Question Set 3 — Parameter Estimation

Question 1

You are a customs agent. Among other things, you are supposed to prevent drugs being smuggled in packages sent into the country. A colleague finds a package containing two toys. Let the number of toys containing drugs be η. Clearly the value of η is either 0, 1, or 2. You drill into one of the toys and find that it does not have drugs. Calculate the posterior distribution for η given that the tested toy did not contain drugs. Assume a (discrete) uniform prior, i.e. $P(\eta = 0) = P(\eta = 1) = P(\eta = 2) = 1/3$.

Question 2

An X-ray source emits photons at a steady rate, but since it's so distant, we only pick up a few photons per second. A standard probabilistic model for the arrival times of the photons is the "Poisson process". A specific prediction of this model is that, if the expected number of photons in a time interval is λ, the (discrete) probability distribution for the actual number of photons x in that interval is a Poisson distribution:

$$p(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \tag{1}$$

Find and plot the posterior distribution for λ given $x = 5$. Use an improper log-uniform prior proportional to λ^{-1}. You can do this numerically or analytically.

Question 3

Use the posterior distribution obtained in the previous question to calculate the predictive distribution for x', the number of photons observed in a different one second interval, given $x = 5$. Plot the predictive distribution and compare it with what you'd get by naively assuming the best fit (maximum likelihood) value $\lambda = 5$ to make the prediction.

Hint 1: Write down the probability distribution for x' given λ and x, and then marginalise out λ.

Hint 2: You may need to do some numerical integration.