Role of dark matter haloes on the predictability of computed orbits

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The predictability of a system indicates how much time a computed orbit is close to an actual orbit of the system, independent of its stability or chaotic nature. We derive a predictability index from the distributions of finite-time Lyapunov exponents. We explore the effect of dark halo shapes on the predictability of computed orbits in a Milky Way mean field model. The results show that not all chaotic orbits have the same predictability and that the predictability of some orbits is more affected than others by the orientation and shape of the dark halo. We show that the lowest predictability may be linked to strong unstable dimension variability.

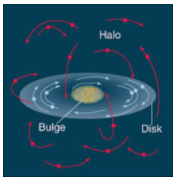
NUMERICAL ASTRONOMY

The gravitational N-body simulation is a common tool to study the evolution of the galaxies and the formation of their features. The galaxy is modelled as a self gravitating system containing stars, gas particles and dark matter, all of them as point-like masses. As alternative, another approach is the use of simulations based in a single mean potential. As there are no collisions among particles, **the dynamics of a galaxy can be considered to be formed by independent trajectories within the global potential**. Some potentials are derived at specific snapshots of the N-body simulations and some others are selected to physically represent desired characteristics.

PREDICTABILITY AND CHAOTICITY

A system is said to be chaotic when it exhibits strong sensitivity to the initial conditions. The predictability of a system is related to, but independent of, its stability or its chaotic nature. The predictability aims to characterize if a numerically computed orbit may be sometimes sufficiently close to another true solution, so may be still reflecting real properties of the model, leading to correct predictions. The real orbit is called a shadow, and within a given distance to the shadow, the observed dynamics can be considered reliable.

TRIAXIAL DARK HALOS



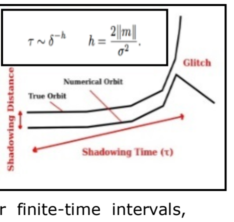
One prediction of the cold dark matter (CDM) models is that galaxy-scale **dark matter haloes are described by a triaxial density ellipsoid**. Cosmological simulations typically lead to triaxial haloes in galaxies that considerably deviate from a spherical shape. The nonlinear coupling introduced by **dark triaxial haloes increases the degree of chaoticity** and may affect the goodness of the computed orbits.

LYAPUNOV EXPONENTS

The ordinary, or asymptotic, Lyapunov exponent λ describes the evolution in time of the distance between two nearly initial conditions, by averaging the exponential rate of divergence of two trajectories starting from a given deviation vector. **Lyapunov exponents detects the presence of strong sensitivity to initial conditions**.

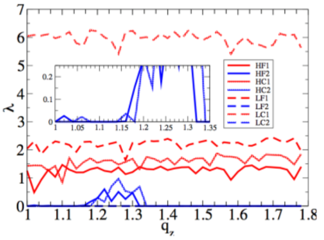
SHADOWING AND PREDICTABILITY INDEX

The shadowing distance is the local phase space distance between a computed trajectory and a true orbit (called shadow). The shadows can exist, but after a given time "τ", they may deviate from the true orbit. Consequently, **this predictability time provides an indication about how reliable a simulation is**.



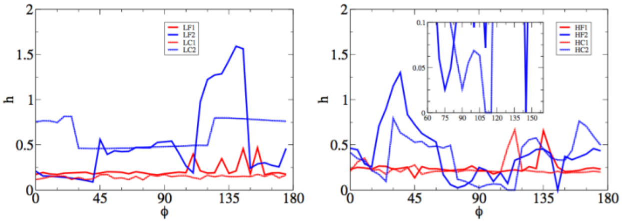
It is possible to compute Lyapunov exponents for finite-time intervals, X(Δt), and to plot the resulting distribution. **A predictability index "h" can be estimated from the mean "m" and standard deviation "σ"** of the distributions of finite-time Lyapunov exponents analysing how their shapes (specifically, the kurtosis) evolve with the interval size Δt.

VARIATION OF CHAOS WITH FLATTENING



We see here the variation in the Maximal Lyapunov exponent with the dark halo flattening q_z. The halo orientation is fixed to φ = 90. **The higher exponent values, corresponding to the most strongly chaotic orbits, are found with the lowest initial velocities** (red curves). The inset shows the range of q_z values where the HC2 and HF2 orbits are chaotic (otherwise they are regular).

VARIATION OF PREDICTABILITY WITH DARK HALO ORIENTATION



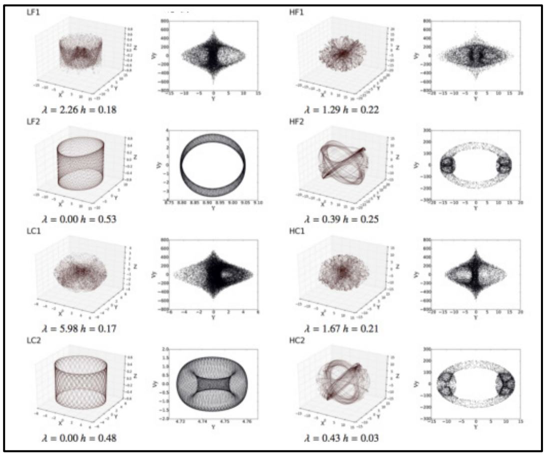
We see here the dependency of the predictability index h on the dark halo orientation φ. The flattening is fixed to q_z = 1.25. A higher h value means a better shadowing, thus a better predictability. (Left) L-orbits (Right) H-orbits. The inset zooms on the orientation angles around 90 degrees. In this interval we see that HC2 and HF2, the blue orbits with **higher initial velocities, have very low h values, therefore low predictabilities**.

REPRESENTATIVE ORBITS IN A MILKY-WAY MODEL

We have selected **a representative set of initial conditions** in the potential described in Law, D. R., Majewski, S. R. & Johnston, K. V., ApJ, 703, L67 (2009). The dynamical system to solve is a particle (star) subject to a potential built upon three components: a Miyamoto-Nagai disc, a Hernquist spheroid, and a logarithmic halo, with **main control parameters the halo orientation φ, and the flattening q_z**. We have analysed how their chaotic nature and predictability changed with the dark halo parameters.

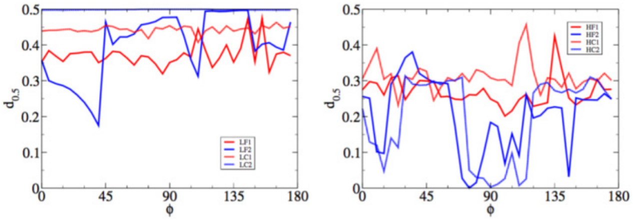
Orbit	Initial location	(x_0, y_0, z_0),	(Vx_0, Vy_0, Vz_0)	Dominant behaviour
HF1	High z, far from centre, low velocity	(10.0, 0.0, 10.0)	(0.0, 50.0, 0.0)	Chaotic
HF2	High z, far from centre, high velocity	(10.0, 0.0, 10.0)	(0.0, 200.0, 0.0)	Mainly regular
HC1	High z, close to centre, low velocity	(5.0, 0.0, 10.0),	(0.0, 50.0, 0.0)	Chaotic
HC2	High z, close to centre, high velocity	(5.0, 0.0, 10.0)	(0.0, 200.0, 0.0)	Mainly regular
LF1	Low z, far from centre, low velocity	(10.0, 0.0, 0.5)	(0.0, 50.0, 0.0)	Chaotic
LF2	Low z, far from centre, high velocity	(10.0, 0.0, 0.5)	(0.0, 200.0, 0.0)	Regular
LC1	Low z, close to centre, low velocity	(5.0, 0.0, 0.5)	(0.0, 50.0, 0.0)	Highly chaotic
LC2	Low z, close to centre, high velocity	(5.0, 0.0, 0.5)	(0.0, 200.0, 0.0)	Regular

The representative set of orbits was divided into orbits with high initial z position, or out-of-disc H-orbits, and those with a lower initial z position, or close-to-disc L-orbits.



Physical trajectories and the corresponding Poincaré sections y – v_y with plane x = 0 and v_x > 0 for the selected orbits. The orientation is φ = 90.0, the flattening is q_z = 1.25. We see a variety of regular and chaotic orbits, as reflected by the Lyapunov exponent λ, and a variety of low- and high- predictable orbits, as reflected by the predictability index h. Low predictability is not always linked to strong chaos.

SOURCES OF LOW PREDICTABILITY



One of the sources of low predictability is the presence of Unstable Dimension Variability, reflected by oscillations around zero of the closest-to-zero finite-time Lyapunov exponent consequence of the existence of both expanding and contracting directions. The distance parameter $d_{0.5}$ ≈ 0.0 indicates these oscillations. (Left) Variation in $d_{0.5}$ for L-orbits. (Right) Variation for H-orbits. The flattening is fixed as q_z = 1.25. **The H-orbits show the lowest values**.

FURTHER DETAILS?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The forecast of predictability for computed orbits in galactic models

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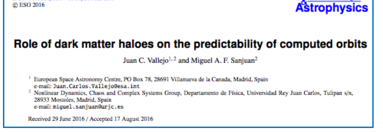
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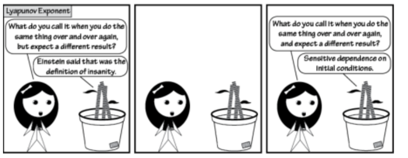
LOST?



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