# Question Set 2 — Probability Distributions

## Question 1

Photons arrive from a source at a typical rate of $\lambda = 10$ per second. In any time interval of length $t$, the number $x$ of photons has a Poisson distribution with parameter $\lambda t$. Non-overlapping time intervals are independent.

(a) Numerically verify that the expected value of $x$ for a one-second time interval is 10 and the standard deviation is $\sqrt{10}$.

(b) Find the probability that there are exactly 10 photons in each of two consecutive one-second intervals.

## Question 2

A "standard normal" distribution (with mean zero and standard deviation 1) has probability density function

$$p(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right). \quad (1)$$

(a) Make a grid in Python (or whatever language you like) and plot the PDF.

(b) Numerically calculate $P(x \in [-1, 1])$. You might recognise the number as a 'one-sigma confidence level'.

(c) Numerically calculate $P(x \in [-2, 2])$. You might recognise the number as a 'two-sigma confidence level'.

(d) Numerically calculate the mean and standard deviation and make sure you get something close to 0 and 1 respectively.

A $t$ distribution has a PDF proportional to

$$p(x|\nu) \propto \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}} \quad (2)$$

The parameter $\nu$ controls the shape of the distribution (see the plot on the Wikipedia page, which shows the effect of $\nu$).

(e) Let $\nu = 3$. Numerically calculate $P(x \in [-2, 2])$.