ASTROMETRIC LIMITS SET BY SURFACE STRUCTURE, BINARITY, MICROLENSING

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ABSTRACT

We give an assessment of the significance of various known effects which may produce genuine fluctuations of star positions comparable to or larger than Gaia's measurement noise, and which thus may limit the ultimately reachable precision of the determination of parallaxes and proper motions. Stellar granulation is found to be no problem except for a small number of cool supergiants. It will be a serious problem for e.g., Mira star parallaxes. Star spots are hard to assess quantitatively. They, too, may be problematic for supergiants and very cool giants. Binarity is a major problem for the astrometry of bright stars. It will significantly impair the kinematics of stellar aggregates with small velocity dispersion. It will also produce a small proportion of grossly wrong parallaxes. Gravitational microlensing will not be a problem, although thousands of highly significant microlensing events will be detected by Gaia. Gravitational macrolensing of quasars will add some noise to their (otherwise 'zero') proper motions.

Key words: Gaia; Astrometry; Gravitational lensing; Star spots; Granulation; Quasar astrometry.

1. INTRODUCTION

Gaia has an extremely broad variety of scientific goals, see e.g., ESA 2000, and many papers in the present volume. The central ones, however, are

- measuring absolute parallaxes for the radius and luminosity calibration of stars and for the cosmic distance scale;

- measuring space velocities of many stars for the investigation of the structure and evolution of our galaxy, the Milky Way.

The major limitation of Gaia's ability to reach these objectives is technical measurement noise. It is determined by the size of the optics, the mission duration, the efficiency of Gaia's CCDs, the data reduction procedures, etc.

But in addition to this purely technical noise there are also astronomical noise sources, i.e., genuine fluctuations of star positions. There are three major categories of known effects that may create positional noise:

- surface structure of observed objects, including spots and outer convection zones (granulation) in the case of stars, and asymmetric shapes and albedo structures in the case of asteroids;

- gravitational lensing;

- stellar multiplicity, including planetary systems.

Detecting and measuring such astrometric fluctuations constitutes an important part of Gaia's scientific motivation and outputs: Binary motion measured by Gaia is the most powerful foreseeable method to find a big unbiased sample of extra-solar giant gas planets, gravitational microlensing observed by Gaia is the only known way to precisely determine the low-mass end of the stellar mass function in the solar neigbourhood (Belokurov & Evans 2002), etc.

But viewed from the basic goal of measuring star parallaxes and space velocities in our Galaxy, such effects are just noise which may limit the ultimately attainable precision of the target quantities parallax and proper motion. Thus, if we were to assign a motto to the present paper, it would be: "One person's signal is another person's noise". In the following sections we try to give an assessment of the significance of the various known effects that may produce astrometric fluctuations. In some circumstances they can become really disturbing, as we will see.

Before discussing the above-mentioned effects we present a few general considerations (Section 2), and in closing we very briefly discuss possible other effects (Section 7).

2. GENERAL CONSIDERATIONS

All three categories of effects mentioned in the introduction produce photometric disturbances as well as astrometric ones. This paper concentrates on the astromet-

ric effects. But it should be kept in mind that the quasi-simultaneous photometric measurements of Gaia will in general give important clues to the underlying cause of any detected astrometric noise.

Parallax measurements are disturbed by any astrometric noise that can be interpreted as a Fourier component at 1 year period (and at the correct phase and position angle aspect ratio for a parallax ellipse). It is not needed that the true astrometric displacement has a component at that frequency, due to the convolution of the true motion's Fourier spectrum with the strange sensitivity function generated by Gaia's sky scanning law. Truly 1-year periodic motions will be the worst case, of course.

Similarly, space velocity measurements are disturbed by anything that mimics a mean motion over Gaia's mission duration. In other words, anything that gives a zero-frequency component after convolution with the sensitivity function. Again, truly constant motions will give the biggest effects.

We stated that the goal of the present paper is to give an 'assessment' rather than a full quantitative analysis. This is for two reasons: Firstly, for some of the effects a precise quantitative estimation is presently not possible. Secondly, it would be misleading to simply give a median or rms value. The size distribution of the angular disturbances does not fall off steeply at large values, unlike e.g., a Gaussian: A given linear displacement of the photocentre in space leads to an angular displacement that is inversely proportional to distance. Thus, for a roughly homogeneous distribution of the observed stars in space, the size distribution of the angular displacements falls off as their fourth power. Such a distribution has 0.15 per cent remaining cases (i.e., ∼1.5 million stars out of Gaia's one billion) beyond 5 times its rms value, while a Gaussian distribution has only about 10^{-7} cases (i.e., about 100 stars for Gaia) beyond 5 times its rms value. The 'tail' of large values is caused by nearby stars.

3. STELLAR GRANULATION

Stellar convection zones reaching up to the photosphere cause the irregular and time-varying surface brightness pattern called granulation. It produces a random offset of the centre of light of a stellar disc (which is observed by Gaia) from the centre of mass of the star (which would define an undisturbed parallax ellipse and galactic space velocity). The size of the astrometric effect is easy to quantify, as shown by Svensson & Ludwig (2004). Its time scale is the convection cell lifetime at the surface. Fig. 1 shows a simulation of the astrometric offset, derived from a radiation-hydrodynamical model of a red giant star by Svensson & Ludwig.

The surface of a convective star is fully covered with convection cells. Each individual cell produces an rms photocentre offset from the star's centre of gravity of $\sqrt{1/6}$ times the radius R_{star} of the star. So, for the average over N_{cell} statistically independent cells of typical size r_{cell}
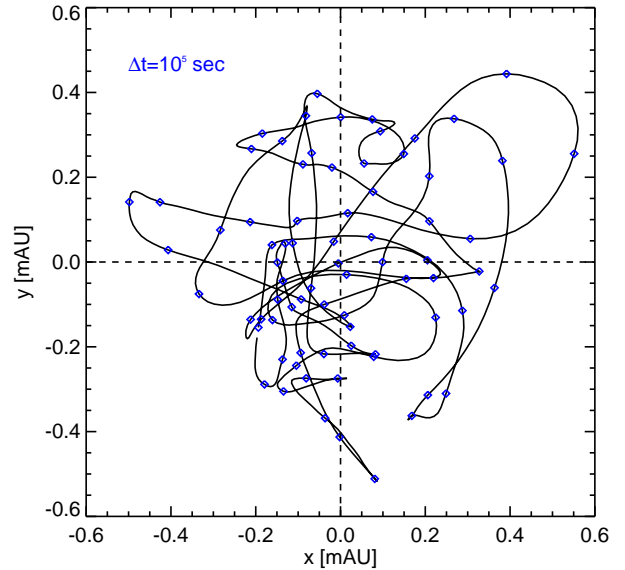

Figure 1. Simulated astrometric displacement of the photocentre of a red giant (see text) from its centre of mass, with ticks added to the curve every 10^5 seconds (from Svensson & Ludwig 2004).

and brightness contrast C we have approximately

$$\sigma_{astrom} \sim C \frac{1}{\sqrt{6}} R_{star} N_{cell}^{-1/2} \qquad (1)$$

$$\propto C \frac{1}{\sqrt{6}} R_{star} (r_{cell}^{-2} R_{star}^2)^{-1/2} \qquad (2)$$

$$\propto C \frac{1}{\sqrt{6}} r_{cell} \qquad (3)$$

Therefore the radius of the star cancels out: the astrometric disturbance depends on r_{cell} only (the contrast C in the optical range always being of the order of unity). The cell size is proportional to the surface pressure scale height, which in turn is inversely proportional to the surface gravity (with a small dependence on the mean molecular weight). Thus the rms astrometric displacement for any star with granulation can be expected to be inversely proportional to log g. This simple analytical consideration of Svensson & Ludwig is fully confirmed by their numerical simulations which are summarized in Figure 2.

That figure clearly shows that the granulation noise is negligible for white dwarfs (log g = 8), main-sequence stars (log g = 3...5) and Cepheids (log g = 2). For a red giant star like that of Figure 1 (T_{eff} = 3676 K, log g = 1) the effect is of the order of 1 mAU = 10^{-3} astronomical units, and its timescale is of the order of a day. This is measurable for nearby giants (since 1 mAU translates into 10 microarcsec (μas) at 100 pc), but is still irrelevant. However, for the parallaxes of Mira stars and cool supergiants (log g around zero) it becomes problematic: Firstly, the rms displacement approaches a considerable fraction of an astronomical unit. But even worse, its typical time scale gets close to one year. So it may create significant parallax errors. For space velocity studies it is ir-
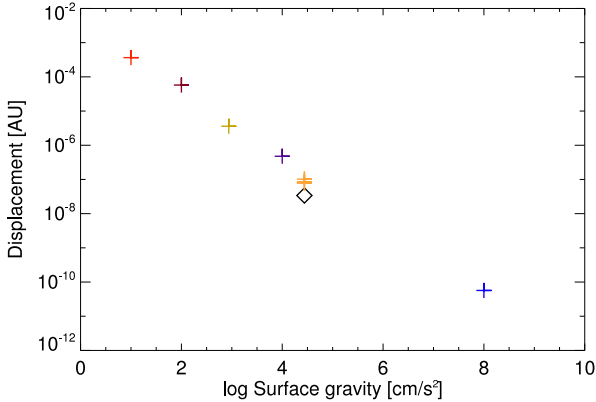
Figure 2. Root-mean-square astrometric displacements of stars due to granulation as a function of surface gravity (from Svensson & Ludwig 2004). For details see text. The diamond symbol shows the weak effect of metallicity ([Fe/H]=−2 instead of zero) for the case of solar-type stars.

relevant for all types of stars because even 0.1 AU/5 years is only $100\,\mathrm{m\,s^{-1}}$.

4. STAR SPOTS

The astrometric effects of star spots (and other magnetic surface features) are hard to quantify statistically, simply because the statistics of star spots are insufficiently known. The most specific statements on the distribution of numbers, sizes etc versus star type that we could find in the literature (e.g. in various papers in Strassmeier et al. 2002):

- "... are common among cool stars with outer convection zones."

- complaints: all available data are strongly biased towards very active stars;

- discussions about supergiants: Do we see dark spots or bright cells?

In the absence of good statistics one can still get a semi-quantitative assessment by looking at some representative cases.

- For the Sun, a spot covering f=1 per cent of a hemisphere is unusually large. Such a spot can produce a *maximum* astrometric offset of $0.5fr_\odot = 2.5 \times 10^{-5}$ AU; this is completely negligible.

- For a solar-type star with a huge spot the maximum displacement may reach up to $0.5r_\odot = 2.5 \times 10^{-3}$ AU. For the sake of parallax or space velocity measurements this is still negligible, but it is clearly measurable with Gaia in

some cases: 2.5×10^{-3} AU translate into $100\,\mu$as at 25 pc. The periodic displacement of the centre of light of a rotating spotted star might then be mistaken as the gravitational pull of a low-mass companion.

- K giants are roughly ten times as big as solar-type stars. Large spots on their surface will thus be readily detectable astrometrically by Gaia. They will still be quite harmless for the measurement of parallaxes and space motions. But the danger of confusion with low-mass companions will be more severe.

- For supergiants and M giants, having radii of the order of $100\,R_\odot$ (and more), the effect may reach 0.25 AU (or more). If its time dependence should happen to mimic a parallax motion, very significant distance errors may result. The maximum possible effect will rarely occur, but a noticeable extra parallax noise can be expected for these stars if they carry large spots (or large convection cells, see previous section). 0.25 AU/5 years translate into $50\,\mu$as per year or $250\,\mathrm{m\,s^{-1}}$ at 1 kpc.

In closing this section let us add three remarks: 1) Big star spots are connected with brightness changes, so the possible confusion with low-mass companions or parallax motion can be (largely) avoided by Gaia's parallel photometry. 2) Plagues tend to be larger than spots (at least in the case of the Sun), but have much smaller contrast; so they should have smaller effects. 3) The magnetically most active star types known, BY Dra stars and RS CVn stars, are small in size and are quickly rotating; so their spots are astrometrically irrelevant.

5. GRAVITATIONAL LENSING

5.1. Basics

Gravitational lensing occurs when a gravitating body (the 'lens') comes close to the line of sight between a light-emitting body (the 'source') and an observer. It generally leads to a splitting and focussing of the light path between source and observer. The observer then sees two (or more) images of the source, usually at higher total brightness than without the lens. The brightness increase due to gravitational lensing has been systematically searched for and detected by large specialized surveys like OGLE and MACHO. For a point-like lens the (two-dimensional) astrometric displacement of the centre of light $\vec{\theta}(t)$ at time t is given by (see e.g., Walker 1995):

$$\vec{\theta}(t) = \frac{\vec{\theta}_{\mathrm{SL}}(t)}{[\theta_{\mathrm{SL}}(t)/\theta_{\mathrm{E}}]^2 + 2} \qquad (4)$$

where the arrows indicate small tangential vectors on the celestial sphere, $\vec{\theta}_{\mathrm{SL}}(t)$ is the angular separation of the (unlensed) source from the lens at time t, and $\theta_{\mathrm{SL}}(t)$ its absolute value. The Einstein radius θ_{E} depends on the lens mass M by

$$\frac{\theta_E}{1\,\mathrm{mas}} = \left(\frac{M}{0.12M_\odot}\right)^{1/2}\left(\frac{\varpi_{\mathrm{L}} - \varpi_{\mathrm{S}}}{1\,\mathrm{mas}}\right)^{1/2} \qquad (5)$$

where ϖ_S and ϖ_L are the parallaxes of the source and lens, respectively, and 'mas' is the abbreviation for milliarcsecond. The time dependence of $\vec{\theta}$ is due to the difference of proper motion and parallactic shift between source and lens.

Gravitational lensing comes in three observational 'flavours':

- Macrolensing: The effect is called macrolensing if the separation between the split images is large enough that they can be directly resolved, i.e., if it approaches the level of an arcsec. In practice this happens only if the lens is of galaxy-sized mass. It leads to the well-known multiple images of quasars behind foreground galaxy clusters, of which Gaia will observe a considerable number. The opposite case, i.e., when the split images are unresolved, is called microlensing.

- Strong microlensing: The effect is called strong microlensing when θ_{SL} is of the order of the Einstein radius θ_E. The displacement of the centre of light then is of the order of θ_E as well.

- Weak microlensing: When θ_{SL} is much larger than θ_E the effect is called weak microlensing. In this case the displacement of the centre of light is of the order of $\theta_E(\theta_E/\theta_{SL})$.

5.2. Stars

The subject of astrometric microlensing with Gaia has been thoroughly investigated by Belokurov & Evans (2002). From extensive simulations they find that strong microlensing will produce very significant and astrophysically extremely useful effects, but at the same time they find that it is very rare. They define a significance level by requesting that the maximum astrometric displacement be $7\sigma_{FoV}$, where σ_{FoV} is Gaia's (magnitude-dependent) measuring precision from a single field-of-view transit of a star, and that this maximum displacement occurs within Gaia's mission duration of five years.

From this criterion they find that one out of every 40 000 Gaia stars will be significantly lensed during the mission (in the microlensing jargon: the optical depth is 2.5×10^{-5}). In other words: Gaia will observe about 25 000 highly significant events, with about 2500 of them yielding precise lens masses. The most important lenses are found to be low-mass stars within a few hundred pc from the Sun. The most important sources are disc and bulge stars several kpc away.

To give just a few specific numbers: The maximum positional shift during a microlensing event occurs when $\theta_{SL}(t) = 1.41\theta_E$. Its absolute value then is $0.35\theta_E$, which for a distant source amounts to about 1100 μas for an 0.12 solar-mass lens at 100 pc, and to 350 μas for an 0.12 solar-mass lens at 1 kpc. The astrometric appearance of a microlensing event is as follows (Figure 3): Over a few years the photocentre of the source moves away from the unlensed position with increasing speed, then within
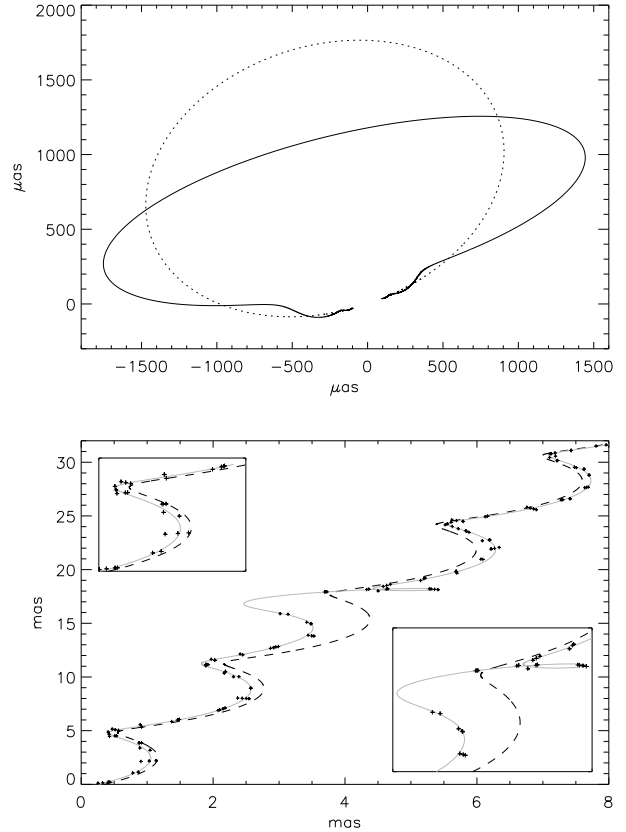

Figure 3. Upper panel: astrometric shift of a microlensing event by a lens of 0.5 solar masses at 150 pc and a source at 1.5 kpc, with 70 km/s transverse velocity, as seen by a barycentric observer (dotted line) and Gaia (solid line). The difference is due to the differential parallactic shift. Lower panel: The same, but showing the actual path of the source's centre of light on the sky, including proper motion and parallax (grey line). The dashed line indicates the unlensed path of the source. The tiny crosses show simulated Gaia measurements, with σ_{FoV}=300 μas corresponding to a 17th-magnitude star. The insets show enlarged views of the beginning and the midpoint of the event (from Belokurov & Evans 2002).

a few months it performs a quick 'swing' to the opposite side, and finally it moves back towards the unlensed position with quickly decreasing speed.

Belokurov & Evans find that strong microlensing, being so rare, is of negligible effect for the overall error budget of Gaia. In the few cases where its size is significant it will rarely be mixed up with a proper motion or parallax effect[1].

Weak microlensing, on the other hand, is ubiquitous, but small. The typical angular displacement of a Gaia star by weak microlensing is of the order of 1 μas, with a secular change of the order of 0.01–0.1 μas per year.

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[1]Less significant events and incomplete events (i.e., events with the maximum displacement outside Gaia's mission lifetime) can be mistaken as binary motion, however.

5.3. Quasars

Quasars were once thought to be perfect inertial reference points, with zero parallaxes and proper motions. But in fact they are not. A proper-motion scatter somewhere in the range 10–100 μas per year must be expected for them, from a variety of causes (quite a number of publications relevant for high-precision astrometry of quasars are cited and summarized in the Gaia Concept and Technology Study Report (ESA 2000), p. 109-119):

- random proper motions due to relativistic jets, up to 500 μas per year, but mostly small (extra noise; no bias);

- systematic proper motions of 3 μas per year on average due to the galactocentric acceleration of the Sun (bias, but no problem; will be modelled);

- image centroiding problems due to underlying galaxy (extra noise; no bias);

- image centroiding problems due to non-stellar spectra (imperfect instrumental chromaticity correction, extra noise; no bias);

- random proper motions due to weak microlensing by galactic stars (typically 0.01–0.1 μas per year, negligible; no bias).

The above items are present for all quasars. Those quasars which are macrolensed by intervening galaxies (about one per cent of all, i.e., several thousand out of Gaia's total of 500 000 or so) suffer additional astrometric noise:

- image centroiding problems due to non-pointlike appearance (extra noise; no bias);

- time variability of the macrolensing due to relative tangential motion of quasar and lens, leading to changes of the relative brightness of the quasar images (random proper motions, often one or a few μas per year, up to dozens of μas per year; no bias);

- time variability of the macrolensing due to strong microlensing by individual stars in the lens galaxy, again leading to changes of the relative brightness of the quasar images (character and size of the effect as in the previous item).

Quasar astrometry will be the central tool to make Gaia's rigid astrometric sphere kinematically non-rotating, by using the quasars as a grid of sources with 'zero' proper motion. All the above effects (except the galactocentric acceleration of the Sun, see Bastian 1995) will add to the measurement noise in quasar proper motions, making the quasars less favourable as might naively be expected. Fortunately, the extra noise is at most as large as the measurement noise (the latter being of the order of 100–200 μas per year for quasars of 18th to 20th magnitude), and will cause no bias.

Concerning parallaxes, Gaia intends to produce absolute parallaxes without external reference. However, the quasars will provide a useful data set to check the success in this respect. Again, the above-listed effects will cause extra noise, but essentially no bias. It should be mentioned that Sazhin et al. (2001) recently discovered that on parallaxes the weak microlensing effect does not create just an extra noise, but also a bias. Due to this effect all quasars are expected to exhibit very small (typically a few nanoarcsec) negative parallaxes. Sazhin et al. demonstrate that in exceptional cases (perhaps one per cent of all quasars, maybe less) this negative pseudo-parallax may reach 1 μas. However, its typical value will be irrelevant for Gaia. The average value can be roughly estimated to be between 10 and 30 nanoarcsec. With 500 000 quasar parallaxes individually measured to 100 μas precision (optimistic for Gaia), the random error of their weighted average will be of the order of 150 nanoarcsec. So the bias due to weak microlensing is negligible[2].

6. STELLAR MULTIPLICITY

Orbital motion of double and multiple stars causes measurable deviations of the observed stellar positions from a pure proper motion and parallax model which, if undetected or at least unmodelled, will create errors in the measured parallaxes and space velocities. As we can safely assume binary orbits to be randomly oriented in space and randomly phased in time, we need not fear any biases to be introduced in this way. If the binary nature of the motion is detected and correctly modelled it will not even create extra noise (but will still weaken the determination of parallaxes and proper motions due to the increased number of unknowns to be solved for).

6.1. Space Velocities

The effects of stellar multiplicity are being carefully studied by the Gaia Double and Multiple Stars Working Group, see the paper by Söderhjelm in this volume and references therein. Extensive simulations of Gaia measurements on double stars, and of binary detection and orbit modelling performed on those measurements, have been carried out. They resulted in the general picture summarized in Figures 4 and 5.

Figure 4 shows the statistics of errors in the measured tangential velocities (of the simulated stars) created by undetected binarity as function of orbital period, for bright stars only (magnitude 10 to 12.5). The median error due to measurement noise is indicated as a thin horizontal line. At very long periods Gaia cannot detect the orbital motion (but generally sees the two binary components as separate light sources). The extra space velocity errors are not important because the orbital velocities are very small. At successively shorter periods (down to about one

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[2]in the oral presentation of the present paper during the conference we claimed it to be relevant; but this was due to a misunderstanding of some statements in Sazhin et al. 2001.
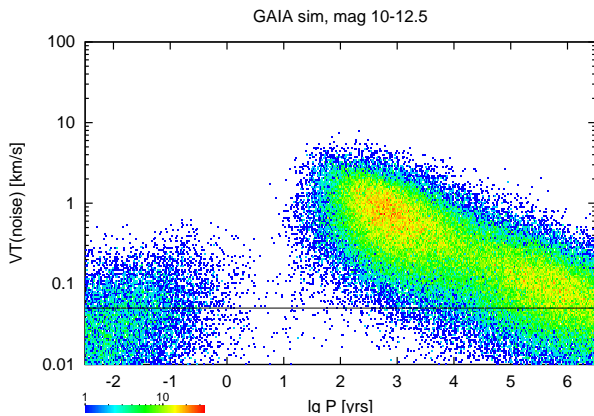
Figure 4. Space velocity errors induced by undetected binarity, for bright stars, plotted versus binary orbital period. The dense parts of the cloud of points are encoded in colour scale. Simulations by S. Söderhjelm, copied from ESA (2000), Figure 1.27.

Figure 5. Same as Figure 4, but for faint stars. Simulations by S. Söderhjelm, copied from ESA (2000), Figure 1.27.

hundred years) the orbital motion increases on a Keplerian slope ($v \propto P^{-1/3}$), but still cannot be distinguished from rectilinear motion in space. The typical error now is much larger than the measurement noise. At periods shorter than a few dozen years, Gaia quickly starts to recognize the orbital acceleration. This creates the prominent gap in the cloud of points in Figure 4. For periods around the mission lifetime of Gaia there are practically no undetected binaries.

At still shorter periods the detection efficiency becomes worse again because the astrometric size of the orbits shrinks below the detectability limit. The space velocity errors introduced by the binarity are nevertheless small because Gaia sees the motion averaged over many orbital revolutions.

Figure 5 shows an analogous plot for faint stars (magnitude 17.5 to 20). The same mechanisms as in Figure 4 are in effect, but the median measurement error (thin horizontal line) is about two powers of ten higher than for the bright stars. This has two independent reasons: Firstly, the (angular) measurements of the faint stars are about an order of magnitude less precise. Secondly, the faint stars are typically about an order of magnitude farther away from the Sun. As a consequence the binarity-induced errors are almost negligible compared to the measurement noise.

We note that Figures 4 and 5 are based on a now obsolete Gaia hardware configuration and on a slightly too optimistic binary detection efficiency estimate. An updated study by S. Söderhjelm (private communication) yielded differences in the details, but no changes in the basic aspects and conclusions. Updated graphics may possibly be included in the paper by Söderhjelm in this volume.
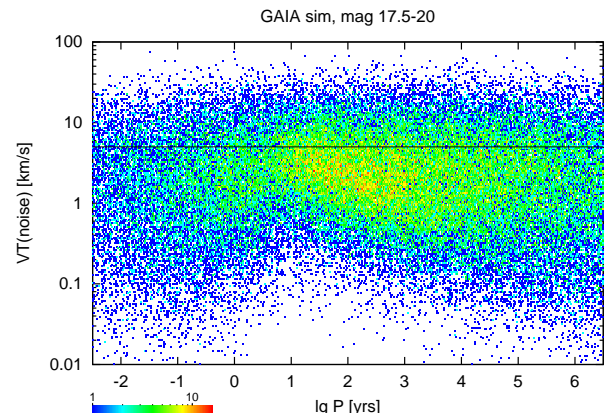
6.2. Parallaxes

Binary motion will likewise produce extra noise in the determination of parallaxes. This is relevant for fairly short orbital periods only. Recent simulations by S. Söderhjelm (unpublished) show that for moderately bright stars (brighter than magnitude 15.5) the typical effect of undetected binarity is of the order of one or two dozen μas, i.e., comparable to the measurement noise. For periods close to one year (± 10 per cent) some parts of the orbital motion become indistiguishable from parallactic shifts. Very big parallax errors (up to hundreds of μas) occur in such cases. The big errors occur even for *detected* binaries of about 1 year period, because in these cases the derived orbital parameters become strongly correlated with parallax.

7. OTHER EFFECTS

There are a number of other physical effects that can create fluctuations of star positions. In this section we very briefly list them and comment on them.

- Asymmetric outbursts, gas jets etc: Rare, except for nearby quasars, where motions of a few hundred μas in a few months may occur.

- Interstellar and interplanetary scintillation: Important in radio astronomy but negligible in the optical (inversely proportional to the frequency of the observed radiation).

- Primordial gravitational radiation: Predicted to have been produced by cosmic inflation, but at amplitudes many orders of magnitude too small to be observed by Gaia.

- Local gravitational radiation, e.g., from neutron star or black-hole binaries: Highly improbable; would require such a binary to move extremely close to the line of sight of a background source.

- Unexpected effects? Going to entirely new realms of precision and/or numbers of measurements always bears the possibility to discover completely unexpected phenomena. If this should happen for Gaia, it might — depending on the size of the unexpected effect — either be welcomed as a great scientific achievement, or regarded as a nuisance for measuring stars and the Galaxy, or even both.

8. CONCLUSION

In summary, there are a number of astronomical phenomena that produce genuine fluctuations of star positions comparable to or larger than Gaia's measurement noise, at least for parts of Gaia's target celestial objects.

- Binarity is a major problem for the astrometry of bright stars. It will significantly impair the kinematics of stellar aggregates with small velocity dispersion (small star clusters, associations, star-forming regions). It will also produce a small proportion of grossly wrong parallaxes.

- Stellar granulation is no problem except for a small number of cool supergiants. It will be a serious problem for e.g., Mira star parallaxes.

- Star spots are hard to assess quantitatively. They may be problematic for supergiants and very cool giants.

- Gravitational microlensing will not be a problem, although thousands of highly significant microlensing events will be detected by Gaia.

- Gravitational macrolensing of quasars will add some noise to their (otherwise 'zero') proper motions and parallaxes.

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