Frequency and Velocity Transforms for HIFI

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Abstract

This document describes the frequency and velocity transforms applied to calibrated instrumental HIFI spectra, taking them from the instrument frame to other observer frames of interest.

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1 Introduction

This document describes the frequency and velocity transforms applied in the HIFI data reduction pipeline. These transformations take instrumental frequencies, which we assume to be properly calibrated, to other reference frames, for instance the Solar Local Standard of Rest. For information on instrumental calibration of the HIFI WBS spectrometer see [1], and for HRS see [2].
In section 2 we describe the HIFI frequency accuracy and set the precision desired of our velocity and frequency handling algorithms. In section 3 we define our frames of reference and the algorithms for frame transformation in `HIPE`. In section 4 we mention some frequency effects considered insignificant enough to disregard.

2 HIFI Instrumental Frequency Accuracy

2.1 Master Oscillator

All frequencies on HIFI are derived from a temperature-controlled 10 MHz reference oscillator (OCXO), except for the local oscillator (LO) in bands 6 and 7 which includes a contribution from a second, free-running 10.4 GHz oscillator. The nominal purity of the reference is [3]:

$$\delta\nu/\nu \approx 10^{-8},$$

so this would be the highest absolute accuracy possible with HIFI (*ie* 5 kHz at 500 GHz, 20 kHz at 2 THz). In fact the requirement on the stability of the final LO is somewhat less, 10^{-7}[4]. The HEB bands have an additional free-running LO with expected stability 3-4 ppm, therefore adding 30 kHz of uncertainty in bands 6 and 7.

2.2 Spectrometer Resolution

The WBS has a native resolution of about 1.1 MHz ([5]). The CCD pixels are about 0.55 MHz wide, but the frequency-width of a pixel changes across each CCD (of which there are four per polarization). The mapping from pixel-number to IF frequency is calculated by injecting fixed tones (the "COMB"), each a multiple of the OCXO, and fitting the response spectrum. In this way, the frequency labeling of each spectral bin at IF is accurate to 100kHz ([6]).
The HRS, in High Resolution Mode, has a spectral response of 81.6 kHz (FWHM, non-apodized)([7]). The accuracy of the labeling of frequency bins at the IF is determined mainly by the accuracy of the OCXO, as all offsets are referenced to the master oscillator and there is only a small jitter added by the up-convert multipliers ($\sim$ 8 kHz). Obviously, HRS drives the software requirements for precision in treatment of velocity and frequency transforms.
An important point to make is that the spectrometer resolution is not the final word in frequency precision. Once you embark upon fitting line profiles, the signal-to-noise ratio (SNR) of the spectrum determines with what precision attributes such as flux centroid

can be measured. For example, the centroid of a COMB line is determined to much higher precision than the WBS native resolution or even CCD pixel width. A spectral line of moderate SNR detected in HRS High Resolution mode could bump against the fundamental OCXO limit of $\sim$ 20 kHz.

2.3 Desired Algorithmic Precision

Our goal in data reduction is that observations of a target taken at two or more epochs show no measurable spectral changes that are not intrinsic. In practice, the spacecraft motion is the changing factor. We would like algorithms which transform data from the spacecraft frame to any other frame to be accurate to less than the OCXO limit, say $\delta\nu/\nu \sim 10^{-9}$. The speed of the Herschel Space Observatory (HSO) as it orbits the Solar System Barycenter at L2 is about 30 km/s. Expanding the relativistic Doppler formula in a series ($\beta \equiv v/c \sim 10^{-4}$):

$$\frac{\nu - \nu_0}{\nu_0} = \underbrace{-\beta}_{\sim 10^{-4}} + \underbrace{\frac{1}{2}\beta^2}_{\sim 10^{-8}} - \underbrace{\frac{1}{2}\beta^3}_{\sim 10^{-12}} + \cdots$$

Two conclusions: Lorentz transforms are necessary, and the accuracy of the HSO velocity used must be $\lesssim 1$ m/s.
We can ignore the acceleration of HSO during integration of a single spectrum, and also General Relativistic effects; see section 4.

3 Reference Frames

From now on, when we refer to a frame we mean an inertial frame in which observables can be transformed to another frame via the Lorentz transform.

3.1 HSO Frame

The internal frequency calibration schemes for HRS and WBS, executed within the HIFI level 0.5 pipeline, produce observed frequencies in the spacecraft frame ("HSO"). They are in the IF scale; the frequency of the detected photon is simply IF + LO. There are two observables of interest: the frequency of the incident wave $\omega = 2\pi\nu$, and its direction $\hat{k}$. In fact, we don't know the direction from which the radiation was incident because the HIFI beam is of finite size ($\theta_{\mathrm{FWHM}} \in (20'', 90'')$) and additionally has a small pointing uncertainty ($\approx 2''$). Often the signal will come from a resolved source. We assume the direction of incidence is the boresight of the beam as reconstructed in the pointing product (see section 4 for an estimate of the uncertainty this contributes in transformed frequencies). Note that because of aberration, the direction of incidence in the HSO frame differs from that observed in, say, the SSBC frame, by $\lesssim 22''$. In practice, the pointing information provided in the pointing product has been de-aberrated, and so is in the SSBC frame.
Only the frequency ω remains known in the HSO frame alone; everything else, including the HSO motion, is known in the SSBC frame.

3.2 SSBC frame

The Solar System Barycenter is the fundamental inertial frame for calculations involving the motion of HSO. The state vectors $(\mathbf{r}, \mathbf{v})$ of solar system objects (SSOs), including HSO, expressed in this frame have as origin the SSBC and as reference directions the International Celestial Reference Frame axes [8].
The motion of HSO with respect to the Geocenter is determined to an accuracy $\lesssim$5 cm/s by the usual tracking techniques [9]. The Geocenter is tied to the Solar System Barycenter through the JPL DE405 planetary ephemerides to a precision of mm/s. Because the HSO motion, target coordinates, and telescope pointing are all defined in this frame, the transformation from the HSO frame to any other passes necessarily through the SSBC.
It's important to keep in mind in which frame observables are defined; in the equations below we use the covention that subscripts refer to the object of interest, and superscripts to the frame in which the observable is measured. For example, the direction of a SSO as seen by the telescope is $\hat{\mathbf{p}}^{\mathrm{HSO}}_{\mathrm{SSO}}$; as seen by an observer at rest in the SSBC it is $\hat{\mathbf{p}}^{\mathrm{SSB}}_{\mathrm{SSO}}$.
A signal is incident upon the spacecraft and detected at frequency ω^{HSO}. The transformation of HSO-centric frequencies to SSB-centric is described by the relativistic Doppler formula:

$$\frac{\nu^{\mathrm{HSO}}}{\nu^{\mathrm{SSB}}} = \gamma^{\mathrm{SSB}}_{\mathrm{HSO}}(1 + \boldsymbol{\beta}^{\mathrm{SSB}}_{\mathrm{HSO}} \cdot \hat{\mathbf{p}}^{\mathrm{SSB}}) \tag{1}$$

where $\boldsymbol{\beta} = \boldsymbol{v}/c$, $\gamma = 1/\sqrt{1-\beta^2}$, and $\hat{\mathbf{p}}^{\mathrm{SSB}}$ is the de-aberrated direction of telescope pointing (the J2000 coordinates of the beam boresight at the time of observation).

3.3 LSR Frame

It might be useful to review the definition of the Local Standard of Rest [10]. Take any point in the Galactic plane, and imagine there exists a circular orbit about the Galactic Center that passes through that point. The circular velocity defines the Local Standard of Rest for that position. Such a point coincident with the Sun defines the Solar Local Standard of Rest (LSR). The Sun has a peculiar velocity with respect to the LSR, which can be estimated in different ways. The LSR so defined is also called the Dynamic LSR, refering as it does to the rotation curve of the Galaxy. In practice, the Sun's peculiar motion with respect to the LSR has also been inferred from the mean motion of bright stars in catalogs or in the solar neighborhood. The LSR defined by this calculation of peculiar motion is called the kinematic LSR (LSRk) and is the more commonly used convention, however there is not a single standard value for the LSRk. Further, it is not very close to any physical velocity of interest[11]; it is only a common convention.
We take for our definition of the LSRk frame that:

> the motion of the SSBC with respect to the LSRk is
> 20.0 km/s toward $(\alpha, \delta) = 18\mathrm{h}03\mathrm{m}50.29\mathrm{s}, +30°00'16.8''$(J2000)

This is a common observatory standard [12] and adopted by many astronomical software suites such as `casa, slalib`, and `class`.

Frequencies in the LSR frame are derived from the SSB frame by a Lorentz transform

$$\frac{\nu^{\rm LSR}}{\nu^{\rm SSB}} = \gamma^{\rm SSB}_{\rm LSR}(1 + \boldsymbol{\beta}^{\rm SSB}_{\rm LSR} \cdot \hat{\mathbf{p}}^{\rm SSB})$$

And using equation 1, LSR frequencies can be calculated directly from observed HSO frequencies and SSB-centric known quantities:

$$\nu^{\rm LSR} = \nu^{\rm HSO} \frac{\gamma^{\rm SSB}_{\rm LSR}}{\gamma^{\rm SSB}_{\rm HSO}} \frac{(1 + \boldsymbol{\beta}^{\rm SSB}_{\rm LSR} \cdot \hat{\mathbf{p}}^{\rm SSB})}{(1 + \boldsymbol{\beta}^{\rm SSB}_{\rm HSO} \cdot \hat{\mathbf{p}}^{\rm SSB})} \tag{2}$$

3.4 Source (nonSSO) Frame

Because the 3-velocity of a star or other such target is unknown, a transformation to its comoving frame is impossible. What may be known about the object is that a spectral line appears shifted from its expected rest frequency, and that shift can be interpreted as a radial velocity. One would like to see the relative velocities of other spectral lines, with a view to constraining a dynamical model of the object. In this context, transforming frequencies to the source frame is only a shift of the frequency axis already defined in an intertial frame (*eg* SSB, LSR) and then expressing the frequencies as velocities according to some convention. There are three operative conventions for expressing redshift as a velocity:

convention	$\beta \to \nu$	$\nu \to \beta$
radio	$\frac{\nu_{\rm rec}}{\nu_{\rm emit}} = 1 - \beta$	$\beta = \frac{\nu_{\rm emit} - \nu_{\rm rec}}{\nu_{\rm emit}}$
optical	$\frac{\nu_{\rm rec}}{\nu_{\rm emit}} = \frac{1}{1+\beta}$	$\beta = \frac{\nu_{\rm emit} - \nu_{\rm rec}}{\nu_{\rm rec}}$
relativistic	$\frac{\nu_{\rm rec}}{\nu_{\rm emit}} = \sqrt{\frac{1-\beta}{1+\beta}}$	$\beta = \frac{\nu^2_{\rm emit} - \nu^2_{\rm rec}}{\nu^2_{\rm emit} + \nu^2_{\rm rec}}$

The relativistic definition would be correct if the relative velocity between observer and source were purely radial. The radio and optical definition are two linearizations of the relativistic equation, and their difference is quite large at HIFI bandwidth $\Delta\nu \approx 4$ GHz and frequencies $\nu_{\rm rec} \approx 500$ GHz:

$$\Delta\beta \equiv |\beta_{\rm optical} - \beta_{\rm radio}| \approx (\Delta\nu)^2/\nu^2_{\rm rec} \sim 5 \times 10^{-5} \Rightarrow \Delta v \approx 16 \text{ km/s}$$

The HIFI Pipeline task `DoVelocityCorrection` will, if requested, recast the frequencies in the data to the "source frame" of a nonSSO target, but it is not a real frame change and a Lorentz transform is not performed.
It is often interesting to view spectra plotted on an axis representing speed. In the present version of `HIPE` ($\leqslant$ 6.0.1360), the SpectrumExplorer display tool uses the relativistic convention when displaying frequencies as velocties; however, the `ConvertFrequencyTask` uses the radio definition.

3.5 Source (SSO) Frame

When observing objects within the solar system, we know or can estimate their full 3-velocity, and so a real Lorentz transform to the object rest frame is performed. The manner is thus: the state $(\mathbf{r}, \mathbf{v})$ of Herschel is known as a function of time, as is the SSO's. A photon received by HSO at time t was emitted by the SSO at time $(t - \mathrm{LT})$, where LT is the light-travel-time between the two. The frame to which we transform is the comoving frame of the SSO at the retarded time $(t - \mathrm{LT})$.

$$\mathrm{LT} = \frac{|\mathbf{r_{SSO}}(t - \mathrm{LT}) - \mathbf{r_{HSO}}(t)|}{c}$$

LT is computed iteratively; three iterations are sufficient to achieve a precision of less than a millisecond within the orbit of Pluto [13], which is less than the error due to ignoring General Relativity. Once the SSO state at the retarded time is known, frequencies are transformed by equation 2 (with SSO subsituted for LSR).

4 Errors

In this section we mention some frequency-changing effects which can be disregarded in the HIFI pipeline.

4.1 Pointing or Source Coordinate Errors

Because of Herschel's finite beamsize and uncertainties in pointing or source coordinates, a spectral line is likely to be due to emission offset from its assumed coordinates. In computing the frequency shift from HSO to some other frame a dot product of Herschel's velocity with an assumed signal arrival direction is made, and the error just described will produce an erroneous frequency shift of the observed spectral line.
A signal is observed at frequency ν. The rest-frame frequency is calculated through the Doppler relation $\nu/\nu_0 = \gamma(1 + \boldsymbol{\beta} \cdot \hat{\mathbf{p}})$, where $\hat{\mathbf{p}}$ is a unit vector in the direction of the source. If the true direction to the source is $\hat{\mathbf{p'}} = \hat{\mathbf{p}} + \epsilon\hat{\boldsymbol{\sigma}}$, where $\epsilon << 1$ and $\hat{\boldsymbol{\sigma}} \perp \hat{\mathbf{p}}$, then the difference in estimated rest frequency will be

$$\frac{\nu_0' - \nu_0}{\nu_0} \approx \frac{\epsilon\boldsymbol{\beta} \cdot \hat{\boldsymbol{\sigma}}}{1 + \boldsymbol{\beta} \cdot \hat{\mathbf{p}}} \sim \epsilon\beta \sim 5 \times 10^{-10}$$

per arcsecond of pointing error ϵ.

4.2 HSO Acceleration

We can also ignore the fact that HSO is not an inertial frame, that it accelerates during the integration of a single spectrum. The integration time dt is less than ten seconds; over that time the HSO velocity will change by an amount dv:

$$\frac{d\nu}{\nu} \approx \frac{dv}{c} \approx \frac{1}{c}\frac{GM_\odot}{r^2}dt \sim \frac{1}{c}(0.06 \text{ m/s}) \sim 10^{-9}$$

which is less than our required velocity accuracy.

4.3 General Relativity

Another effect of HSO's orbital motion that comes to mind is the gravitational redshift due to changing potential. As seen in the SSB frame, with respect to its mean rate a clock on HSO will run fast at aphelion and slow at perihelion. The magnitude of this effect for an orbit of semimajor axis a and eccentricity e is

$$\frac{\Delta\nu_{\text{a-p}}}{\nu_0} \approx \frac{2GM_\odot}{c^2}\frac{1}{a}\frac{e}{1-e^2} \sim 3 \times 10^{-10}$$

So, we can ignore GR corrections due to HSO motion, being two orders of magnitude below instrumental precision. We will also ignore GR effects on objects whose lines-of-sight pass near Solar System bodies such as Jupiter (HSO cannot observe near the Sun). The gravitational redshift of a line in Jupiter's atmosphere is of order $\delta\nu/\nu \approx GM_{♃}/R_{♃}c^2 \sim 10^{-8}$ but doesn't depend on HSO state and so satisfies our repeatability criterium. In such observations the interpretation of spectral line frequencies is complicated and we can consider it the observer's responsibilty to account for the physics of interest.

References

[1] WBS Software Requirements, HIFIKOSMSPSA400002, 2005
[2] HRS Calibration Products, CESR-HRS-SP-3F11-380, 2005
[3] HIFI LocalOscillator Specification, JPL/HIFI/SP/1998-001, 1998
[4] HIFI Instrument Specification, SRON-G/HIFI/SP/1998-001, 2004 (see table 4.5.2-1)
[5] WBS Specifications and Interfaces (SID), HIFI-KOSM-SP-SA100-001, 2006
[6] On the Predictability of the WBS Frequency Scale, V. Ossenkopf, 2008
[7] HRS Design Description, CESR-HRS-SP-3162-045, 2002
[8] IERS Technical Note No. 36, p. 28.
[9] Orbit Determination of the Planck Satellite, Bernard Godard *et al*, http://www.mediatec-dif.com/issfd/OrbideII/Godard.pdf
[10] Galactic Astronomy, W. H. Freeman and Co., New York, 1981.
[11] McMillan, P. J. & Binney, J. J., MNRAS, 402:934, 2010
[12] Reid *et al*, ApJ 700:137, 2009
[13] NAIF SPICE Package Documentation (module spkezr)