Characterizing the astrometric errors in the Gaia catalogue

Berry Holl

Thanks to Lennart Lindegren and David Hobbs

Seminar ESAC, Villafranca, 20 June 2012

LUND UNIVERSITY

Introduction to astrometry & Gaia

Importance of error characterization

Results of my work

What is ‘Astrometry’?

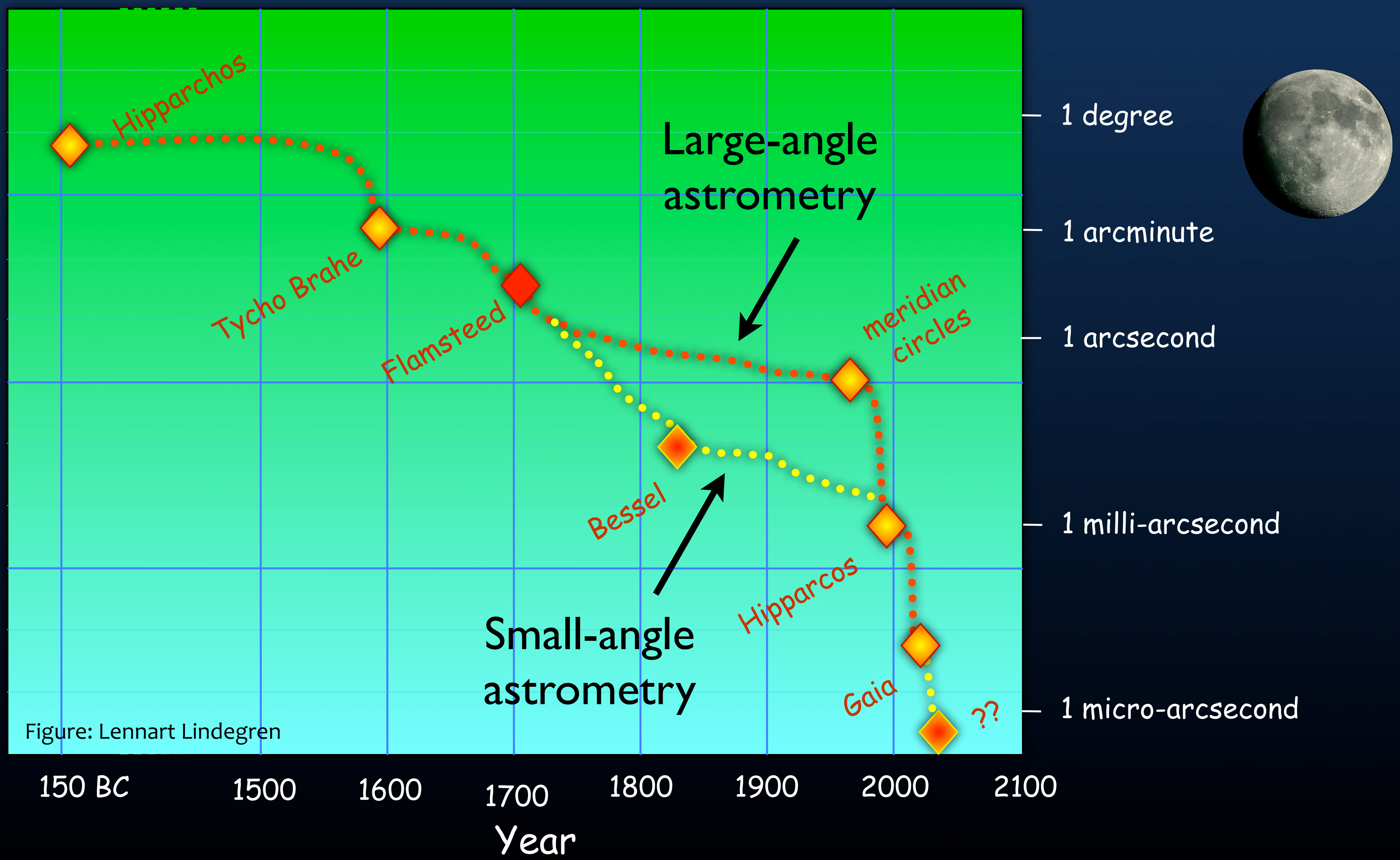
Etymology:

- From Greek *astron* “star”, and *-metria* “a measurement of”
- Used since ~1800 meaning all sorts of measurements
- Since ~1900 meaning the measurement of (angular) positions and motions

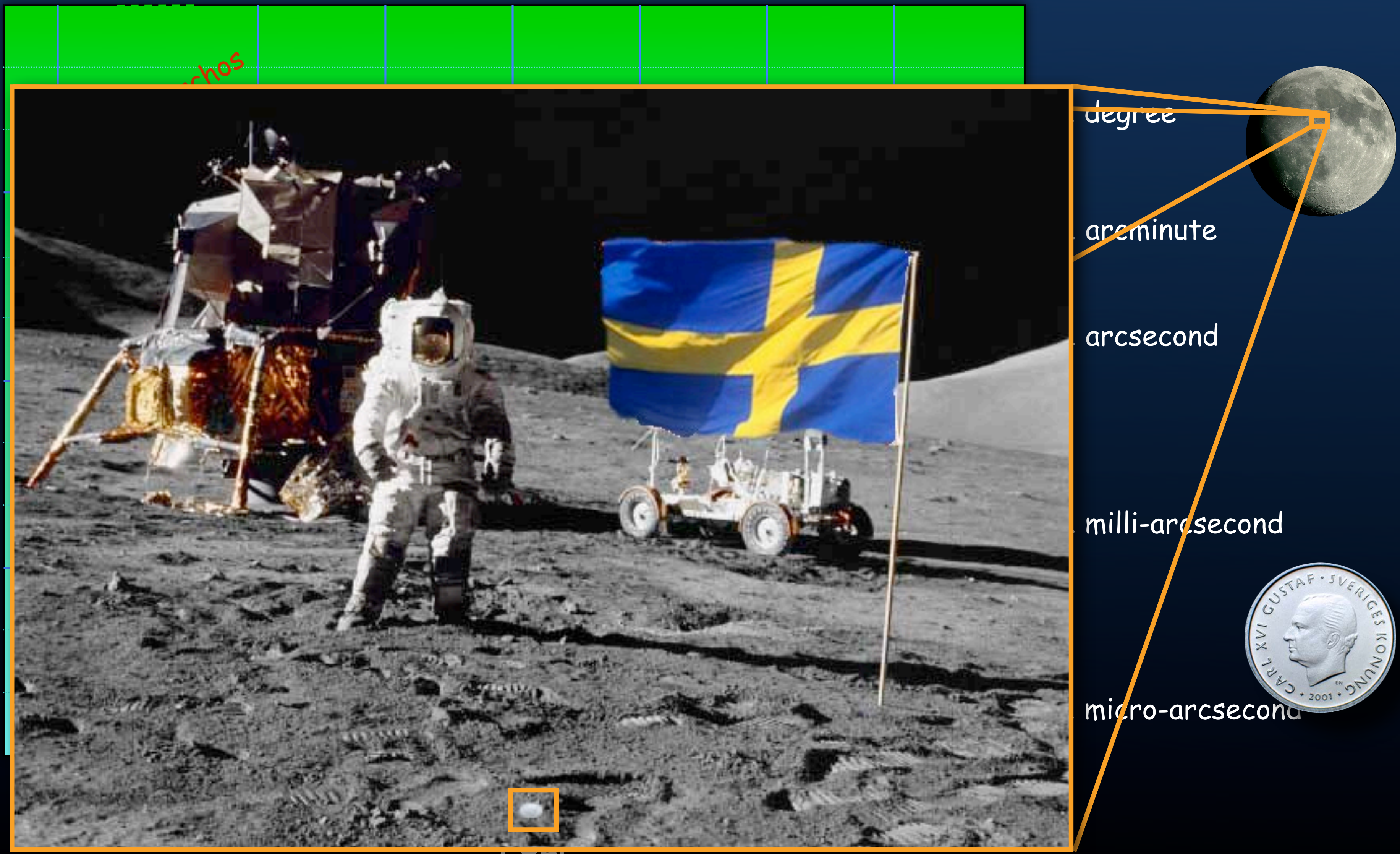
Some definitions:

- ”The scientific measurement of the positions and motions of celestial bodies”
  (The American Heritage® Dictionary of the English Language, Fourth Edition,2009)
- ”Astrometry is the branch of astronomy that involves precise measurements of the positions and movements of stars and other celestial bodies.”
  (Wikipedia)

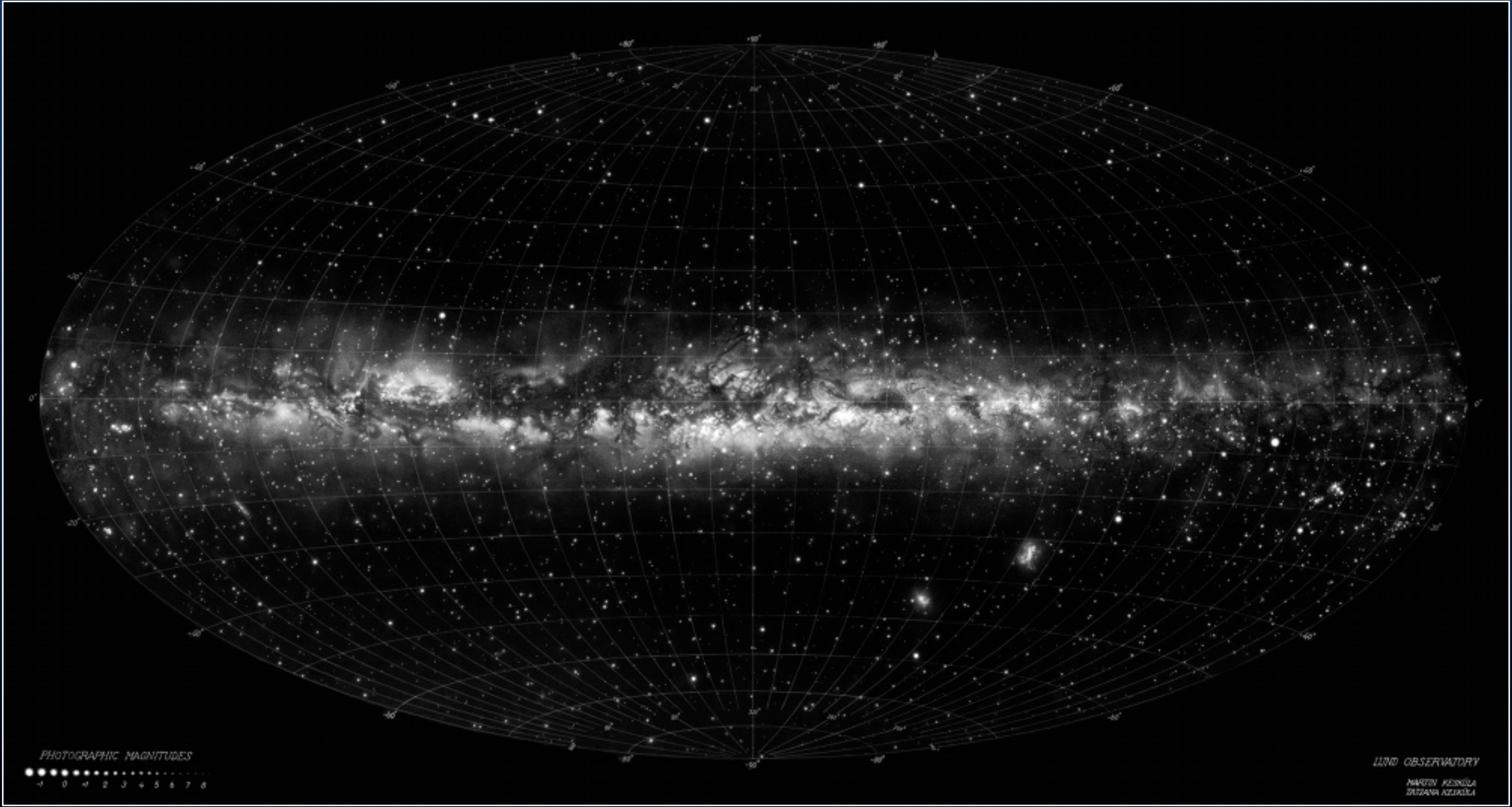
Angular accuracy in optical astrometry



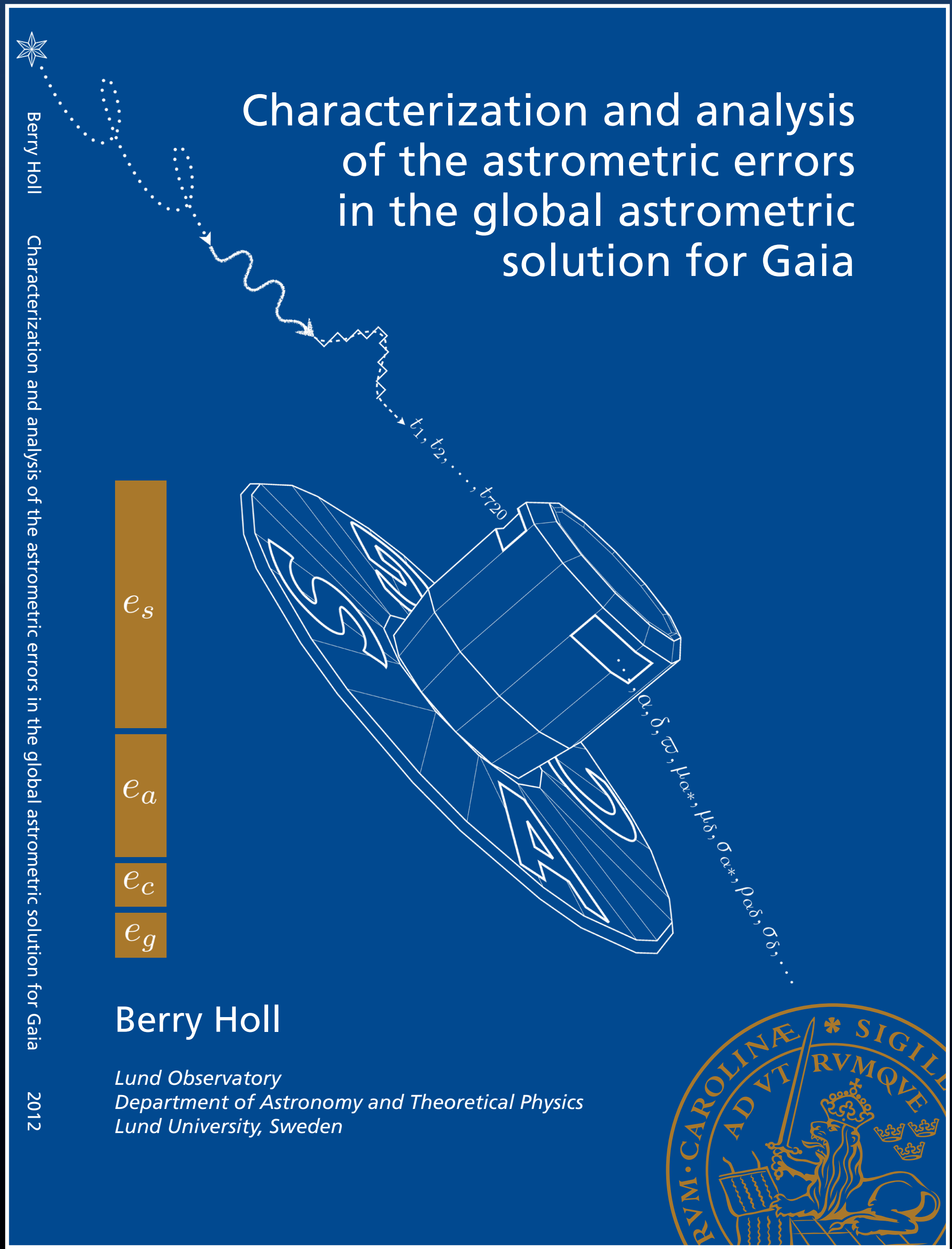
Angular accuracy in optical astrometry

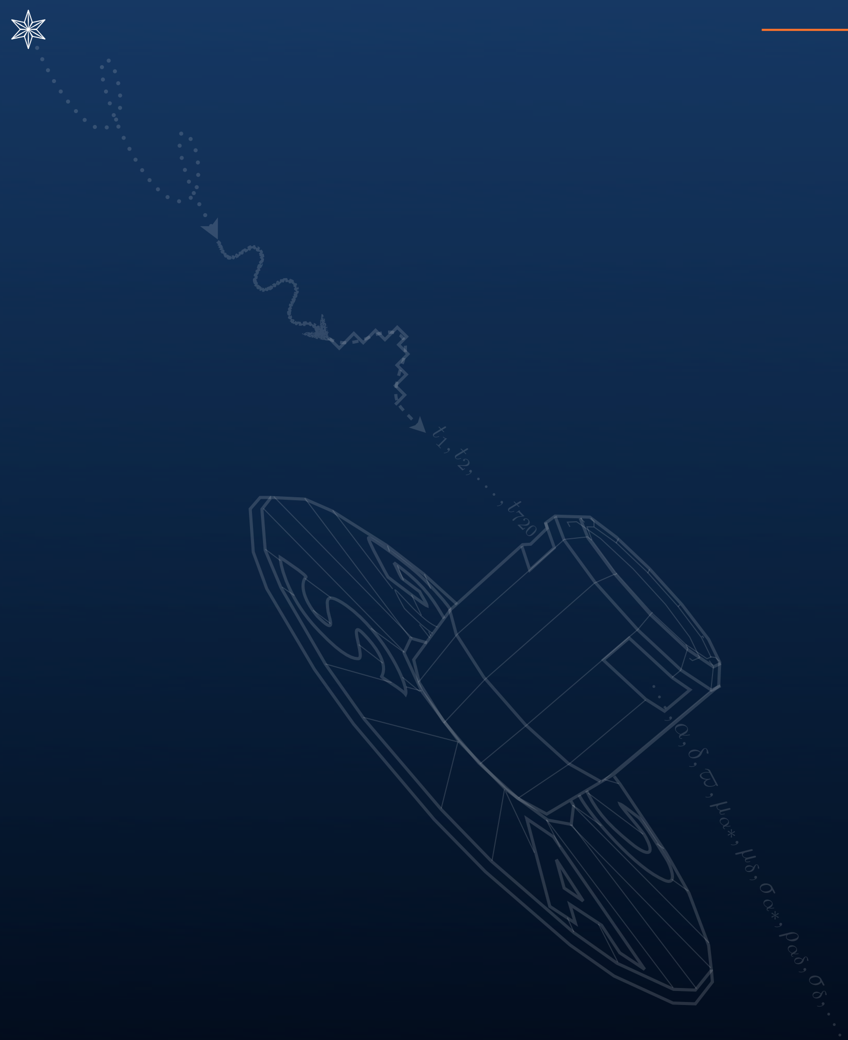


The Lund panorama of the Milky Way (1955)



7000 stars and the Milky Way (2x1m) painted by Martin and Tatjana Kesküla

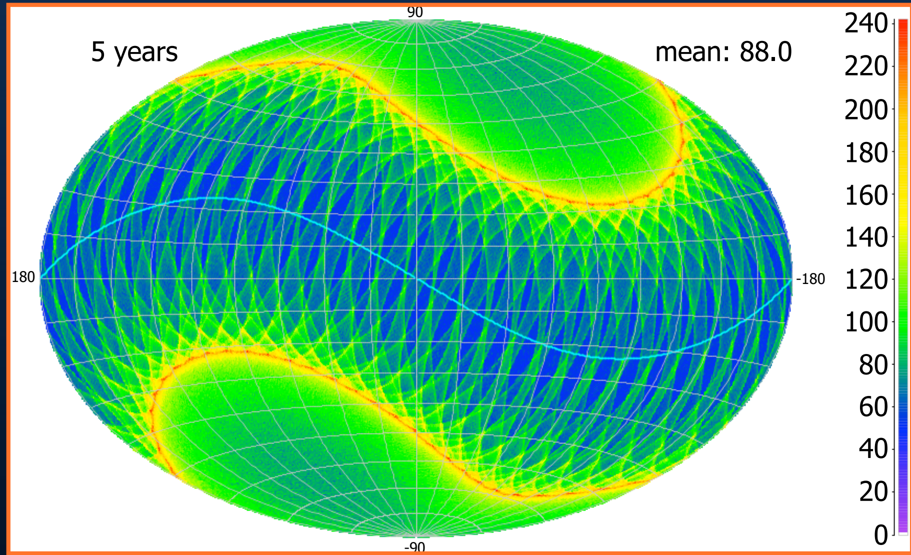

Berry Holl
Characterization and analysis of the astrometric errors in the global astrometric solution for Gaia
2012
Characterization and analysis of the astrometric errors in the global astrometric solution for Gaia
$t_1, t_2, \ldots, t_{20}$
$\alpha, \delta, \varpi, \mu_{\alpha*}, \mu_\delta, \sigma_{\alpha*}, \rho_{\alpha\delta}, \sigma_\delta, \ldots$
e_s
e_a
e_c
e_g
Berry Holl
Lund Observatory
Department of Astronomy and Theoretical Physics
Lund University, Sweden



Gaia

during 5 year mission from L2:

- observing 1 billion sources (mag 6-20),
- on average 72 times (between 40-240),



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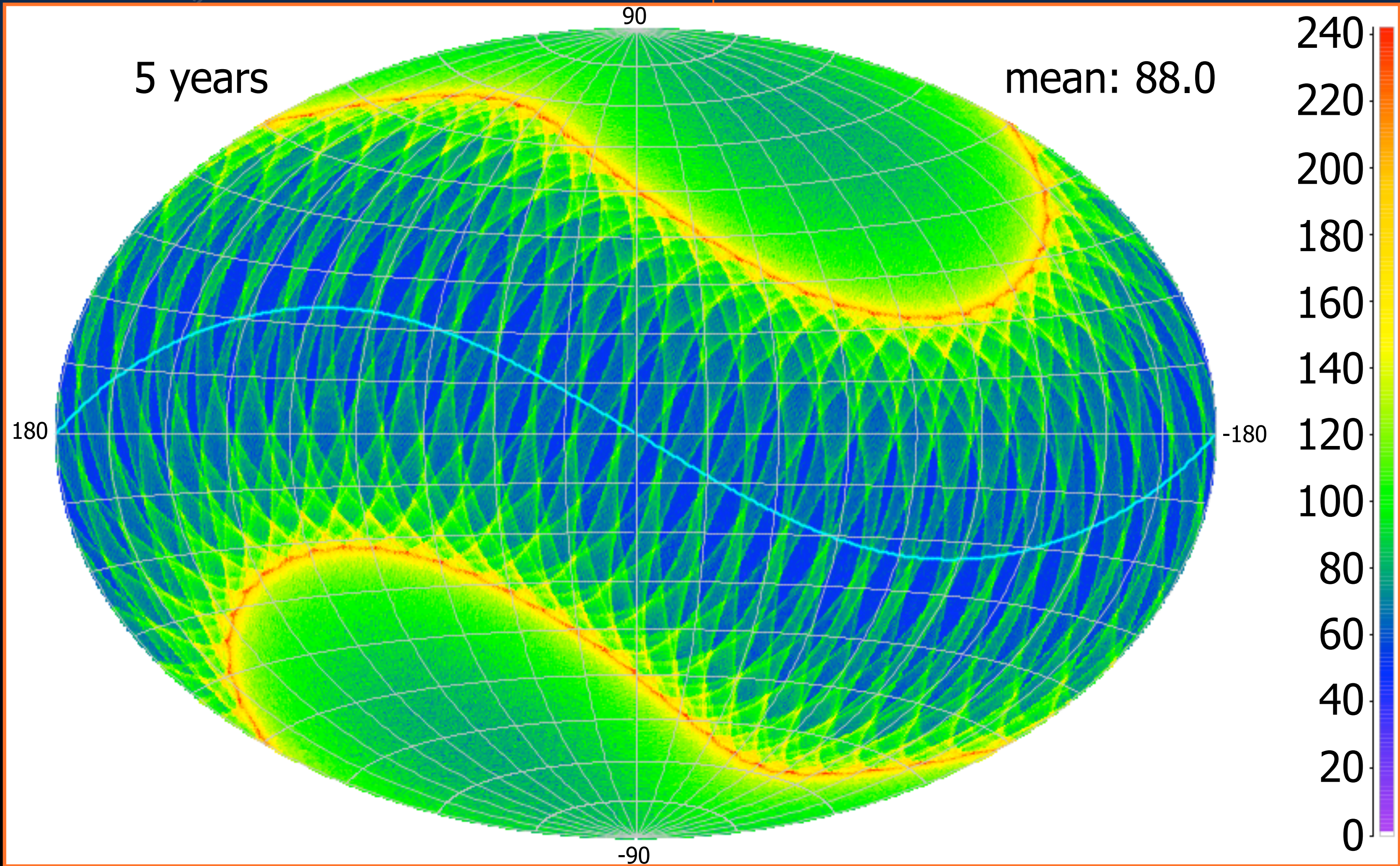

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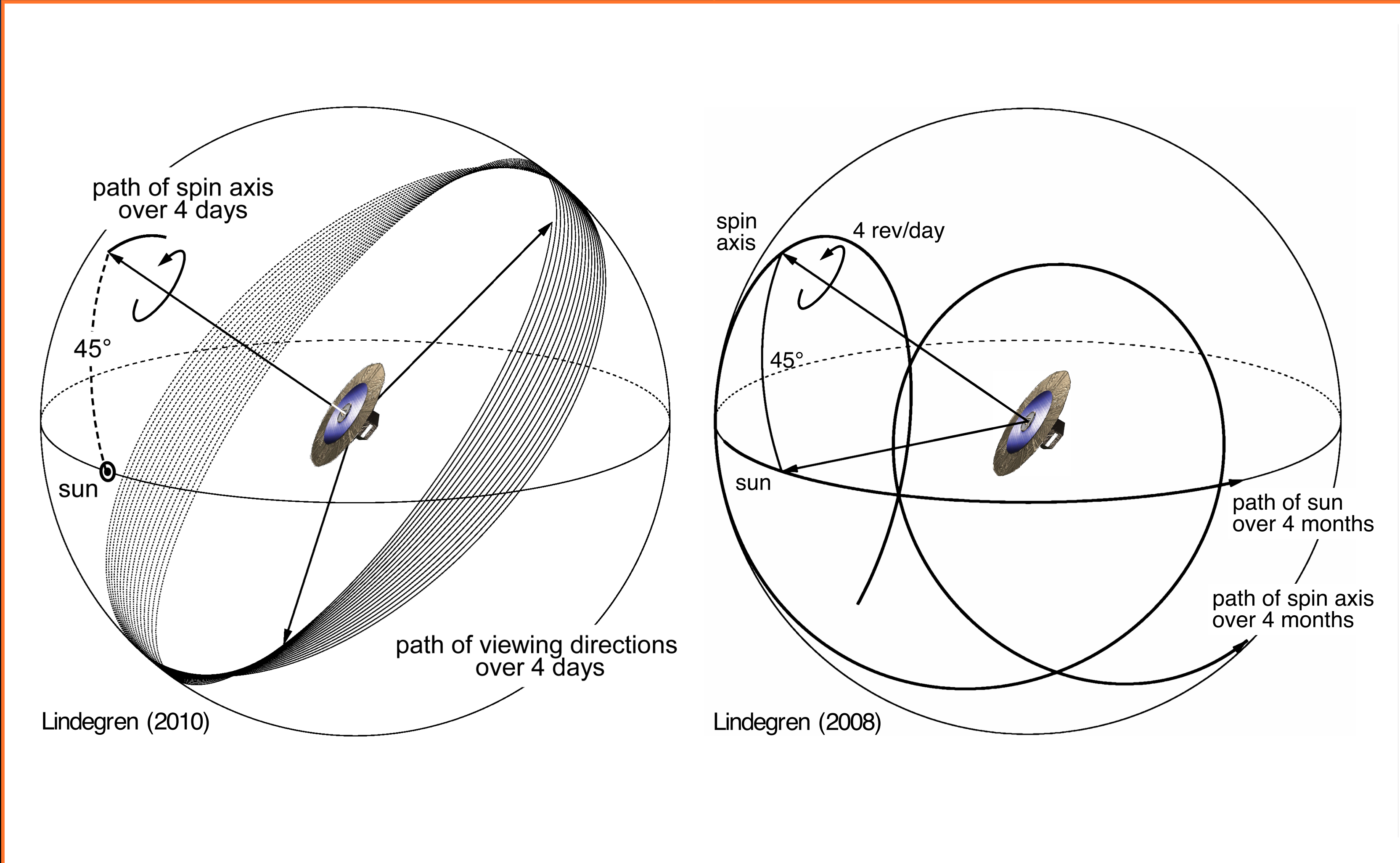

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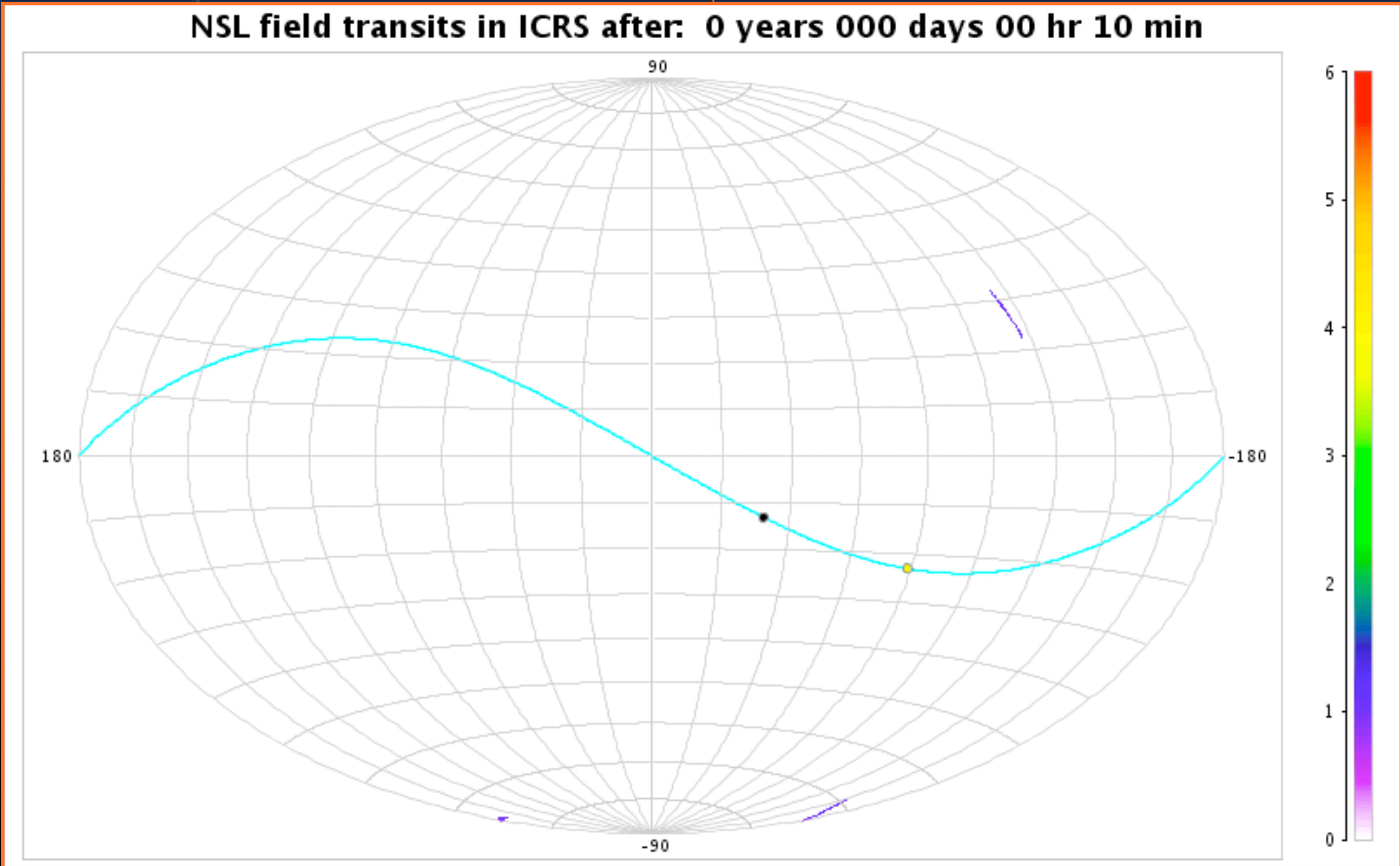

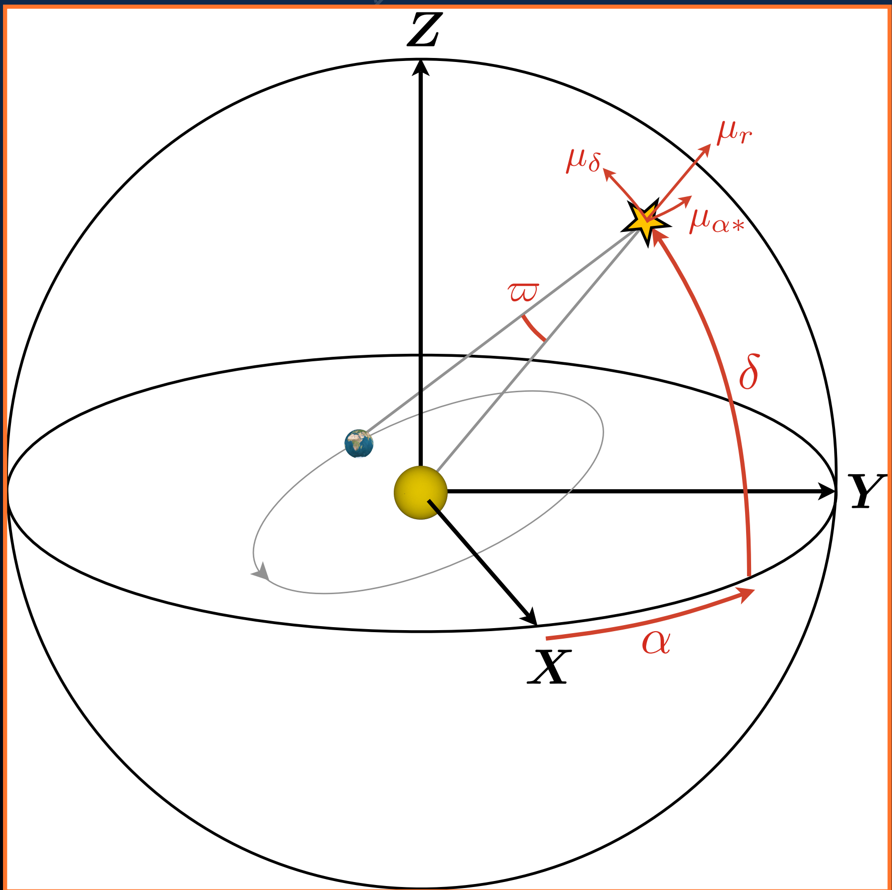

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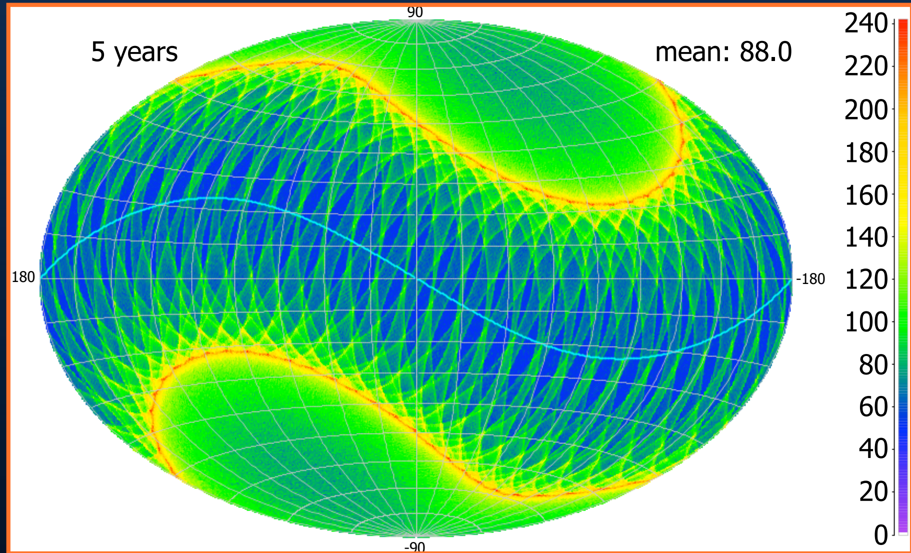
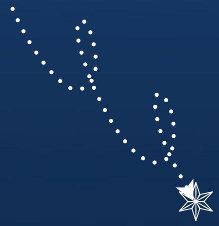
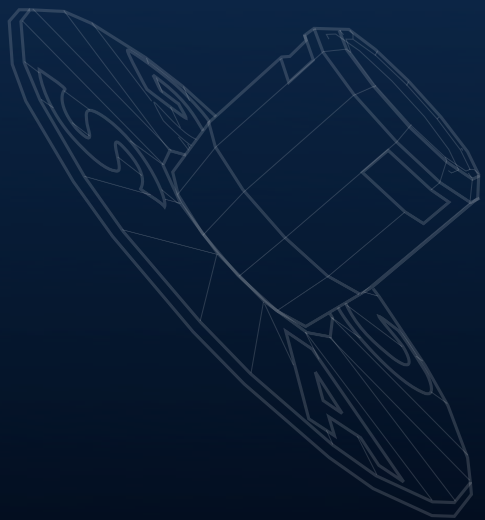


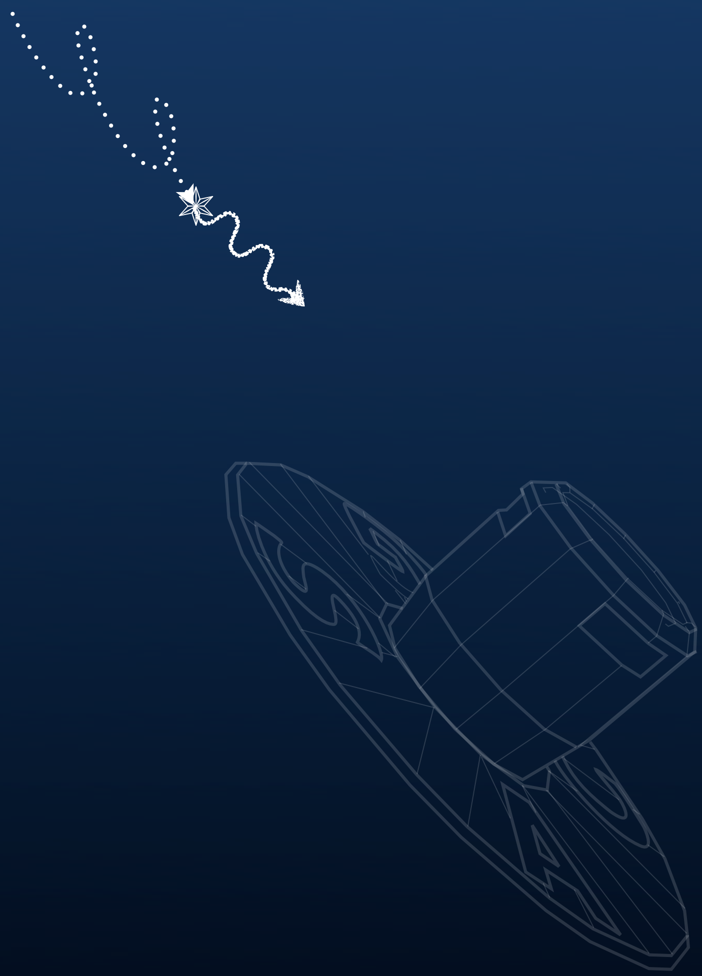
- positions, **parallax**, and proper motion with 20 µas(/yr) precision at V= 15 mag,
- astrometric motion of ~80% of sources can be described using this 5 parameter model.





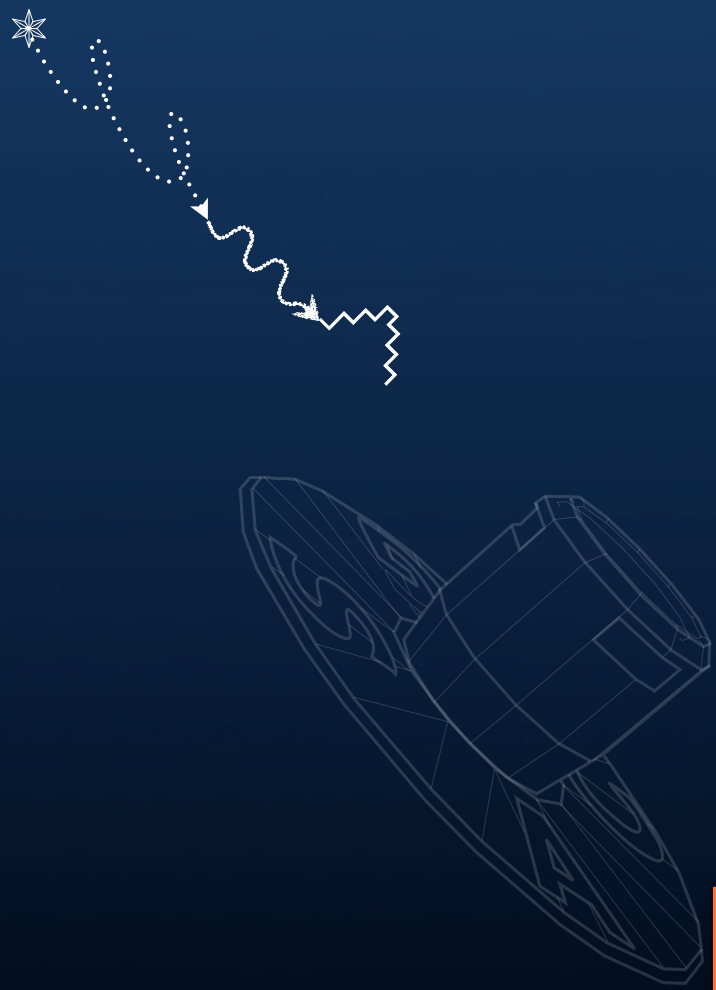
Observing process:

- Apparent source motion on the sky,



Observing process:

- Apparent source motion on the sky,
- propagation of light to Gaia,



Observing process:

- Apparent source motion on the sky,
- propagation of light to Gaia,
- Astrometric CCDs readout,
  typically 6x1 binned counting profile

 $\times\ 720 \times 10^9 = \mathbf{10^{12}}$

Forward modelling:

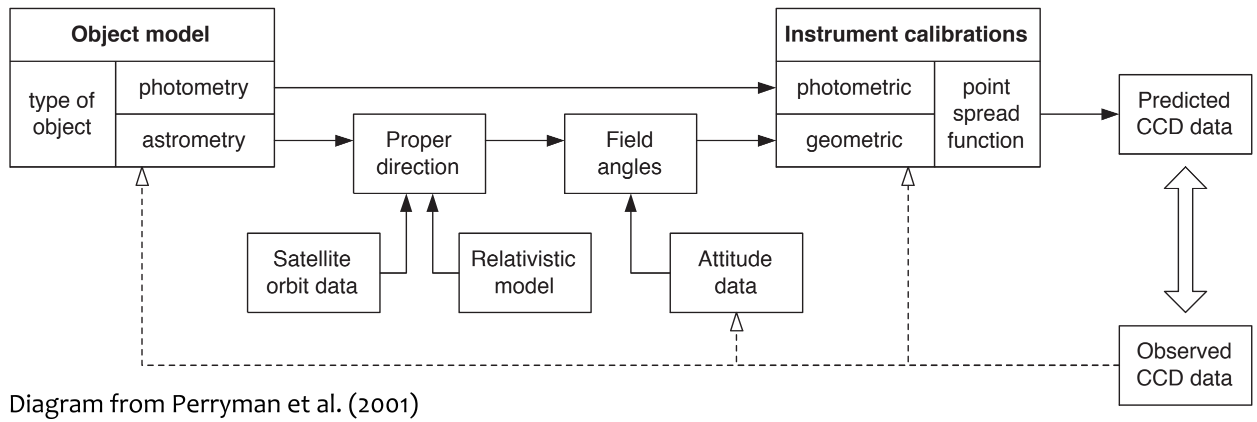
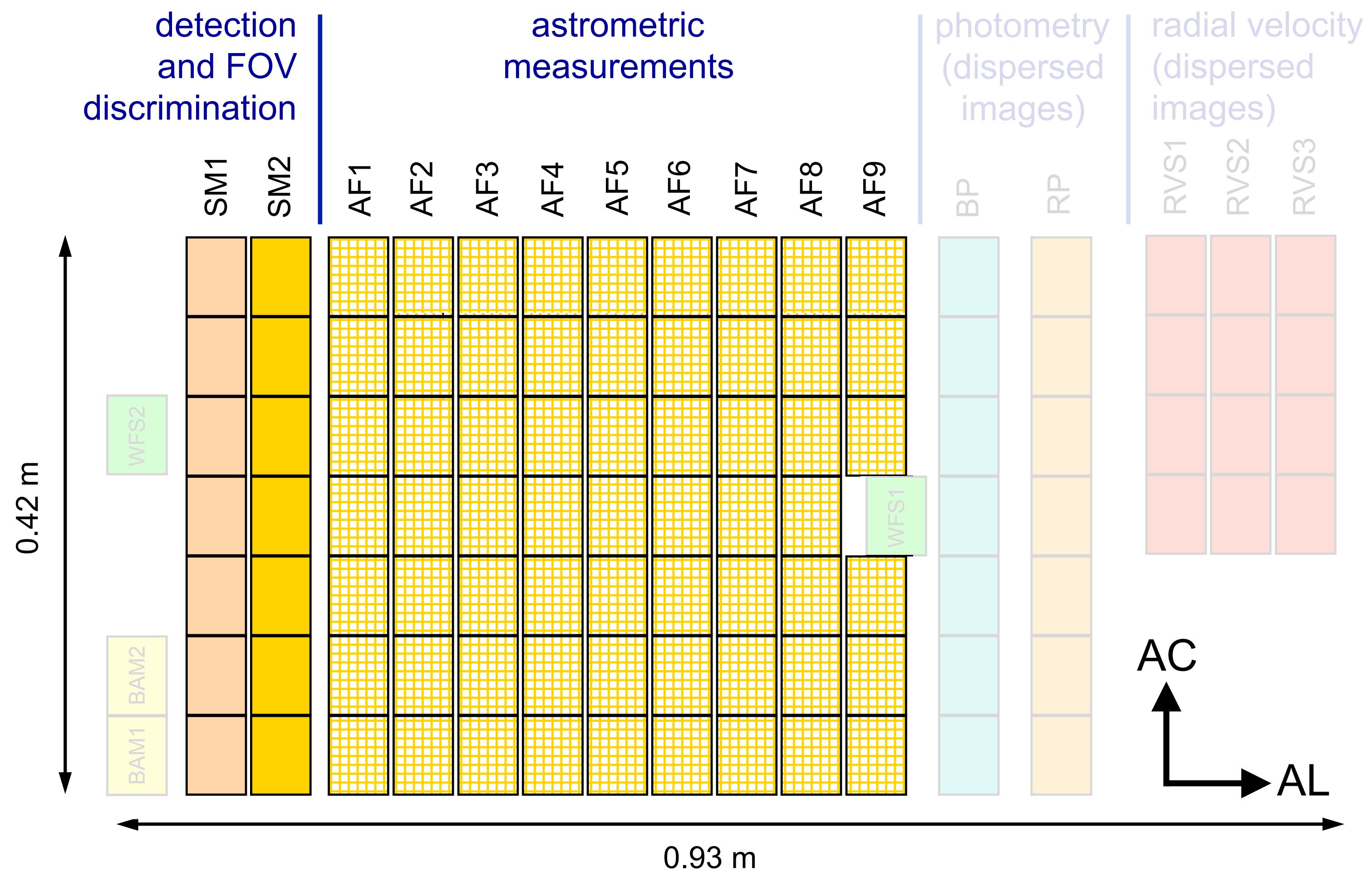
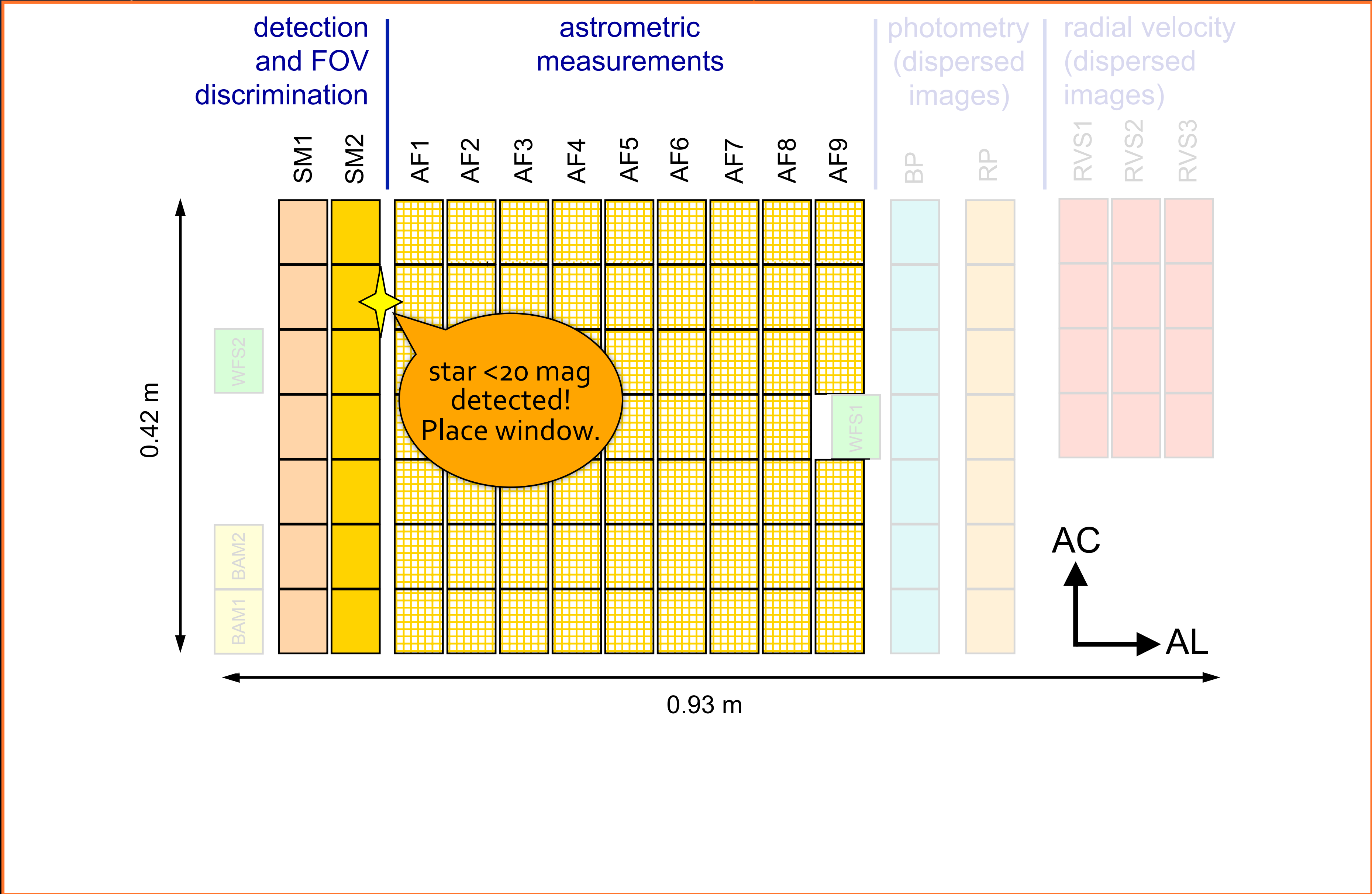


Diagram from Perryman et al. (2001)

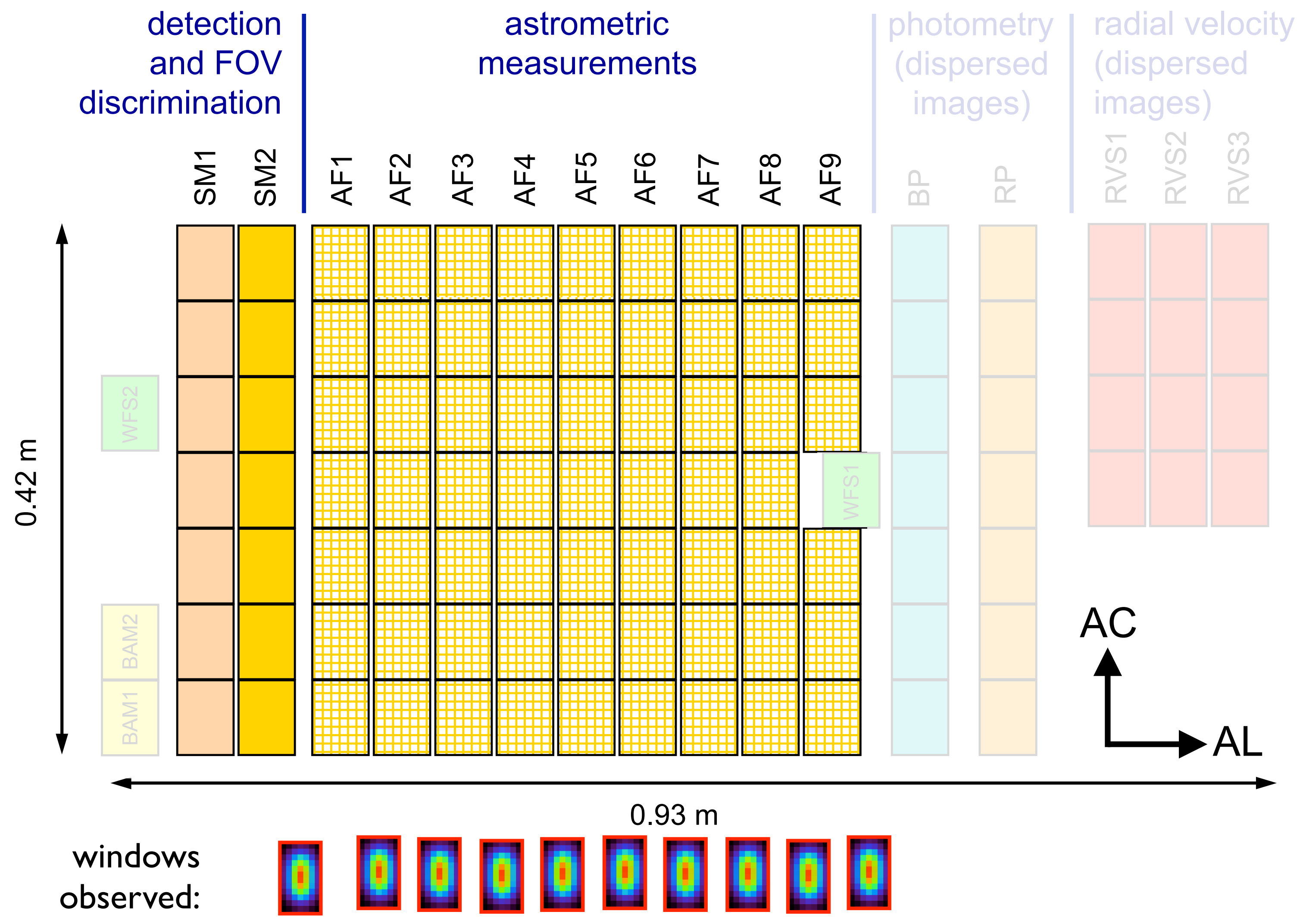
Observing process:



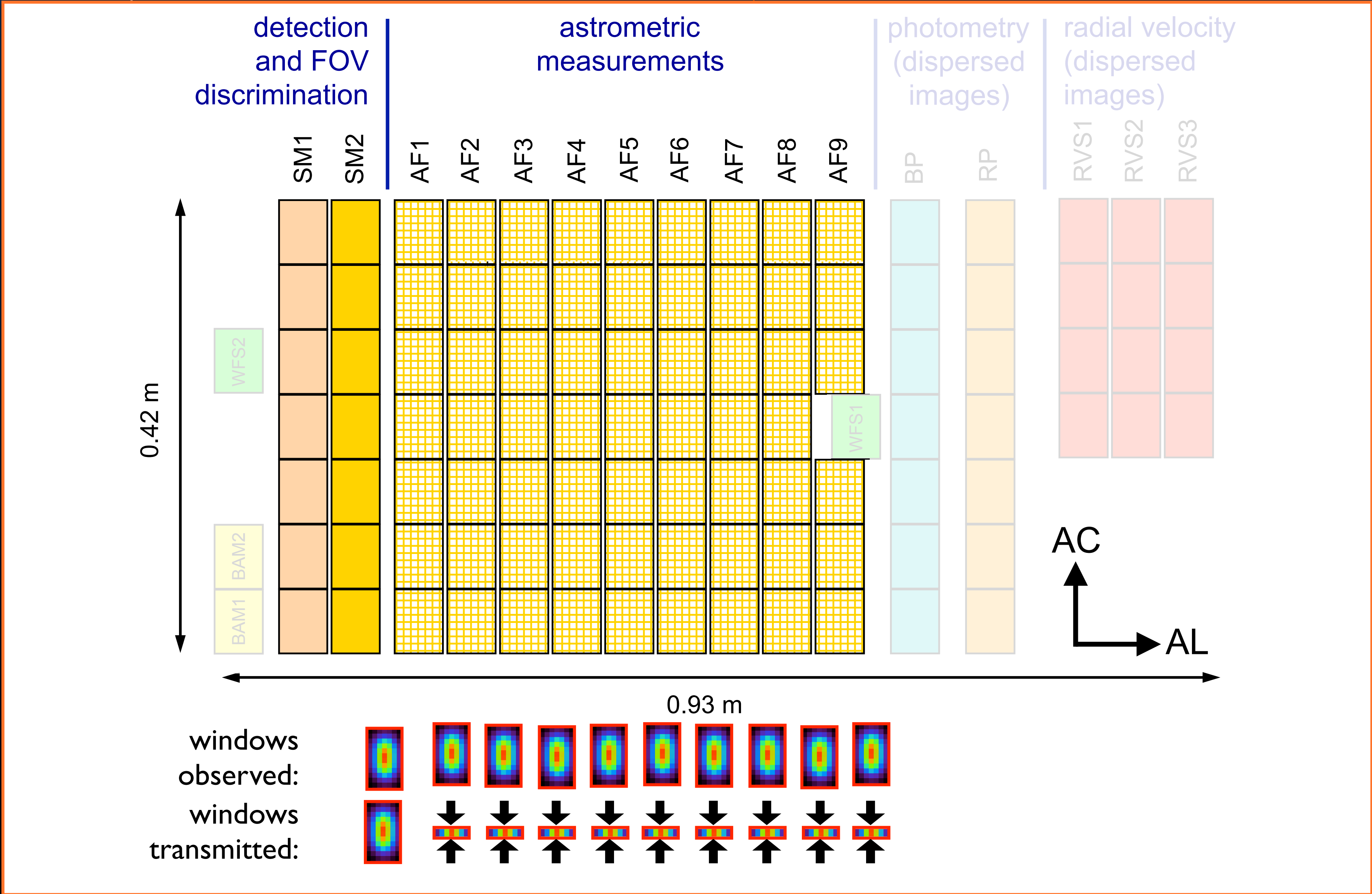
Observing process:



Observing process:

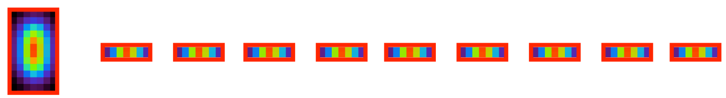


Observing process:

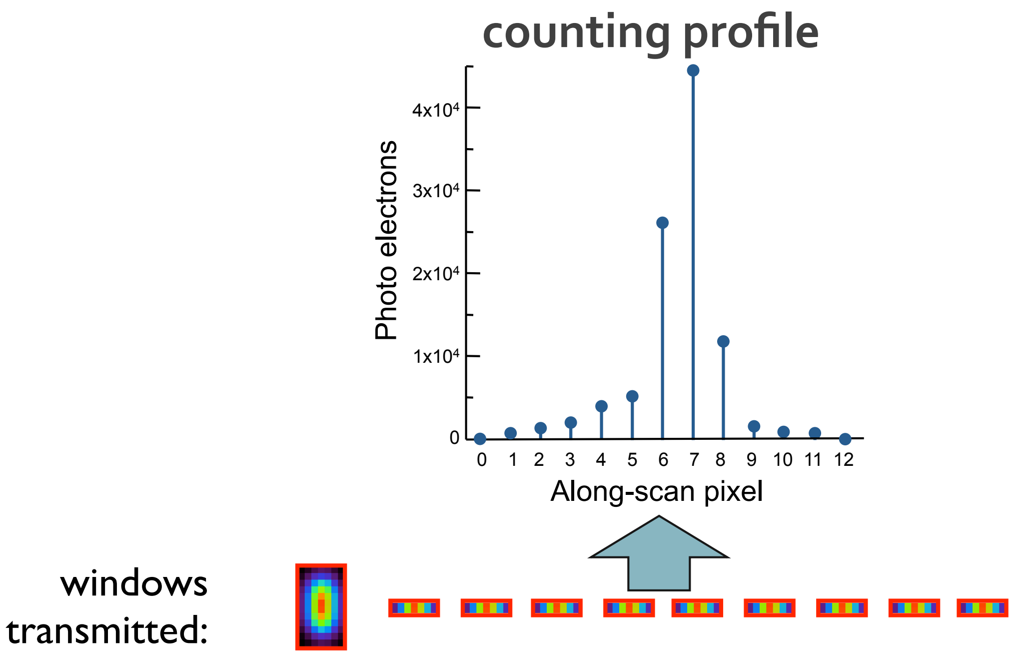


Observing process:

windows transmitted:



Observing process:



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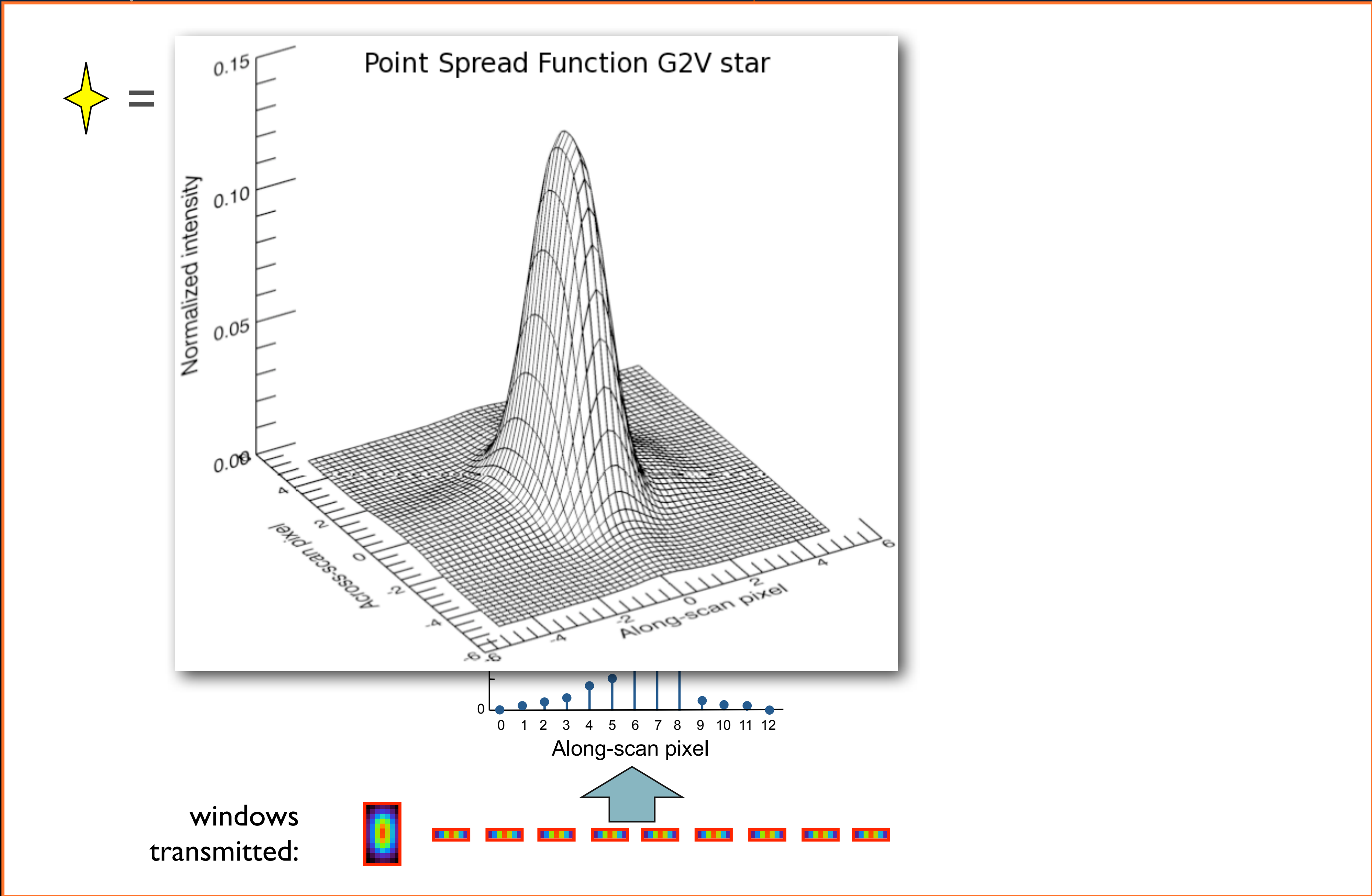

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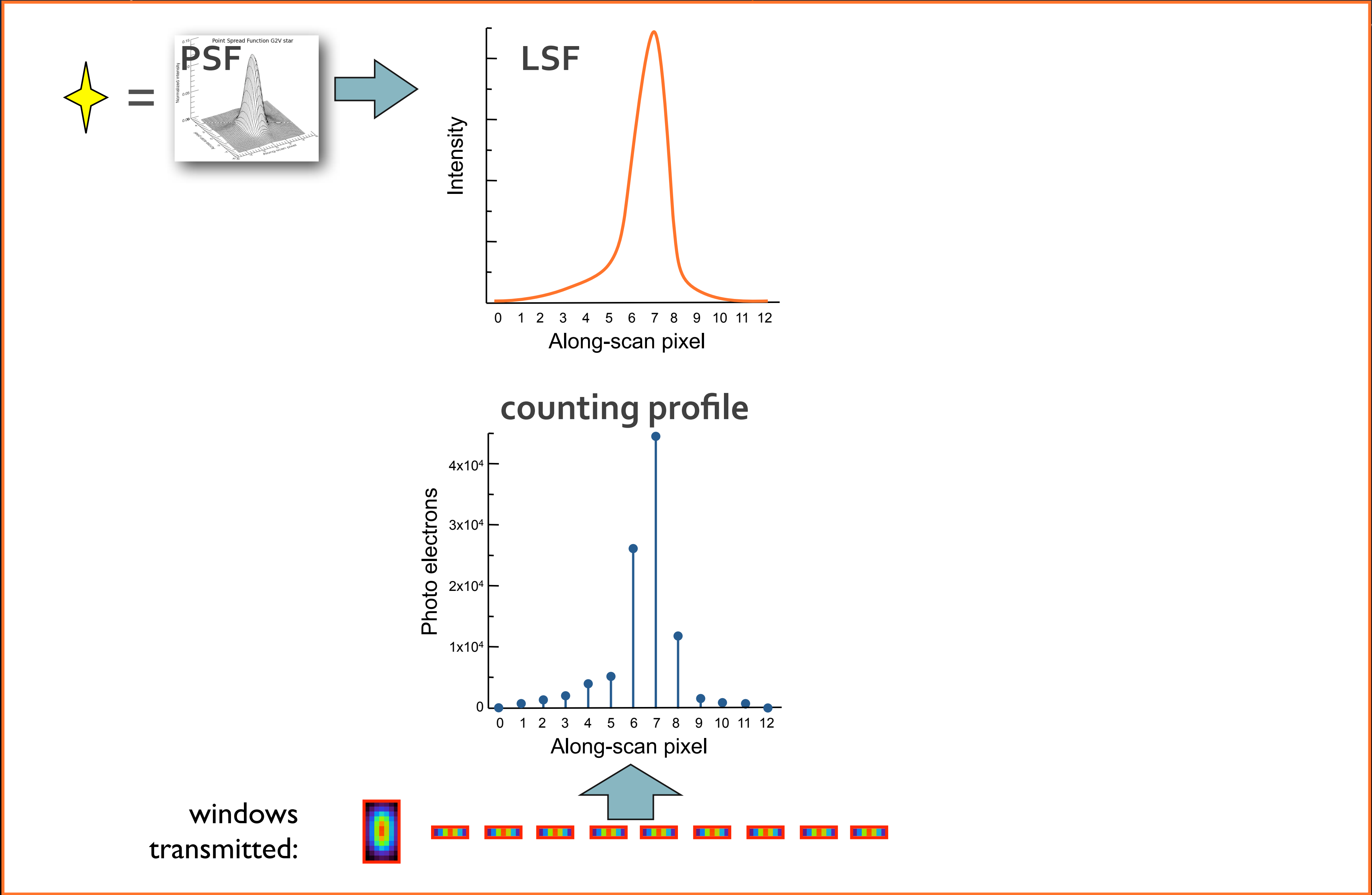

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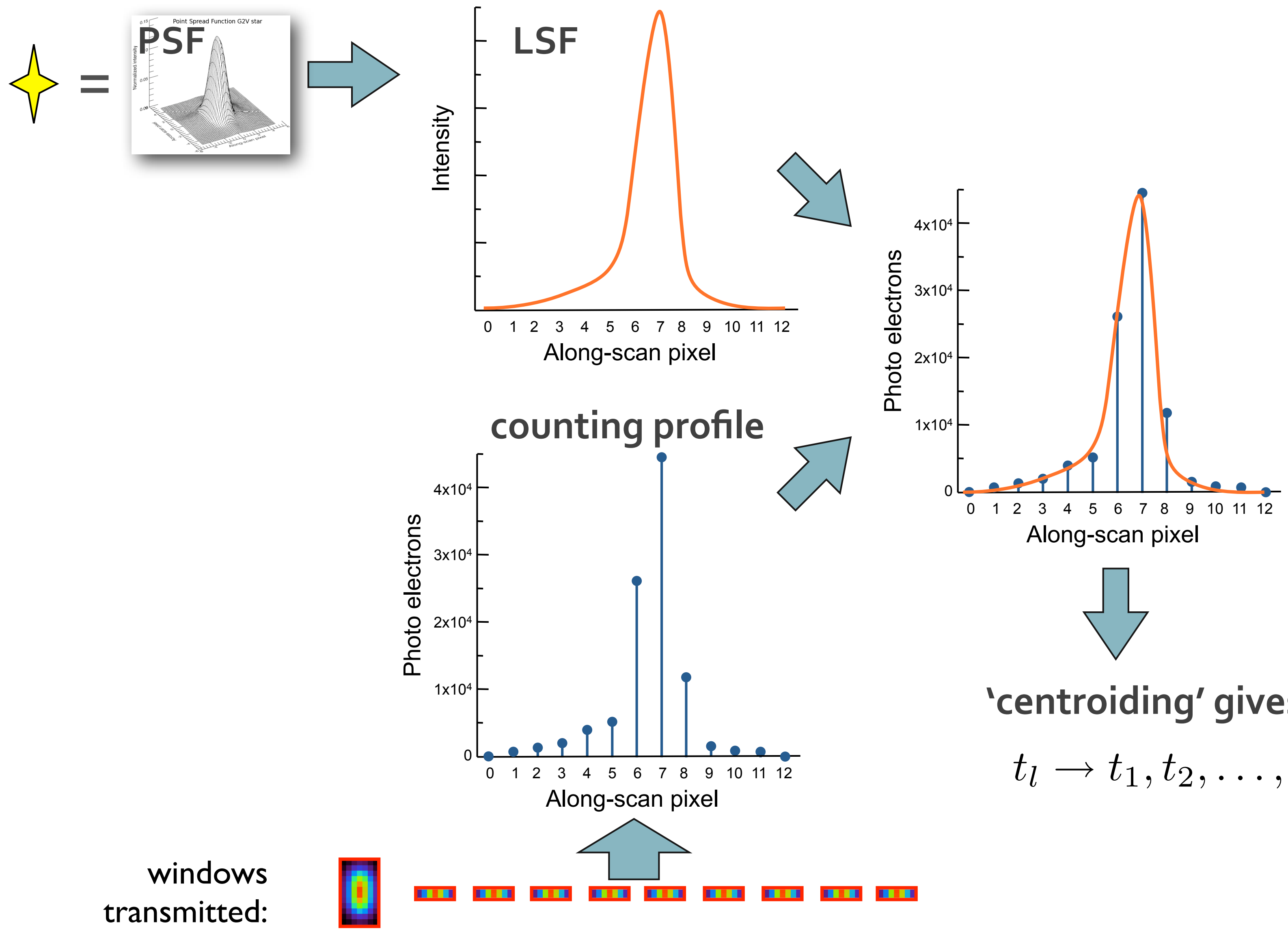
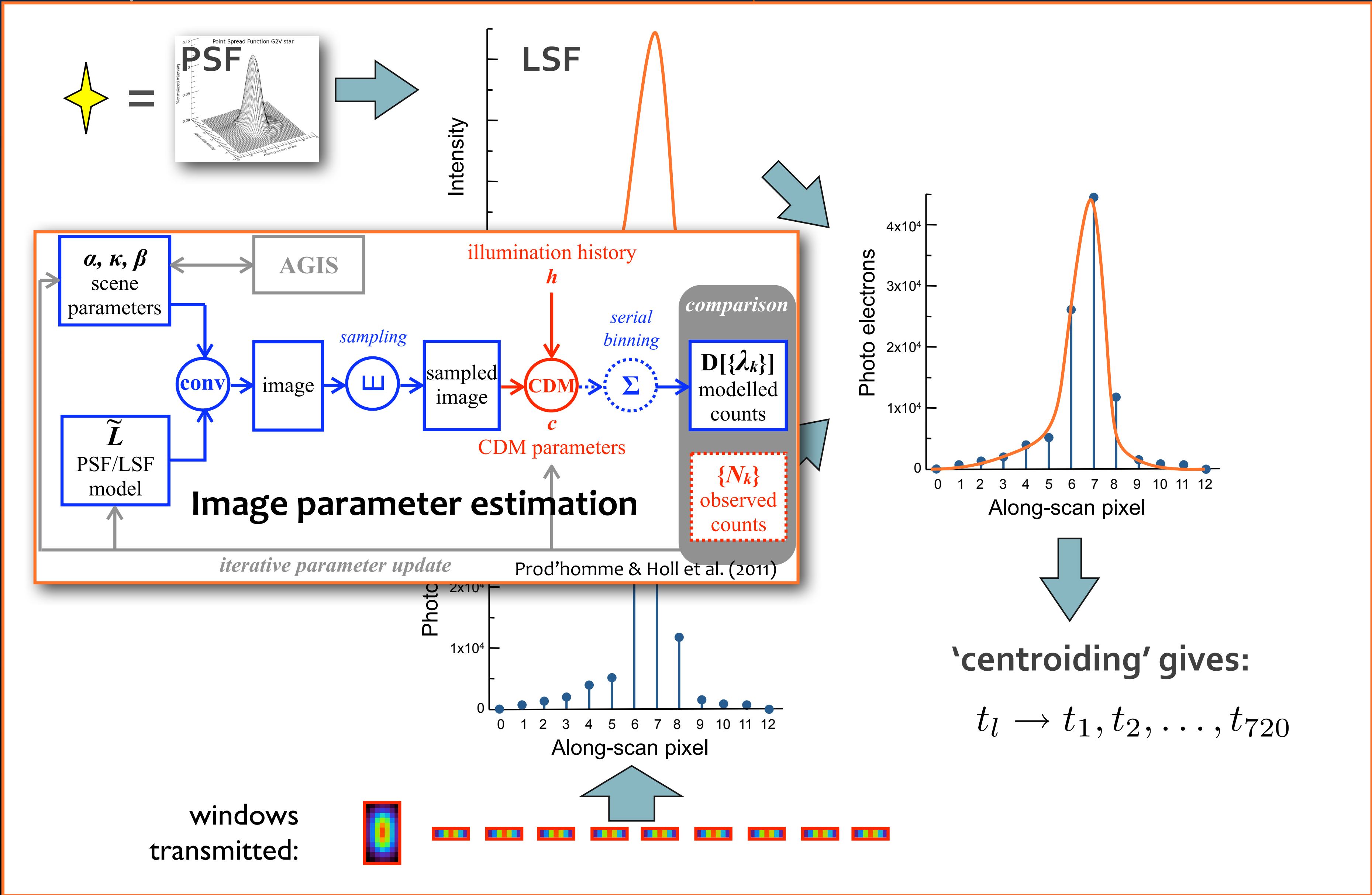
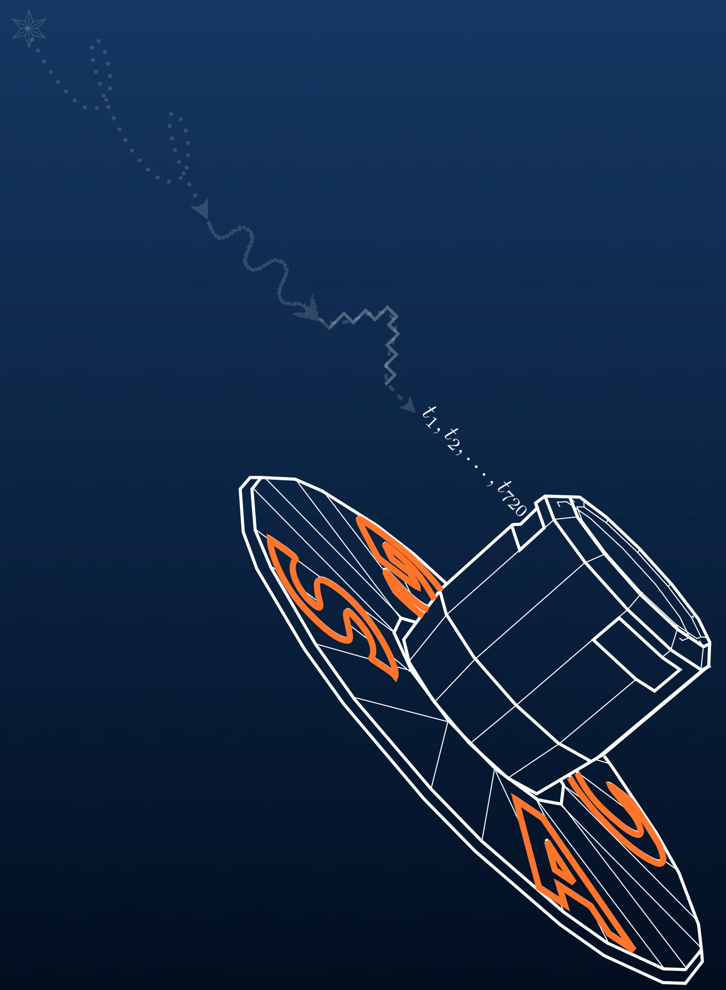


'centroiding' gives:

$$t_l \rightarrow t_1, t_2, \ldots, t_{720}$$

Observing process:





Astrometric solution

Estimates parameters of 4 models:

- (S) Source — 5×10^9 param
- (A) Attitude — $\sim 10^8$ param
- (C) Calibration — $\sim 10^6$ param
- (G) Global — $< 10^2$ param

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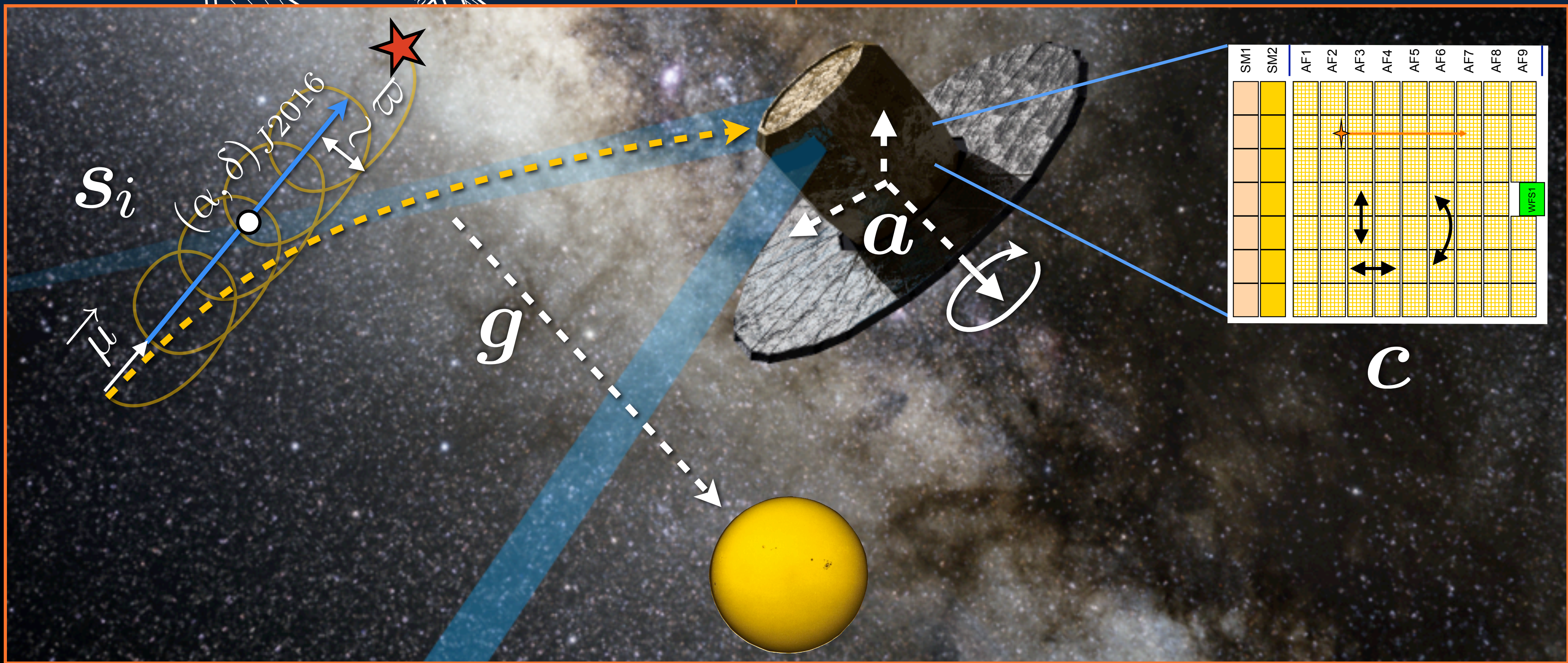

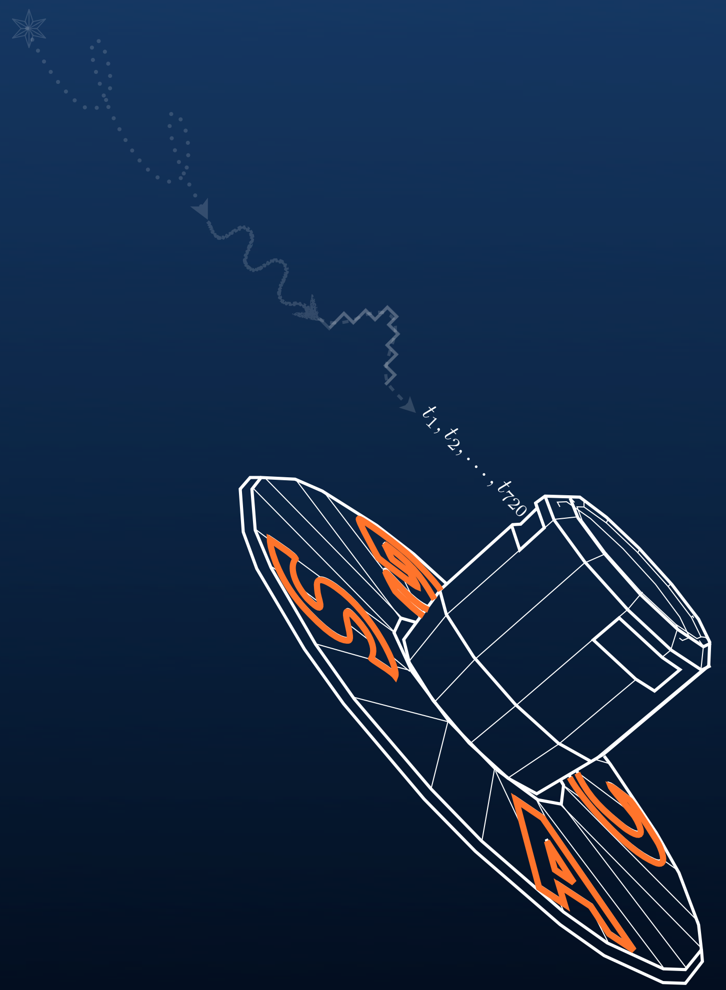

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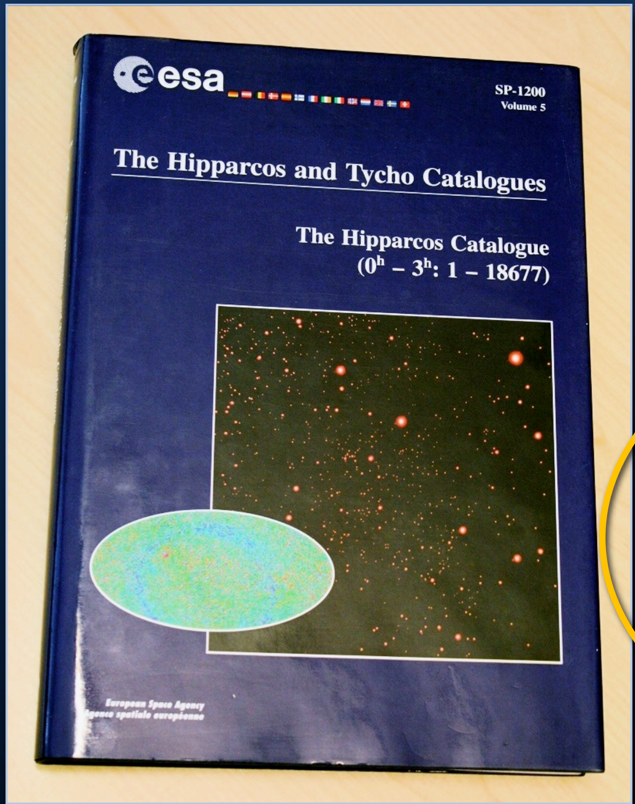
$$\min_{\boldsymbol{s},\,\boldsymbol{a},\,\boldsymbol{c},\,\boldsymbol{g}} \sum_l \left[\frac{t_l - f_l(\boldsymbol{s},\boldsymbol{a},\boldsymbol{c},\boldsymbol{g})}{\sigma_l} \right]^2$$

- Least squares solution:
  10^{10} parameters using 10^{12} observations,
- direct solution unfeasible
  (Bombrun et al. 2011),
- use Astrometric Global Iterative Solution:
  AGIS (Lindegren et al. 2012)

Hipparcos vs Gaia catalogue



5 astrometric parameters

- estimated values and their standard errors:

$$\alpha \ \delta \ \varpi \ \mu_{\alpha*} \ \mu_\delta$$

$$\sigma_\alpha \ \sigma_\delta \ \sigma_\varpi \ \sigma_{\mu_{\alpha*}} \ \sigma_{\mu_\delta}$$

- Correlation between astrometric parameters of each star,

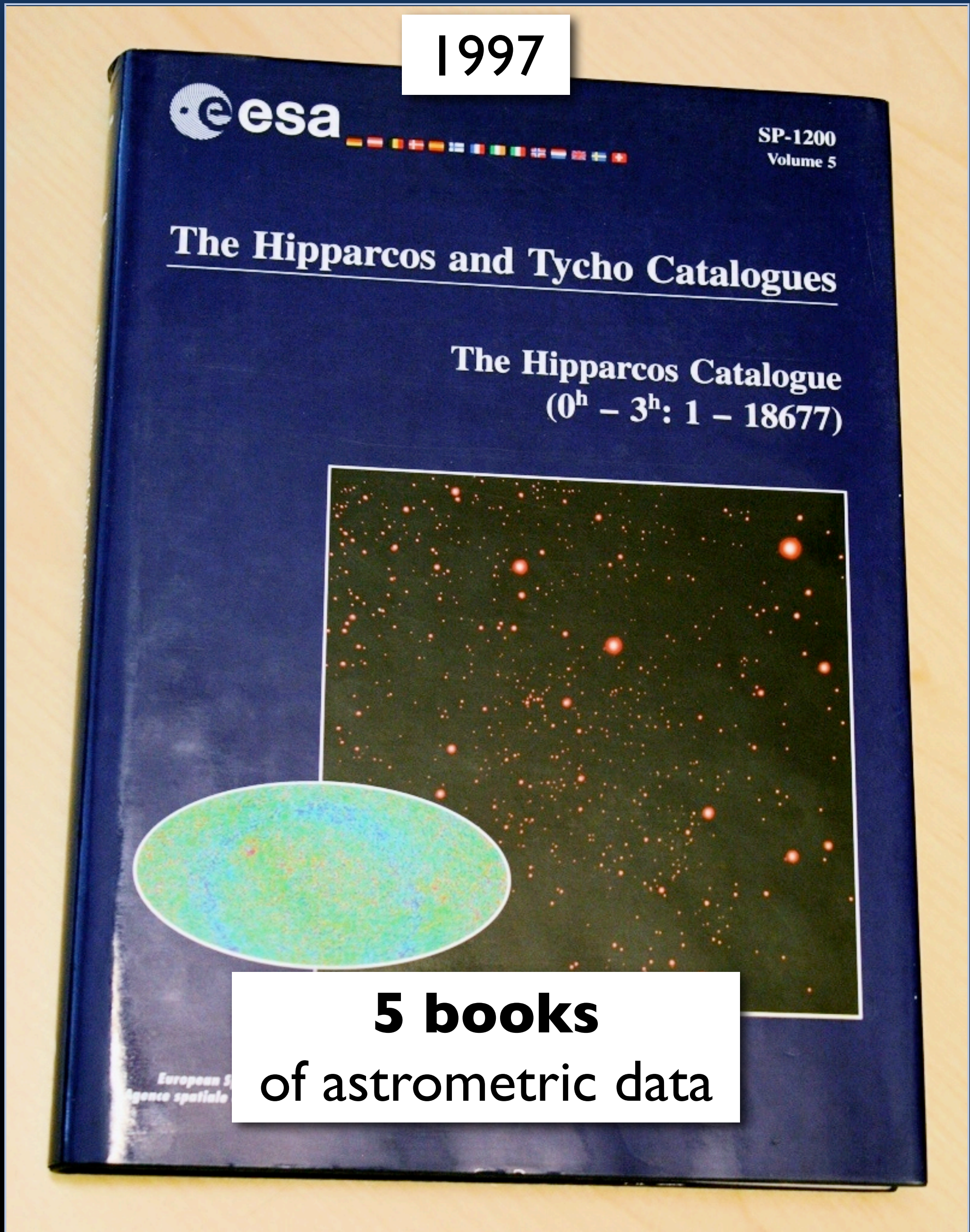
all 10 combinations

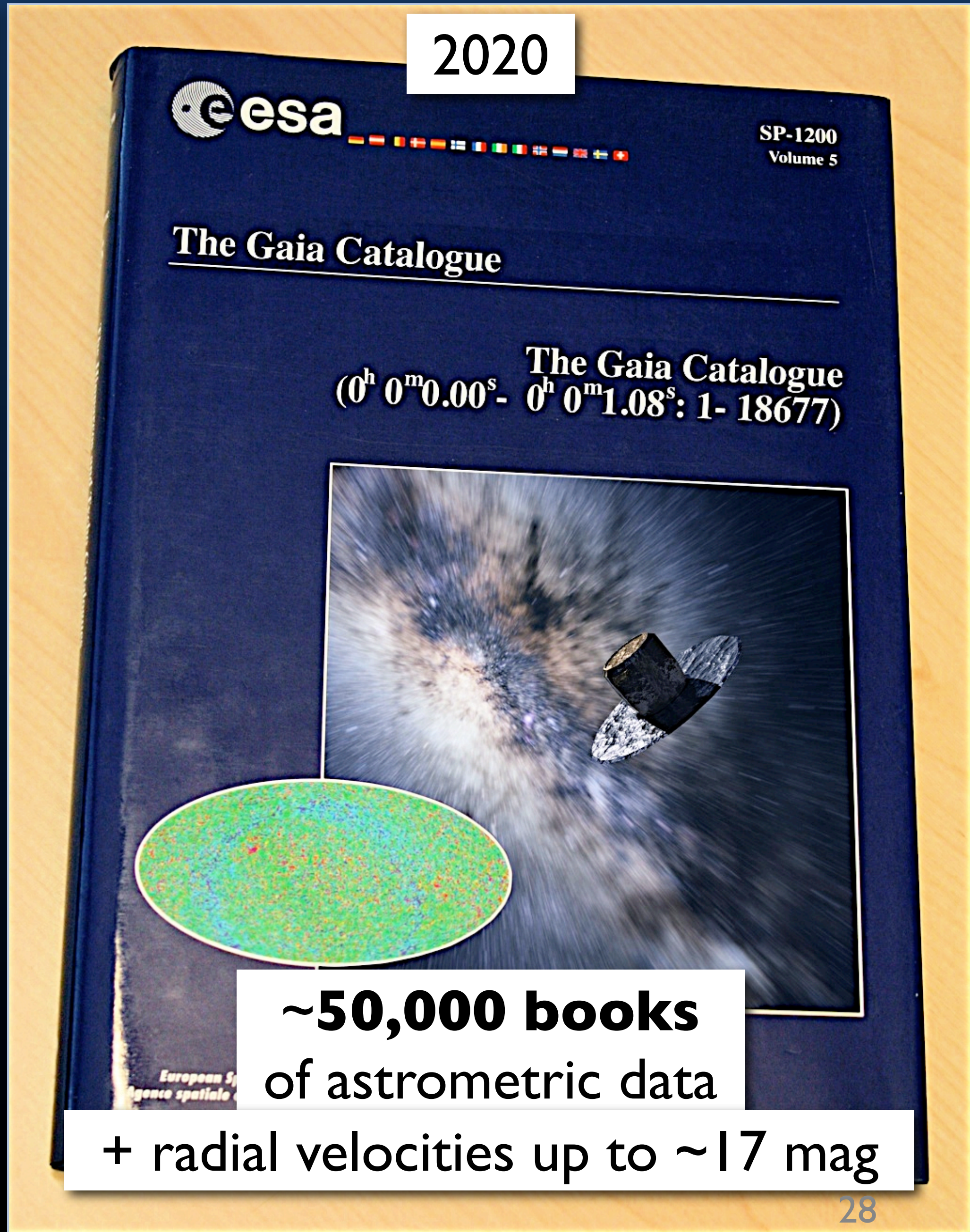
Covariances

Number HIP		Descriptor: epoch J1991.25 RA h m s	Dec ±° ′ ″	V mag			Position: epoch J1991.25 α (ICRS) deg	δ deg		Par. π mas	Proper Motion $\mu_{\alpha*}$ mas/yr	μ_δ
1	2	3	4	5	6	7	8	9	10	11	12	13
1		00 00 00.22	+01 05 20.4	9.10		H	0.000 911 85	+01.089 013 32		3.54	−5.20	−1.88
2		00 00 00.91	−19 29 55.8	9.27		G	0.003 797 37	−19.498 837 45	+	21.90	181.21	−0.93
3		00 00 01.20	+38 51 33.4	6.61		G	0.005 007 95	+38.859 286 08		2.81	5.24	−2.91
4		00 00 02.01	−51 53 36.8	8.06		H	0.008 381 70	−51.893 546 12		7.75	62.85	0.16
5		00 00 02.39	−40 35 28.4	8.55		H	0.009 965 34	−40.591 224 40		2.87	2.53	9.07

HIP	Standard Errors α* mas	δ mas	π mas	$\mu_{\alpha*}$ mas/yr	μ_δ mas/yr	Astrometric Correlations (%) δ α*	π α*	π δ	$\mu_{\alpha*}$ α*	$\mu_{\alpha*}$ δ	$\mu_{\alpha*}$ π	μ_δ α*	μ_δ δ	μ_δ π	μ_δ $\mu_{\alpha*}$	Soln F1 %	F2
1	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1.32	0.74	1.39	1.36	0.81	+32	− 7	−11	−24	+ 9	− 1	+10	− 1	+ 1	+34	0	0.74
2	1.28	0.70	3.10	1.74	0.92	+12	−14	−24	−29	+ 1	+21	− 2	−19	−28	+14	2	1.45
3	0.53	0.40	0.63	0.57	0.47	+ 6	+ 9	+ 4	+43	− 1	− 6	+ 3	+24	+ 7	+21	0	−0.45
4	0.53	0.59	0.97	0.65	0.65	−22	− 9	− 3	+24	+20	+ 8	+18	+ 8	−31	−18	0	−1.46
5	0.64	0.61	1.11	0.67	0.74	+10	+24	+ 6	+26	−10	+20	−16	−30	−19	+ 6	0	−1.24

Hipparcos vs Gaia catalogue





Introduction to astrometry & Gaia

Importance of error characterization

Results of my work

Terminology

x is a measured or derived quantity (e.g. parallax of a star):

error	$e = x - x_{true}$	unknown
uncertainty	Random variability in measurement.	~known

Described by **probability distribution**
characterized by e.g. mean, standard deviation, skewness etc.

Often confusingly called ‘error’, ‘mean error’
or (slighly better) ‘standard error’, ‘RMS error’, etc.

I will use **‘standard uncertainty’**
for the uncertainty of a Gaussian error distribution,

Observations are often influenced by many random processes
approaching Gaussian error distribution (central limit theorem).

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Normally estimated quite well:
- by repeating (assuming identical experiments),
- observation process knowledge (e.g. photons Poisson statistics).

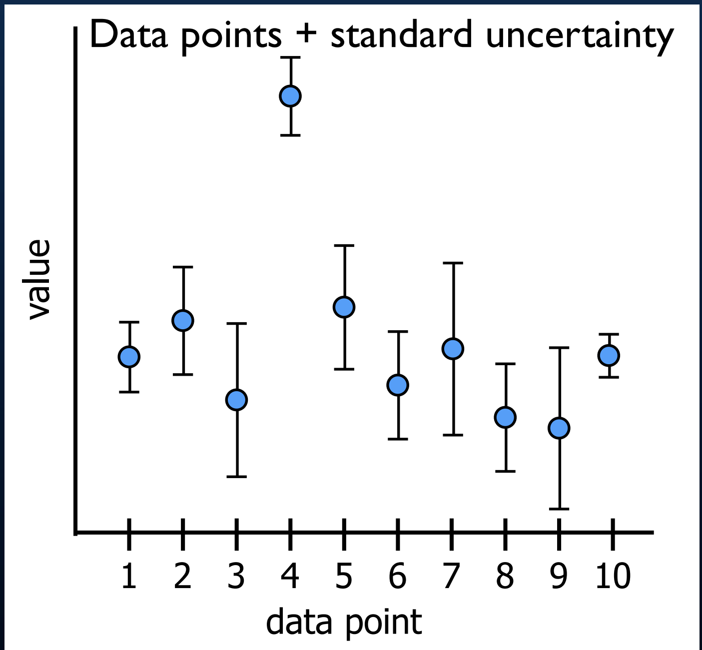
bias Nonzero mean in probability distribution. often unknown

If assumed zero leads to **systematic errors**.
These are difficult to determine by repeating the experiment
(e.g. observing Castor while you should be observing Pollux)

Why we need to understand errors

Essential for interpreting data, examples:

proper motion of stars in cluster



Determine membership

$$y = \frac{\mu_i - < \mu >_{\text{excluding } i}}{\sigma_{diff}}$$

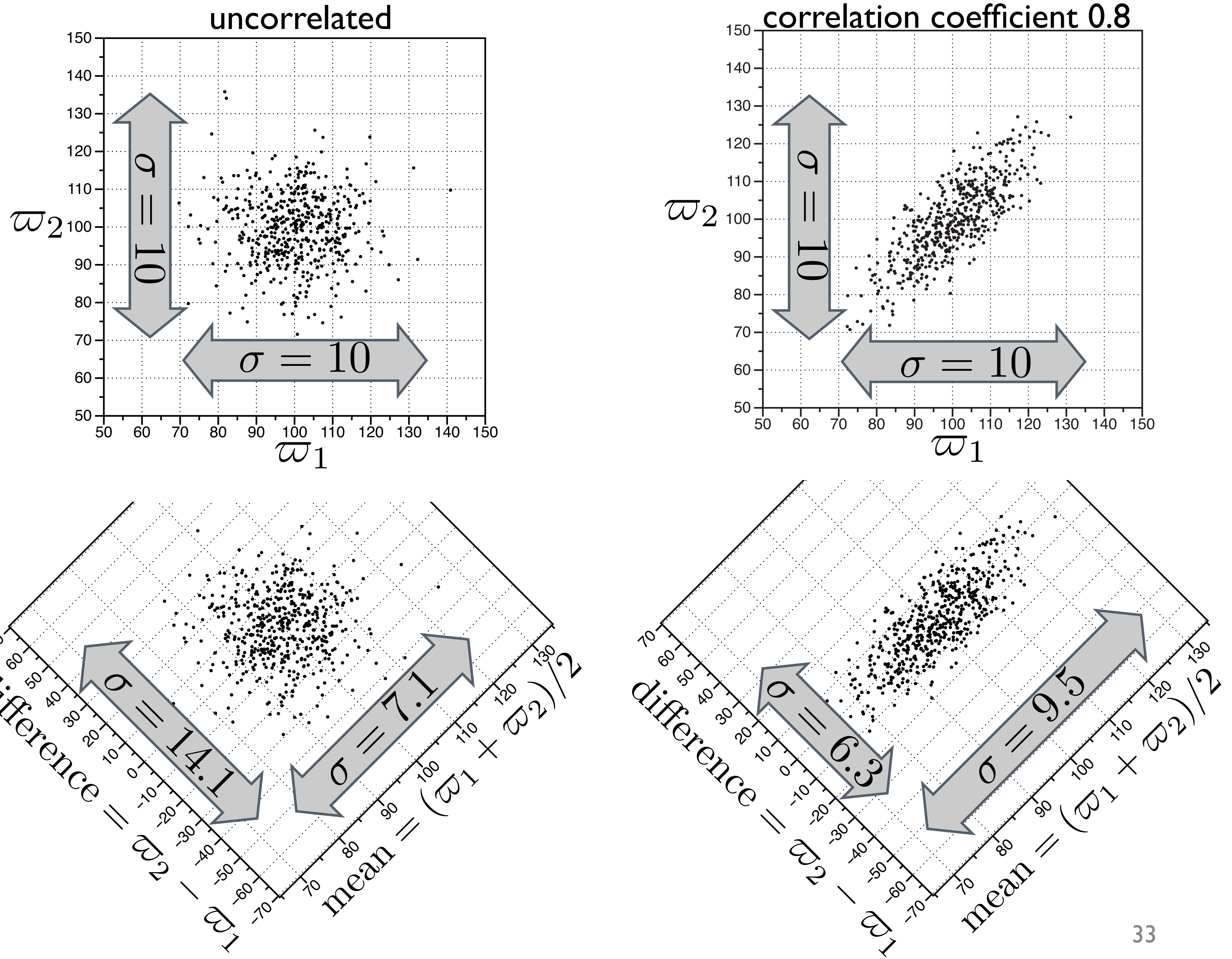
Outcome does not critically depend on uncertainty (e.g. +/- 10% does not change result)

Compute velocity dispersion (excl. 4)

$$\sigma^2_{\text{computed}} = \sigma^2_{\text{intrinsic}} + \sigma^2_{\text{data}}$$

If the latter two are of similar size, the data uncertainties need to be very well known.

Depending on the application uncertainties can be of crucial importance!


uncorrelated
correlation coefficient 0.8
ϖ_1
ϖ_2
$\sigma = 10$
$\sigma = 10$
$\sigma = 14.1$
$\sigma = 7.1$
$\sigma = 6.3$
$\sigma = 9.5$
difference = $\varpi_2 - \varpi_1$
mean = $(\varpi_1 + \varpi_2)/2$

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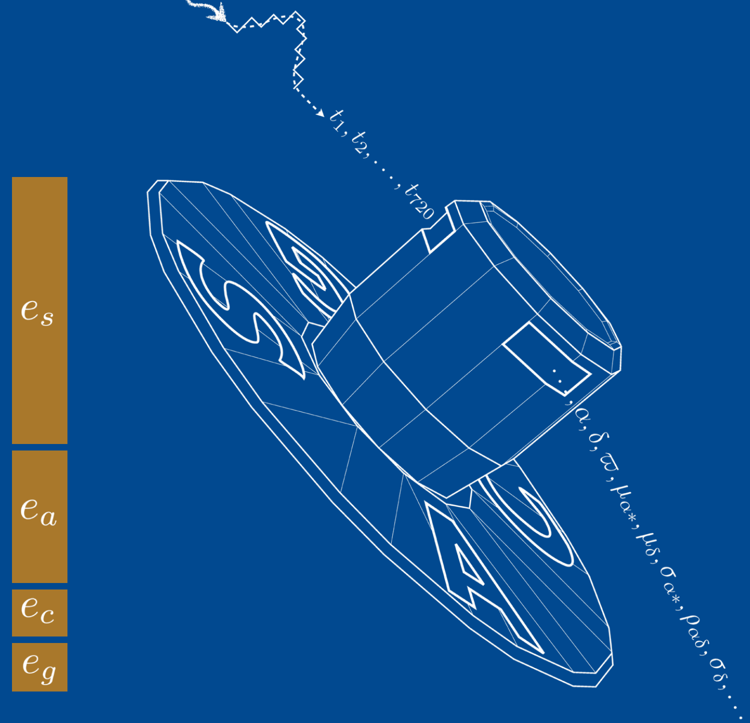
Characterization and analysis of the astrometric errors in the global astrometric solution for Gaia



Berry Holl

Lund Observatory
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Lund University, Sweden

Berry Holl Characterization and analysis of the astrometric errors in the global astrometric solution for Gaia 2012

Questions

How do random errors propagate in the astrometric solution?

- Holl, Lindegren, Hobbs (2010, 2011, 2012a, 2012b)

How can the covariance between any pair of astrometric parameters be estimated?

- Holl, Lindegren, Hobbs (2012a, 2012b)

What is the impact of CCD radiation damage on Gaia astrometry?

- Prod'homme & Holl et al. (2011), Holl & Prod'homme et al. (2012)

Random errors in Gaia catalogue

Measurement process includes effects like:

- diffraction of light through telescope,
- effective integration time,
- motion of the satellite during integration,
- CCD readout noise,
- CCD charge transfer inefficiencies (due to radiation damage).

Random variability is dominated by photon fluctuations:

- raw photo-electron accurately follow Poisson statistics,
- so measurements are (largely) **unbiassed and uncorrelated**,
- number of electrons per pixel is >>10 (even for G=20)
  it is similar to a **Gaussian distribution** sampled at discrete points.

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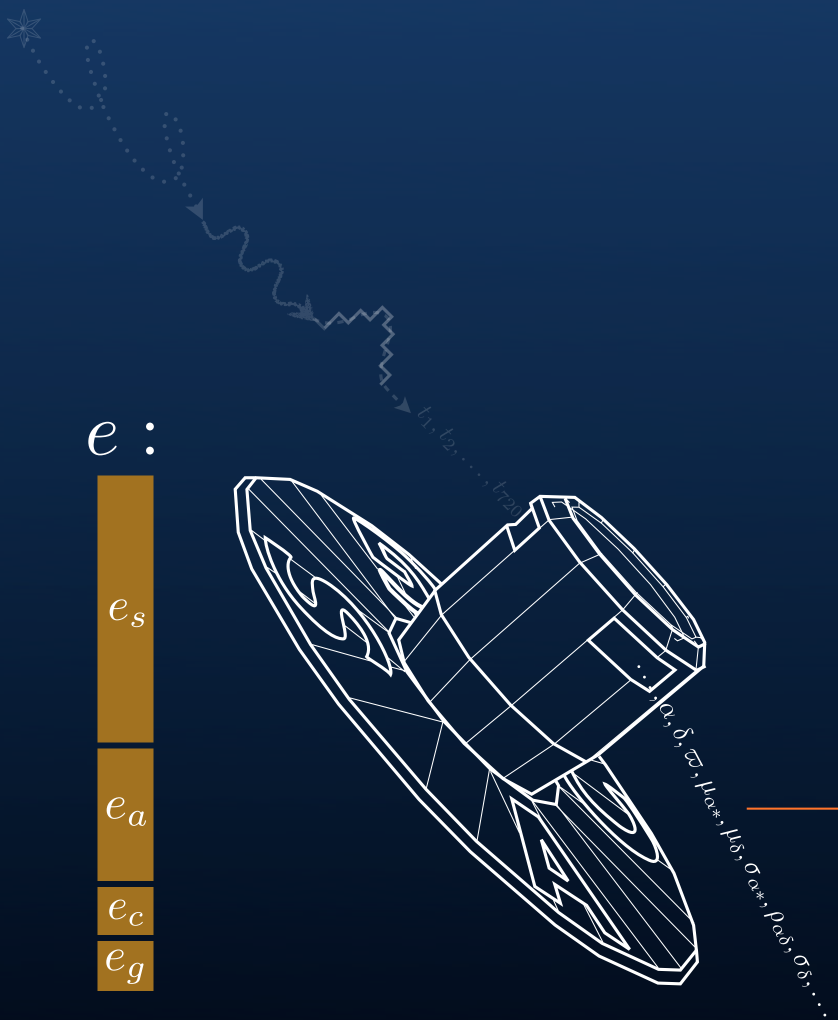
In absence of radiation damage:

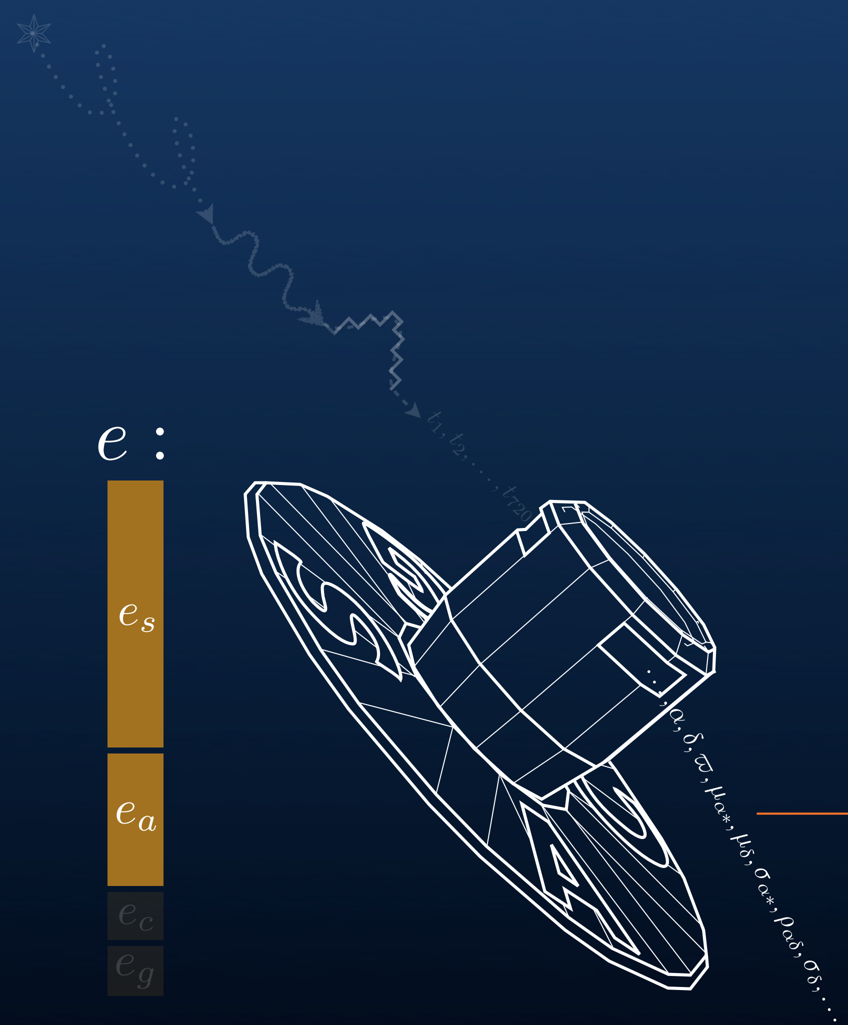
- the ***observation times*** estimated by the **centroiding** process
  will also be **Gaussian, unbiassed, and uncorrelated**,
- therefore, to characterize astrometric errors
  we only need to study error propagation through AGIS!



Astrometric errors

- (S) Source 5×10^9 param
- each parameter error can be decomposed into contributions from estimated models:

$$e_\varpi = e_s + e_a + e_c + e_g$$



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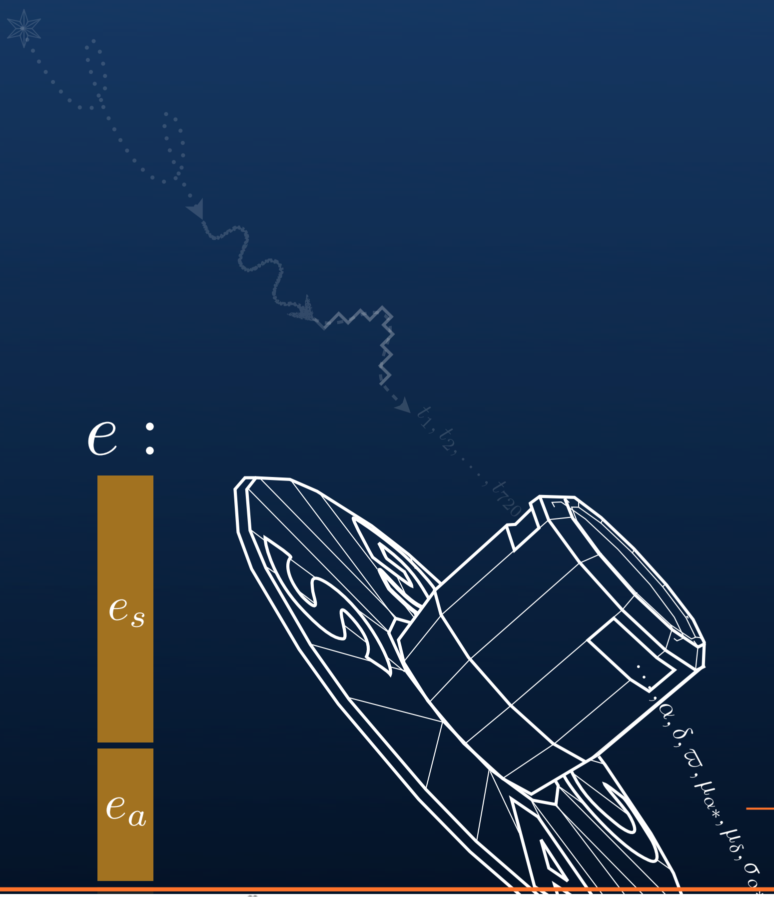
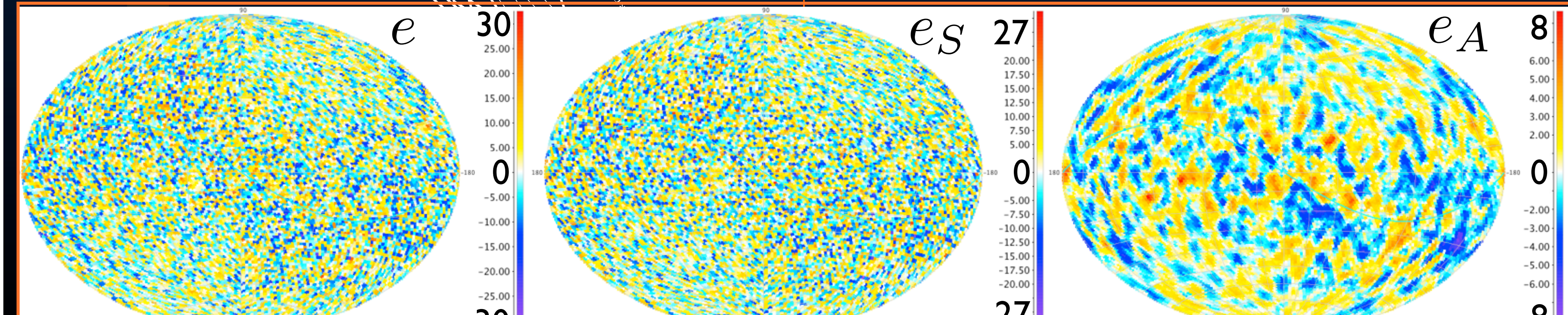


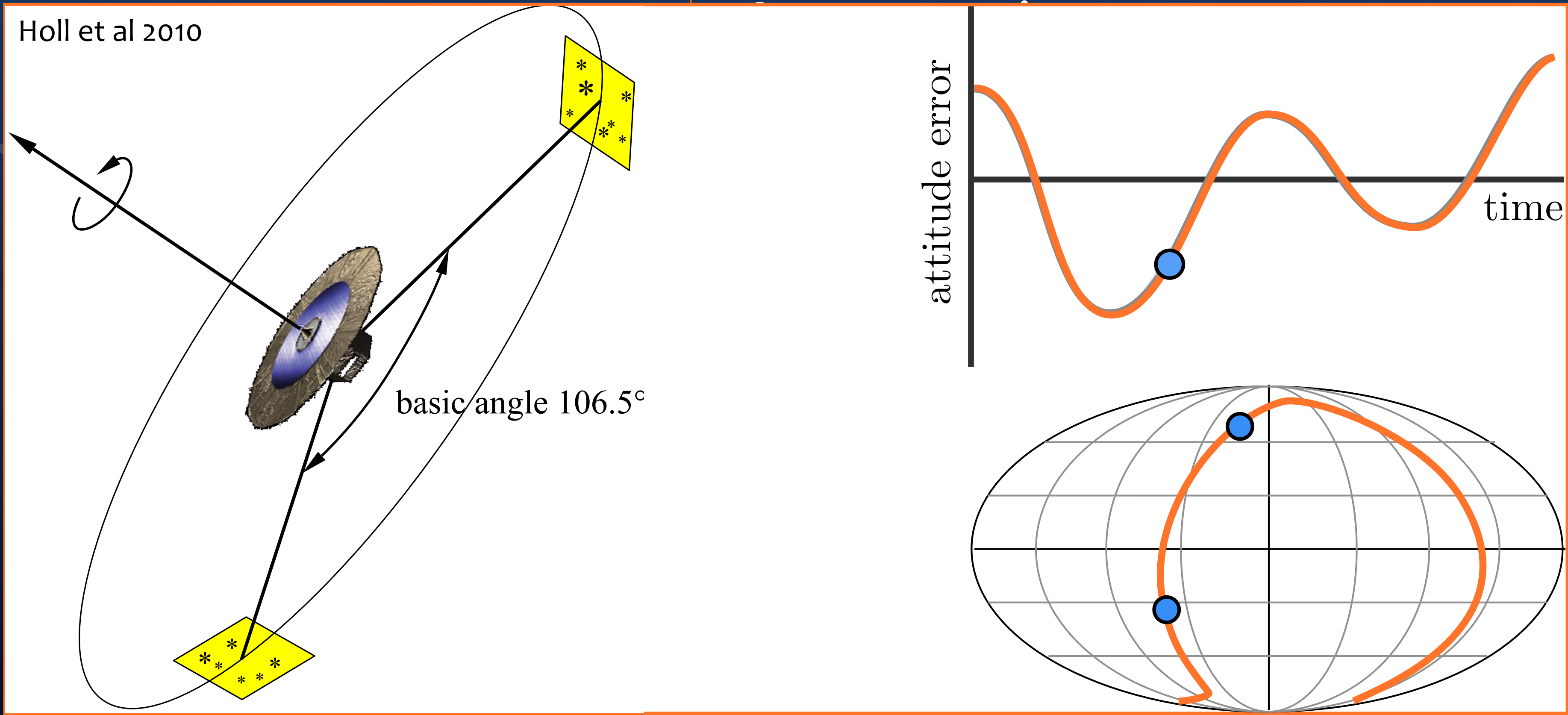


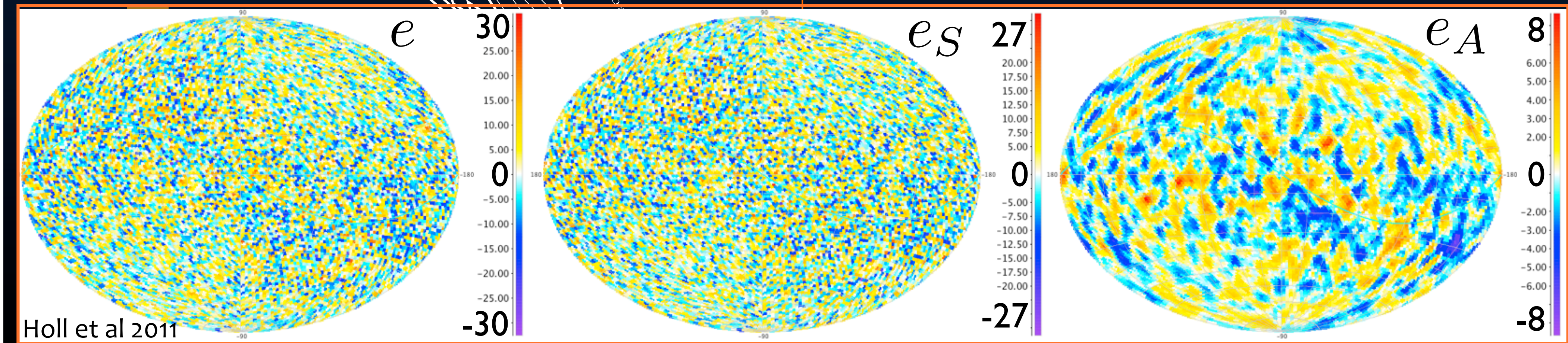
Holl et al 2011

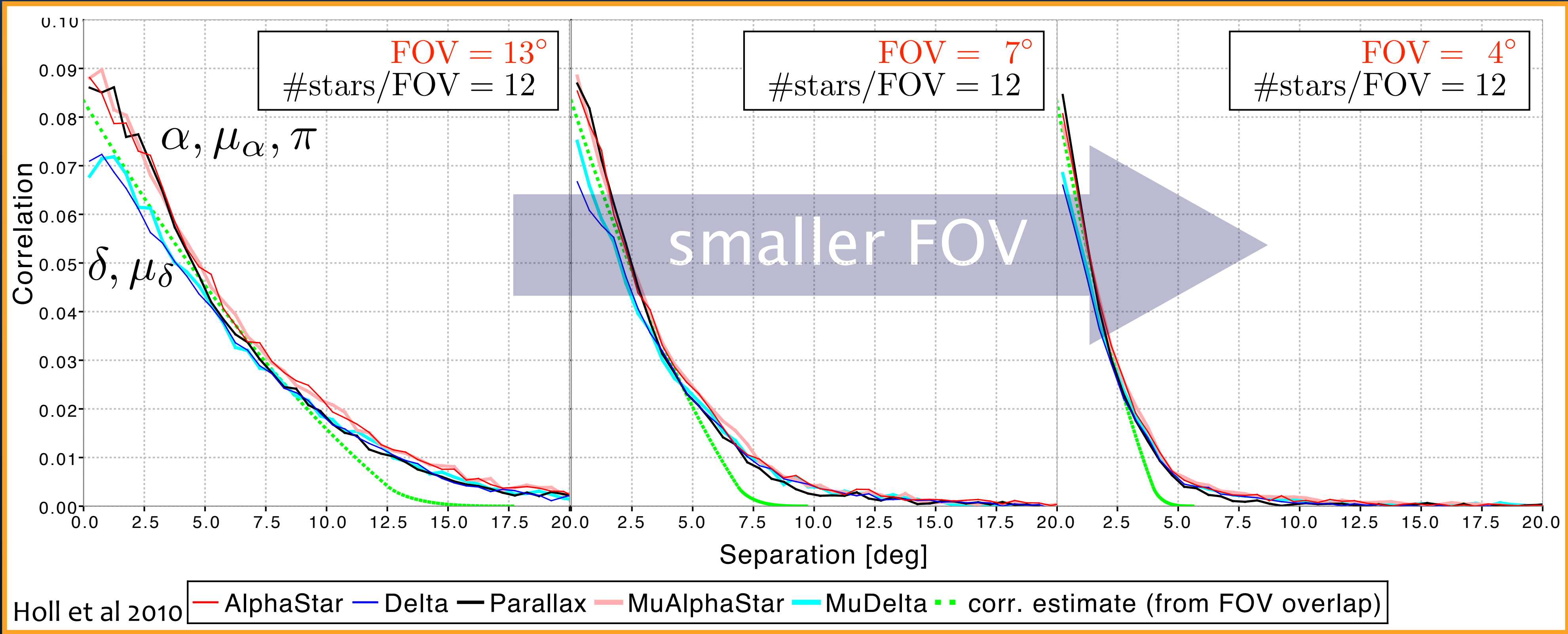
e :

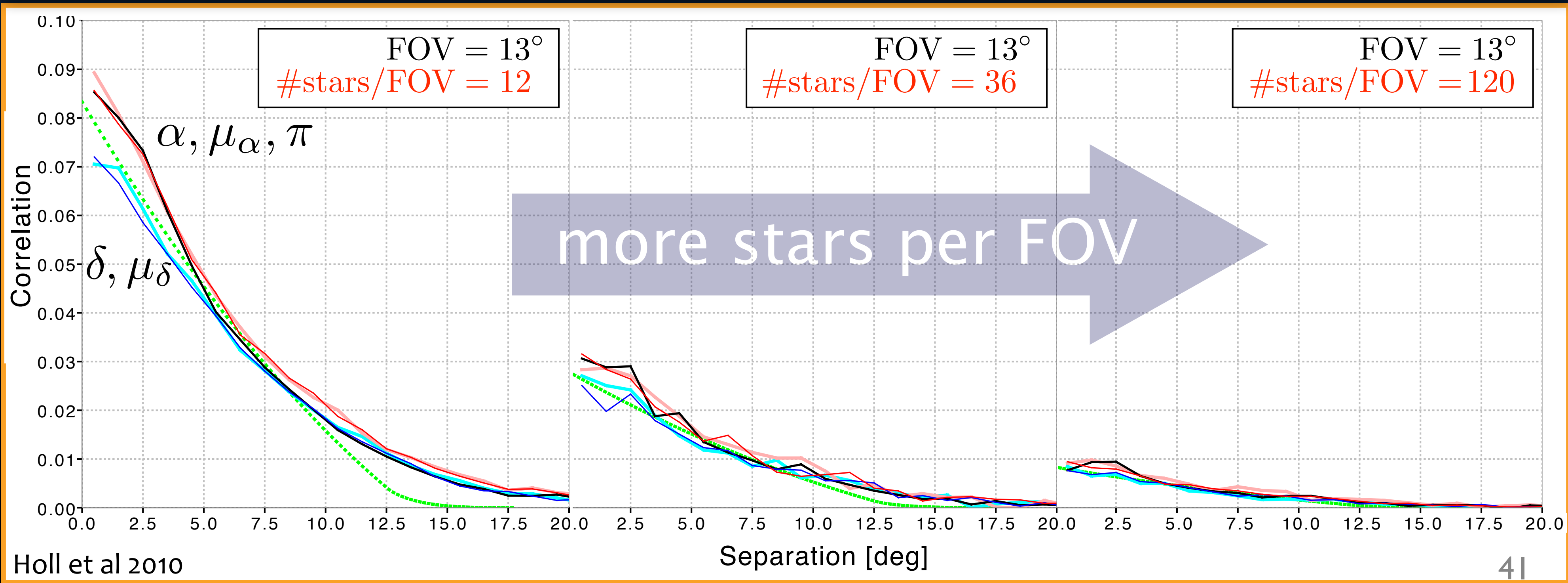
e_s

e_a


Holl et al 2010
basic angle 106.5°
attitude error
time


e
30
0
-30
e_S
27
0
-27
e_A
8
0
-8
Holl et al 2011


FOV = 13°
#stars/FOV = 12
FOV = 7°
#stars/FOV = 12
FOV = 4°
#stars/FOV = 12
α, μ_α, π
δ, μ_δ
smaller FOV
Correlation
Separation [deg]
Holl et al 2010
AlphaStar
Delta
Parallax
MuAlphaStar
MuDelta
corr. estimate (from FOV overlap)


FOV = 13°
#stars/FOV = 12
FOV = 13°
#stars/FOV = 36
FOV = 13°
#stars/FOV = 120
α, μ_α, π
δ, μ_δ
more stars per FOV
Correlation
Separation [deg]
Holl et al 2010

Questions

How do random errors propagate in the astrometric solution?

- Holl, Lindegren, Hobbs (2010, 2011, 2012a, 2012b)

How can the covariance between any pair of astrometric parameters be estimated?

- Holl, Lindegren, Hobbs (2012a, 2012b)

What is the impact of CCD radiation damage on Gaia astrometry?

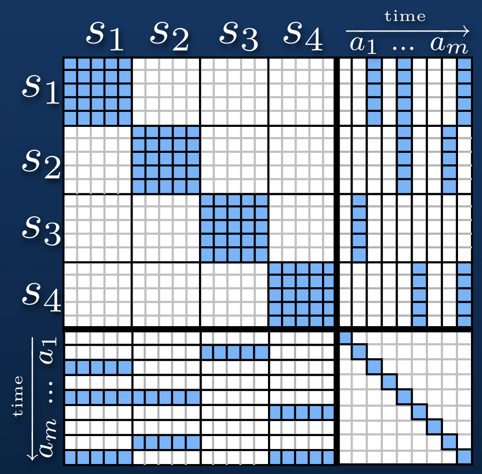
- Prod'homme & Holl et al. (2011), Holl & Prod'homme et al. (2012)

Covariance expansion model

Holl & Lindegren (2012)

normal matrix:

$$N = \left[\begin{array}{cccc|c} S_1'S_1 & 0 & \cdots & 0 & S_1'A_1 \\ 0 & S_2'S_2 & \cdots & 0 & S_2'A_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & S_n'S_n & S_n'A_n \\ \hline A_1'S_1 & A_2'S_2 & \cdots & A_n'S_n & \sum_i A_i'A_i \end{array}\right] = \begin{bmatrix} P & R \\ R' & Q \end{bmatrix}$$



covariance matrix:

$$C \equiv \begin{bmatrix} U & W \\ W' & V \end{bmatrix} = \begin{bmatrix} P & R \\ R' & Q \end{bmatrix}^{-1}$$

Inverse infeasible! (Bombrun et al. 2010)

source covariance series expansion:

$$\begin{aligned} U &= P^{-1} + \\ &\quad P^{-1}RQ^{-1}R'P^{-1} + \\ &\quad P^{-1}RQ^{-1}R'P^{-1}RQ^{-1}R'P^{-1} + \\ &\quad P^{-1}RQ^{-1}R'P^{-1}RQ^{-1}R'P^{-1}RQ^{-1}R'P^{-1} + \cdots \\ &\equiv U^{(0)} + U^{(1)} + U^{(2)} + U^{(3)} + \cdots \end{aligned}$$

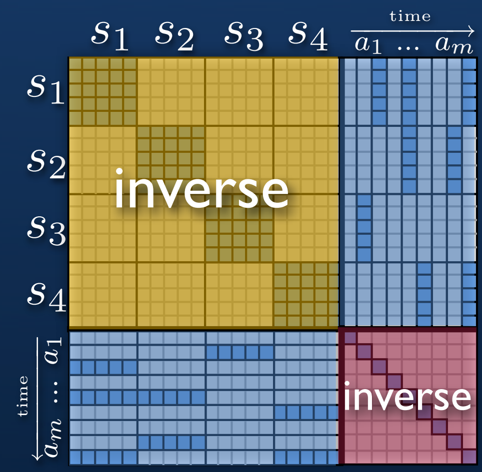
Size $U = 10^{19}$ elements $\sim 10^{8}$ TeraByte = utterly impracticable!

Covariance expansion model

Holl & Lindegren (2012)

normal matrix:

$$N = \left[\begin{array}{cccc|c} S_1'S_1 & 0 & \cdots & 0 & S_1'A_1 \\ 0 & S_2'S_2 & \cdots & 0 & S_2'A_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & S_n'S_n & S_n'A_n \\ \hline A_1'S_1 & A_2'S_2 & \cdots & A_n'S_n & \sum_i A_i'A_i \end{array}\right] = \begin{bmatrix} P & R \\ R' & Q \end{bmatrix}$$



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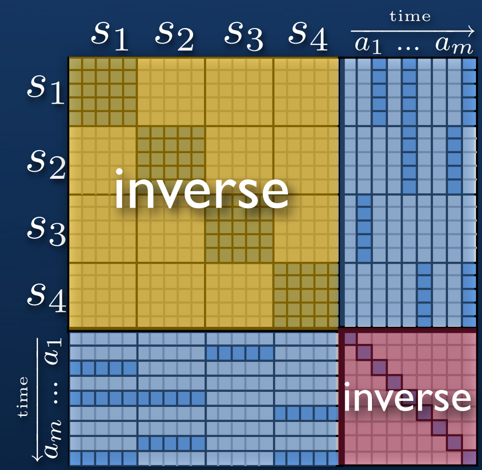
Covariance expansion model

Holl & Lindegren (2012)



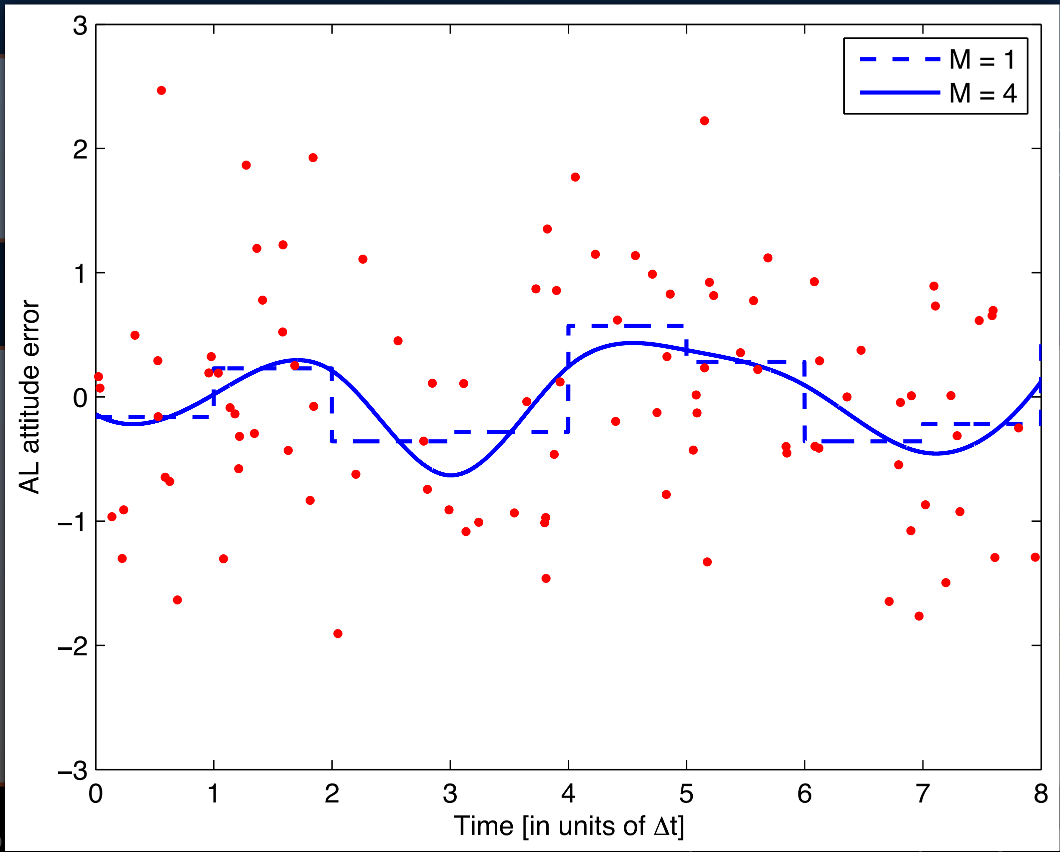
Practical computational model:

Kinematographic approximation
Replaces band-diagonal structure of Q by a diagonal structure:
efficiently compute single elements, or any statistic for any given Jacobian!

covariance matrix:

Bombrun et al. 2010)

source covariance series expansion:



$+ \cdots$

S

cticable!

Questions

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- Prod'homme & Holl et al. (2011), Holl & Prod'homme et al. (2012)

Systematic errors in Gaia catalogue

Measurement process includes effects like:

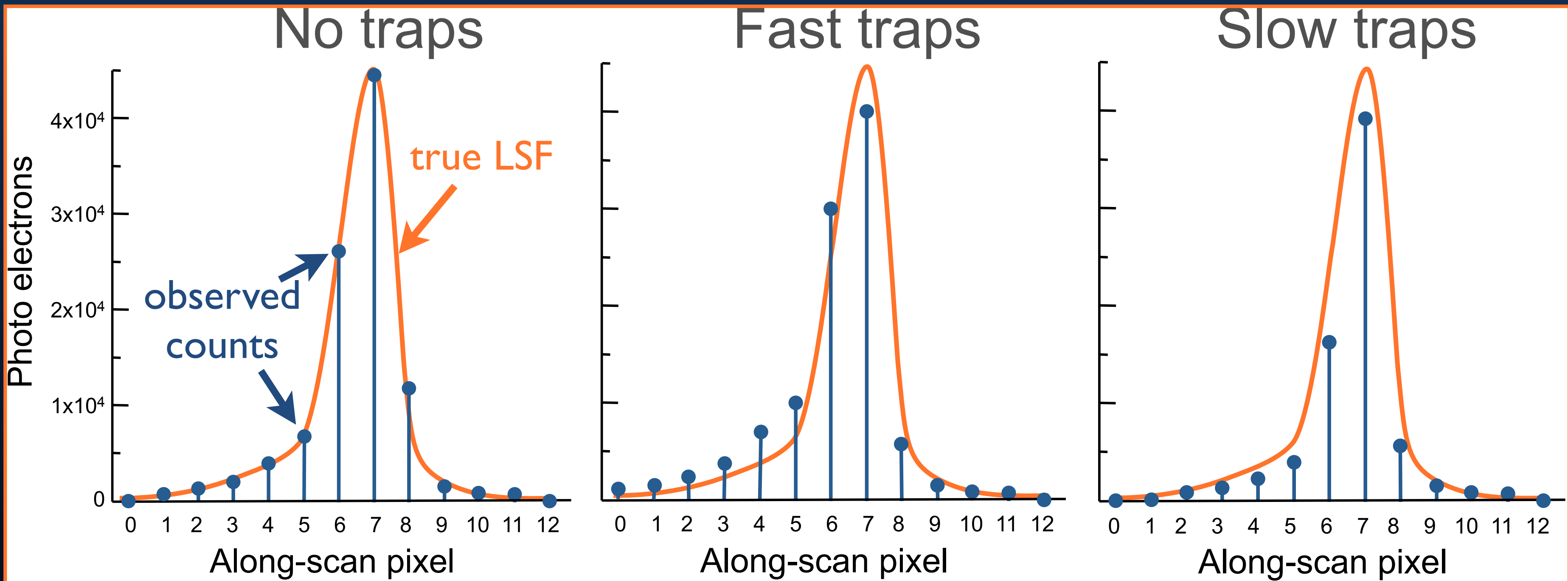
- diffraction of light through telescope,
- effective integration time,
- motion of the satellite during integration,
- CCD readout noise,
- **CCD charge transfer inefficiencies (due to radiation damage).**

CCD Charge Transfer Inefficiencies (CTI):

- causes nonlinear distortions of the observed photoelectron counts,
  (this is partly mitigated for faint stars by the Supplementary Buried Channel),
- introduces correlations between observations through the illumination history,
  (this is largely mitigated by regular Charge Injections.)

Location estimation in presence of radiation damage

 $t_l \longrightarrow t_1, t_2, \ldots, t_{720}$

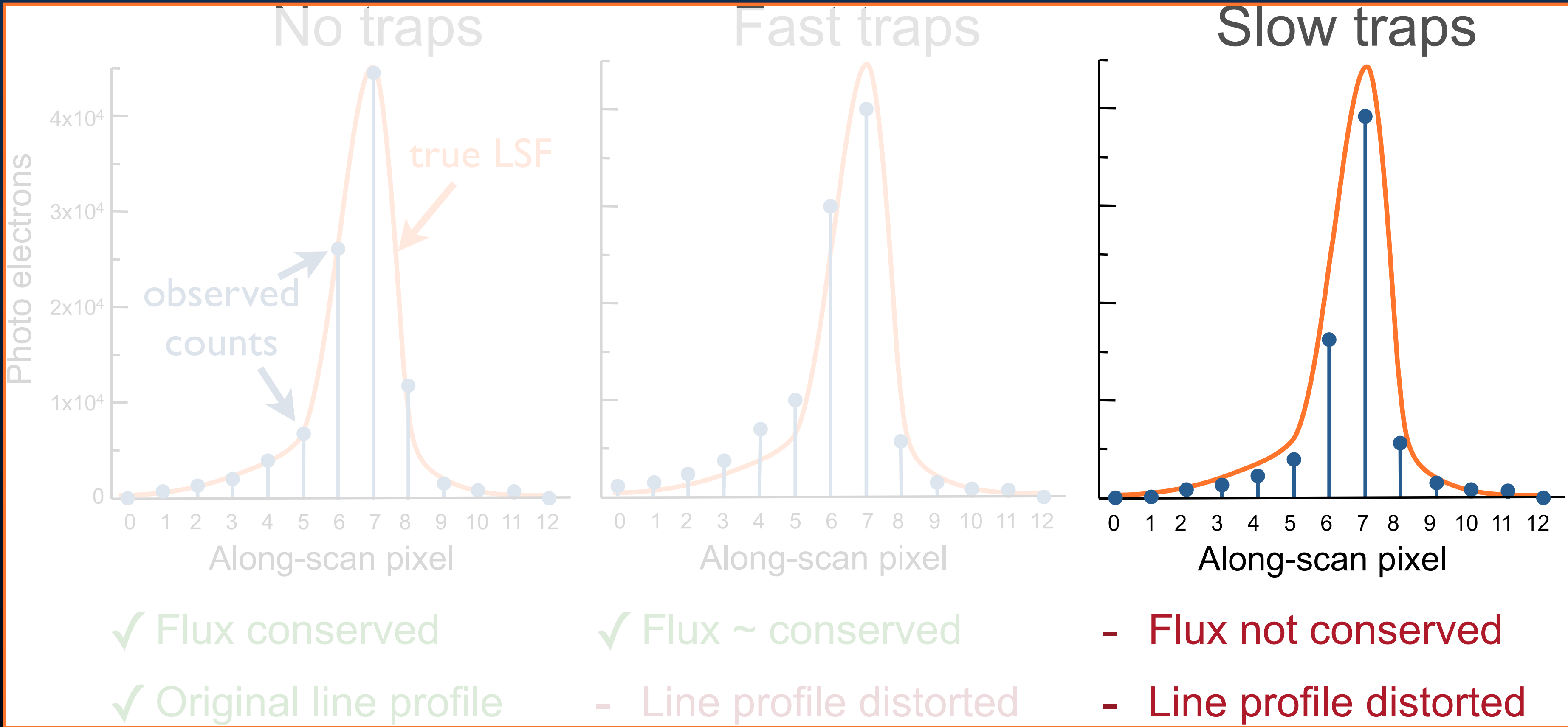


No traps	Fast traps	Slow traps
✓ Flux conserved	✓ Flux ~ conserved	- Flux not conserved
✓ Original line profile	- Line profile distorted	- Line profile distorted

Prod'homme & Holl et al. (2011)

Location estimation in presence of radiation damage

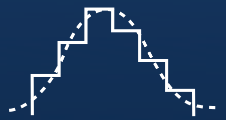
$$t_l \longrightarrow t_1, t_2, \ldots, t_{720}$$



Using **slow** trap with time release constant of ~90 ms

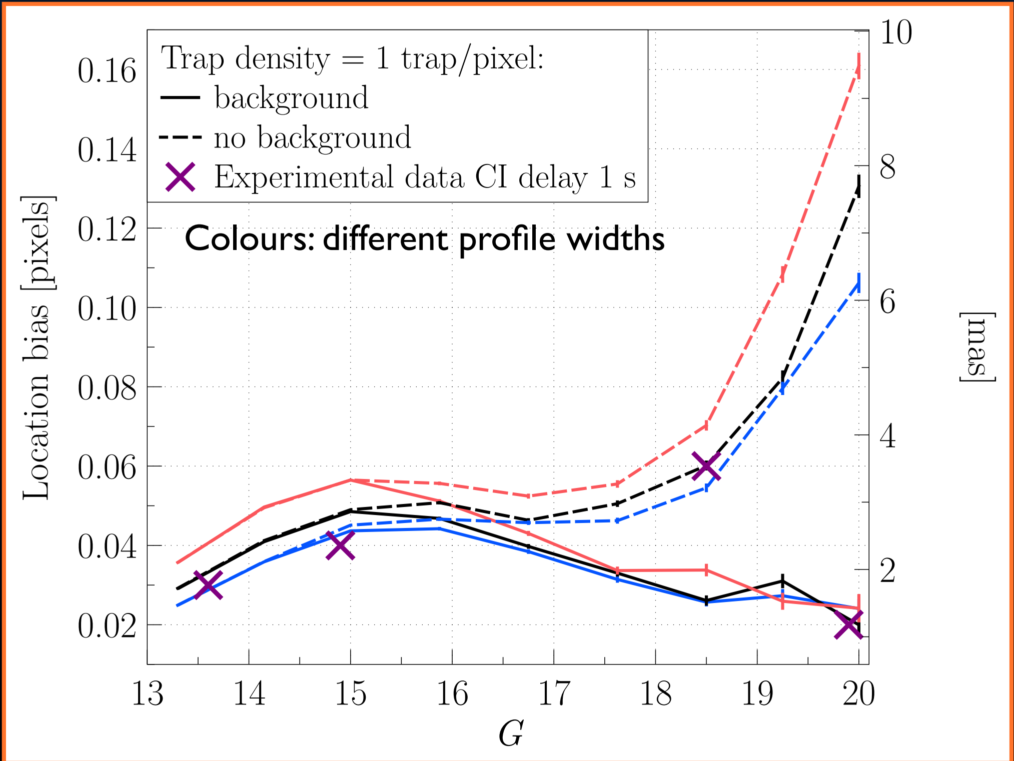
Prod'homme & Holl et al. (2011)

Location estimation in presence of radiation damage

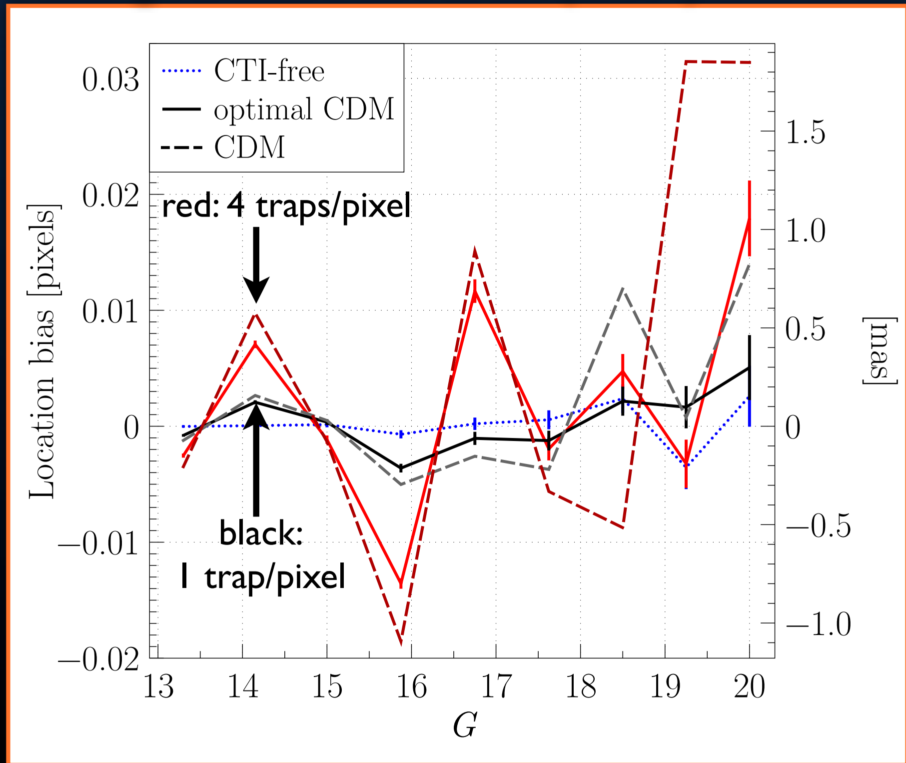
 $t_l \longrightarrow t_1, t_2, \ldots, t_{720}$

Fitting ‘damaged’ observations with:

undamaged LSF



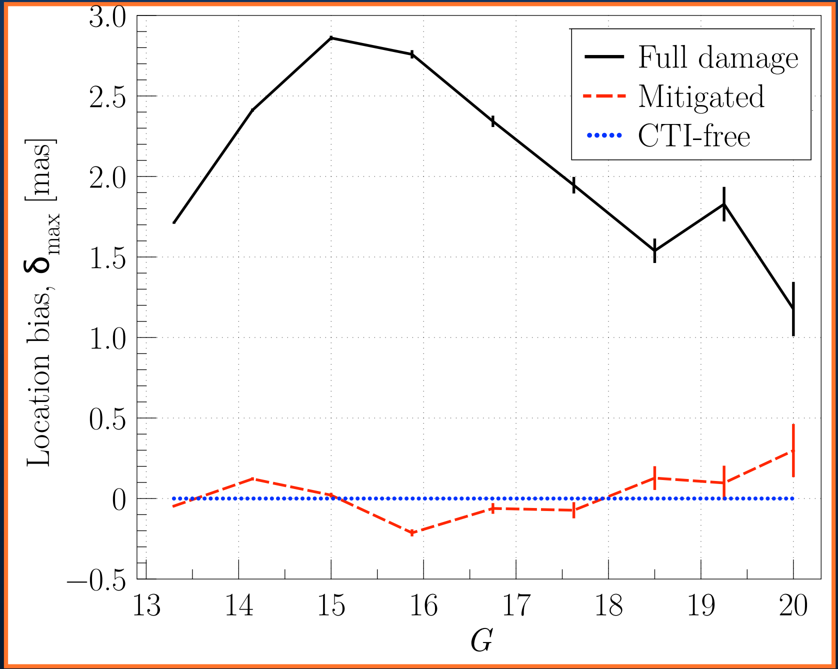
undamaged LSF perturbed by Charge Distortion Model (CDM)



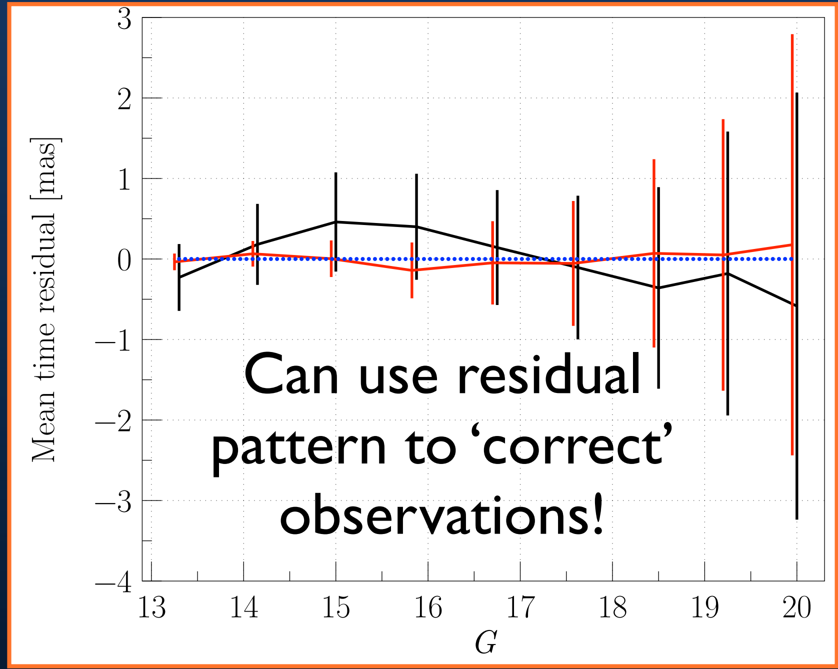
Prod’homme & Holl et al. (2011)

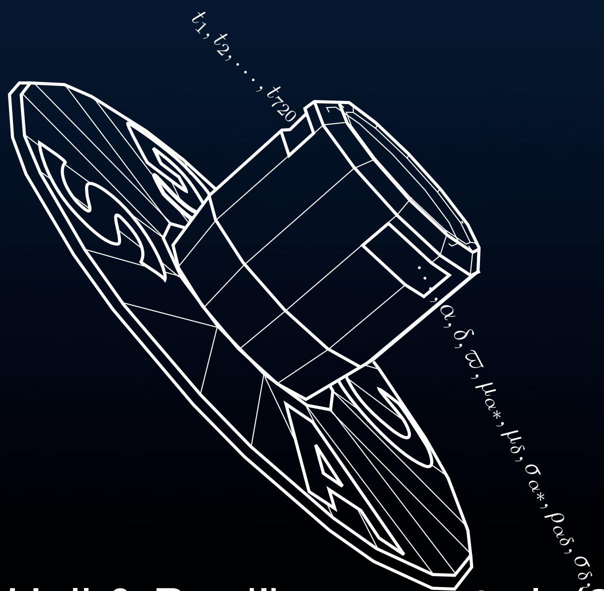
Effect of image location errors on astrometry

Input pattern observation **bias**:



Residuals pattern after AGIS:





Holl & Prod'homme et al. (2012)

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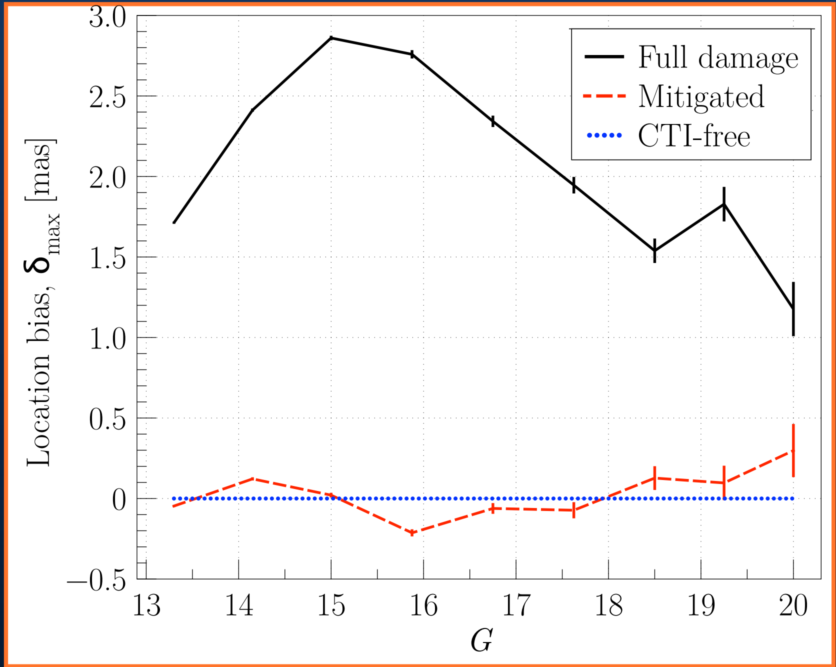
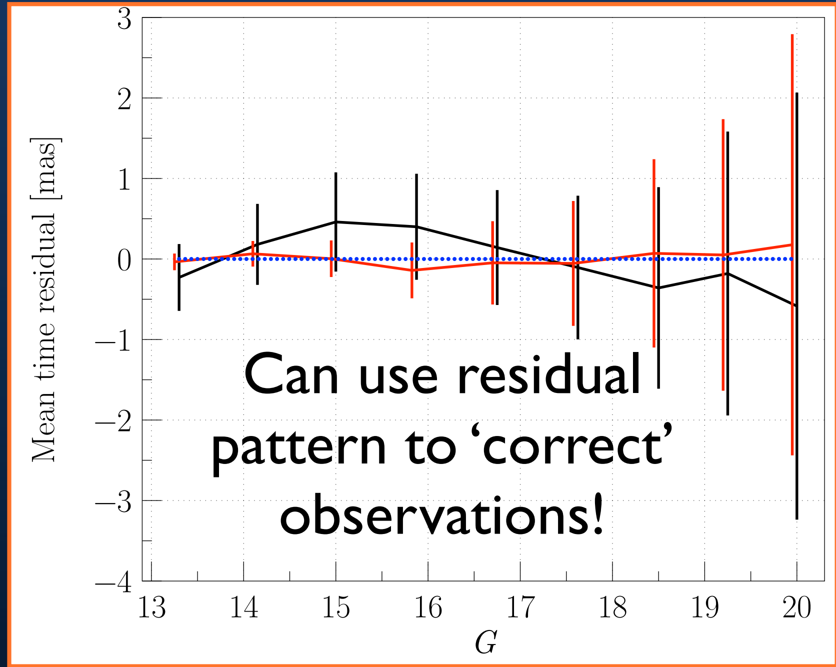
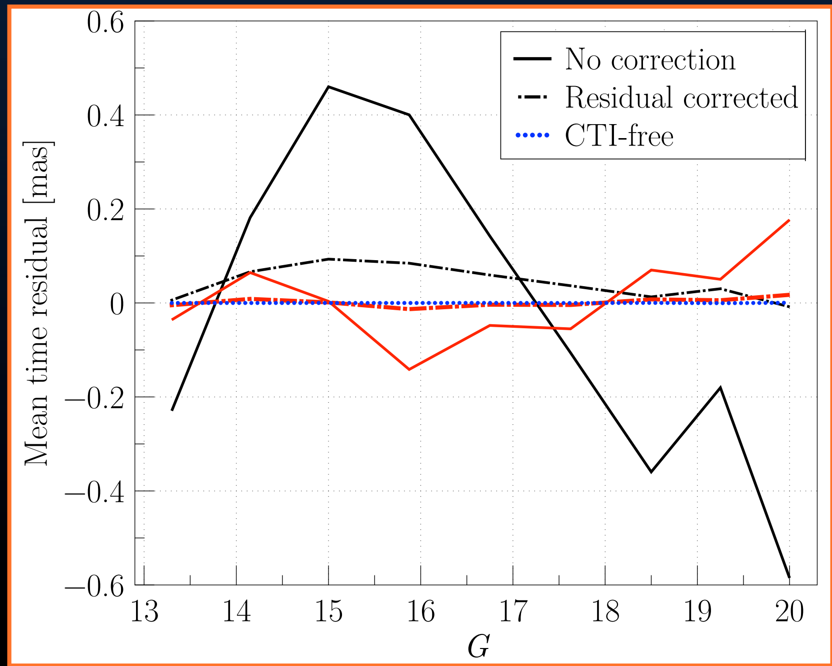


Residuals pattern after AGIS:



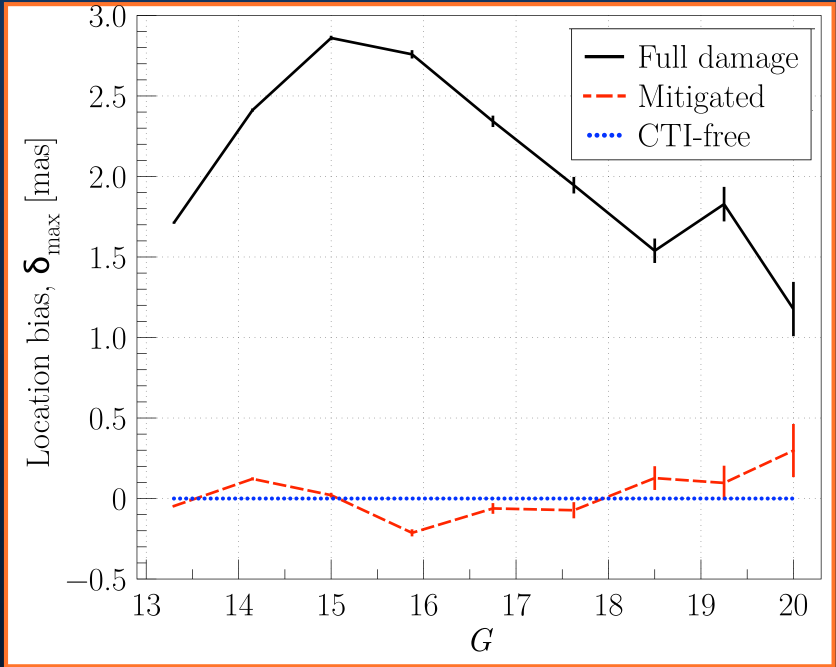
Residuals after subtracting the residual pattern: (note the scale change!)



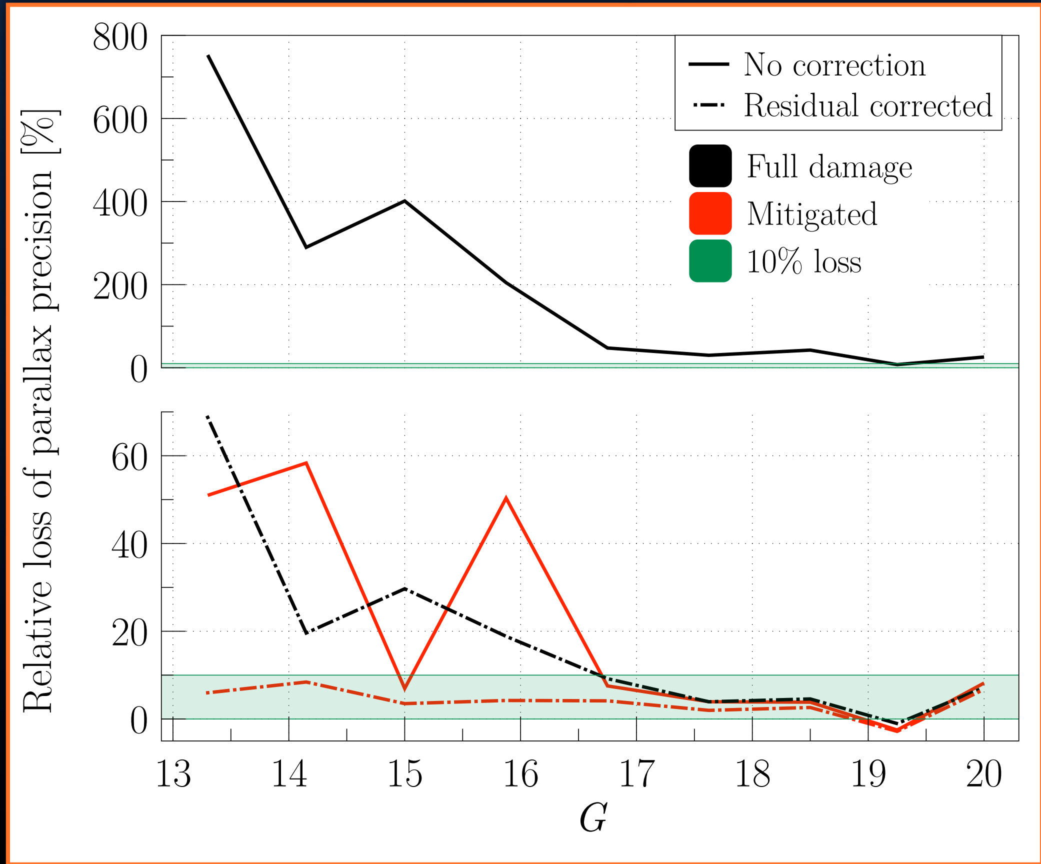
Holl & Prod'homme et al. (2012)

Effect of image location errors on astrometry

Input pattern **observation bias**:



Resulting increase of standard uncertainty in estimated astrometric parallax:



Holl & Prod'homme et al. (2012)

Study of **random** errors

- Errors due to the attitude nuisance parameters can be accurately analysed and characterised using (small scale) Monte-Carlo simulations,
- covariances can be directly computed using Kinomatographic approximation to <0.1% accuracy with reasonable computational effort.
- Note that before this work, parameter variances estimates were underestimated and correlations between sources not known.

Study of **systematic** errors

Impact of radiation damage on astrometric performance of Gaia:

- Bias is likely to be detectable in AGIS residuals and can be calibrated to required levels.
- Modelling errors are likely to add no more than 10% to the standard uncertainty of the parameter estimates (correlations were not examined), which is within the 20% contingency budget in the performance analyses.

Depending on the application uncertainties can be of crucial importance!

Hipparcos 1000 brightest stars: positions + proper motions