# Pseudo-Marginal MCMC

- This exercise is designed to illustrate the pseudo-marginal MCMC algorithm (i.e., the use of unbiased random estimates within a Monte Carlo accept-reject rule)

- ***scenario: Bayesian synthetic likelihood***

$L(y|\theta)$ known to be of form $y \sim \text{Normal}\big(\mu(\theta), \Sigma(\theta)\big)$ but $\mu(\cdot), \Sigma(\cdot)$ unknown

however, we can generate mock data $y \sim L(\cdot\,|\theta)$ and want to do MCMC on $\pi(\theta|y)$

<u>pseudo-marginal MCMC with Normal Likelihoods</u>

(i) from $\theta_i$ with current likelihood estimate, $\widehat{L_i}$

(ii) propose $\theta_{i+1} \sim q(\cdot\,|\theta_i)$

(iii) draw $n$ mock datasets $y_{j=1:m} \sim L(\cdot\,|\theta_{i+1})$

(iv) estimate $\widehat{L_{i+1}}$ by plugging $y_{j=1:m}$ into the unbiased estimator from Ghurye and Olkin (1969)

(v) accept $\theta_{i+1}$ if $p \sim \text{Uniform}(0,1)$ such that $p < \frac{\pi(\theta_{i+1})q(\theta_i|\theta_{i+1})\widehat{L_{i+1}}}{\pi(\theta_i)q(\theta_{i+1}|\theta_i)\hat{L}_i}$

else $\theta_{i+1} \leftarrow \theta_i$

- we'll start with a model that is readily tractable (mu linear, sigma fixed) and compare against the truth with exact likelihood MCMC and against Heavens & Sellentin's t-distribution method
- then we'll modify our code to accept a block box for generating mock data