SELF-CONSISTENT DISTANCE DETERMINATIONS FOR LUTZ-KELKER-LIMITED SAMPLES

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ABSTRACT

We present a method designed to correct for Lutz-Kelker effects in distance-limited samples. The method allows for the calculation of distances to individual objects and, at the same time, provides a fit to a parameterized, self-consistent spatial distribution of the population. An example using Hipparcos data is presented and the relevance to Gaia is also discussed.

Key words: Astrometry; Galaxy: structure; Gaia; Methods: numerical.

1. WHY?

The measurement of distances from trigonometric parallaxes is complicated by the existence of the Lutz-Kelker bias (Lutz & Kelker 1973), which generates selection effects when analyzing a given stellar population and also requires the use of correction factors for the distance to a given star derived from its parallax (Smith 2003). Standard Lutz-Kelker corrections become significant for observed parallaxes with $\varepsilon_\pi \equiv \sigma_\pi/\pi_{\rm o} \geq 0.05$ and diverge for $\varepsilon_\pi \geq 0.175$ (Figure 1, left panel). Ignoring them can introduce gross errors in any derived quantity. For stars observed with Gaia, unobscured G dwarfs are expected to have $\varepsilon_\pi = 0.05$ at $\approx$ 2 kpc; that value is achieved for unobscured M dwarfs at distances of $\sim$ 500 pc. It is clear that Lutz-Kelker corrections will have to be applied to Gaia parallaxes for those objects, among others.

The problem is even more serious than that: if we assume a constant underlying constant spatial distribution $\rho(r)$ and a Gaussian distribution for the parallax uncertainty (i.e., the standard assumptions for Lutz-Kelker corrections), the real distance r probability distributions for individual stars, $p_i(r)$, will always be ill-behaved for $r \rightarrow \infty$, even for small values of ε_π (Figure 1, right panel). This characteristic precludes a precise statistical analysis of parallaxes unless cutoffs are specified for the Gaussian distribution.

The situation described in the previous paragraph is actually not a realistic representation of the Galactic stellar populations that will be sampled by Gaia. $\rho(r)$ is not expected to be constant; on the contrary, in most cases it is expected to drop significantly beyond a relatively short distance. This alleviates or eliminates altogether the ill behavior of $p_i(r)$ for $r \rightarrow \infty$ but requires the use of new techniques for its precise calculation. That is the purpose of this work.

2. HOW?

The following procedure can be used for a simple conical or biconical volume centred on the position of the Sun. It can also be adapted with some modifications to other simple geometries involving cone or cylinder sections (see Figure 2).

We start by selecting a distance-limited (not magnitude-limited) population along a given direction. It is possible (and in most cases expected) for a good fraction of the sample to have large values of ε_π. Negative values of $\pi_{\rm o}$ for part of the sample are also possible and, again, expected if the sample is severely Lutz-Kelker-limited (for example, if we are dealing with Gaia data for dim, low-mass stars).

We then assume that $\rho(r)$ is not constant for that population and provide a description in terms of a reasonable functional form that goes to zero as $r \rightarrow \infty$. For example, one can assume a Galactic disc with constant surface density and a Gaussian profile in the vertical direction and derive the expected $\rho(r)$ along a given direction. More complex distributions with multiple components can also be used.

For the next step, an educated guess for the free parameters of the functional form is provided. This can be deduced from pre-existing data. Then, the probability distribution $p_i(r)$ for each star i given $\pi_{{\rm o},i}$, and $\sigma_{\pi,i}$ as:

$$p_i(r) = A\, r^2\, e^{-\frac{1}{2}\left(\frac{1-r\pi_{{\rm o},i}}{r\sigma_{\pi,i}}\right)^2} \rho(r), \qquad (1)$$

where A is a normalization factor.

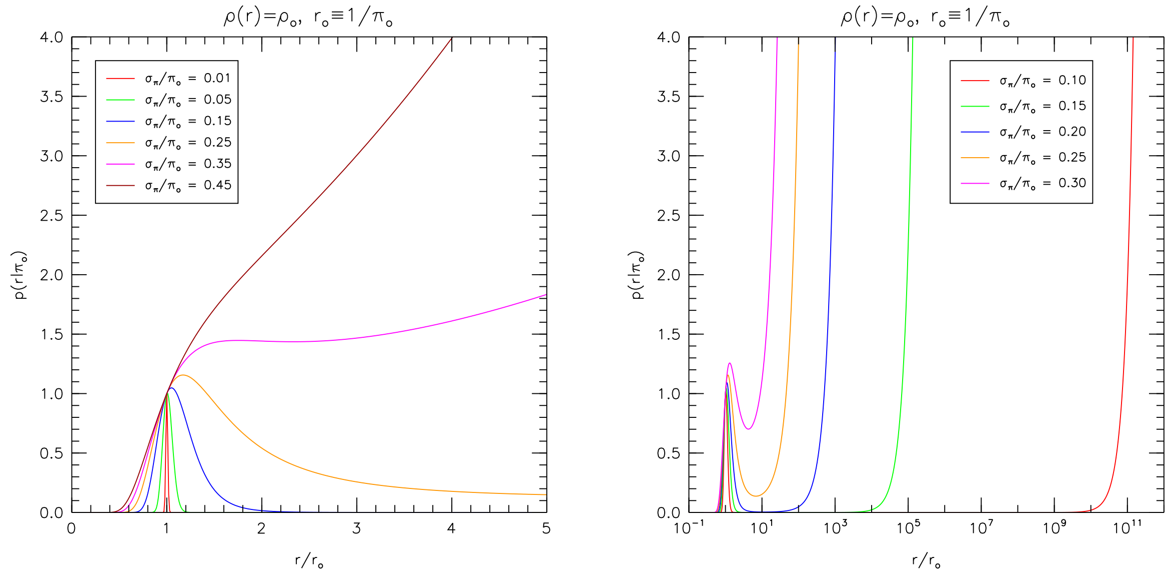

Figure 1. Real distance probability distributions for Gaussian uncertainties and an underlying constant spatial distribution assuming different values of ε_π. The left panel has a linear horizontal scale and the right one a logarithmic one.

Then we sum over all the stars in that particular direction to obtain the total probability distribution $p(r)$ and, if different directions are sampled, a 3-D $p(x, y, z)$ is derived. The total probability distribution is then used to derive a $\rho(r)$ by χ^2 fitting of the free parameters in the chosen functional form.

The procedure is then iterated until convergence. At the end, the residuals of the fit can be analyzed to check whether the selected functional form is capable of producing a reasonable representation of the data. If that is not the case, a new functional form can be selected and the process is repeated.

The expected quality and magnitude completeness of Gaia data will allow the spatial distribution of different stellar populations to be analyzed with this method.

3. AN EXAMPLE

We have used a variation of this method with a somewhat more complex geometry (Figure 2) to determine the vertical structure of the spatial distribution of early-type stars in the solar neighbourbood from Hipparcos parallaxes. The full analysis is available in Maíz-Apellániz (2001a). Here we summarize the most important results:

The Sun is located above the plane of the Galaxy at a distance of $z_\odot = 24.7 \pm 1.7$ (random) $\pm$ 0.4 (systematic) pc. This value is consistent with most of the ones obtained by other authors using similar or different populations (Chen et al. 2001).

The scale height of the disc early-type stellar population assuming a self-gravitating, isothermal, single-mass disc is $h_s = 34.2 \pm 0.8$ (random) $\pm$ 2.5 (systematic) pc. The data does not have a good enough precision to differentiate between that model and one in which the gravitational field is provided by a constant-density background population, since both functional forms yield similar good values in the χ^2 fit. The value for h_s is slightly lower than the one obtained by other authors: the likely reason for this effect is the absence of a thick disc/halo component in previous works (see below).

The local disc surface density for O–B5 stars is $\sigma_\star = (1.62 \pm 0.04$ (random) $\pm$ 0.14 (systematic)$)\times$ 10^3 stars kpc^{-2}. This is in reasonable agreement with previous studies.

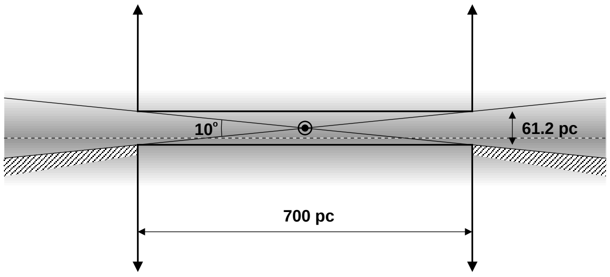

Figure 2. The double semi-infinite cylinder used to study the vertical distribution of O-B5 stars is represented by the two regions contained inside the thick lines. The cylinder has a radius of 350 pc and the gap is defined by excluding the stars within 5° of the Galactic equator. The shaded area represents the Galactic disc with the dashed line being the Galactic plane and the Sun symbol marking the position of our star. The hatched regions mark the area excluded to estimate the bias induced by extinction.

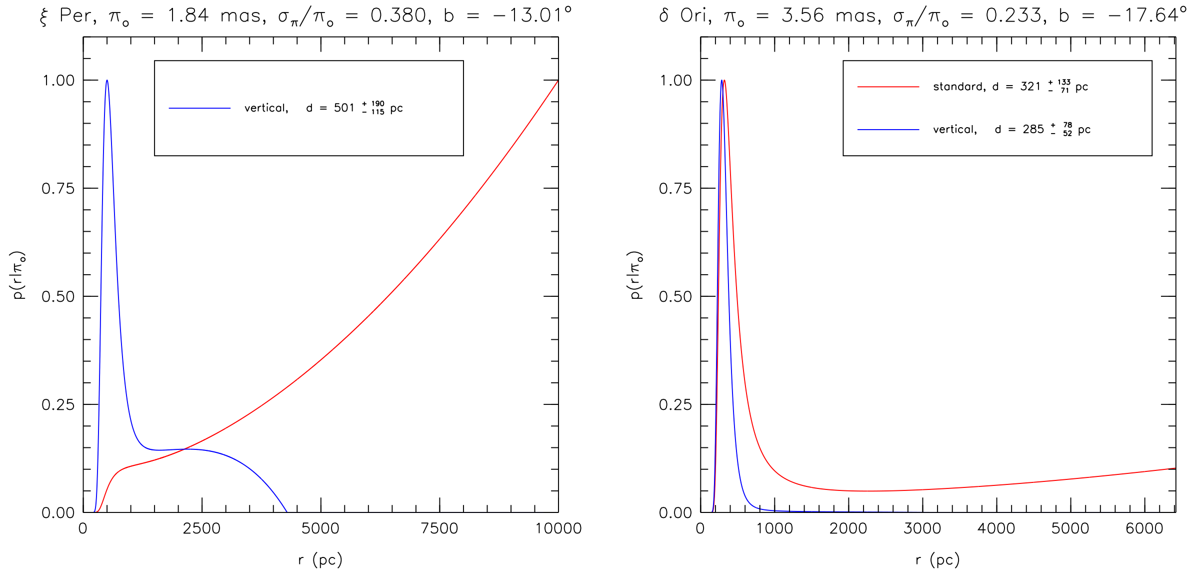

Figure 3. Real distance probability distributions for two Galactic O stars derived from Hipparcos parallaxes. The red curve assumes an underlying constant spatial distribution while the the blue curve uses the Galactic vertical structure model of Maíz-Apellániz (2001a).

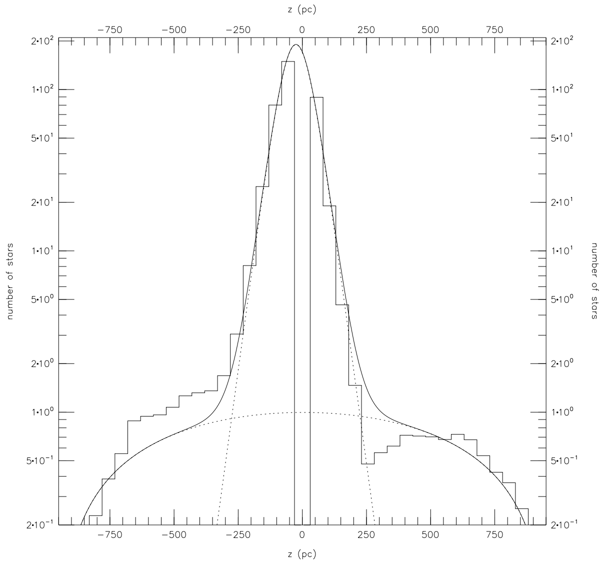

Figure 4. Observed (histogram) and model fit frequency distributions as a function of the vertical coordinate for O–B5 stars using a self-gravitating isothermal disc plus parabolic halo distribution. The dotted lines represent each individual component and the continuous one the sum of the two. The displayed bin size is uniform and equal to 50 pc, but is not used for the fit. The region immediately surrounding $z = 0$ is not considered for the fit, as it corresponds to the space between the two semi-infinite cylinders in Figure 2. Note that the observed distribution shows 'fractional stars' due to the procedure used to derive the histogram.

A halo/thick disc component is clearly detected at large distances from the plane (Figure 4). The data set is not good enough to provide detailed information on that population except to detect its presence, infer that it contributes with at least 5% of the total O–B5 stellar population in our region of the Galaxy, and deduce that it may extend beyond a distance of 500 pc from the Galactic plane. The origin of such a component is still a matter of debate: most of those stars appear to be runaways (Rolleston et al. 1999, Hoogerwerf et al. 2000) but some could have been formed *in situ* (Conlon et al. 1992).

The volume within $\sim$ 100 pc of the Sun is deficient in early-type stars. Beyond there, we start to find OB associations, such as Scorpius-Centaurus, that compensate the local deficiency (see also de Zeeuw et al. 1999, Maíz-Apellániz 2001b).

We have used these results to include distance information derived from Hipparcos parallaxes for some of the stars in our Galactic O Star Catalog (Maíz-Apellániz et al. 2004). Two examples are shown in Figure 3. We would like to point out that the discrepancies between trigonometric and spectroscopic parallaxes for early-type stars claimed by Skórzyński et al. (2003) and Patriarchi et al. (2003) are due to an incorrect treatment of Lutz-Kelker corrections; no discrepancies are found for the distances derived from our method.

A colour version of this poster is available at `http://www.stsci.edu/~jmaiz`.

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