SPIRE FTS: converting integrated line fluxes to alternative units

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1 How spectral lines are measured in FTS spectra

Some useful concepts, taken out of the SPIRE Handbook (2018) and the SPIRE Data Reduction Guide (2016). For more details please read both documents.

- The SPIRE FTS instrumental line profile is a sinc function:

$$f(\nu, p) = p_0 \, \sin(x)/x; \;\; x = (\nu - p_1)/p_2 \tag{1}$$

- This definition assumes the continuum is already subtracted and that the spectra are already in LSR frame (local standard of rest).

- Here p_0 is the amplitude, p_1 is the line centroid, p_2 is the characteristic line width (the sinc width), which is the distance from the peak to the first zero crossing.

- For unresolved lines $p_2 = \Delta\nu/\pi$, where $\Delta\nu$ is the spectral resolution. For Fourier-Transform spectrometers $\Delta\nu$ is dictated by the maximum optical path difference created by the scan mirror travel. Thus, $\Delta\nu(\text{GHz}) = c \times 10^{-7}/(2\,L_{\text{max}})$, where L_{max} is the maximum optical path difference in [cm]. For the SPIRE FTS $\Delta\nu = 1.2$ GHz for high resolution spectra. Most lines (except some lines in the highest frequency range of FTS) are expected unresolved and p_2 is usually kept fixed for the line fit.

- The FWHM of a sinc line with a width p_2 is $\text{FWHM} = 1.20671 \times p_2$.
- The analytical integral of a sinc function (Eq. 1) is

$$F_{\text{line}} = \int_{-\infty}^{+\infty} f(\nu, p)\mathrm{d}\nu = \int_{-\infty}^{+\infty} p_0 \frac{\sin\left[(\nu - p_1)/p_2\right]}{(\nu - p_1)/p_2}\,\mathrm{d}\nu = \pi\, p_0\, p_2. \quad (2)$$

  For unresolved lines this simplifies to $F_{\text{line}} = 1.2 \times p_0$.

1.1 Lines in point-source calibrated spectra

Point-source calibrated spectra S_ν are in units of Jy, so the amplitude p_0 is in Jy too. The integrated line flux is therefore:

$$F_{\text{line}} = \int_{-\infty}^{+\infty} S_\nu \mathrm{d}\nu = \pi\, p_0\, p_2 \;[\mathrm{Jy\,GHz}] \quad (3)$$

or in SI units

$$F_{\text{line}} = \int_{-\infty}^{+\infty} S_\nu \mathrm{d}\nu = \pi \times 10^{-17} \left(\frac{p_0}{\mathrm{Jy}}\right)\left(\frac{p_2}{\mathrm{GHz}}\right)\;[\mathrm{W\,m^{-2}}] \quad (4)$$

and in *cgs* units

$$F_{\text{line}} = \pi \times 10^{-14} \left(\frac{p_0}{\mathrm{Jy}}\right)\left(\frac{p_2}{\mathrm{GHz}}\right)\;[\mathrm{erg\,s^{-1}\,cm^{-2}}], \quad (5)$$

because $1\;[\mathrm{W\,m^{-2}}] = 10^3\;[\mathrm{erg\,s^{-1}\,cm^{-2}}]$.

For unresolved lines (i.e. $p_2 = 1.2/\pi$) this simplifies to:

$$F_{\text{line}} = 1.2 \times 10^{-17} \left(\frac{p_0}{\mathrm{Jy}}\right)\;[\mathrm{W\,m^{-2}}] = 1.2 \times 10^{-14} \left(\frac{p_0}{\mathrm{Jy}}\right)\;[\mathrm{erg\,s^{-1}\,cm^{-2}}]. \quad (6)$$

The standard SPIRE FTS integrated line flux output from the line fitting script in HIPE is in $[\mathrm{W\,m^{-2}}]$ for point-like sources.

1.1.1 Integrated line flux in Jy km/s

This is straightforward to derive, usually done for a single line, when the line region is converted from the natural frequency units to relative velocity, relative to the expected line centroid ν_0. That is: $\Delta v = \mathrm{c}(\nu - \nu_0)/\nu_0$, where c is the speed of light in vacuum. Therefore, fitting the function from Eq. 1 gives:

$$F_{\text{line}}(\nu_0) = \pi \left(\frac{p_0}{\mathrm{Jy}}\right)\left(\frac{p_2}{\mathrm{km/s}}\right)\;[\mathrm{Jy\,km/s}], \quad (7)$$

for unresolved line $p_2 = c\Delta\nu_{\rm HR}/\nu_0$, where $\Delta\nu_{\rm HR} = 1.2$ GHz is the instrumental resolution for high resolution mode, and ν_0 is the line centroid in GHz. Note that $\Delta\nu_{\rm HR}$ is constant throughout the whole FTS frequency range.

Given the integrated line flux of an unresolved FTS line at ν_0, then we can rewrite Eq. 7:

$$F_{\rm line}(\nu_0) = \pi\left(\frac{p_0}{\rm Jy}\right) c\Delta\nu_{\rm HR}/\nu_0 \;\; = 1.13\times 10^6 \left(\frac{p_0}{\rm Jy}\right)\left(\frac{\nu_0}{\rm GHz}\right)^{-1} \; [\mathrm{Jy\,km/s}], \tag{8}$$

or at *wavelength* λ_0:

$$F_{\rm line}(\lambda_0) = 3.77\times\left(\frac{p_0}{\rm Jy}\right)\left(\frac{\lambda_0}{\mu\rm m}\right) \; [\mathrm{Jy\,km/s}]. \tag{9}$$

1.1.2 Convert line fluxes from [W m^{-2}] to [Jy km/s]

Let's assume *unresolved line* and let

$$Q\times F_{\rm line}([\mathrm{W\ m^{-2}}]) = F_{\rm line}([\mathrm{Jy\ km/s}]) \tag{10}$$

Combining Eqs 6 and 7 we derive

$$\begin{aligned} Q &= \pi\times c\times 10^{11}\left(\frac{\nu_0}{\rm GHz}\right)^{-1} = 9.418\times 10^{19}\left(\frac{\nu_0}{\rm GHz}\right)^{-1} \quad \text{or} \\ &= \pi\times 10^{17}\left(\frac{\lambda_0}{\mu\rm m}\right). \end{aligned} \tag{11}$$

So, the output line fluxes from the line fitting script (in units of [W m^{-2}]) must be multiplied by Q in order to convert them to [Jy km/s].

For example, an unresolved line at $\lambda_0 = 300\,\mu$m ($\approx$ 1000 GHz) with peak flux density of 1 Jy corresponds to an integrated line flux of $F_{\rm line} = 1.2\times 10^{-17}$ W m^{-2} which is 1130 Jy km/s. Note that the unresolved line sinc width is 360 km/s and the corresponding FWHM is $1.20671\times 360 = 434$ km/s.

1.2 Lines in extended-calibrated sources

For extended source calibration, the spectrum I_ν and consequently p_0 are in units of W m^{-2} Hz^{-1} sr^{-1}, so the integrated line flux is directly:

$$F_{\rm line}(\nu_0) = \pi\times 10^9 \frac{p_0}{\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}}\left(\frac{p_2}{\rm GHz}\right) \; [\mathrm{W\,m^{-2}\,sr^{-1}}]. \tag{12}$$

For unresolved lines at the FTS HR this simplifies to

$$F_{\text{line}}(\nu_0) = 1.2 \times 10^9 \frac{p_0}{\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}}\ [\mathrm{W\,m^{-2}\,sr^{-1}}]. \tag{13}$$

For resolved sources, one can convert from $[\mathrm{W\,m^{-2}\,sr^{-1}}]$ to [Jy] by multiplying with the beam solid angle at ν_0 in steradians:

$$F_{\text{line}}(\nu_0)[\mathrm{Jy}] = 10^{26} \left(\frac{\theta_b(\nu_0)}{\mathrm{sr}}\right) F_{\text{line}}[\mathrm{W\,m^{-2}\,sr^{-1}}]. \tag{14}$$

1.3 Upper limits

If no line is detected, then the estimated continuum *rms* (σ) at the position of the line can be used as an upper limit at e.g. 5σ by setting $p_0 = 5\sigma$. As a proxy of the continuum *rms* one can use the average error at the position of the line, using the error column in the FTS spectra.

2 Intensities in Kelvins

Due to historical reasons, in the FIR/submm and radio astronomy, some alternative units, based on concepts such as antenna temperature (T'_A), are used (see e.g. Wilson et al. 2009). This is the case for *Herschel* HIFI instrument, which is calibrated in antenna temperature using internal calibrators (see Mueller & Jellema 2014, section 3.2). The flow in this case is usually $T'_A \Rightarrow T_b \Rightarrow I_\nu$ (or S_ν), where T_b is the brightness temperature (see below). SPIRE FTS, however, is calibrated in such a way that this notion of antenna temperature is not applicable.

In the case of the SPIRE FTS, we already have I_ν (or S_ν) properly calibrated using the *Herschel* telescope (I_ν) or Uranus (S_ν) and thus the flow is I_ν (or S_ν) $\Rightarrow T_b \Rightarrow T'_A$. The step $T_b \Rightarrow T'_A$ is not really necessary, unless one wants to derive T'_A for a particular telescope. Then the knowledge of the aperture efficiency η_A and the main beam efficiency η_B are needed, as well as the beam coupling with the source T_b distribution (see e.g. Wilson et al. 2009 for details). These parameters were not derived for the SPIRE FTS and even if we derived them then T'_A would have been only applicable as antenna temperature for *Herschel*-SPIRE FTS, which is not very useful. Consequently we will not provide any guidelines on how to do $T_b \Rightarrow T'_A$.

The transformation I_ν (or S_ν) $\Rightarrow T_b$ is straightforward. Let's assume a *total equilibrium* of the source radiation field with the surroundings, then its intensity is that of a black-body and is represented by the Planck function:

$$I_\nu \equiv B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}. \tag{15}$$

This is also true for the case of a *local thermodynamic equilibrium* in the optically thick material (i.e. with large optical depth).

In radio astronomy, $h\nu/kT \ll 1$ and then Eq. 15 becomes:

$$I_\nu = \frac{2\nu^2}{c^2}kT_b = \frac{2}{\lambda^2}\,kT_b, \tag{16}$$

this is the so called Rayleigh-Jeans (RJ) law, or RJ approximation. The usefulness of the RJ law is that the intensity I_ν and the brightness temperature T_b are equivalent in describing the source radiation properties. We can rewrite Eq. 16 and obtain the conversion:

$$T_b = \frac{c^2}{2k\nu^2}I_\nu = \frac{\lambda^2}{2k}\,I_\nu. \tag{17}$$

In particular, using this equation we can convert the FTS level-2 *extended* source calibrated spectrum I_ν in units $[\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$ to brightness temperature in Kelvins.

For point sources, i.e. source size much smaller than the beam, we have:

$$T_b = \frac{c^2}{2k\nu^2}\frac{S_\nu}{\Omega_\nu} = \frac{\lambda^2}{2k}\,\frac{S_\nu}{\Omega_\nu}, \tag{18}$$

where Ω_ν is the beam area and S_ν is the FTS level-2 *point* source calibrated spectrum in units of [Jy].

For spectral lines we just have to replace p_0 in the equations in the previous section with the one converted to Kelvins at $\nu = \nu_0$, using either Eq. 17 or Eq. 18, depending on the source size.

The RJ approximation is widely used even in cases when $h\nu \approx kT$! But note that in this case the interpretation of T_b as a representative temperature is incorrect as it does not represent the real radiation temperature T_R of the source.

2.1 Convert from [W m^{-2} sr^{-1}] to [K]

This is straightforward, substituting p_0 in Eq. 12 with $p_0 \times c^2/(2k\nu_0^2)$, we derive the conversion factor Q':

$$Q'(\nu_0) = 1.04\times 10^{12}\left(\frac{\nu_0}{\mathrm{GHz}}\right)^{-2}\ [\mathrm{K\ per\ W\ m^{-2}\ sr^{-1}}], \tag{19}$$

i.e.

$$F_{\mathrm{line}}[K] = Q' \times F_{\mathrm{line}}[\mathrm{W\ m^{-2}\ sr^{-1}}]. \tag{20}$$

For example, an unresolved line at $\nu_0 = 1000$ GHz with peak intensity $p_0 = 10^{-16}$ $\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$ corresponds to an integrated line intensity of 1.2×10^{-7} W m^{-2} sr^{-1}, which in brightness temperature is 0.125 K.

2.2 Convert from [Jy km/s] to [K km/s]

This is applicable for point sources and if F_{line}[K km/s] $= Q'$ F_{line}[Jy km/s] then following Eqs. 18 and 7, we obtain

$$\begin{aligned} Q'(\nu_0) &= \frac{c^2}{2k\nu_0^2}\frac{1}{\Omega(\nu_0)} = \frac{c^2\,4\ln 2}{2\pi k\nu_0^2}\,\theta_b^{-2} = \\ &= 1.22 \times 10^6 \left(\frac{\theta_b}{\mathrm{arcsec}}\right)^{-2} \left(\frac{\nu_0}{\mathrm{GHz}}\right)^{-2} \ [\text{K km/s per Jy km/s}], \end{aligned} \tag{21}$$

where we used $\theta_b(\nu)$ – the beam FWHM as provided in the calibration context. Note that $\Omega_\nu = \pi\theta_b^2/(4\ln 2)$, with the approximation that the beam is Gaussian, which is good for SSW but not a good approximation for SLW.

So, the same example as at the end of Section 1.1.2, the integrated line of 1130 [Jy km/s] corresponds to 3.65 [K km/s], using the beam FWHM of $19.45''$ at 300 μm (1000 GHz). The same integrated line flux of 1130 [Jy km/s] but observed with a telescope with $40''$ beam will be 0.86 [K km/s], i.e. 4 times smaller as the beam is two times larger.

2.3 Convert from [W m^{-2}] to [K km/s]

As we know how to convert [W m^{-2}] to [Jy km/s] (see Sec. 1.1.2) then it's just the product of Eq. 11 and Eq. 21, namely

$$\hat{Q} = 1.149 \times 10^{29} \left(\frac{\theta_b}{\mathrm{arcsec}}\right)^{-2} \left(\frac{\nu_0}{\mathrm{GHz}}\right)^{-3}. \tag{22}$$

As a consistency check we see that the integrated line with 1.2×10^{-17} [W m^{-2}] corresponds to 3.64 [K km/s], i.e. the same result as the conversion from [Jy km/s].

2.4 Semi-extended sources

Note that the same considerations as for the point sources are applicable for semi-extended sources, once their S_ν is corrected for the source size and normalised at a particular reference beam $\Omega_{\rm ref}$ and θ_b, which should be used for the conversions.

3 Apodized lines

The apodization of the FTS spectra is a smoothing operation that suppresses the sinc function sidelobes but increases the line width, i.e. decreases the spectral resolution. Apodized lines can be represented with a Gaussian:

$$f(\nu, p) = p_0 \exp(-x^2/2);\; x = (\nu - p_1)/p_2. \tag{23}$$

In this case the integrated line flux is:

$$F_{\rm line} = \int_{-\infty}^{+\infty} f(\nu, p) \mathrm{d}\nu = \sqrt{2\pi} p_0 p_2. \tag{24}$$

Following the same approach as in the previous section one can derive the conversions for apodized lines. We leave this exercises for the reader.

4 Summary

This is the list of the conversions from the standard units used in the SPIRE FTS line fitting script to some alternative units. Note that we only show the conversions for unresolved lines.

$$F_{\rm line}\,[\mathrm{W\ m^{-2}}] \times \begin{cases} 10^3 & \Rightarrow [\mathrm{erg\ cm^{-2}\ s^{-1}}] \\ 9.418 \times 10^{19}\,(\nu_0/\mathrm{GHz})^{-1} & \Rightarrow [\mathrm{Jy\ km/s}] \\ \pi \times 10^7\,(\lambda_0/\mu m) & \Rightarrow [\mathrm{Jy\ km/s}] \\ 1.149 \times 10^{29}\,(\theta_b/'')^{-2}\ (\nu_0/\mathrm{GHz})^{-3} & \Rightarrow [\mathrm{K\ km/s}] \end{cases}$$

$$F_{\rm line}\ [\mathrm{Jy\ km/s}] \times 1.22 \times 10^6 \left(\frac{\theta_b}{''}\right)^{-2} \left(\frac{\nu_0}{\mathrm{GHz}}\right)^{-2} \Rightarrow [\mathrm{K\ km/s}]$$

For extended sources:

$$F_{\rm line}\ [\mathrm{W\ m^{-2}\ sr^{-1}}] \times 1.04 \times 10^{12} \left(\frac{\nu_0}{\mathrm{GHz}}\right)^{-2} \Rightarrow [\mathrm{K}]$$

Appendinx: radiation temperature

Note that if we directly solve the Planck function (Eq. 15) for the temperature, we obtain the so called radiation temperature T_R:

$$T_R(\nu) = \frac{h\nu}{k} \frac{1}{\ln\left(h\nu^3/c^2/I_\nu + 1\right)}. \tag{25}$$

This T_R is the temperature of a black-body that has intensity I_ν at the particular frequency ν.

In the SPIRE FTS band, 447-1550 GHz, for sources with temperatures 100 K we have $h\nu/kT \approx 0.2$ at 450 GHz and ≈ 0.7 at 1550 GHz. For colder sources, with $T = 40$ K, the results are 0.5 and 1.9 at the two frequency limits. This means that the Rayleigh-Jeans brightness temperature T_b derived using Eq. 16 will not be equivalent to the actual radiation temperature T_R.

Here is an example of the difference. For the same source with $I_\nu = 7.85 \times 10^{-16}\ \mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$ at $\nu_0 = 450$ GHz we obtain $T_b = 13$ K and $T_R = 22$ K. With a source with $I_\nu = 5.0 \times 10^{-15}\ \mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$ at $\nu_0 = 1550$ GHz we have $T_b = 8$ K and $T_R = 32$ K, which are significant differences!

References

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