Report: Gyro-based Attitude Reconstruction Software

Craig Stephenson

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Overview of method

To estimate the spacecraft attitude $\mathbf{q}_{\text{aca}}(t)$ at time $t = t'$:

- Select a reference attitude $\mathbf{q}_0(t')$ close to $\mathbf{q}_{\text{aca}}(t')$ and consider the rotation $\mathbf{q}_r(t)$ between $\mathbf{q}_0(t')$ and $\mathbf{q}_{\text{aca}}(t)$ over interval $t \in [t' - T, t' + T]$.
- Express $\mathbf{q}_r(t)$ in terms of the small-angles rotations $\theta_x(t)$, $\theta_y(t)$ and $\theta_z(t)$ about the spacecraft axes.
- Assume the linear model:

$$\begin{aligned} \theta_x(t) &= \psi_x(t) + b_x\, t + c_x, \\ \theta_y(t) &= \psi_y(t) + b_y\, t + c_y, \qquad -T \le t - t' \le T \\ \theta_z(t) &= \psi_z(t) + b_z\, t + c_z, \end{aligned} \tag{1}$$

where $\psi_x(t)$, $\psi_y(t)$ and $\psi_z(t)$ are angles related to gyro output.

Requires: θ_x, θ_y, θ_z small and gyro drift rates $\approx$ constant.

Overview of method (cont.)

- Construct measurements: $\theta_x^{(i)} - \psi_x^{(i)}$, $\theta_y^{(i)} - \psi_y^{(i)}$ and $\theta_z^{(i)} - \psi_z^{(i)}$
  - $\theta_x^{(i)}$, $\theta_y^{(i)}$, $\theta_z^{(i)}$, $(i = 1, \ldots, n_{\mathrm{s}})$ from star tracker att. estimates $\widehat{\mathbf{q}}^{\mathrm{s}}_{\mathrm{aca}}(t_{\mathrm{s}}^{(i)})$
  - $\psi_x^{(i)}$, $\psi_y^{(i)}$, $\psi_z^{(i)}$, $(i = 1, \ldots, n_{\mathrm{g}} \approx 4\, n_{\mathrm{s}})$ from gyro output
- Estimate b_x, b_y, b_z, c_x, c_y, c_z using linear least-squares.
- Then

$$\widehat{\mathbf{q}}_r(t') = \left[\widehat{\theta}_x(t')/2 \quad \widehat{\theta}_y(t')/2 \quad \widehat{\theta}_z(t')/2 \quad 1\right]^{\mathsf{T}}, \tag{2}$$

where

$$\begin{aligned} \widehat{\theta}_x(t') &= \psi_x(t') + \widehat{b}_x\, t' + \widehat{c}_x, \\ \widehat{\theta}_y(t') &= \psi_y(t') + \widehat{b}_y\, t' + \widehat{c}_y, \\ \widehat{\theta}_z(t') &= \psi_z(t') + \widehat{b}_z\, t' + \widehat{c}_z. \end{aligned} \tag{3}$$

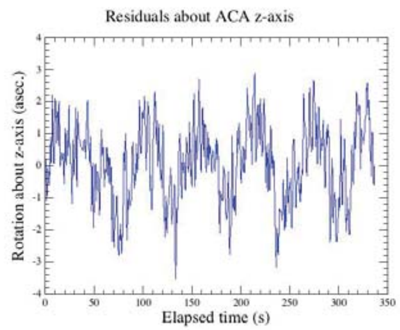
- And finally:

$$\widehat{\mathbf{q}}_{\mathrm{aca}}(t') = \mathbf{q}_0(t')\, \widehat{\mathbf{q}}_r(t'). \tag{4}$$

Main problems (1 of 3)

1. A fixed reference attitude $\mathbf{q}_0$ is currently used for each observation [HCSS-18779]. For large scan maps, assumption that θ_x, θ_y, θ_z are small is violated and large errors will result.
   (e.g. $\sim 5''$ error in approximating $\sin\theta$ by θ when $\theta = 3^\circ$.)

2. Large oscillations have been observed in the central residual from the least-squares fit [HCSS-18766]. E.g. $\theta_z - \widehat{\theta}_z$ for a scan map of Arcturus (1342262519). (Here, rotations from $\mathbf{q}_0 < 4'$.)



Main problems (2 of 3)

3. To detect and investigate such problems we need a quality indicator [HCSS-18840]. For example, look at the minimized cost functions:

$$\chi^2_{x,\mathrm{min}} = \sum_{i=1}^{n_\mathrm{s}} \frac{[\theta_x^{(i)} - \psi_x^{(i)} - \widehat{b}_x\, t_\mathrm{s}^{(i)} - \widehat{c}_x]^2}{P_{\theta\theta}^{(i)}[1,1]},^{1} \tag{5}$$

$\chi^2_{y,\mathrm{min}} = \ldots, \chi^2_{z,\mathrm{min}} = \ldots$

- Need to calculate Cartesian attitude covariance matrix $P_{\theta\theta}^{(i)}$ of star tracker attitude measurements (included in [HCSS-19291]).
- Need to interpolate $\psi_x^{(i)}$, $\psi_y^{(i)}$, $\psi_z^{(i)}$ to times of $\theta_x^{(i)}$, $\theta_y^{(i)}$, $\theta_z^{(i)}$, not the other way around (as currently performed) [HCSS-18777]. Otherwise, the noise on the combined measurements will be highly correlated and the goodness-of-fit test invalid.

[1] $t_\mathrm{s}^{(i)}$ is the time of measurement i.

Main problems (3 of 3)

4. The software outputs no information on accuracy of new attitude estimates [HCSS-19291].
   - Cartesian attitude covariance matrix $P^{(i)}_{\theta\theta}$ of star tracker attitude measurements may be calculated using equations found in [Shuster and Oh, 1981].[2]
   - Covariance of fitted parameters in (5) is inverse of $D^\mathsf{T} D$, where D is design matrix for least-squares problem.
   - And then: $\mathrm{E}[\tilde{\theta}_x(t')^2] = t'^2\,\mathrm{E}(\tilde{b}_x^2) + 2\,t'\,\mathrm{E}(\tilde{b}_x\tilde{c}_x) + \mathrm{E}(\tilde{c}_x^2)$ etc.
5. When no STR-specific diagnostics data available, the software derives $\theta^{(i)}_x$, $\theta^{(i)}_y$, $\theta^{(i)}_z$ using attitude estimates from on-board filter [HCSS-19234]. This is completely incorrect.
6. Each old attitude estimate (from on-board filter) is simply overwritten by new attitude estimate with closest matching time stamp. The software does not check by how much these times differ [HCSS-19201].

[2] M. D. Shuster and S. D. Oh. Three-Axis Attitude Determination from Vector Observations. *Journal of Guidance and Control*, 4(1):70–77, Jan–Feb 1981.

Future work

Gyro-based attitude reconstruction method **assumes**:

- s/c attitude and gyro drift rates $\approx$ constant over $\sim$ 400 s intervals.[3]
  - In line scans (rate $\leq 60''/\text{s}$) attitude may change by 6.7° in 400 s.
  - Drift rates believed $\approx$ constant over interval [Tuttlebee, 2013, p. 68].[4]

To overcome these limitations:

- Develop a conventional (batch or recursive) attitude estimator which makes use of improved star tracker measurements from `calcStrAttitude`.
- Proposed implementation: smooth data by combining output from a forward and a backward filter—possibly use on-board filter with retuned gains [see Tuttlebee, 2013]. Would need to find gains suitable for non-staring modes.

In addition:

- Improve residual distortion maps by using best estimate of attitude (iteratively) to calculate expected star coordinates [`HCSS-19122`].

[3] Current implementation requires s/c attitude $\approx$ constant over entire observation.

[4] M. Tuttlebee. PT-CMOC-OPS-RP-6435-HSO-GF, Aug 2013.

Future work (cont.)

In addition:

- Complete implementation of gyro-based method.
- Improve residual distortion maps by using best estimate of attitude (iteratively) to calculate expected star coordinates [HCSS-19122].

Bonus slides: TN on the APE_{68} measurement (estimation)

Definition of APE_{68}:

1. Inconsistency between definition (req.) and how measured
    - Two components (req.) vs. one component (measured).
    - Percentile w.r.t. time (requirements) vs. w.r.t. observations (measured). [Single APE / observation $\Longrightarrow$ jitter excluded.]

APE_{68} estimated from sample of observations (by a 2-stage process)

2. Sample unlikely to be representative.

Stage 1: Assume offsets distributions normal, $\Delta Y \sim N(\mu_y, \sigma_y^2)$, $\Delta Z \sim N(\mu_z, \sigma_z^2)$, and estimate μ_y, μ_z, σ_y^2, σ_z^2.

3. Assumption of normality is unrealistic.
4. Estimates $\widehat{\mu}_y$, $\widehat{\mu}_z$, $\widehat{\sigma}_y^2$, $\widehat{\sigma}_z^2$ are subject to large uncertainties.

Bonus slides (cont.)

Stage 2: Calculate $\widehat{\text{APE}}_{68} \equiv \text{APE}_{68}(\widehat{\mu}_y, \widehat{\mu}_z, \widehat{\sigma}^2_y, \widehat{\sigma}^2_z)$. But $\widehat{\text{APE}}_{68}$ has been approximated by $\text{APE}^{\dagger} = \sqrt{\widehat{\sigma}^2_y + \widehat{\sigma}^2_z}$.

5. Mean offsets $\widehat{\mu}_y$, $\widehat{\mu}_z$, have been ignored.
6. $\text{APE}^{\dagger} \neq \text{APE}_{68}(0, 0, \widehat{\sigma}^2_y, \widehat{\sigma}^2_z)$.

In TN `HERSCHEL-HSC-TN-2059`:

- Estimate uncertainties in $\widehat{\mu}_y$, $\widehat{\mu}_z$, $\widehat{\sigma}^2_y$, $\widehat{\sigma}^2_z$ and look at effect on $\widehat{\text{APE}}_{68}$:
  - Errors in $\widehat{\text{APE}}_{68}$ of $0.2 - 0.3''$ seem likely.
- Show how to calculate $\widehat{\text{APE}}_{68}$ with mean offsets included and avoiding approximations:
  - Including mean offsets changes $\widehat{\text{APE}}_{68}$ by $\sim 0.3''$ (period 3) (occasionally far worse)
  - Use of $\text{APE}^{\dagger}$ introduces further error of $\sim 0.1''$.
- 'Suggest' alternative way of measuring APE_{68} using reconstructed attitude.