Poincaré, Heisenberg, Gödel.
Some limits of scientific knowledge.

Fernando Sols
Universidad Complutense de Madrid

Henry Poincaré (1854-1912)

nonlinear
dynamics

Werner Heisenberg (1901-1976)

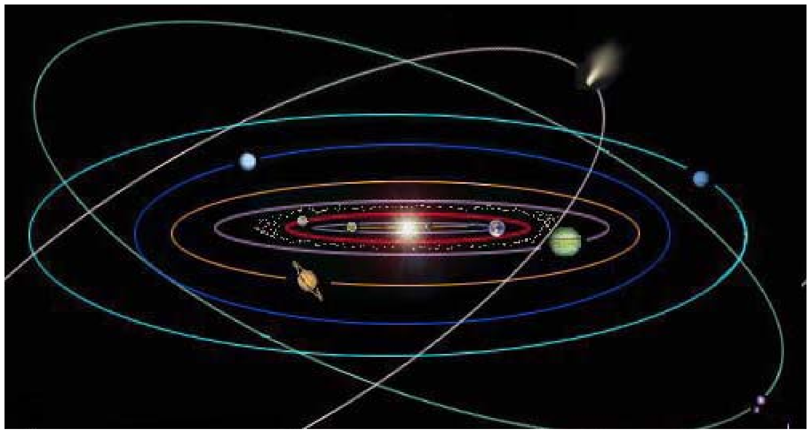
uncertainty
pricniple

Kurt Gödel (1906-1978)

incompleteness
theorems

Isaac Newton (1643-1727)

classical mechanics
deterministic universe



$$p(0), x(0)$$

determines

$$p(t), x(t)$$

Henry Poincaré (1854-1912)

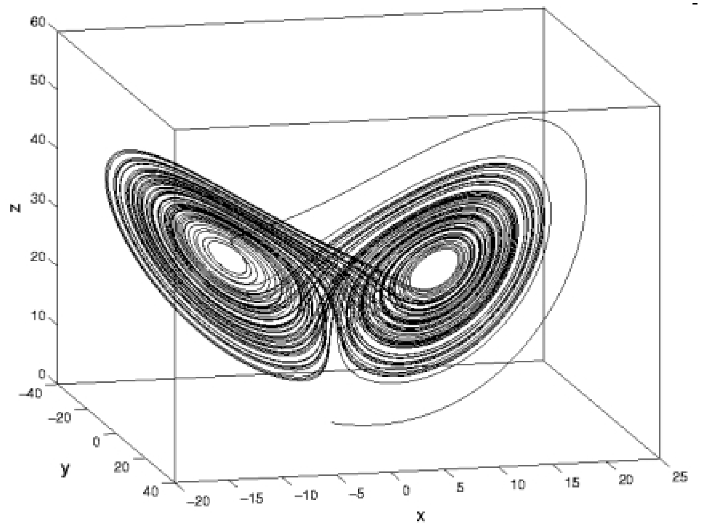
nonlinear dynamics
theory of chaos

• planetary system (2-body problem) is an exception

• most dynamical systems are chaotic

• long term evolution very sensitive to initial conditions

• trajectories in phase space strongly intertwined



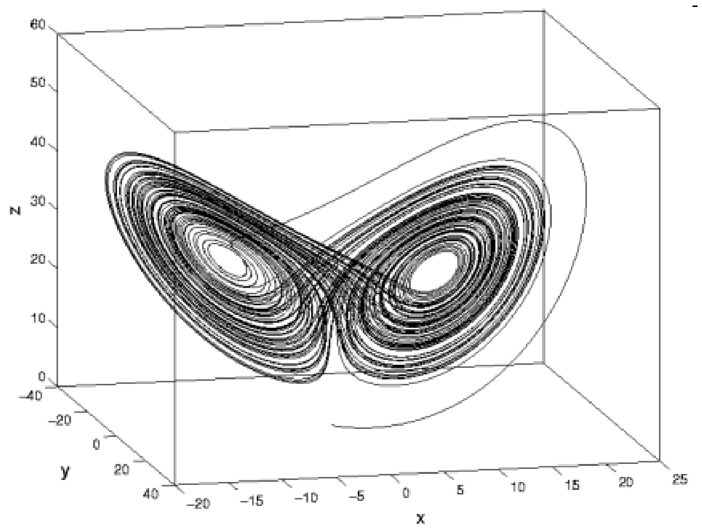
Chaotic systems

the greater the precision with which we wish to know the future

or

the farther the future we wish to predict

the greater the accuracy with which we need to know the initial conditions



Prediction of the future requires knowledge of the present with infinite accuracy

$$\Delta p(0) \rightarrow 0$$

$$\Delta x(0) \rightarrow 0$$

Heisenberg uncertainty principle

position (x) and momentum (p = mv) cannot be known simultaneously with arbitrary precision.

$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

→ The precise prediction of the future is forbidden by quantum mechanics

[local hidden-variable theories rejected by experiment (Bell's theorem …)]

Poincaré: practical indeterminacy

Poincaré + Heisenberg $\rightarrow$ intrinsic indeterminacy

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Future is not determined

There is room (although it does not prove) the existence of

- Free will (human design)
- Providence

Quantum measurement. "Wave-packet collapse"

$$\underbrace{\left(|x\rangle + |y\rangle + |z\rangle\right)}_{\text{quantum system}} \quad \underbrace{|A\rangle}_{\substack{\text{macroscopic}\\ \text{apparatus}}} \quad \underbrace{\rightarrow}_{\substack{\text{measurement}\\ \text{(interaction)}}} \quad \underbrace{|x\rangle|X\rangle + |y\rangle|Y\rangle + |z\rangle|Z\rangle}_{\text{entangled state system-apparatus}}$$

$|X\rangle, |Y\rangle, |Z\rangle$ are macroscopically distinct

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→ in practice only one of the possible states is observed:

e.g. $|y\rangle|Y\rangle,$ o $|z\rangle|Z\rangle$ (“collapse”)

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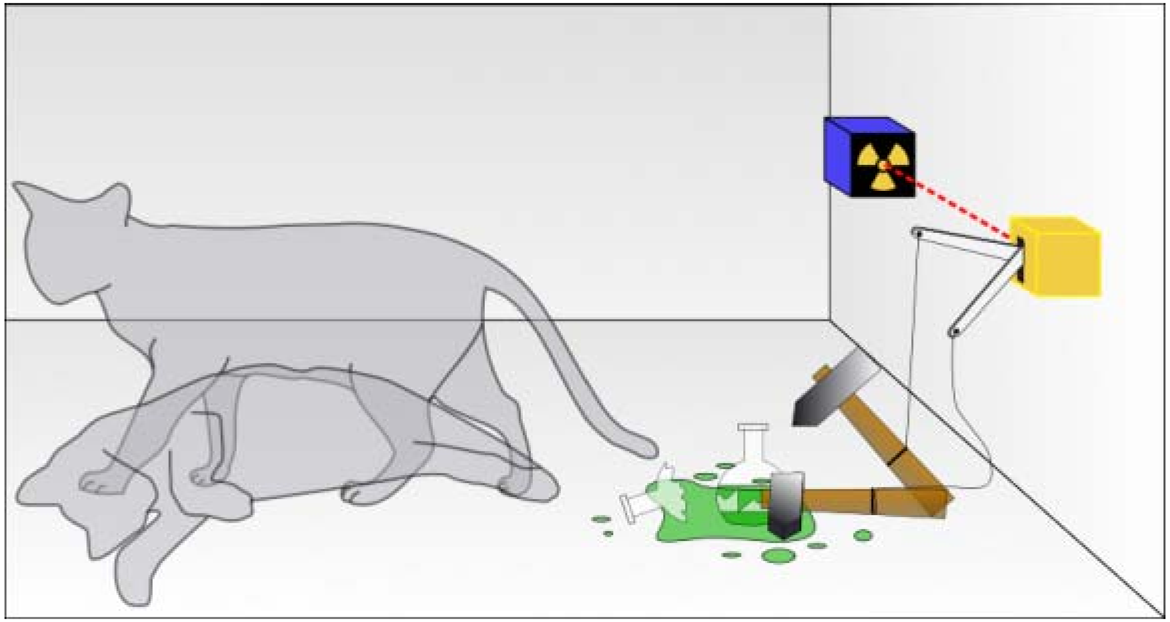
Quantum mechanics only predicts (and very well) the statistics of results in identically prepared experiments.

→ *cannot predict the result of an individual experiment* ←

The “collapse” or “reduction” of the wave function is essential to convert uncertainty about the present → unpredictability of the future



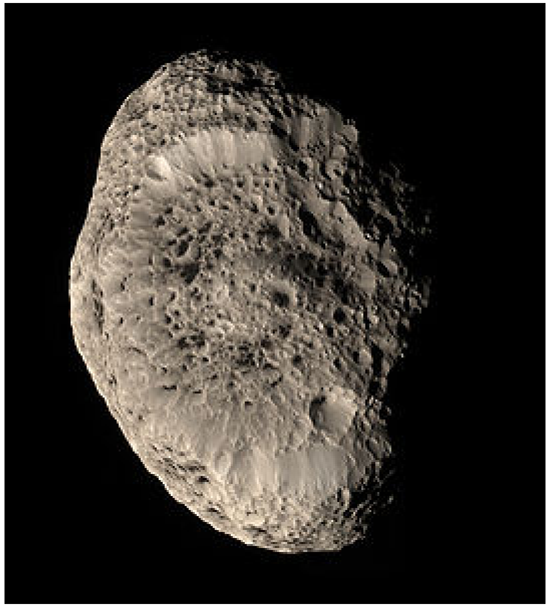
Schrödinger's cat paradox

$$\underbrace{\left(|x\rangle + |y\rangle\right)}_{\text{atom}}\underbrace{|A\rangle}_{\text{cat}} \underbrace{\rightarrow}_{\text{mechanical detector}} \underbrace{|x\rangle|X\rangle + |y\rangle|Y\rangle}_{\text{entangled state atom-cat}}$$

$$|x\rangle|X\rangle = |\text{excited atom}\rangle|\text{cat alive}\rangle$$

$$|y\rangle|Y\rangle = |\text{decayed atom}\rangle|\text{dead cat}\rangle$$

→ in practice we observe the cat alive or dead



Hyperion
satellite of Saturn

oblong

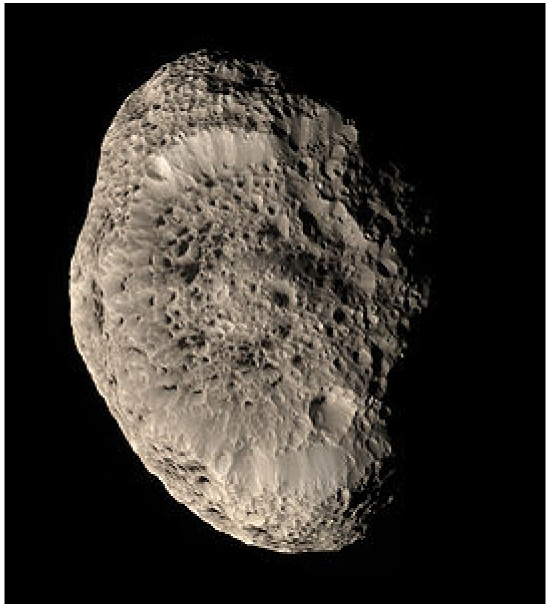
chaotic rotation

mean diameter ~ 300 km

rotation period ~ 21 days

mass ~ 6×10^{18} kg

Lyapunov ~ $(40 \text{ days})^{-1}$



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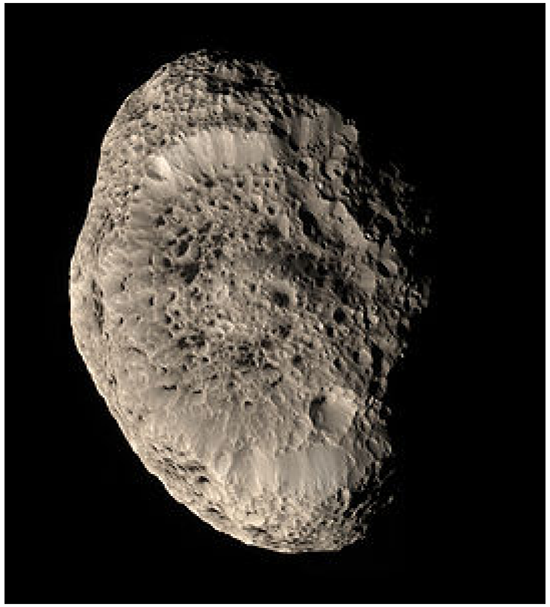
W H Zurek (1998)



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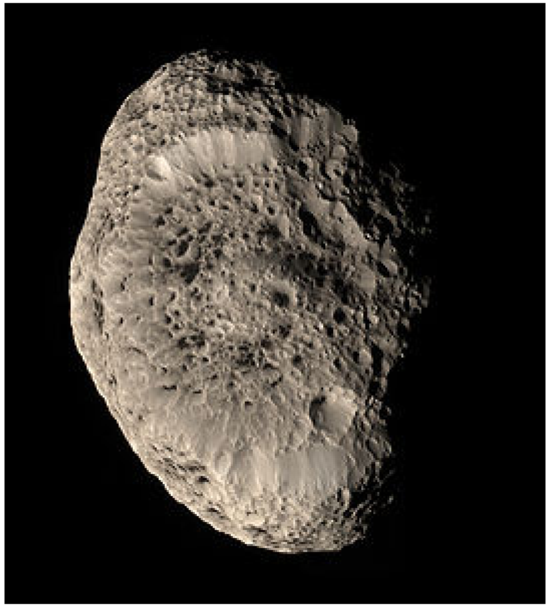
$\mathbf{t > t_\hbar}$ prediction prohibited by QM

information is nowhere



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(solar system: $t_{\hbar}$ ~ 700 million yrs)

Poincaré: practical indeterminacy

Poincaré + Heisenberg $\rightarrow$ intrinsic indeterminacy

Future is not determined

There is room (although it does <u>not</u> prove) the existence of

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What or who determines the future?

Initial conditions (determinism)	ruled out	(except for marginal proposals)
Human design	non-controversial	
External (intelligent) design	very controversial	
Chance, randomness…		

Gödel's incompleteness theorems (1931)

A logical system with a finite number of axioms and rules, sufficiently complex to include arithmetics, and consistent (without contradictions):

is not complete, i.e. contains theorems which are true but unprovable.

We cannot know which ones are those theorems.

Given a non-proved and non-refuted conjecture, we cannot know whether:

- It's true or false
- In any of the two cases:
  - whether the proof exists but we don't find it, or
  - whether the proof does not exist

Undecidability

There are classes of undecidable statements:

Neither “T” nor “no T” can be proved, in the general case

(although, in each particular case, one of the two has to be true)

Example: the halting problem …

Alan Turing (1912-1954)

Turing machine =
universal computer

• The halting problem (1936):

• No general algorithm exists to know if a program will lead the computer to a halt.

Another instance of undecidable problem: the proof of randomness …

What is randomness (chance)?

Algorithmic definition:

A number secuence is random if it *cannot be compressed*, i.e. if no program exists which, *being shorter than the sequence, does completely determine the sequence.*

314159265358979323846264338327950288419716939937510 58209749445923078164062862089986280348253421170679
82148086513282306647093844609550582231725359408128481117450284102701938521105559644622948954930381 96
44288109756659334461284756482337867831652712019091456485669234603486104543266482133936072602491412 73
72458700660631558817488152092096282925409171536436789259036001133053054882046652138414695194151160 94
33057270365759591953092186117381932611793105118548074462379962749567351885752724891227938183011949 12
98336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051 32
00056812714526356082778577134275778960917363717872146844090122495343014654958537105079227968925892 35
42019956112129021960864034418159813629774771309960518707211349999998372978049951059731732816096318 59
50244594553469083026425223082533446850352619311881710100031378387528865875332083814206171776691473 03
59825349042875546873115956286388235378759375195778185778053217122680661300192787661119590921642019 89
38095257201065485863278865936153381827968230301952035301852968995773622599413891249721775283479131 51
5574857242454150695950829533116861727855889075098 381754637464939319255060400927701671139009848824012
85836160356370766010471018194295559619894676783744944825537977472684710404753464620804668425906949 12
93313677028989152104752162056966024058038150193511253382430035587640247496473263914199272604269922 79
67823547816360093417216412199245863150302861829745557067498385054945885869269956909272107975093029 55
321165344987202755960236480665499119881834797753 56

seems random but is not

Gottfried Leibniz (1646-1716)

Infinitesimal calculus
binary system

philosopher, mathematician, jurist, politician

(precursor of information theory)

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \dots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

Leibniz formula

Gregory Chaitin (1947-)

algorithmic
theory of
information

No general algorithm exists to decide if a sequence is random

The random character of a sequence is undecidable
(in the Gödel and Turing sense)

$\Rightarrow$ Randomness cannot be proved

We can’t be sure about randomness …

Facultad de CC. Físicas UCM, 27 octubre 2011

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Randomness	unprovable

Randomness cannot be proved

$\Longrightarrow$ The absence of design cannot be proved

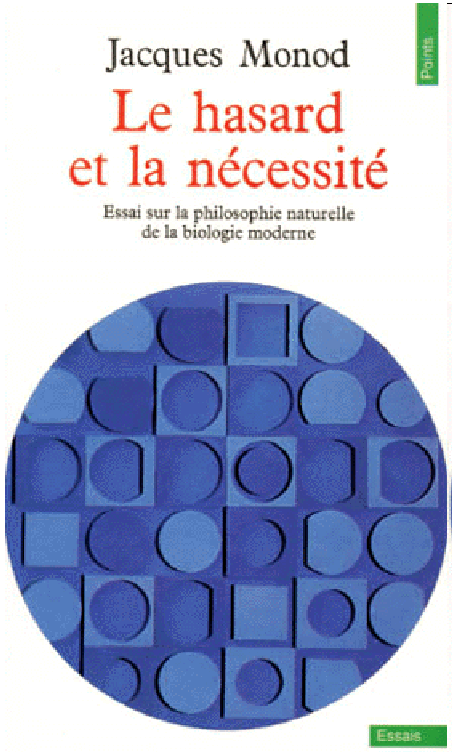
Randomness (understood as *indeterminacy without design*) can be a reasonable, useful hypothesis, essential to make progress on some occasions (e.g. statistical physics, quantum physics, evolution theory),

but not provable *stricto sensu.*

The concept of randomness cannot be ascribed with certainty to any process.

Randomness is a *phenomenological* concept.

Jacques Monod (1910-1976)



(translated from the Spanish version”

“Selection operates with the products of chance indeed, and cannot be fed in any other way; but it acts on a domain of stringent requirements from where randomness is removed [natural selection].”

“Randomness must be here regarded as essential.”

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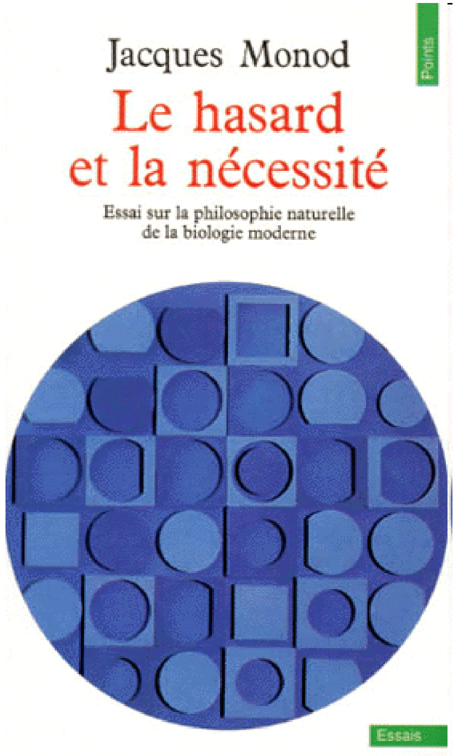

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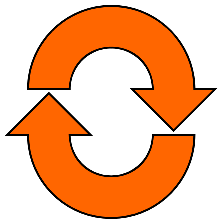
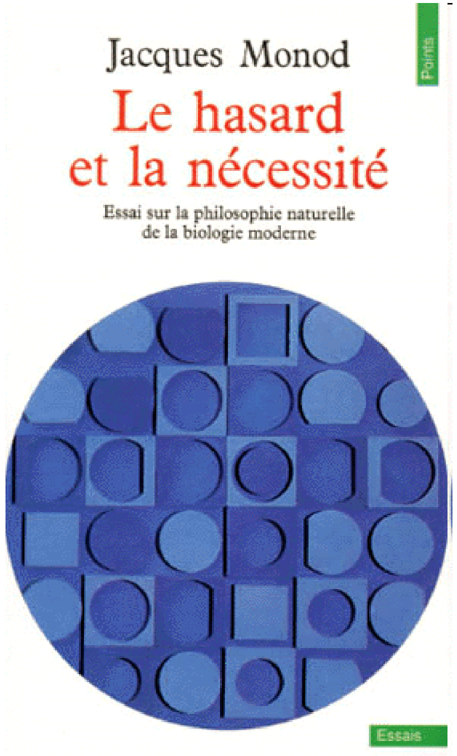


circular argument … if the existence of chance is presented as a scientific <u>conclusion</u>.

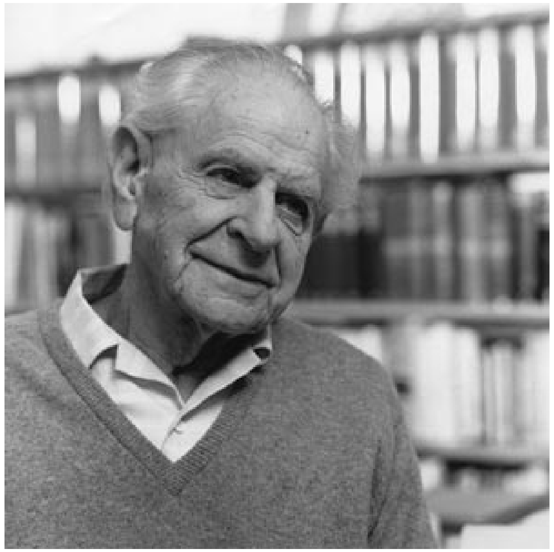
Jacques Monod (1910-1976)



After describing some genetic mutations, he writes:

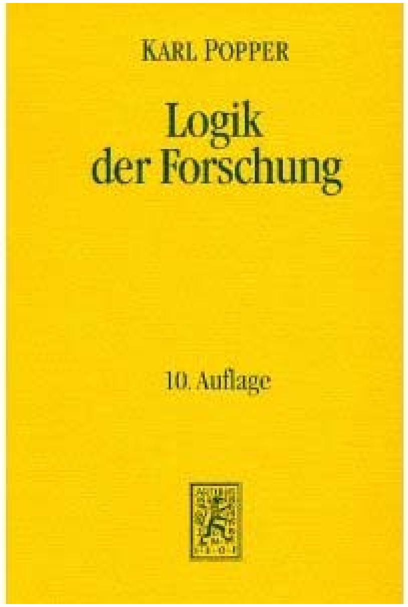
“[mutations] constitute the *only* possible source of modifications in the genetic text, itself the *sole* repository of the organism's hereditary structures, it necessarily follows that chance *alone* is at the source of every innovation, of all creation in the biosphere. Pure chance, absolutely free but blind, at the very root of the stupendous edifice of evolution: this central concept of modern biology is no longer one among other possible or even conceivable hypotheses. It is today the *sole* conceivable hypothesis, the only one that squares with observed and tested fact. And nothing warrants the supposition — or the hope — that on this score our position is likely ever to be revised.”

[Monod, 1970].
Emphases by Monod.



Karl Popper (1902-1994)

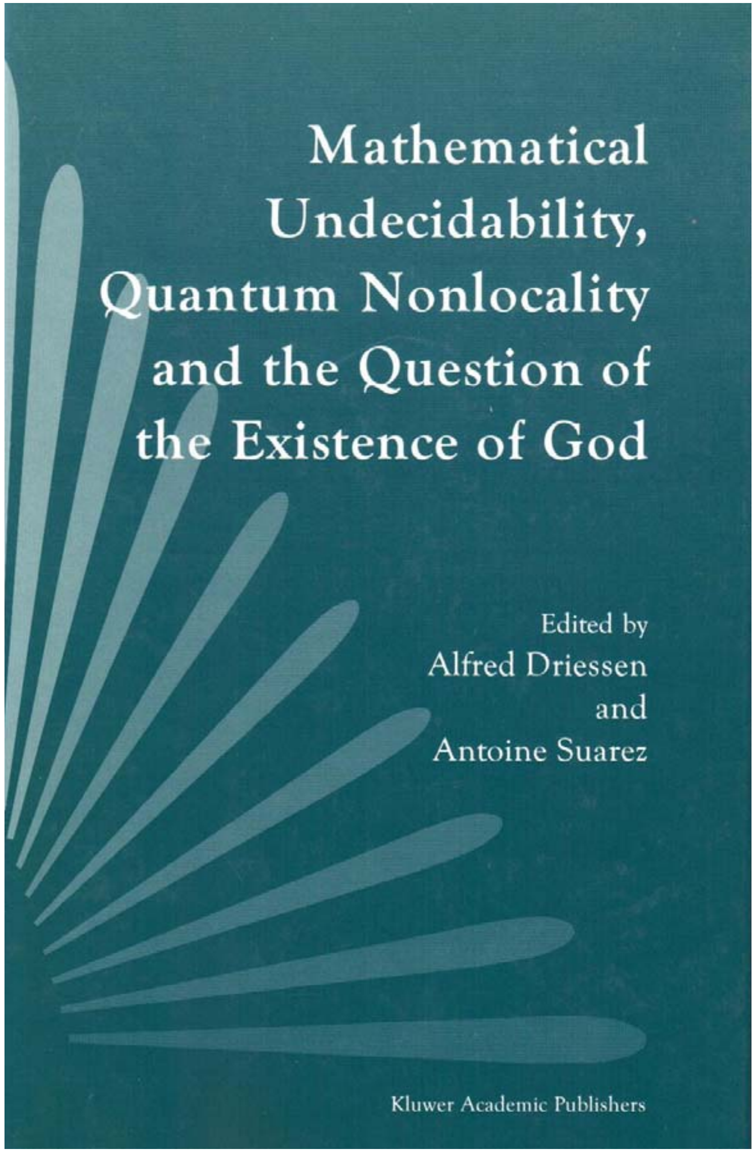
falsability criterion



A theory is scientific if it is refutable by a conceivable event (experiment), if it is "falsable".

Stricto sensu, a scientific theory is never verified in a definite way, but it can successfully pass many probes in each of which it could be refuted (falsified).

Practical certainty about some theories can be reached.



H. C. Reichel:
"The hypothesis of *randomness* is *unprovable* in principle, and conversely, the *teleological* thesis is *unrefutable* in principle.

I add:
In this sense, the teleological hypothesis is not scientific either, since it does not satisfy Popper's falsability criterion.

Statements about the existence of design may be, occasionally, *very reasonable philosophical statements,* but are not scientific in a strict sense.

Statements about the absence of design are not scientific either, since randomness cannot be proved.

Finality hypothesis is not scientific (*stricto sensu*) because it's irrefutable in the general case.

Chance hypothesis is not scientific (*stricto sensu*) because it's unverifiable in each particular case.

What or who determines the future?

Initial condition (determinism)	ruled out
Human design	non polemic
External (intelligent) design	very polemic
Randomness	unprovable

Science may suggest the existence of design, but cannot prove it. It cannot prove its absence either. In particular, it cannot prove the absence of Intelligent Design (or its presence).

Questions about finality may be of high philosophical interest, but must remain outside the scientific debate.

“Provisional” limits of science

- Pushed further by any sound research program.
- Those facts which, being currently unknown, can eventually be unveiled by scientific progress.

Realities *within the domain* of science

and *within the reach* of science

“External” limits of science

- Creation from *metaphysical* nothing
- Conscience as *subjective* experience
- Ethics (human rights, …)
- Aesthetics (artistic experience)
- Concept of God
- …

realities *outside the domain* of science and

therefore *outside the reach* of science

(element of *immateriality*)

(Popper’s net analogy)

“Internal” limits of science

- Incompleteness (Gödel)
- Undeterminacy (Poincaré, Heisenberg)

realities *within the domain* of science

but *outside the reach* of science

future not contained in the present – nonobservable causes

mathematical truths not reachable by human reason

→ The myth of an all-explaining science is unjustifed,
a remnant of 19th century world view.

→ aperture to the idea of trascendence
(God, soul, concepts outside science)