# A reliable methodology to determine fireball terminal heights 

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## Introduction

Previous studies [1, 2] have proved the importance of terminal heights in order to properly characterize fireball flight properties and their degree of atmospheric penetration. However, despite ablation and drag processes associated with atmospheric entry of meteoroids have been widely studied, little attention was devoted to interpret the measured fireball terminal height. This key parameter also provides clues on the bulk physical properties of the meteoroids and, combined with the entry velocity and slope, we can state the deceleration experienced by the meteoroid during the atmospheric flight and thus, the ablation and dynamic pressure held.

## Mathematical Formulae and Results

The classical way of resolving the equations of motion requires knowing beforehand a set of properties and physical variables that we cannot obtain accurately enough with ground-based observations. Alternatively, by using scale laws and dimensionless variables another approach can be introduced. The unknown values of the meteoroid's atmosphere flight motion equations are gathered into two new variables $\alpha$ (ballistic coefficient) and $\beta$ (mass loss parameter), e.g. [3, 4]. The analytical solution of these equations (using dimensionless variables) leads to:

$$
\begin{gathered}
m=\exp \left[-\left(1-v^{2}\right) \beta /(1-\mu)\right] \\
y=\ln 2 \alpha+\beta-\ln \Delta, \quad \Delta=\bar{E} i(\beta)-\bar{E} i\left(\beta v^{2}\right) \\
\bar{E} i(x)=\int_{-\infty}^{x} \frac{e^{t} d t}{t}
\end{gathered}
$$

Where,

$$
\alpha=\frac{1}{2} c_{d} \frac{\rho_{0} h_{0} S_{e}}{M_{e} \sin \gamma} ; \quad \beta=(1-\mu) \frac{c_{h} V_{e}^{2}}{2 c_{d} H^{*}}
$$

Several simplifications of these equations are admitted and studied [5]. We prove that using the results recently obtained in [6] and, suggesting a shift along parameter $\beta$, leads to a higher agreement between observed and calculated terminal height values:

$$
h_{\text {III }}=y_{t} \cdot h_{0}=h_{0} \cdot \ln \left(\frac{2 \alpha(\beta-1.1)}{\left(1-e^{(\beta-1.1)\left(v^{2}-1\right)}\right)}\right)
$$

Results are shown in Fig.1. Despite the small spread in results ( $\sigma=0.75 \mathrm{Km}$ ), there is still a small deviation at low heights. This region usually indicates low $\beta$ values and could be linked to the simplifications assumed in the
calculations [5].


Fig. 1 Observed (MORP) vs. calculated terminal heights. A line representing equal values has been plotted.

## Conclusions

The used parameterization allows describing in detail the meteoroid trajectory in the atmosphere. Besides, new classifications can be set using these parameters which may not be biased by previous assumptions on meteoroid's properties. Furthermore, this methodology can speed up the study of the still large amount of archived data available and of the many new meteor registrations to come.
Finally, a better knowledge of these parameters could get clues on the ability of meteoroids to penetrate into the terrestrial atmosphere and to quantify their capability of being a source of impact hazard for life on Earth.

## References

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