Self-calibration in astronomy

A general principle as applied to Gaia

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Outline

- Classical calibration versus self-calibration:
  An example from computer vision

- Some examples from astronomy

  Meridian circle observations

  Plate overlap technique

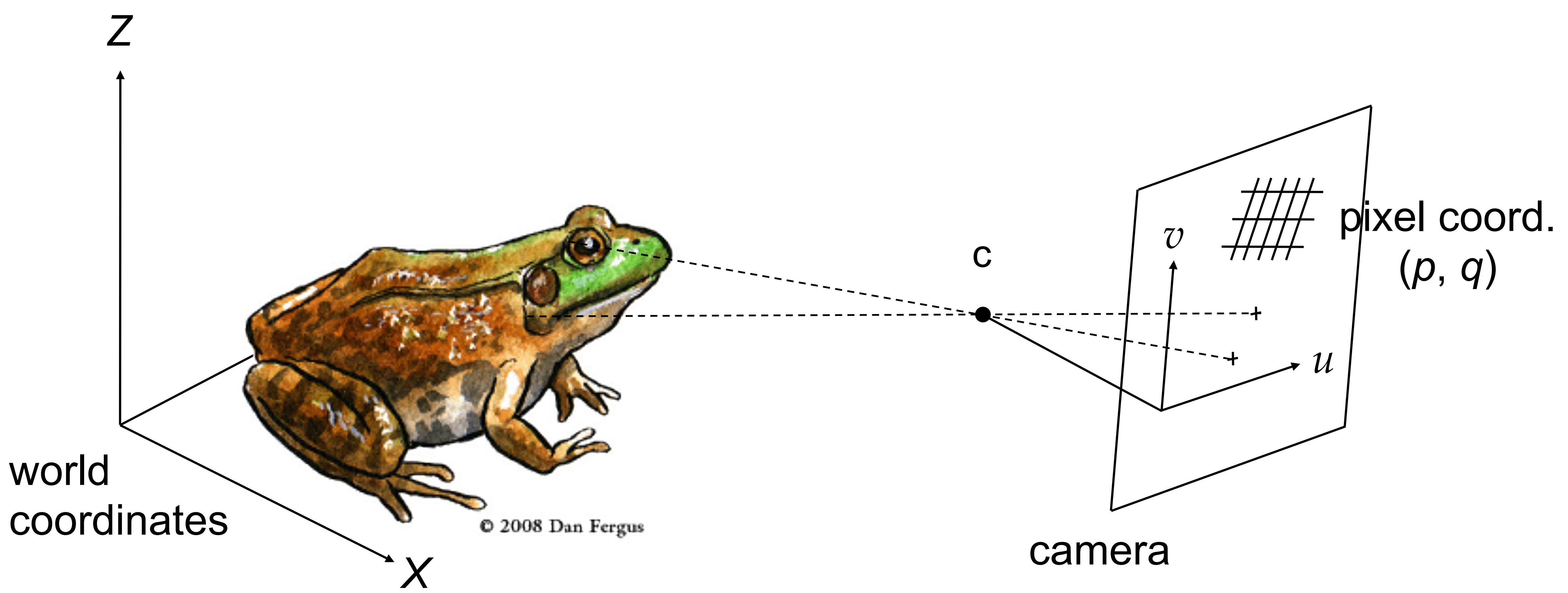
  Hubble Space Telescope

  Radio interferometry

  Gaia

- Summarizing the concept

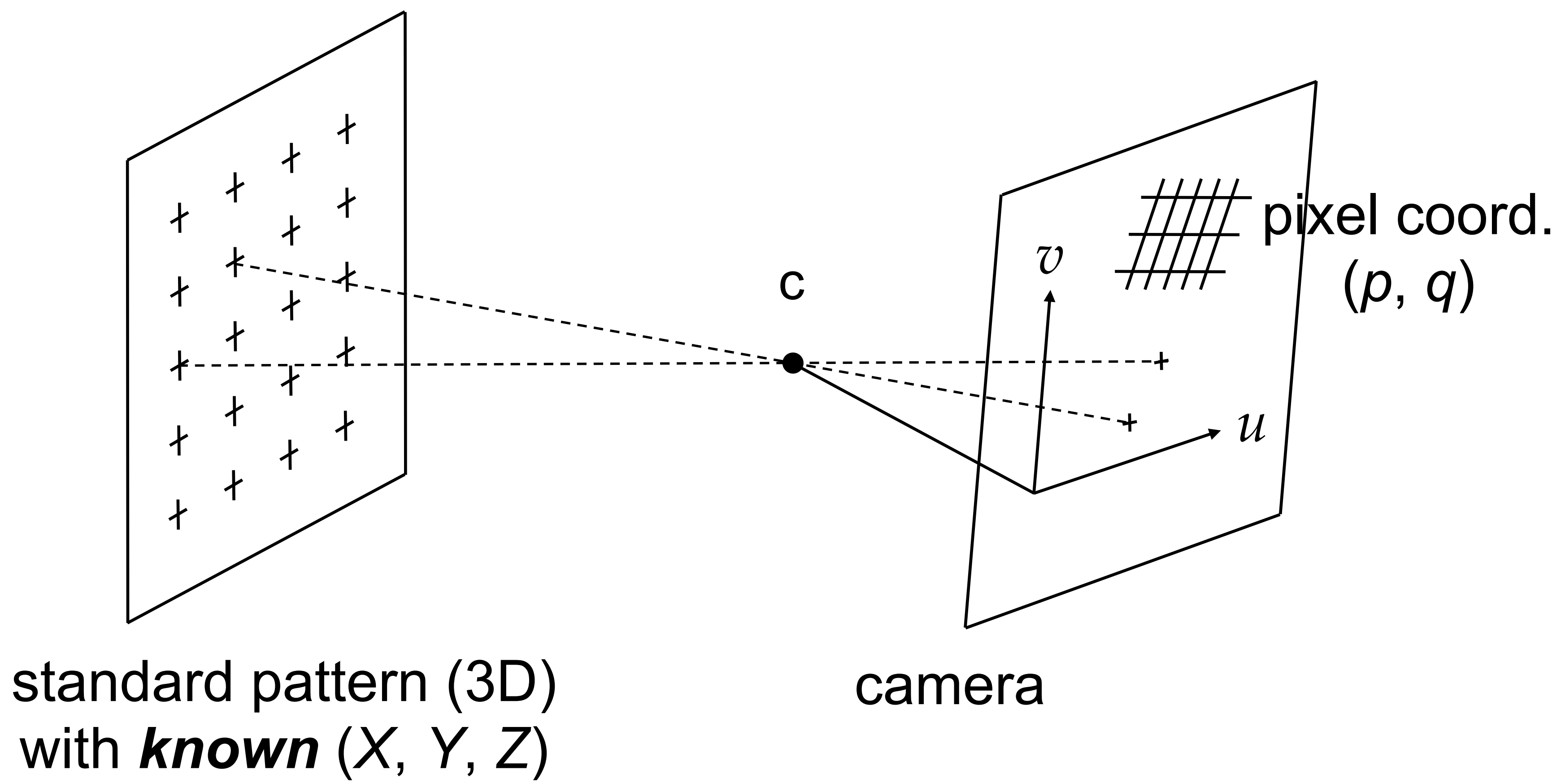
Computer vision: Camera calibration



- For a pinhole camera, the transformation (X, Y, Z) → (u, v) is described by a rotation and translation (six ***extrinsic*** camera parameters)
- The ***intrinsic*** camera calibration (u, v) → (p, q) needs, in the most general linear model, six additional parameters:

$$p = p_0 + p_1 u + p_2 v, \qquad q = q_0 + q_1 u + q_2 v$$

Computer vision: Classical calibration method



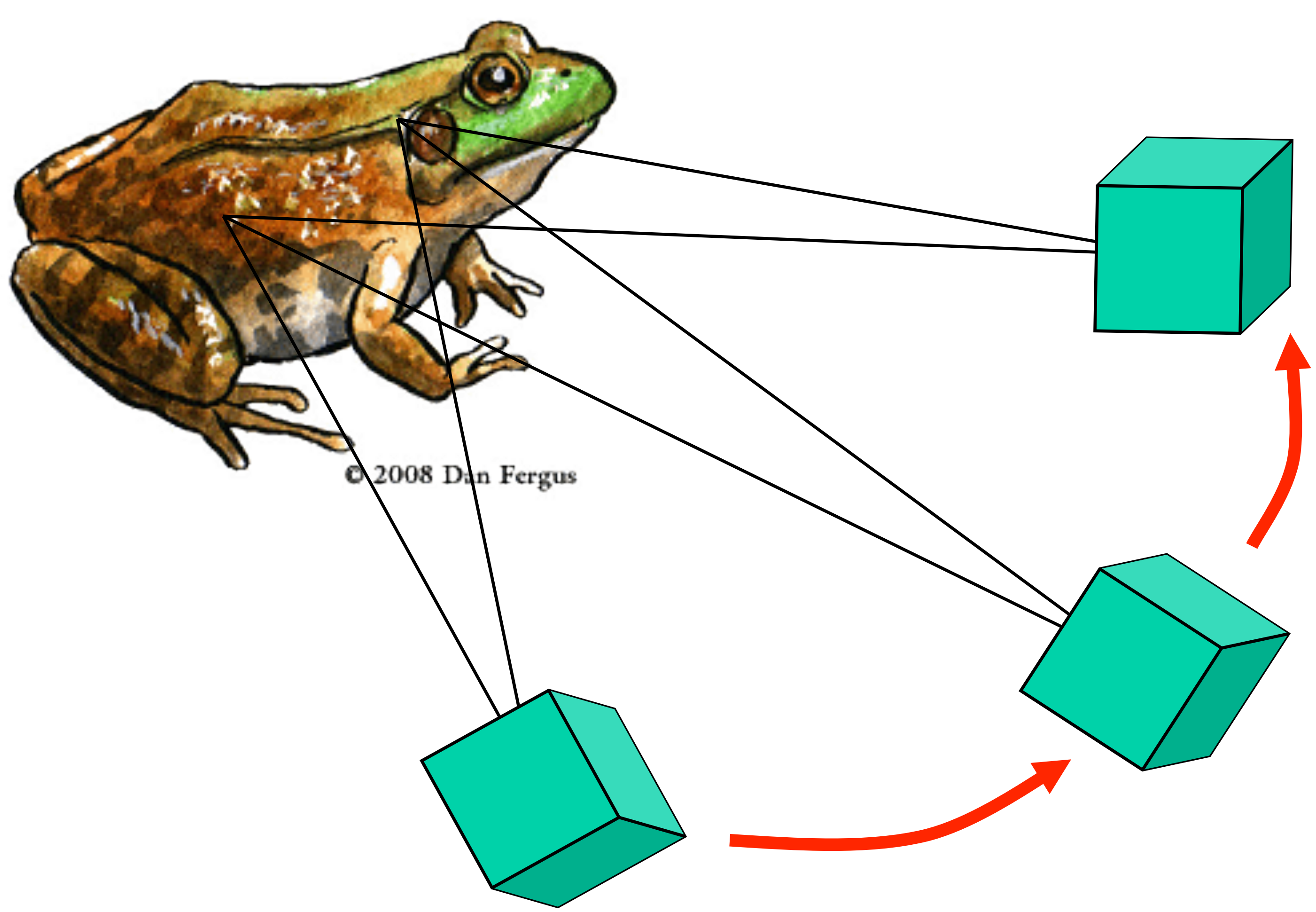
Measuring a sufficient number of correspondences

$(X, Y, Z) \rightarrow (p, q)$

allows to determine the camera parameters

Computer vision: Self-calibration

- Self-calibration method using a moving camera



Maybank & Faugeras (1992) showed that with ≥ 7 point correspondences and ≥ 3 camera positions, the extrinsic and intrinsic camera parameters can be recovered up to a scale factor.

Assumptions:
- intrinsic params constant
- object constant in (X,Y,Z)

Camera self-calibration: The matematician’s approach

- Camera parameters are recovered semi-analytically, e.g. using the invariance of conic sections under the pinhole projection

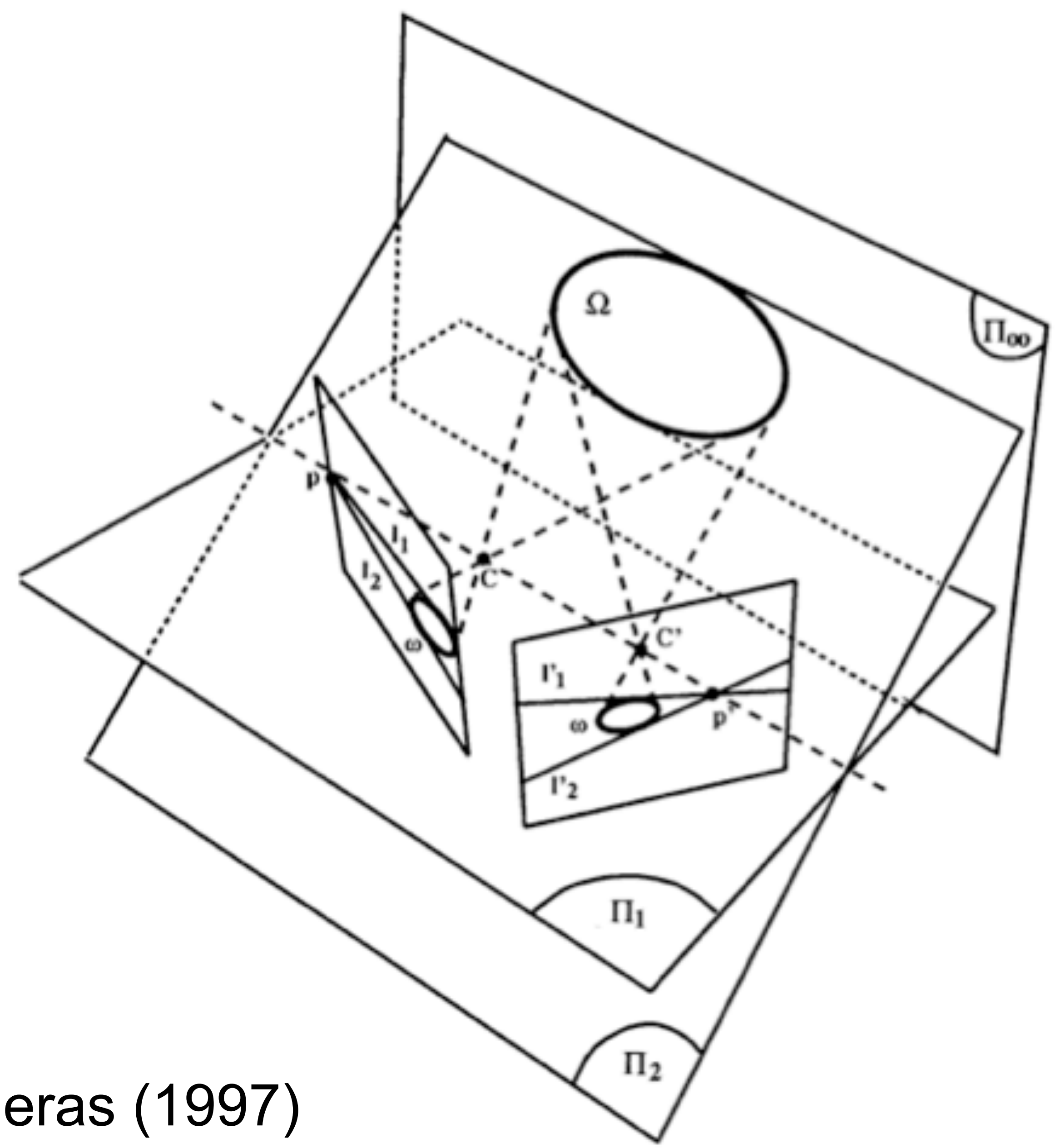


Figure from
Luong & Faugeras (1997)

Camera self-calibration: The simple astronomer's approach

- Unknowns:
  - (X_i, Y_i, Z_i), $i = 1, \ldots, n$ for the n points in the (fixed) object
  - $(x_j, y_j, z_j, \varphi_j, \theta_j, \psi_j)$, $j = 1, \ldots, m$ for the m camera positions/orientations
  - $(p_0, p_1, p_2, q_0, q_1, q_2)$ for the (fixed) intrinsic camera parameters
  - in total $3n + 6m + 6$ unknown parameters $\boldsymbol{x}$
- Observations:
  - $(p_{obs}, q_{obs})_{ij}$ pixel coordinate pairs for the nm point/camera combinations
  - in total $2nm$ measurements (or a subset of them)
- Theoretical model: $p_{obs} \approx p(\boldsymbol{x})$, $q_{obs} \approx q(\boldsymbol{x})$
- Set up linearized observation equations and solve by least-squares:

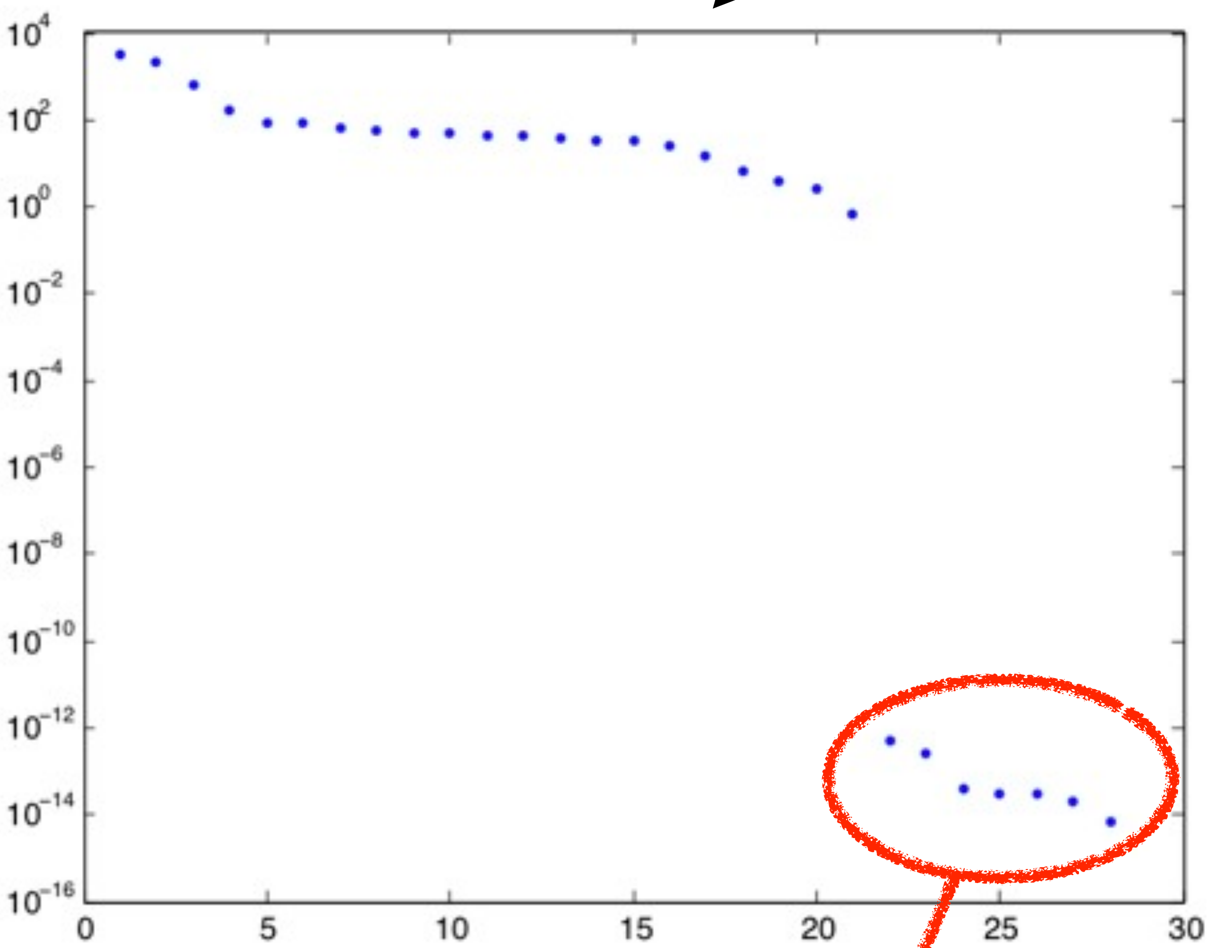
$$\left.\begin{aligned} p_{obs} - p(\boldsymbol{x}) &= \partial p(\boldsymbol{x})/\partial \boldsymbol{x} \,.\, \boldsymbol{\Delta x} \\ q_{obs} - q(\boldsymbol{x}) &= \partial q(\boldsymbol{x})/\partial \boldsymbol{x} \,.\, \boldsymbol{\Delta x} \end{aligned}\right\} \Rightarrow \boldsymbol{\Delta x} \Rightarrow \boldsymbol{x} := \boldsymbol{x} + \boldsymbol{\Delta x}, \quad \text{etc}$$

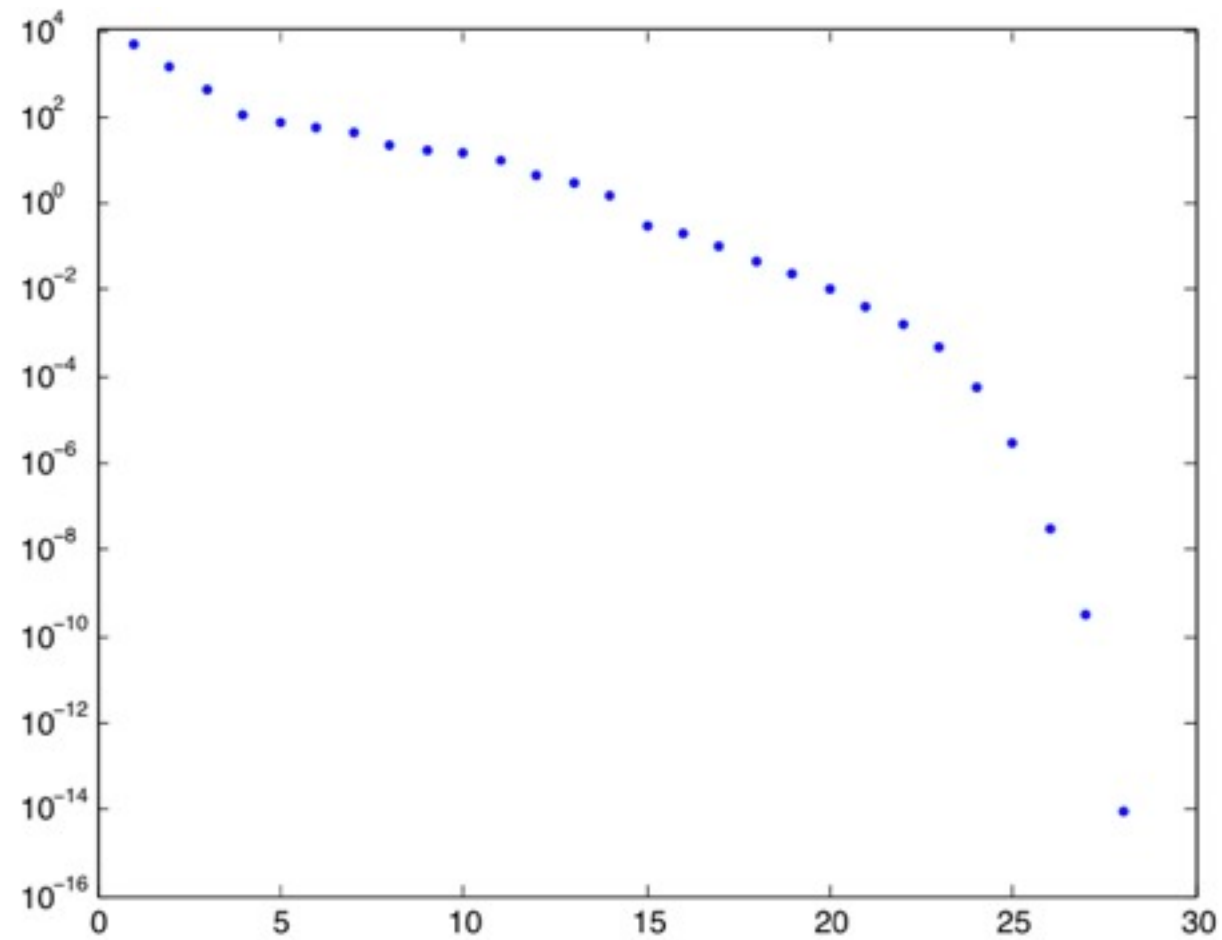
Resulting system of linear equations should be singular due to the undefined spatial origin, axis orientation, and scale!

Singular, well-posed and ill-posed problems

Singular problems are not necessarily bad (if you understand why)

Singular values for a well-posed and ill-posed problem:





Seven d.o.f. for the undefined
origin, orientation, and scale

Some features of the camera self-calibration

- The test object need not be known, but it must be constant
  - Could extend e.g. to case where each point has a uniform motion: $X_i(t) = X_i^{(0)} + X_i^{(1)}t$ (twice as many object parameters to solve)
- The extrinsic camera parameters (translation, rotation) need not be known, and must vary
- The intrinsic camera parameters (scale, distortion) must be constant
  - Much more general models than the pinhole projection + linear transformation are possible, e.g. optical distortion + small-scale irregularities. Cf. human vision!
- Some properties remain undetermined (e.g., a common scale factor)

Amélioration dans les réductions d'un catalogue méridien. Méthode de "synthèse".

par

P. LACROUTE (1964)

Résumé.

On montre qu'il est possible de diminuer notablement les erreurs aléatoires d'un catalogue méridien en tenant compte de la comparaison des résultats obtenus dans différentes soirées pour améliorer les constantes. La méthode permet également l'emploi de poids significatifs et offre de grandes garanties au point de vue des erreurs systématiques.

IMPROVMENT IN THE REDUCTIONS OF A MERIDIAN CATALOGUE. METHOD OF « SYNTHESIS ».

Abstract.

It can be showed that it is possible to reduce notably the random errors of a meridian catalogue, taking account of the comparison between the results obtained in differents evenings to improve the constants. This method also enable us to use significative weights and offers serious surety as far as systematic errors are concerned.

УЛУЧШЕНИЯ В СОКРАЩЕНИЯХ МЕРИАННАГО КАТАЛОГА СПОСОБ «СИНТЕЗА»

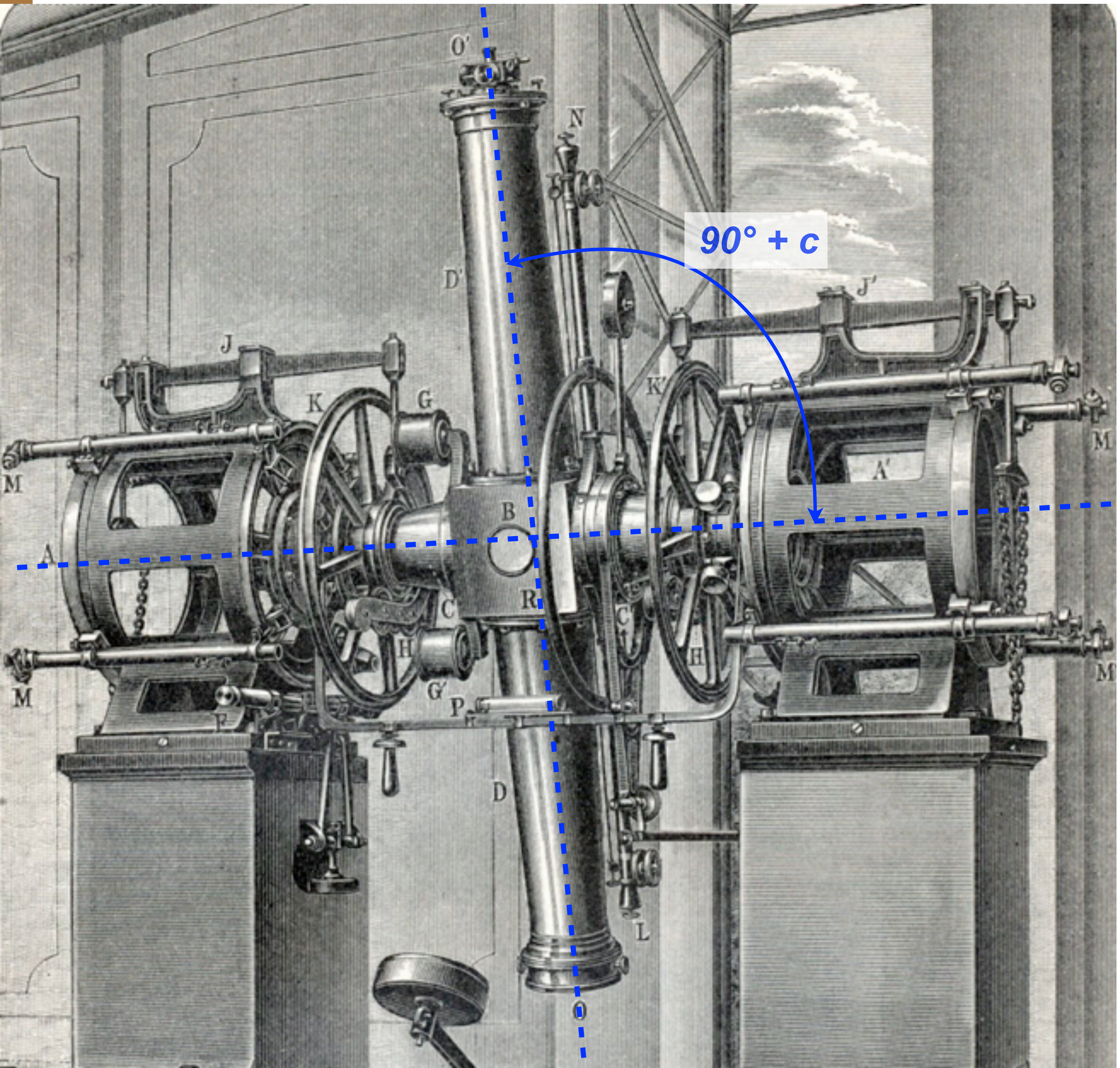
П. ЛАКРУТ.

Резюме.

Существует возможность значительно сократить случайные ошибки в меридианном каталоге, если принять во внимание отношение между результатами полученными во время различных сеансов, чтобы улучшить константы. Этот способ позволяет также применить всю тяжесть его значения и дает большую гарантию в смысле систематических ошибок.

Pierre Lacroute (1906-1993), "Father of space astrometry", director of l'Observatoire de Strasbourg 1946-76

Calibration parameters for a meridian circle



The meridian circle at Strasbourg observatory, ca 1895

a = azimuth error
b = level error
c = collimation error

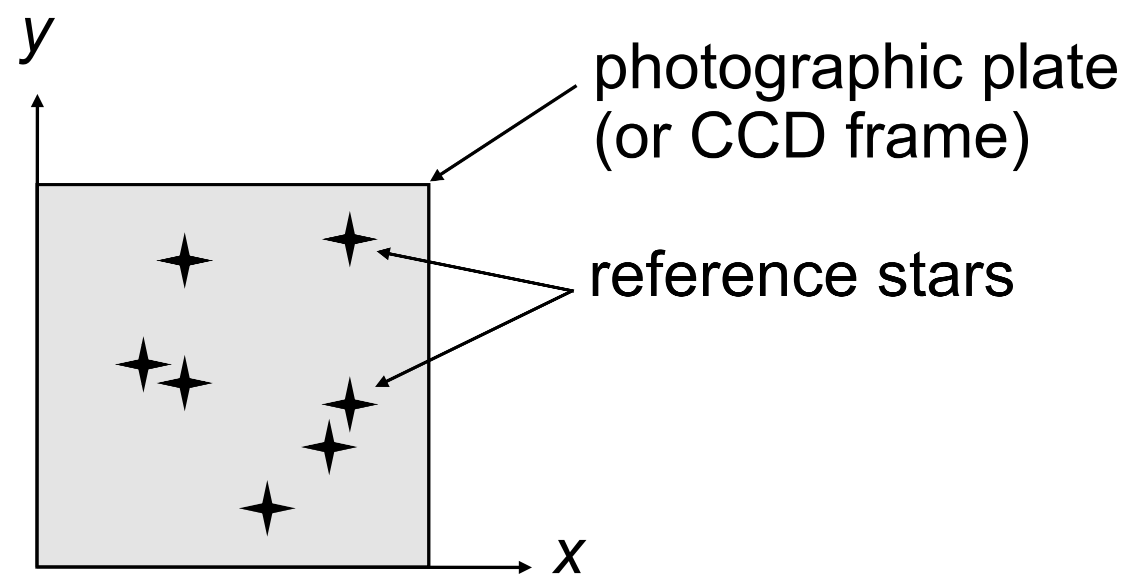
Tobias Mayer's formula (1756):

$$\Delta t \cos \delta =$$
$$= a \sin(\varphi - \delta) + b \cos(\varphi - \delta) + c$$

a, b, c are improved by using constraints set by the same star being observed on consecutive nights

Classical small-field astrometry

- Transformation
  F: (x, y) ↔ (α, δ)
  established by means
  of reference stars
  (with ”known” α, δ)



- (α, δ) for other stars obtained by applying transformation to measured coordinates: (α, δ) = F(x, y)
- Problems:
  - low density of reference stars (e.g. Hipparcos, Tycho-2)
  - measurements often more precise that reference stars
  - essentially an interpolation method: errors increasing towards the edges (extrapolation); systematic catalogue errors cannot be removed

Plate overlap method

Über die Reduktion von photographischen Sternpositionen und Eigenbewegungen

Von HEINRICH EICHHORN, Eau Gallie, Florida*)

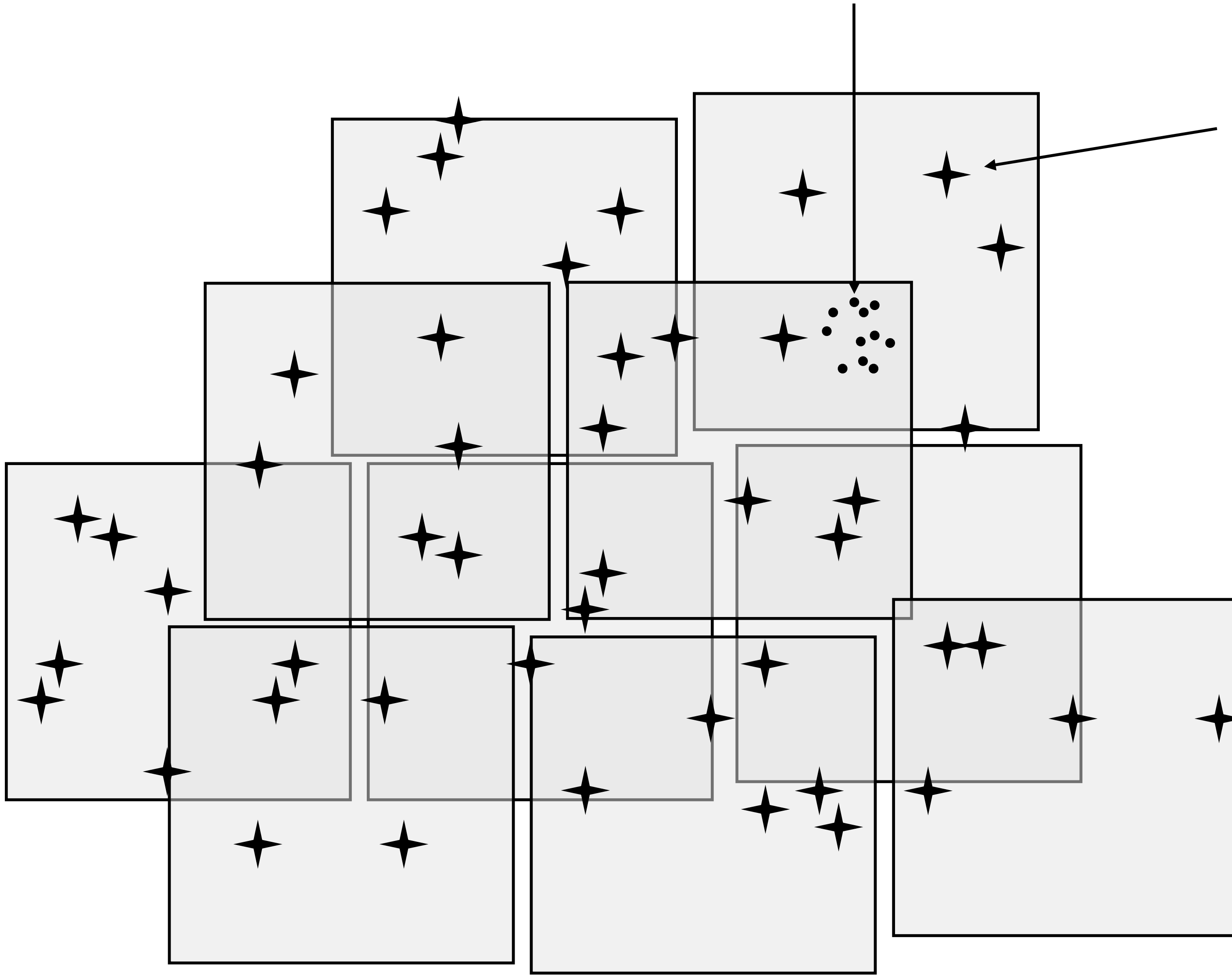
(Eingegangen 1959 September 26)

Summary

The accuracy of the reduction of photographic positions to a given system of reference stars can be considerably improved, or the number of reference stars reduced without resulting loss of accuracy by regarding not only the plate constants, but also the field stars' coordinates (and proper motions, where required) as unknowns in a least squares solution. This means that overlapping plates are "tied together" and that positions of stars resulting from measurements on overlapping plates will not show systematic differences.

Plate overlap method

- Every (non-reference) star that appears on two plates (i, j) provides a constraint: $F_i(x_i, y_i) = F_j(x_j, y_j)$ [= (α, δ)]

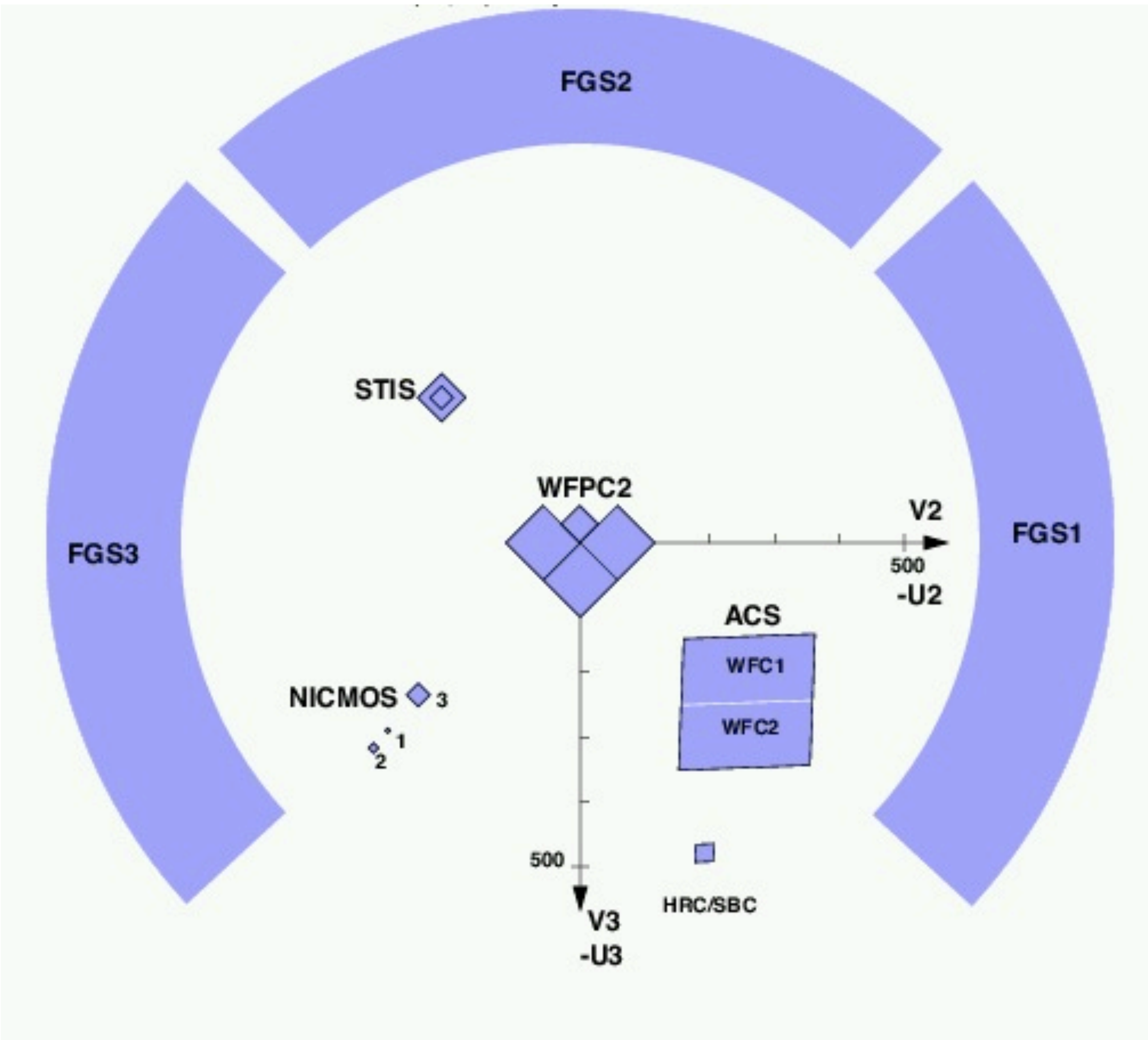


Lots of non-ref stars ⇒ many more constraints than calibration param. ⇒ much better accuracy

Can we do without reference stars altogether?

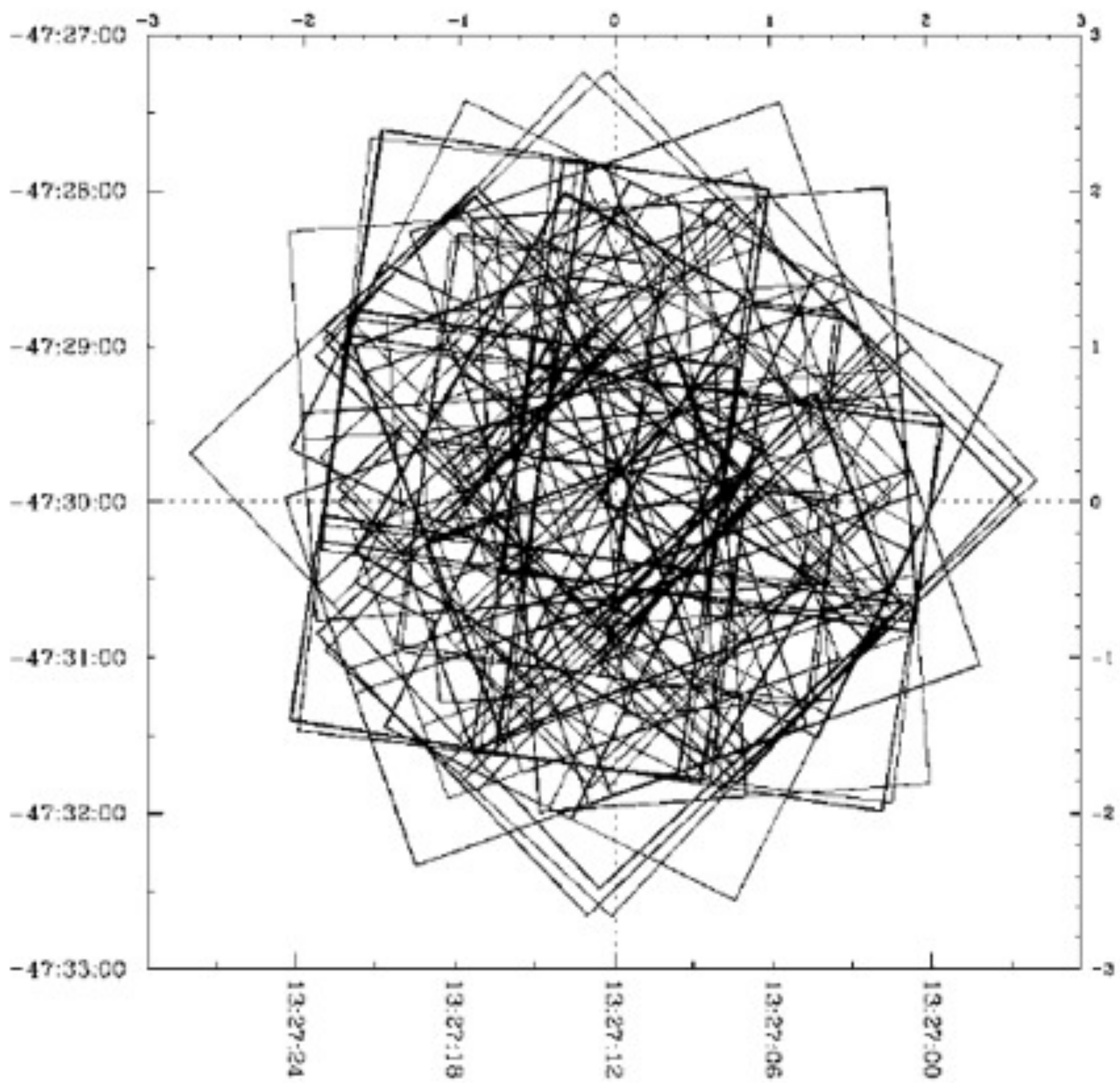
Astrometry with HST: WFPC2, ACS and FGS

- HST focal plane layout before SM4

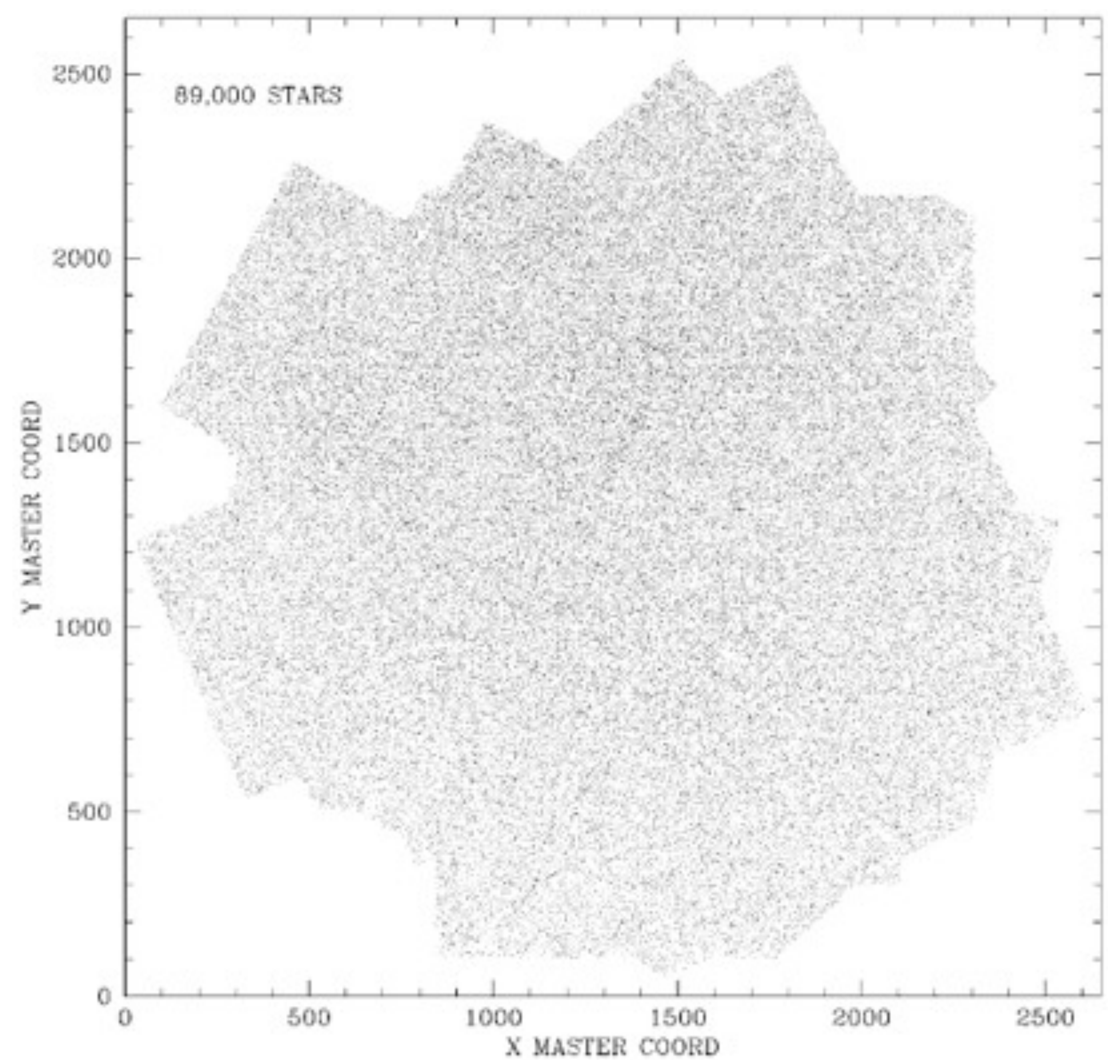


HST calibrations: WFPC2 example

- Anderson & King (2003) PASP 115, 113 (using ω Cen)



Pattern of exposures

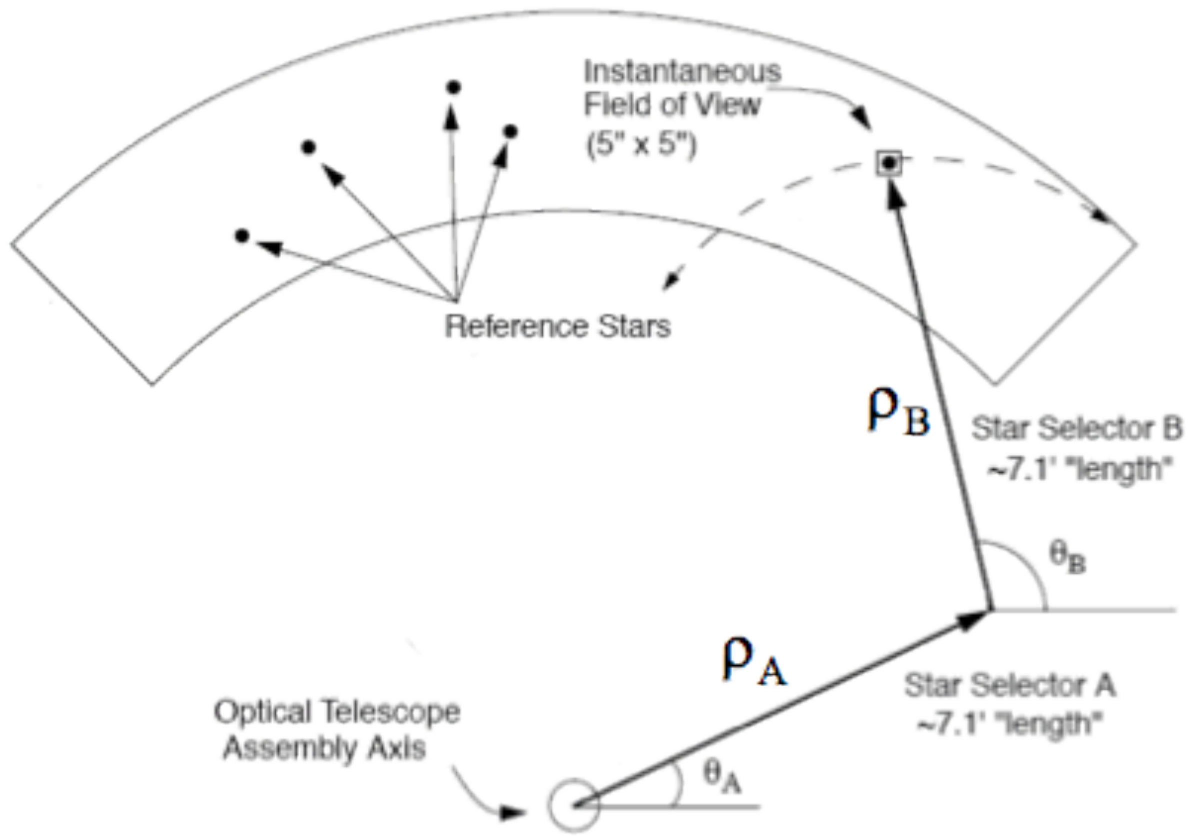


Map of 89,000 stars used

HST calibrations: Fine Guidance Sensors (FGS)

Calibration parameters:

- ρ_A, ρ_B (arm lengths)
- κ_A, κ_B (offsets in θ_A, θ_B)
- a_{ij}, b_{ij} (distortion)



$$
\begin{aligned}
x' = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{02}y^2 + a_{11}xy + a_{30}x(x^2+y^2) + a_{21}x(x^2-y^2)\\
+a_{12}y(y^2-x^2) + a_{03}y(y^2+x^2) + a_{50}x(x^2+y^2)^2 + a_{41}y(y^2+x^2)^2\\
+a_{32}x(x^4-y^4) + a_{23}y(y^4-x^4) + a_{14}x(x^2-y^2)^2 + a_{05}y(y^2-x^2)^2
\end{aligned}
$$

$$
\begin{aligned}
y' = b_{00} + b_{10}x + b_{01}y + b_{20}x^2 + b_{02}y^2 + b_{11}xy + b_{30}x(x^2+y^2) + b_{21}x(x^2-y^2)\\
+b_{12}y((y^2-x^2) + b_{03}y(y^2+x^2) + b_{50}x(x^2+y^2)^2 + b_{41}y(y^2+x^2)^2\\
+b_{32}x((x^4-y^4) + b_{23}y(y^4-x^4) + b_{14}x(x^2-y^2)^2 + b_{05}y(y^2-x^2)^2
\end{aligned}
$$

HST calibrations: Fine Guidance Sensors (FGS)

Rotation and offset of FGS 1R in one calibration set (using M35 cluster)

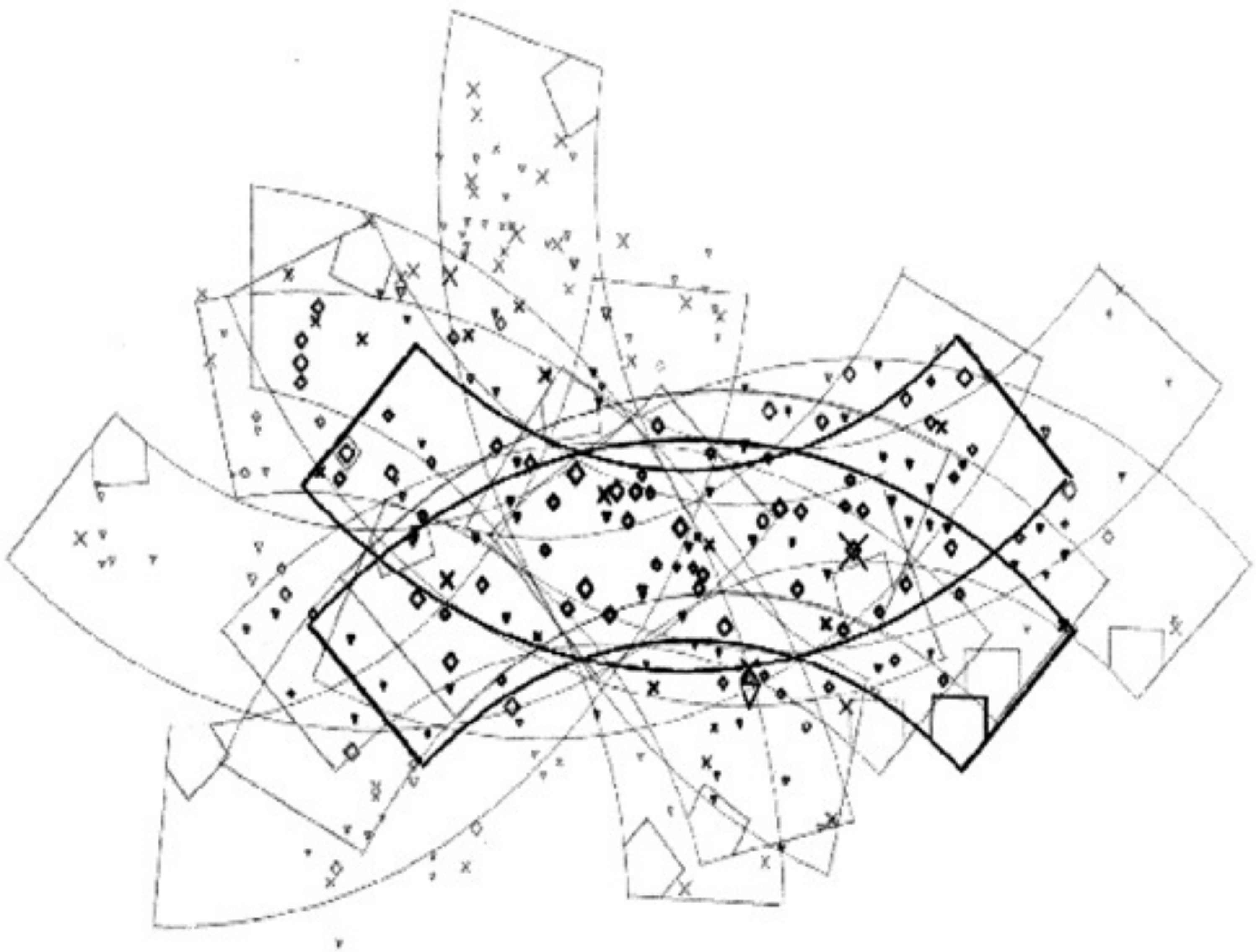
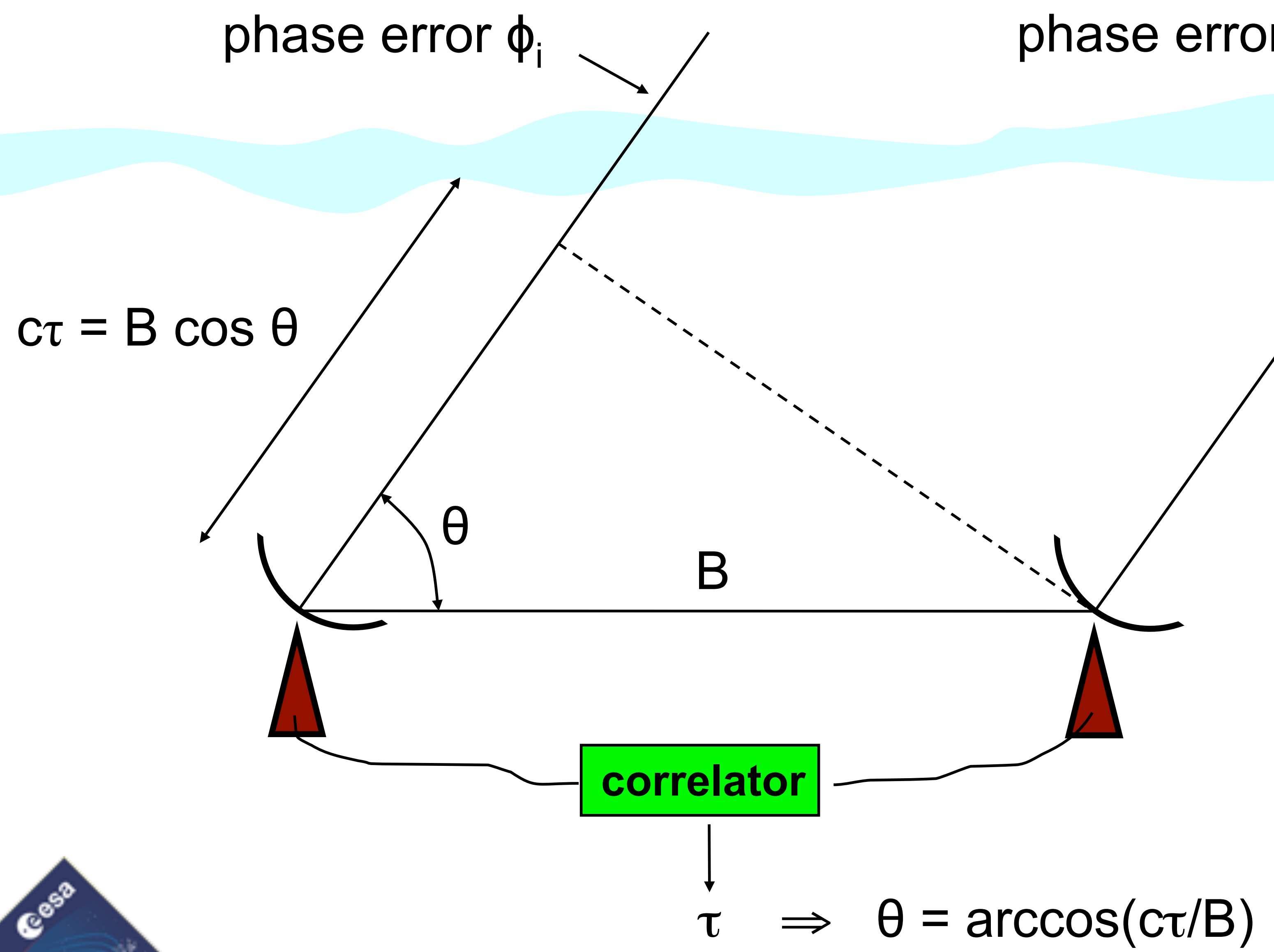


Figure 1. Rotation and Offsets of FGS 1R Winter 2000 OFAD.

McArthur, Benedict & Jefferys, 2002 HST Calibration Workshop, p. 373

Self-calibration in radio interferometry

- An interferometer measures coherence in the electric field between pairs of antennae (baseline)



phase errors

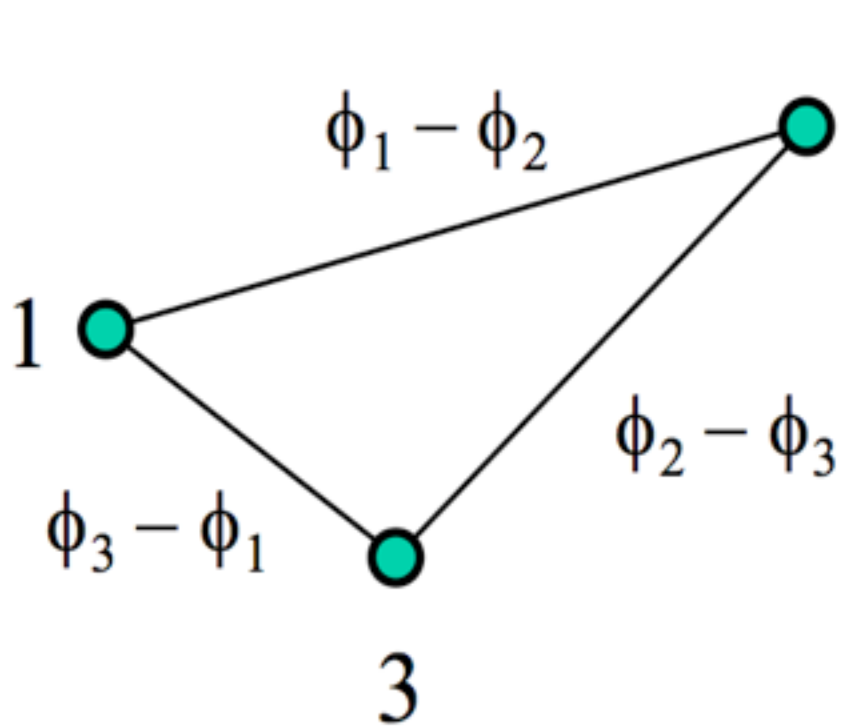
$$\Downarrow$$

$$\Delta\theta \propto \phi_i - \phi_j$$

Self-calibration in radio interferometry

- With N = 3 antennas the instantaneous phase errors ***when observing the same point source*** must satisfy the constraint

$(\phi_1 - \phi_2) + (\phi_2 - \phi_3) + (\phi_3 - \phi_1) = 0$ (closure phase)

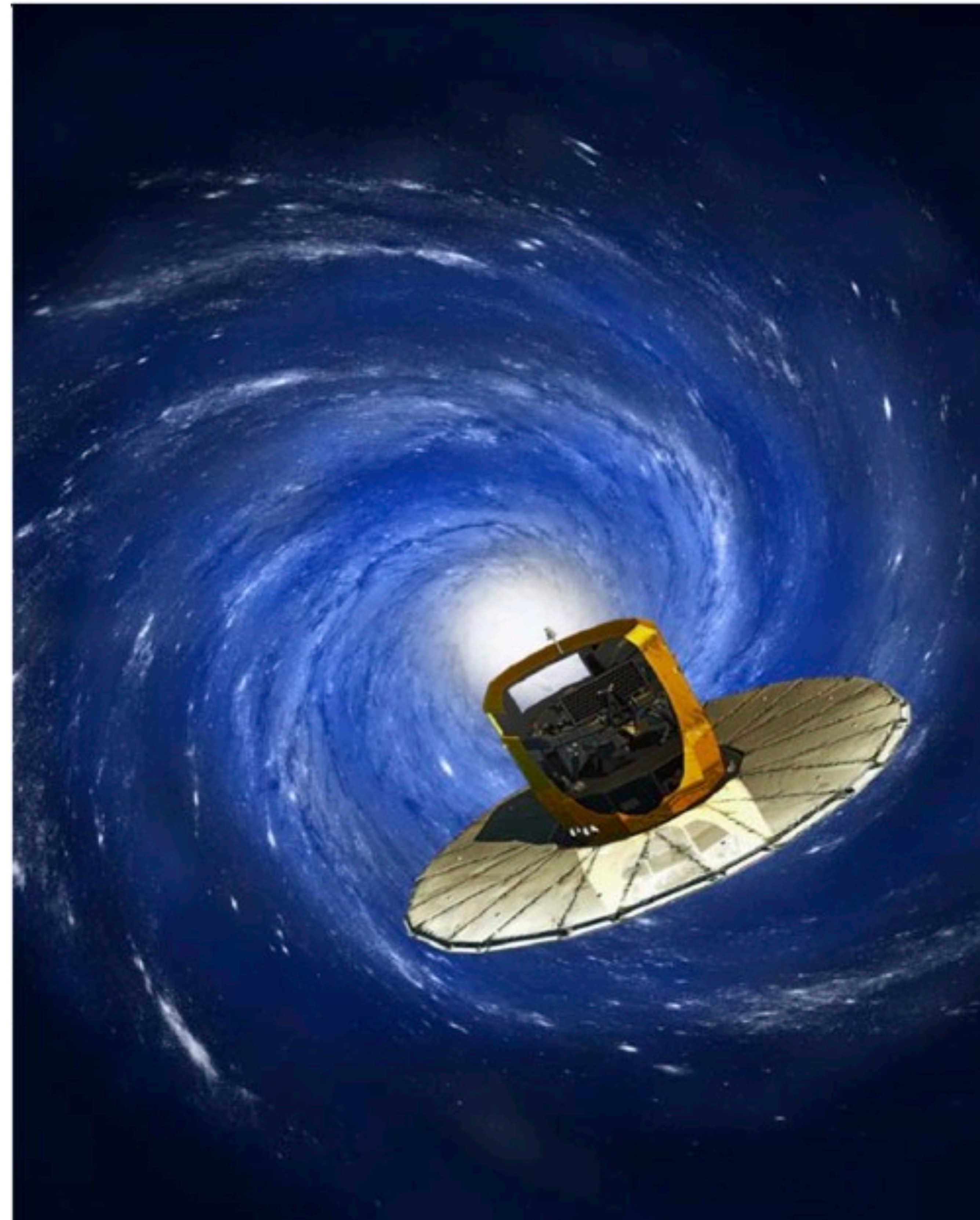


N	N_{constr}
2	0
3	1
4	3
5	6
6	10
7	15
:	:
27	325

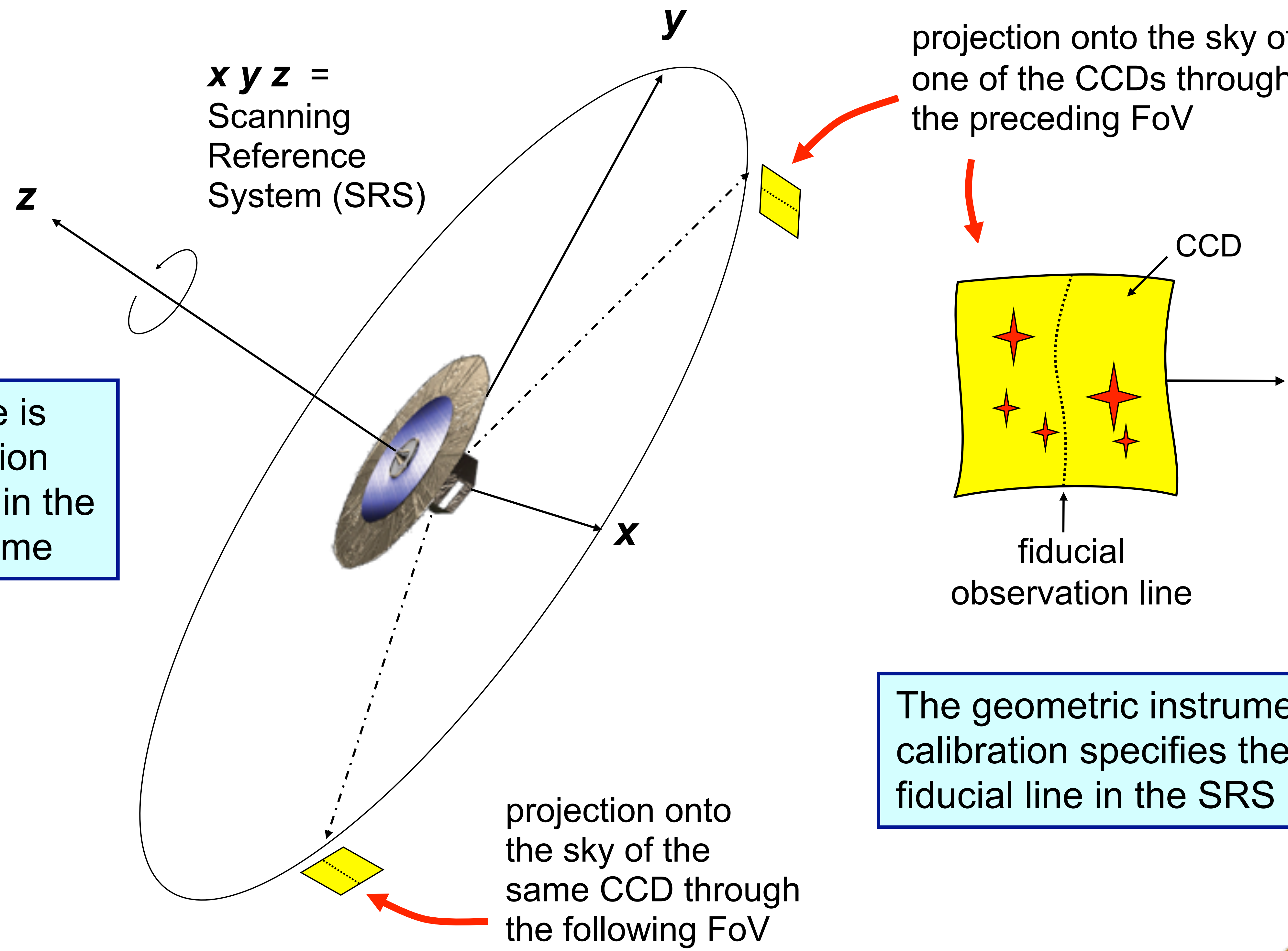
Closure-phase allows to determine $\phi_j(t)$ up to a constant offset (works best for large N)

Gaia

- Launch 2013
- Measures the astrometric parameters (positions, parallaxes, proper motions) for 10^9 stars
- Accuracy ~ 10^{-10} rad (20 μas)
- Gaia is a self-calibrating instrument (it must be!)



How Gaia works



The attitude is the orientation of the SRS in the celestial frame

The geometric instrument calibration specifies the fiducial line in the SRS

How Gaia works

- Elementary observation:
  a star crosses the fiducial line of a CCD at a certain time *t*

- *t* depends on:

  a. the position of the star on the sky (wanted parameters)
  b. the instantaneous pointing of Gaia
  c. the mapping from spherical coordinates in the SRS frame to CCD pixels

  (nuisance parameters)

- b) and c) are the "calibration" of Gaia - they are needed to transform an observation to a celestial direction

Gaia as a self-calibrating instrument

- Pre-launch laboratory calibration is not feasible to required accuracy
- No special "calibration observations" are made - just regular observations
- Gaia does not rely on any previous astrometric measurements for its calibration
- Many (>10%) of the observations contribute to the calibration (AGIS)
- Instrument stability on short time scales (< 6 hr) is essential (cf. BAM)
- The resulting system of positions and proper motions has six d.o.f. corresponding to the orientation and spin of the reference frame - these have to be fixed from external data (VLBI positions of quasars, assumed zero proper motion of quasars)

Why is self-calibration possible for Gaia?

- We use ***models*** of the stars, attitude and geometric calibration:
  - the stars move uniformly through space
  - the attitude is continuous and smooth
  - the optics and CCD layout are very stable on short time scales (< 6 hr)
- Here we are helped by Nature (most stars are benign) and the clever design of Gaia
- The models impose a very large number of constraints on the possible solutions ($N_{constr} \approx N_{obs}$)

Self-calibration: Some common features

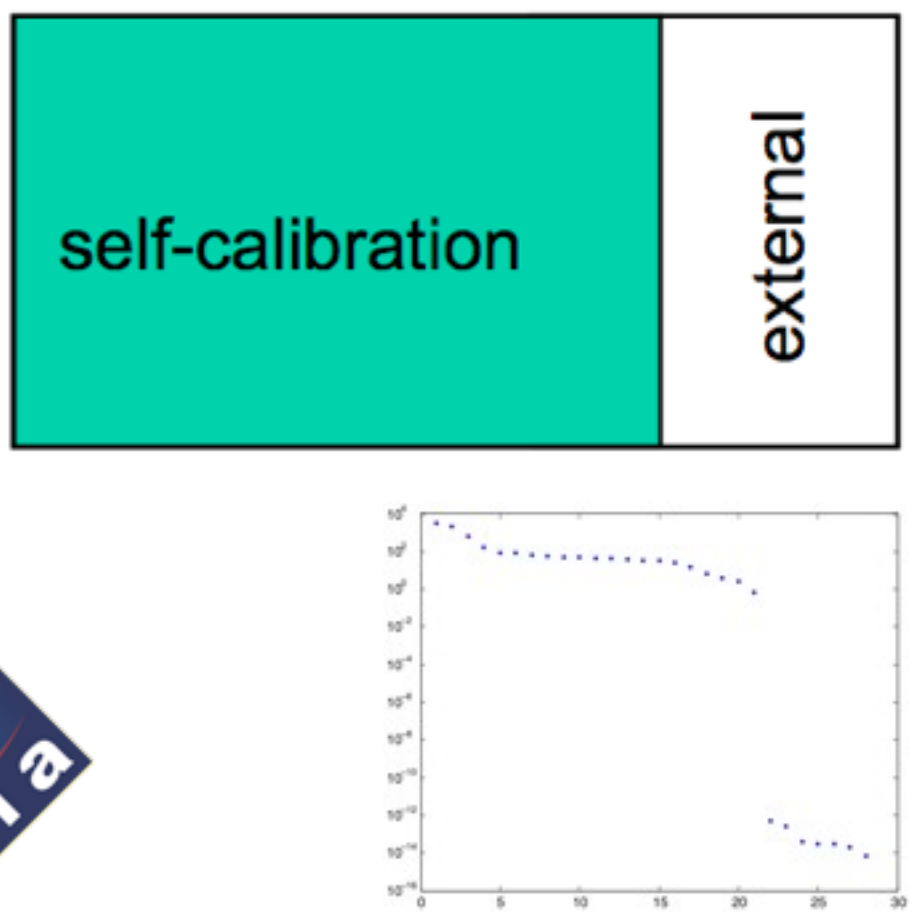
- Self-calibration works by introducing constraints
- A large fraction of the observations produce constraints
- Something is assumed to be constant
- Self-calibration is usually only for a subset of the parameters
  $\Rightarrow$ intrinsic calibration parameters
- Thus external calibration is still needed for some parameters (scale, orientation, arbitrary origin, ...) $\Rightarrow$ extrinsic cal. parameters
- Ideally, there should be a sharp boundary between intrinsic and extrinsic parameters (complete self-calibration = well-posed problem) - next slide
- The constraints interconnect large datasets - hence self-calibration tend to be computationally intensive

Complete and partial self-calibration

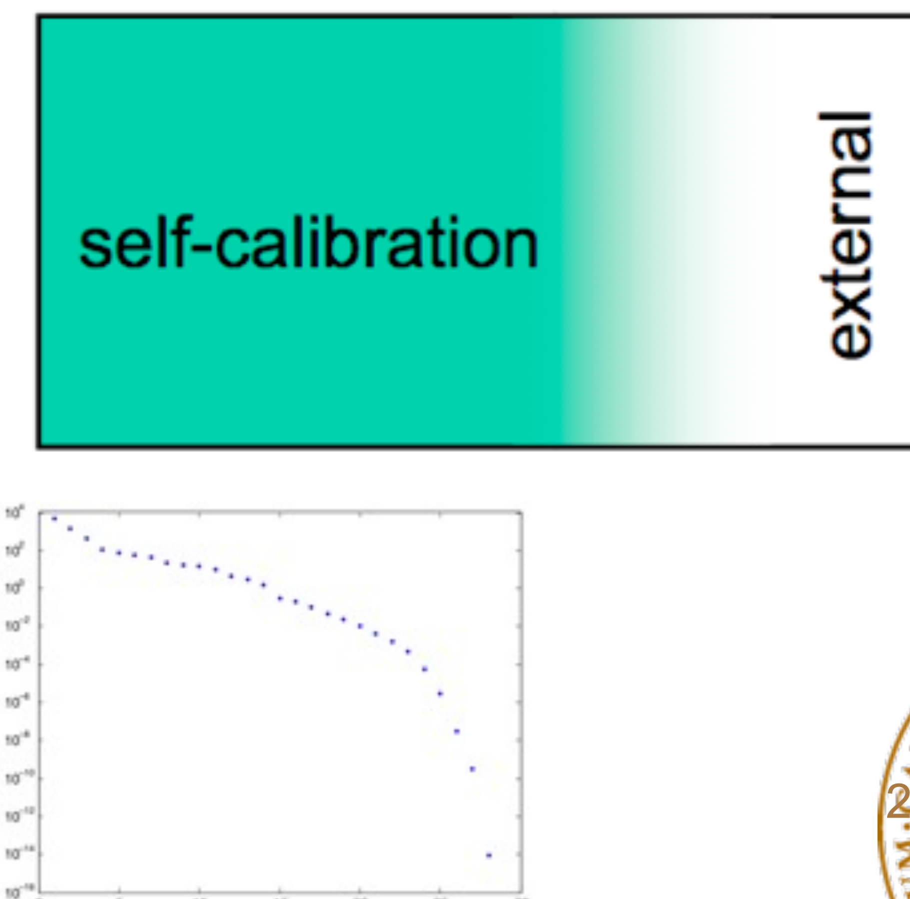
External data are usually needed e.g. to fix scale, orientation, ...

- ***Complete self-calibration:*** external data are only used to fix thos parameters that cannot be determined by self-calibration
- ***Partial self-calibration:*** external data influence also other parameters (example: plate overlap method using reference stars)

Complete self-calibration

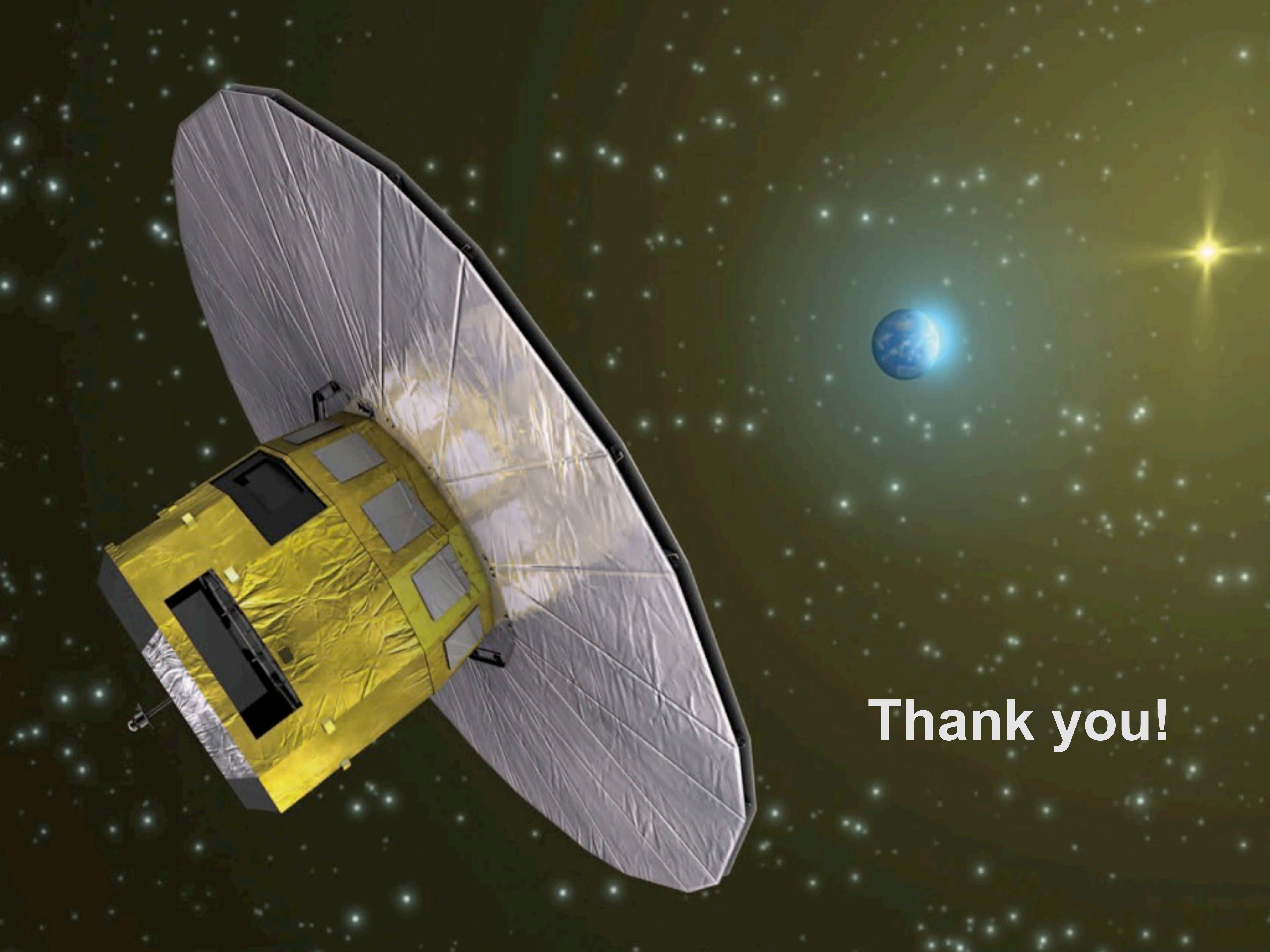


Partial self-calibration



Conclusions

- Self-calibration has been around for some time and is used in many different contexts (under various names)
- Compared with classical techniques it can give a vastly improved calibration
- Understanding the concept may help to apply it more systematically, to new projects, and perhaps in a better way (e.g. complete versus partial self-calibration)
- It is probably a very useful concept, but we still don’t have a clear definition of it and we have only started to explore its full potential ...


Thank you!