OPTIMAL MINIMUM-ENERGY EARTH-MARS TRANSFER TRAJECTORIES

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Introduction: The analysis of space mission costs has become a key factor in mission planning. The determination of optimal trajectories aiming to lower costs in terms of impulses allows for more massive payloads to be transported at a minimum energy cost. In this field, the case of low-thrust transfers Earth to Mars is of great interest.

In this work we analyse the determination of optimal interplanetary trajectories from Earth to Mars by minimizing the total required energy in the major mission phases: Earth departure, interplanetary targeting orbit and Mars arrival. Such analysis is done by solving the Lambert orbital boundary-value problem and investigating the optimal departure and arrival windows. Minimizing the cost function by exhaustive search or analytical methods is infeasible, since for each Earth departure date and Mars arrival date combination a Lambert's problem need to be solved to determine the optimal orbital transfer parameters.

To avoid this difficulty, we will first focus in data visualization of the departure characteristic energy and the hyperbolic arrival velocity, solving the Lambert problem for various combinations of departure and arrival dates in order to reduce the launch and arrival windows. Genetic algorithms are then applied to these reduced windows to choose an optimal solution and follow this with the optimal transfer orbital parameters determination.

Earth-Mars Trajectory Modelling: Transfer orbits are defined by patching a hyperbolic orbit at the boundary of the sphere of influence with respect to Earth, an elliptical heliocentric orbit and an arrival hyperbolic trajectory about Mars. To determine the departure and arrival velocity vectors, and the transfer trajectory orbital parameters it is necessary to solve the Lambert orbital boundary-value problem constrained by two points, P_1 and P_2 and an elapsed time of flight (TOF), t_2-t_1,:

$$\ddot{\vec{r}} = -\mu \frac{\vec{r}}{r^3},$$

$$\vec{r}(t_1) = \vec{r_1},$$

$$\vec{r}(t_2) = \vec{r_2},$$

Solutions to each Lambert problem results in an elliptic conic section connecting P_1 and P_2. We consider the short-way solution that satisfies the boundary conditions. The solution is based in the iterative procedure given in [1] choosing as parameter for the iteration the time transfer function introduced by [2]. Precision ephemerides for the planets are obtained from the JPL HORIZONS system [3].

Trajectory Optimization: In this minimum-energy problem, we search a solution minimizing

$$C = W_{C_3} C_3 + W_{\infty_M} V_{\infty_M},$$

where the characteristic energy at the boundary of the sphere of influence of the Earth, $C_3 = V^2_{\infty_E}$, and the Mars hyperbolical arrival velocity, V_{∞_M}, are weighted according to the values W_{C_3} and W_{∞_M}.

The minimum C tends to give lower values of the total impulsive manoeuvres, $\Delta V = \Delta V_E + \Delta V_M$, required in an Earth-Mars transfer, to first give to the space vehicle an orbital velocity greater than the parabolic Earth escape velocity and after, at the Mars arriving hyperbolic orbit, to reduce the hyperbolic excess velocity.

We applied a heuristic-based approach to identify optimal Earth-Mars trajectories: 1) To reduce the launch and arrival windows by contour plots analysis of the departure characteristic energy and the hyperbolic arrival

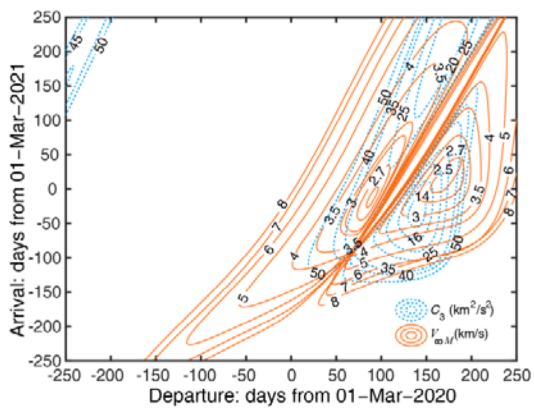

Figure 1: Departure characteristic energy and hyperbolic arrival velocity contour plots for the Earth-Mars transfer from 1 July 2019 to 1 November 2021.

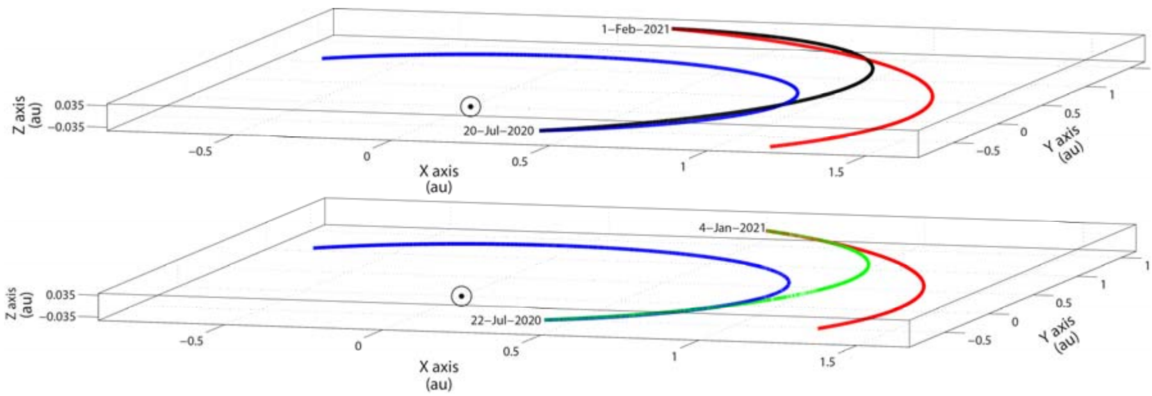

Figure 2: Minimum energy optimal trajectory for the Earth-Mars transfer from 1 July 2019 to 1 November 2021 (black). Comparison with a trajectory with 31 days less of time of flight (green).

velocity, solving the Lambert problem for various combinations of departure and arrival dates; 2) to apply genetic algorithms to these reduced windows to choose an optimal solution; 3) to determine optimal transfer orbital parameters.

Data Visualization. Contour Plots Analysis: In order to decide the best launch and arrival windows, Fig. 1 shows the departure characteristic energy and hyperbolic arrival velocity contour plots for the Earth-Mars transfer from 1 July 2019 to 1 November 2021.

Optimization Results. Genetic Algorithms: We have compared the performance of the Remainder and the Stochastic Uniform functions as selection functions to select the individuals that contribute to the population at the next generation, and the Heuristic, the Scattered and the Single point rules as crossover functions to combine two individuals to form the next generation [4]. The main results (see Table 1) for the different weight functions are: 1) Departure dates change only in 3 days, 2) TOF changes from 197 to 202 days, 3) Mars arrival hyperbolic excess velocity is reduced 0.1 km/s.

Pop.	Crossover	Selection	C.P.U. Time (s)	Departure date (20-Jul-20)	TOF (days)	C_3 (km^2/s^2)	$V_{\infty M}$ (km/s)
100	Heuristic	remainder	10.86	01:19:08	196.9397	13.1268	2.7683
		stoch. unif.	10.36	01:05:27	196.9253	13.1264	2.7686
	Scattered	remainder	10.30	01:04:01	196.9501	13.1265	2.7686
		stoch. unif.	10.49	01:01:17	196.9281	13.1264	2.7687
	Single point	remainder	10.14	01:06:02	196.9657	13.1270	2.7681
		stoch. unif.	10.35	01:14:40	196.9566	13.1270	2.7681
500	Heuristic	remainder	48.30	01:06:27	196.9288	13.1264	2.7686
		stoch. unif.	47.86	01:13:05	196.9420	13.1267	2.7684
	Scattered	remainder	48.31	01:12:22	196.9420	13.1267	2.7684
		stoch. unif.	47.60	01:21:17	196.9537	13.1270	2.7681
	Single point	remainder	47.32	01:24:10	196.9407	13.1268	2.7683
		stoch. unif.	47.19	01:16:41	196.9450	13.1268	2.7683
2000	Heuristic	remainder	188.06	01:04:09	196.9393	13.1266	2.7685
		stoch. unif.	186.57	01:07:45	196.9332	13.1265	2.7686
	Scattered	remainder	222.98	01:16:41	196.9450	13.1268	2.7683
		stoch. unif.	229.03	01:10:38	196.9361	13.1266	2.7685
	Single point	remainder	238.38	01:08:54	196.9343	13.1266	2.7685
		stoch. unif.	237.49	01:26:54	196.9480	13.1270	2.7681

Table 1: Simulation scenarios to compare the genetic algorithms performances minimizing the cost function of the required energy and the arrival velocity with equal weight factors.

The considered selection and crossover functions, do not change significatively the results. C.P.U time depends on the population size, leading to equivalent results.

Optimal Transfer Orbital Parameters: Figure 2 shows the minimum energy optimal trajectory for the Earth-Mars transfer from 1 July 2019 to 1 November 2021, compared with a 31 days less of TOF trajectory. Table 2 lists the orbital parameters for optimal transfer.

Parameter	Minumun energy
Departure Date	20-Jul-2020 01:13:05h
Arrival Date	1-Feb-2021 23:49:34h
Semi-major axis (km)	198179671.95
Eccentricity (rad)	0.23297
Inclination (deg)	1.7206
Ascending node longitude (deg)	297.8072
Argument of the perihelium (deg)	358.9220
True anomaly (deg)	1.0779
$V_{\infty E}$ (km/s)	3.6231
$V_{\infty M}$ (km/s)	2.7684
Time of flight (days)	196.9420

Table 2: Earth-Mars optimal transfer orbital parameters for the 1 Jul. 19 -1 Nov. 21 window.

References: [1] R. H. Gooding, On the solution of lambert's orbital boundary-value problem, Royal Aerospace Establishment (1988).
[2] E. R. Lancaster, R. C. Blanchard, A unified form of Lambert's theorem, Goddard Space Flight Center (1988).
[3] HORIZONS Web-Interface. NASA JPL, http://ssd.jpl.nasa.gov/horizons.cgi.
[4] https://es.mathworks.com/help/gads/genetic-algorithm-options.html#f7820.

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