years—in comparison with the orbital period, as illustrated by the three cases A, B, C. See text for further explanation.

sponding to the different cases, as for instance the detectability of the non-uniform motion depends on many other factors as well (size and eccentricity of orbit, orbital phase, magnitude, etc).

The ‘variability-induced movers’ (Section 4.4) are a different kind of astrometric binary, where the duplicity is revealed through the photometric variability of one of the components rather than the orbital motion (Figure 3).

## 4. THE DOUBLE AND MULTIPLE SYSTEMS ANNEX

The astrometric solutions in the Hipparcos Catalogue were divided into the seven categories listed in Table 1. The first category contains the entries which were treated as single stars, i.e. according to the standard model of stellar motion (Section 2). The last

Table 1. Number of entries in the Hipparcos Catalogue for the different categories of astrometric solutions.

Type of solution	Annex	Entries
Single-star solutions	—	100 038 <sup>1</sup>
Component solutions	C	13 211 <sup>2</sup>
Acceleration solutions	G	2 622
Orbital solutions	O	235
Variability-induced movers	V	288
Stochastic solutions	X	1 561
No valid astrometric solution	—	263 <sup>3</sup>
Total number of entries		118 218
Entries with valid astrometry		117 955

<sup>1</sup> of which 6763 flagged as suspected double

<sup>2</sup> comprising 24 588 components in 12 195 solutions

<sup>3</sup> of which 218 flagged as suspected double

category consists of the entries without associated astrometry, e.g. because of no detectable signal, too few observations, or where the signal could not be adequately interpreted in terms of any of the previous models. The other five categories correspond to the five parts of the Double and Multiple Systems Annex designated C, G, O, V and X, described hereafter.

It should be noted that the Hipparcos Catalogue gives (in the main catalogue) the five astrometric parameters ( $\alpha, \delta, \pi, \mu_{\alpha*}, \mu_{\delta}$ ) for *all* the 117 955 entries with valid astrometry. However, the precise meaning of these data may vary according to the type of solution: the position and proper motion may refer to a specific component of a resolved system, to the photocentre of a resolved or non-resolved system, or to the centre of mass in case of an orbital system. Similarly the photometry may refer to a single component or the combined intensity of two or more components. The relevant flags in the catalogue should be consulted (Fields H10 and H48).

#### 4.1. Part C: Component Solutions

This part of the Double and Multiple Systems Annex gives the five astrometric parameters and a magnitude ( $H_p$ ) for a total of 24 588 resolved components in 12 176 separate systems. The systems are identified by their identifiers in the CCDM (Catalogue of Components of Double and Multiple Stars; Dommange & Nys 1994), which was updated to include 2996 double stars discovered by Hipparcos; the components in a system are identified by letters, usually ‘A’, ‘B’, ... for the primary, secondary, etc. There are 11 969 systems with two resolved components, 181 with three, 24 with four, and one each with five and six components.

While a ‘system’ is the natural entity to consider for multiplicity statistics, a more relevant unit from the data analysis viewpoint was the ‘solution’, comprising those components whose mutual influence had to be taken into account in the analysis of the detector signal. 18 of the systems with four or more components were in fact treated as 37 independent solutions with two or three components each. Thus, the

total number of solutions is 12 195, of which 12 005 with two, 182 with three, and eight with four resolved components. It should be noted that there are also a number of physical triple stars consisting of one ‘single’ component (given in the main catalogue only) and one ‘double’ (in the Annex).

Since Part C gives the five astrometric parameters (plus a magnitude) for each component, partially the same format could be used as in the main catalogue. However, this format does not permit solutions with curved orbital motion: only linear relative motion among the components of a system can be represented by their different proper motions. Most solutions were furthermore constrained to give the same parallax and proper motion for all components (10 895 solutions of type ‘F’ = fixed), or the same parallax but allowing different proper motions (1186 solutions of type ‘L’ = linear); only in 202 solutions were no such constraints introduced (type ‘I’ = individual, i.e. a possible optical pair).

The published component solutions were obtained by combining the results of the independent analyses performed by the reduction consortia FAST and NDAC. For nearly 90 per cent of the solutions the agreement between the two reductions was sufficiently good to permit a straightforward averaging of the results, while the differences were used to calibrate the estimated standard errors. In 1113 cases the two reductions could not be reconciled, as they differed typically by a multiple of the fundamental period of the modulating grid ( $\simeq 1.2$  arcsec), and one of them had to be selected on the basis of various criteria. Such cases are flagged in the catalogue and the main parameters of the alternative solution are given in the Notes to the Annex.

The positions and proper motions for the components are given in the same reference frame as used for the rest of the Catalogue, namely the International Celestial Reference System (ICRS); see Kovalevsky (1997).

#### 4.2. Part G: Acceleration Solutions

Part G of the Double and Multiple Systems Annex gives the non-linear polynomial terms of the solutions corresponding to the astrometric binaries discussed in Section 3.2 as case B (see Figure 2). A quadratic or cubic polynomial of time was fitted to the motion in each coordinate, in addition to the standard five astrometric parameters. In tangential coordinates ( $\xi \sim \Delta\alpha \cos \delta$ ,  $\eta \sim \Delta\delta$ ) the apparent motion—excluding parallax—was modelled as:

$$\begin{aligned} \xi(t) &= \xi_0 + t\mu_{\alpha*} + \frac{1}{2}(t^2 - a)g_{\alpha*} + \frac{1}{6}(t^2 - b)t\dot{g}_{\alpha*} \\ \eta(t) &= \eta_0 + t\mu_{\delta} + \frac{1}{2}(t^2 - a)g_{\delta} + \frac{1}{6}(t^2 - b)t\dot{g}_{\delta} \end{aligned} \quad (1)$$

where  $t$  is the time in years from J1991.25, ( $g_{\alpha*}, g_{\delta}$ ) are the accelerations in  $\text{mas yr}^{-2}$ , and ( $\dot{g}_{\alpha*}, \dot{g}_{\delta}$ ) the rates of change of the accelerations in  $\text{mas yr}^{-3}$ . The constants  $a = 0.81 \text{ yr}^2$  and  $b = 1.69 \text{ yr}^2$  were chosen to make the  $g$  and  $\dot{g}$  terms approximately orthogonal to the preceding terms in Equation 1.

Quadratic or cubic solutions were accepted on the basis of the statistical significance of the  $g$  and  $\dot{g}$  terms.

2163 quadratic and 459 cubic solutions are included in this part of the Annex. The polynomial representation of the motion has no validity outside the mission interval 1989.9–1993.2. The position and proper motion data given in the main catalogue correspond to  $(\xi_0, \eta_0)$  (given as  $\alpha, \delta$ ) and  $(\mu_{\alpha*}, \mu_{\delta})$  in Equation 1. The proper motion components represent the mean motion over the mission interval, and are therefore much safer to extrapolate to other epochs than the non-linear polynomial. However, the positional uncertainty may be considerably greater than computed from the standard errors in proper motion.

Only very few ( $\sim 5$ ) of the entries in Part G were known as astrometric or orbital binaries before Hipparcos.

#### 4.3. Part O: Orbital Solutions

This part of the Double and Multiple Systems Annex contains orbital solutions for the photocentres of 235 astrometric binaries. Seven elements ( $P, T, a_0, e, \omega, i, \Omega$ ) are given for each system in addition to the five astrometric parameters for the centre of mass. Excluding the parallax, the apparent motion in tangential coordinates was therefore expressed as:

$$\begin{aligned}\xi(t) &= \xi_0 + t\mu_{\alpha*} + BX(t) + GY(t) \\ \eta(t) &= \eta_0 + t\mu_{\delta} + AX(t) + FY(t)\end{aligned}\quad (2)$$

where  $A, B, F, G$  are the Thiele-Innes elements (a transformation of  $a_0, \omega, i, \Omega$ ) and  $X(t), Y(t)$  the time-dependent coordinates in the orbital plane (depending on  $P, t - T$  and  $e$ ).

In many cases some of the elements, in particular the period, were adopted from ground-based observations of the astrometric or spectroscopic binaries, or taken as starting values for the solution. However, in nearly half of the cases the period was identified from periodogram analysis of the space data performed at the Astronomisches Rechen-Institut, Heidelberg, and refined elements subsequently determined by least-squares fitting to the Hipparcos intermediate astrometric data (Section 5). For 45 systems all seven orbital elements were determined from the Hipparcos observations.

#### 4.4. Part V: Variability-Induced Movers

Variability-induced movers, or VIMs, are unresolved binaries in which one of the components is variable. As illustrated in Figure 3, the photocentre of a VIM shows a specific motion on the sky, coupled to the variation of the total brightness of the system (Wieken 1996). Given the total magnitude determined by the satellite for each scan across the system, a VIM solution requires two elements ( $D_{\alpha*}, D_{\delta}$ ) in addition to the five astrometric parameters. Excluding the parallax, the apparent motion in tangential coordinates was expressed as:

$$\begin{aligned}\xi(t) &= \xi_0 + t\mu_{\alpha*} + f(t)D_{\alpha*} \\ \eta(t) &= \eta_0 + t\mu_{\delta} + f(t)D_{\delta}\end{aligned}\quad (3)$$

where  $f(t) = 10^{0.4[Hp(t) - Hp_{ref}]} - 1$  is a function of the total magnitude of the binary, and where  $Hp_{ref}$

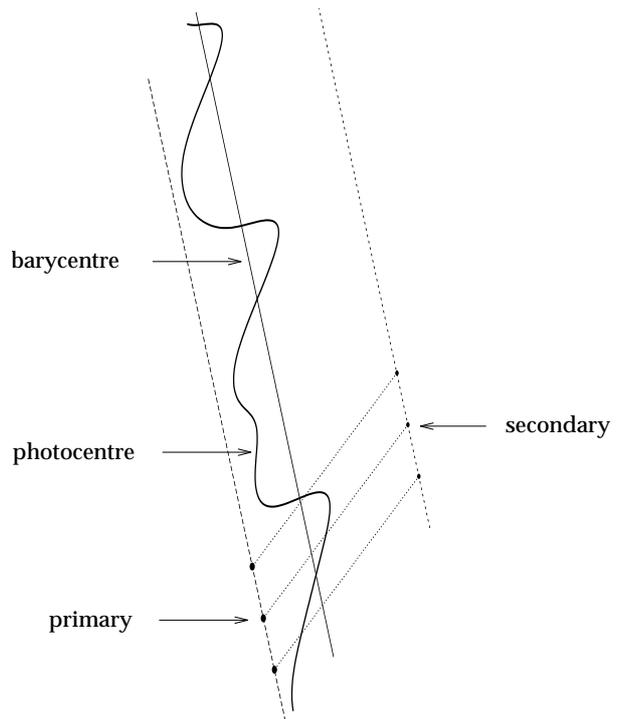


Figure 3. For an unresolved system in which one of the components is photometrically variable, the photocentre (thick curve) moves back and forth between the components in phase with the variations of the total intensity of the system. Such a system can be detected as an astrometric binary even if there is no significant orbital motion during the interval of observation: it is a ‘variability-induced mover’, or VIM.

is an adopted reference magnitude. The position and proper motion given in the main catalogue refer to the photocentre for a total magnitude equal to  $Hp_{ref}$ .

From the VIM elements ( $D_{\alpha*}, D_{\delta}$ ) one can derive the position angle of the constant component with respect to the variable component, and a lower limit for the separation. The latter is typically in the range 10 to 90 mas. The VIM solutions were derived at the Astronomisches Rechen-Institut, Heidelberg, by a critical examination of the variable stars, using FAST intermediate astrometry.

#### 4.5. Part X: Stochastic Solutions

For some objects it was not possible to find an acceptable single or double star solution in reasonable agreement with the statistical uncertainties of the individual measurements. Such objects could be short-period unresolved binaries, or resolved objects where the secondary could not be located or was perturbed by variability or edge effects of the instantaneous field of view. Lacking an acceptable deterministic model for these objects, a reasonable alternative is to adopt a stochastic model for the displacements relative to the centre of mass. The astrometric parameters were derived exactly as for the single stars, i.e. from the one-dimensional positions (abscissae)

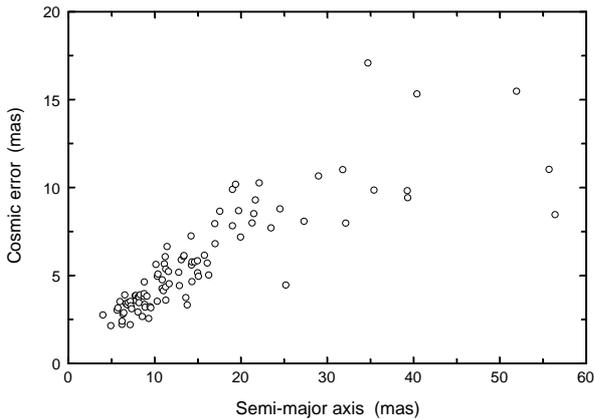


Figure 4. Cosmic error ( $\epsilon$ ) versus the semi-major axis ( $a_0$ ) of the barycentric orbit of the photocentre, for 95 systems published as orbital astrometric binaries in the Hipparcos Catalogue (Annex Part O). Stochastic solutions were also calculated for these systems in order to show that the cosmic error gives some indication of  $a_0$  for an unrecognised orbital astrometric binary.

according to the standard model of stellar motion. However, the *a priori* standard errors of the abscissae were increased according to the formula:

$$\sigma_{\text{adopted}}^2 = \sigma_{\text{a priori}}^2 + \epsilon^2 \quad (4)$$

where  $\epsilon$  is the ‘cosmic error’ of the solution.  $\epsilon$  was chosen to give a unit weight residual exactly equal to one (the sum of the squares of the normalised residuals equal to the number of degrees of freedom). In the accepted stochastic solutions the cosmic error typically ranges from 3 to 30 mas. Solutions with a cosmic error greater than 100 mas were rejected as this was normally an indication of grid-step errors, or of an insignificant stellar signal.

The stars which received a stochastic solution may be astrometric binaries for which no acceleration or orbital solution could be found, e.g. because the period could not be identified. In such cases  $\epsilon$  should give an order-of-magnitude estimate of the size of the orbit of the photocentre. To demonstrate this, stochastic solutions were also computed for a number of the orbital binaries in Part O of the Double and Multiple Systems Annex. Figure 4 is a comparison of the cosmic error for these solutions with the semi-major axis ( $a_0$ ) of the photocentre orbit in the corresponding orbital solution. The relatively good correlation for small  $a_0$  indicates that  $2.4\epsilon$  could be taken as a rough estimate of  $a_0$  for such systems.

## 5. ADDITIONAL DATA

In addition to the entries included in the Double and Multiple Systems Annex, 6763 entries solved as single stars and 218 entries without an astrometric solution were flagged as suspected double stars. This means that (some of) the detection criteria discussed in Section 3.1 indicated that the entry was non-single, although no convincing component solution was found.

The Notes to the Double and Multiple Systems Annex contain additional information on alternative interpretations of the data, and in some cases probable corrections derived after the completion of the Annex. The general notes and photometric notes also provide valuable information pertaining to double and multiple systems.

In view of the many possible alternative object models that might apply to the non-single stars, and the limitations set by the adopted format of the Double and Multiple Systems Annex, two major intermediate data sets have been included on the CD-ROM version of the Hipparcos Catalogue. The first is the ‘Hipparcos Intermediate Astrometry’, consisting of the one-dimensional positions (or abscissae) for all Hipparcos entries, that were used to derive the astrometric parameters of all single stars and for the entries in Parts G, O, V and X of the Annex. The second is the ‘Hipparcos Transit Data’, summarising the detector signals for almost a third of the entries, including all confirmed or suspected non-single stars. The transit data can be used to derive component solutions for all resolved systems, if necessary using different or more general models than in the Annex (e.g. including curved orbital motion). Examples of the use of the intermediate data are given in several contributions to this symposium (see contributions by Bernstein; van Leeuwen; van Leeuwen & Hansen Ruiz; Quist et al.; Söderhjelm et al.; Söderhjelm & Lindegren).

## ACKNOWLEDGMENTS

The Hipparcos Catalogue Double and Multiple Systems Annex was produced through the collaboration of several groups within the INCA, FAST, NDAC and TDAC Consortia. This presentation was made on behalf of the many individuals who over the years participated in this difficult process.

## REFERENCES

- Dommanget, J., Nys, O., 1994, Comm. Obs. R. Belg., Ser. A, No. 115 (with updates in Bull. CDS No. 46 and 48)
- ESA, 1997, The Hipparcos and Tycho Catalogues, ESA SP-1200
- Kovalevsky, J., 1997, Connection of the Hipparcos Catalogue to the Extragalactic Reference Frame, in these proceedings
- Perryman, M.A.C., 1997, The Hipparcos Catalogue Contents, in these proceedings
- Wielen, R., 1996, A&A, 314, 679