RELATIVISTIC ASTROMETRY: THE RAMOD PROJECT

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ABSTRACT

The technology of modern astrometric satellites like Gaia assures an accuracy of one microarcsecond in the measurements of angles. At this level, one also has to take into account the general relativistic effects due to most of the Solar System planets and their largest satellites. The aim of the RAMOD project is the construction of a fully general relativistic data reduction scheme consistent with these expectations. The project consists of the development of subsequent models having increasing accuracies and complications. Each model has been used as a testbed for comparison with the successively more advanced models. Here we illustrate only some of the problems already solved or presently under investigation for the fulfillment of our task.

Key words: Relativity; Reference frames; Gaia.

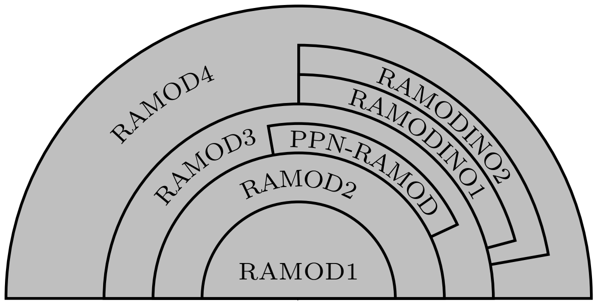

Figure 1. Graphical representation of the evolution of the RAMOD project.

1. INTRODUCTION

Modern space technology will soon be able to provide measurements of stellar directions to an accuracy of one microarcsecond (μas). At this level, one has to take into account the general relativistic effects on light propagation arising from metric perturbations due not only to the bulk mass but also to the rotational and translational motion of the Solar System bodies and their multipole structure. The aim of our project is to develop a Relativistic Astrometric MODel (RAMOD) which enables us to deduce to microarcsecond accuracy the astrometric parameters of a point-like source from observations taken by an astrometric satellite like Gaia. Up to now we have produced several models of increasing intrinsic accuracy (see Figure 1). The first, called RAMOD1, was a non-perturbative model of the static celestial sphere where the background metric was described by the exact Schwarzschild solution (de Felice et al. 1998), with the Sun as the only source of gravity. In RAMOD2 (de Felice et al. 2001) we extended the previous version to the non-static case to include the determination of parallaxes and proper motions. RAMOD2 has been fully tested on an end-to-end simulation of a Gaia-like astrometry mission and proved capable of estimating positions, parallaxes and annual proper motions with an accuracy of ~ 15 μas for stars of $V \sim 17$ mag. The implementation of such models has been exploited to investigate the potential accuracy of astrometric observations in the determination of the PPN parameter γ; a deviation of the latter from unity, which holds in General Relativity, would signal the existence of a new fundamental interaction at the elementary particle level. In a model called PPN-RAMOD (Vecchiato et al. 2003) we showed that the factor γ can be measured with an accuracy of 10^{-7}, i.e., two orders of magnitude better than the most recent estimation by Cassini (Bertotti et al. 2003).

All these formulations have been essential touchstones of comparison for the more advanced many-body model RAMOD3 (de Felice et al. 2004a) where the astrometric problem was tackled in the presence of geometry perturbations due to the bodies of the Solar System. Here we considered a static case in which the bodies of the Solar System are at rest during the light transit from the boundaries of the Solar System to the orbiting telescope; this model has been fully developed and tested up to the order of $1/c^2$, c being the velocity of light in vacuum. Although accurate to the milliarcsecond level as expected, and therefore not suitable for a realistic data reduction procedure, such a formulation is essential for providing a necessary test case for comparison with a more comprehensive model. We have in fact developed an extension of RAMOD3 into a multi-modular dynamical model accurate to one microarcsecond which means retaining terms of the order of $1/c^3$. This is called RAMOD4, now ready to be tested (de Felice et al. 2004b).

The basic mathematical problem which underlies all RAMOD models is that of inverse ray tracing which amounts to reconstructing the space-time trajectory of light from the event of observation at the satellite to that of emission at the star. Evidently one has to solve a system of four coupled second order partial differential equations, the unknowns being the stellar space-time coordinates. Essential requirements for solving this system are the definition of a coordinate parameterization over a domain at least as large as our Galaxy and a suitable solution of the boundary value problem.

2. THE MANY FRAMES OF THE RAMOD SCHEME

The most convenient and generally accepted choice of coordinates is one which allows the space-time generated by the Solar System bodies to be described by the following form of the metric:

$$g_{\alpha\beta} = \eta_{\alpha\beta} + \sum_a h^{(a)}_{\alpha\beta} + O(h^2) \tag{1}$$

where the sum is extended to all Solar System bodies. In this approximation, the metric tensor (1) has in general a non-vanishing term $g_{0i} = O[(1/c)^3]$ and the non-linearity of the gravitational field is confined to terms $O[(1/c)^4]$ in the metric coefficients.

In August 2000 the General Assembly of the IAU stated that a solution like (1) has to be adopted to define the reference frames and time scales in the Solar System. At the first Post-Newtonian level of approximation, the metric tensor (1) takes the form:

$$\begin{aligned} g_{00} &= -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + O(c^{-5}) && (2)\\ g_{0i} &= -\frac{4w_i}{c^3} + O(c^{-5}) && (3)\\ g_{ij} &= \delta_{ij}\Big(1 + \frac{2w}{c^2}\Big) + O(c^{-4}) && (4) \end{aligned}$$

with i, j = 1, 2, 3. In the equations above, w represents a generalization of the (total) Newtonian potential and w_i is a vector potential describing the dynamical contribution to the background geometry due to the relative motion of the gravitating sources as well as the peculiarities of their extended structures.

The spatial coordinates which underly this form of the metric are Cartesian-like, taken along axes pointing to far distant sources and having the origin at the centre of mass of the Solar System; this frame will be termed the Barycentric Celestial Coordinate System (BCRS). The time coordinate has been chosen according to the most recently IAU recommendations.

The surfaces at t =constant are the set of points having as time coordinate the one measured by an observer at rest at the barycentre of the Solar System. These are not surfaces of simultaneity for observers at rest with respect to the spatial BCRS grid because of the presence of mixed terms, i.e., g_{0i}, in the background metric. The rest *space* of these observers is locally different from the t =constant surfaces but, since this *difference* contributes to terms of order higher than $1/c^3$, it can be neglected without loss of precision. Hence, to the required accuracy, the rest space of the locally static observers will coincide with the coordinate space of the BCRS. The proper time of these observers is proportional to the coordinate time by a correction factor which depends on the gravitational potential at their location. Therefore, the history of each of these observers is described by a line parallel to the (local) coordinate time direction and the tangents to these lines form a vector field given by the following expression:

$$\begin{aligned} \hat{u}^\alpha &= \frac{dx^\alpha}{d\hat{\sigma}} = \mathrm{e}^\psi \delta^\alpha_0 \\ \hat{u}_\alpha &= g_{0\alpha}\mathrm{e}^\psi \end{aligned} \tag{5}$$

where $\mathrm{e}^\psi = (dt/d\hat{\sigma}) = (-g_{00})^{-1/2}$. The integral curve of $\hat{\boldsymbol{u}}$ through each space-time point identifies a *local barycentric observer*.

It is clear that while the coordinate system fixed at the Barycentre of the Solar System provides the basic coordinate representation of whatever tensorial quantity we shall be dealing with, the physical interpretation of any local measurement is only possible if we refer it to a local barycentric frame. Consider a light ray propagating in space-time from a distant star to an orbiting satellite. Its motion in space-time is described mathematically by a null vector, say $\boldsymbol{k}$, which satisfies an appropriate differential equation written in terms of coordinate components referred to the BCRS. However, we can associate a physical meaning only to its *spatial* direction, identified by the local barycentric observer as the line of sight to the observed object. This can only be defined by a suitable projection of the general light-like vector $\boldsymbol{k}$ into the rest space of the local barycentric observer. In this way we identify a new vector field which can be given a direct physical meaning and can be expressed in terms of the solutions of our integration procedure.

There are other reference frames which enter our model. We shall only mention here the one comoving with the satellite. This frame is identified by means of tensorial quantities (tetrad) whose components are always referred to the BCRS but physically describe the rest space and proper time of the satellite-observer. Light reaching the satellite will carry information which will be encoded in the satellite frame and then related by a suitable non trivial mathematical procedure to the boundary values of our ray tracing problem. Fixing the satellite frame is in itself a rather awkward procedure since the satellite attitude is the result of several independent states of motion, as we show next.

3. THE BOUNDARY VALUE PROBLEM AND THE SATELLITE ATTITUDE

The problem of fixing the boundary conditions has been tackled and fully solved analytically up to $1/c^3$ in two specialized models termed RAMODINO1 (Bini & de Felice 2003) and RAMODINO2 (Bini et al. 2003). The first treats the satellite-observer as a point orbiting the Sun with a generic equation of motion. The second describes the satellite-observer's attitude and motion strictly following Gaia's specifications, namely: the centre of mass moves on a Lissajous-type orbit around the Sun-(Earth-Moon) Lagrangian point L2; the satellite spins with a proper time rate of one turn every 6 hours about its $X-$axis. This axis has a 50° inclination with respect to the Earth-Sun direction and precesses about the Sun direction with a period of 70 days. Fixing a reference frame which is adapted to this more realistic configuration required a substantial mathematical complication which, however, has been treated analytically and is in a form ready to enter a numerical implementation. As already stated, this frame (which we call the *attitude* frame) is comoving with the satellite and identifies in a natural way the rest space and proper time of the satellite-observer. In our case the boundary value problem is solved when one is able to express the space-time coordinate components of the line-of-sight four-vector relative to the local barycentric observer in terms of the *observations* made in the satellite attitude frame. The observations consist in the cosine directions that the spatial line-of-sight of the incoming light signal makes with the spatial directions of the attitude frame at the time of observation. RAMODINO2 solves this problem to the order of $1/c^3$. We verified that, by dropping all $1/c^3$ terms, the solutions of Bini et al. (2003) obey to the obvious requirement of reproducing the case of RAMOD3 which is accurate to $1/c^2$.

We have assumed up to now that the absolute satellite attitude is known with an arbitrary accuracy. However, this will not be true in practice. The expected attitude error, in fact, will be propagated in our formulation of the boundary conditions inducing in such a way an error in the solution of the light path integration. However, in the foreseen data reduction scheme, the satellite attitude will be part of the unknowns, as it is expected to be reconstructed using the satellite observations. Thus, an ongoing effort of the RAMOD project is the set-up of a specific data reduction model which includes the treatment of the attitude itself.

4. THE RETARDED TIME CORRECTIONS

It is clear that the motion of the Solar System planets with respect to the BCRS cannot be ignored during the flight time of a light signal from a star to the satellite, hence our relativistic treatment of light propagation requires that one takes into account the retarded time corrections. This means that at any point P along the light path and at a coordinate time t, the metric coefficients $h^{(a)}_{\alpha\beta}(t)$ generated by the $a^{\rm th}$ source of the Solar System, are determined when that source was located at a point Q of its trajectory at a coordinate time t' where $t' = t - r^{(a)}/c$ where c is the velocity of light in vacuum. Here $r^{(a)}$ is the *spatial* distance calculated in the rest frame of the local barycentric observer at P between P and the point Q' fixed by the intersection of the coordinate time-line stemming from Q with the surface t =constant. Clearly Q and Q' have the same spatial coordinates. The point-events P and Q' are simultaneous to each other only up to terms of the order of $(1/c^2)$ however this accuracy is sufficient for our purposes since the retarded distance $r^{(a)}(t)$ will enter terms of the order of $(1/c^2)$ at least. The $a^{\rm th}$ source of gravity has a velocity $v(t)$ with respect to the local barycentric observer therefore it will cross the surface t =constant at a point $\tilde{Q}$ which is at a distance $r_0^{(a)}(t)$ from P (see Figure 2). Since the quantity $r_0^{(a)}(t)$ at each time t is known from the ephemerides and t is our integration variable, our task is to express the retarded distance $r^{(a)}(t)$ in terms of $r_0^{(a)}(t)$. This relation is obviously not trivial and depends on the planet's velocity and on the spatial distance $\tilde{r}^{(a)}(t)$ between the points $\tilde{Q}$ and Q'. This relation takes the general form, for any (a): $r(t) = r[r_0(t), \tilde{r}(t), v(t)]$. Clearly $\tilde{r}(t)$ depends implicitly on $r(t)$ hence the above equation is of the Kepler type and requires a numerical treatment.

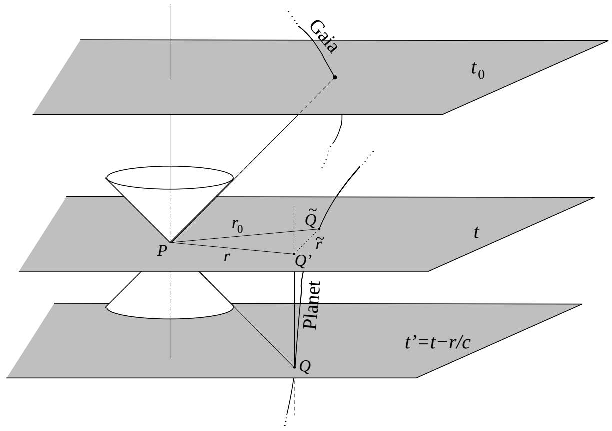

Figure 2. Pictorial representation of the problem of the retarded distance.

5. CONCLUSION

The final product of our RAMOD project is a structured general relativistic astrometric model, accurate to one microarcsecond, which can be adapted to many different satellite settings and orbital specifications. The theoretical construction is supported by a software package which will serve for testing the reliability of the model.

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