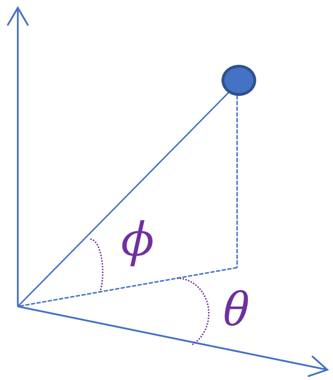
Sequential Importance Sampling

- This exercise is designed to illustrate online updating of particle based representations of probability distributions

- ***scenario: tracking a randomly moving object against a crowded field***



system equation

$$\begin{pmatrix}\theta\\ \phi\end{pmatrix}_{i+1} = \begin{pmatrix}\theta\\ \phi\end{pmatrix}_{i} + \begin{pmatrix}\dot\theta\\ \dot\phi\end{pmatrix}_{i} + \frac{1}{2}\begin{pmatrix}\Delta\dot\theta\\ \Delta\dot\phi\end{pmatrix}_{i} \qquad \begin{pmatrix}\dot\theta\\ \dot\phi\end{pmatrix}_{i+1} = \begin{pmatrix}\dot\theta\\ \dot\phi\end{pmatrix}_{i} + \begin{pmatrix}\Delta\dot\theta\\ \Delta\dot\phi\end{pmatrix}_{i}$$

observation errors

$$\theta_{\text{observed},i} \sim \text{VonMises}(\theta_i, \kappa)$$

$$(\phi_{\text{observed},i} - \frac{pi}{2})/\pi \sim \text{Beta}\left(N(\phi_i - \frac{\pi}{2})/\pi, N(1 - (\phi_i - \frac{\pi}{2})/\pi)\right)$$

random noise

$$\begin{pmatrix}\Delta\dot\theta\\ \Delta\dot\phi\end{pmatrix}_{i} \sim \text{Normal}(\mathbf{0}, \sigma_v^2 \boldsymbol{I})$$

crowded field

$$N_{\text{noise}} \sim \text{Poisson}(N_{\text{exp}})$$

$$\begin{pmatrix}\theta\\ \phi\end{pmatrix}_{\text{noise},i} \sim \text{Uniform}\begin{pmatrix}0, 2\pi\\ -\pi, \pi\end{pmatrix}$$

- Method: we will imagine exact knowledge of the object's initial location and that it begins with zero velocity

we will track it via a set of particles iterated (randomly) via the system equation, weighted via the observational likelihood, and resampled to avoid particle degeneracy