GAREX: A RELATIVITY EXPERIMENT WITH GAIA

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ABSTRACT

The possibility to observe close to Jupiter's edge with Gaia offers a chance to test General Relativity through the detection of relativistic effects experienced by photons while propagating in the gravitational field produced by the planet and to demonstrate for the first time the contribution of the quadrupole. We derive the main formulas relevant for this experiment and describe its main features.

Key words: Gaia; Science; General Relativity (GR); Solar System.

1. INTRODUCTION

Due to its high precision and multi-epoch astrometry Gaia will be able to detect the relative positional change of a star resulting from the tiny curvature of the light ray brought about by the gravitational field of the Sun, and to a lesser extent by the giant planets. In the former case, the science case has included from the outset an investigation of the value of the PPN parameter γ, considered as a global unknown in the astrometric model. The analysis indicates that Gaia measurements should provide a precision of 5×10^{-7} for this parameter (Mignard 2002; Vecchiato et al. 2003), an improvement of more than two orders of magnitude to the current best estimate (Bertotti et al. 2003). Besides this, it will be also possible to process the observations performed close to the surface of a giant planet to detect the light bending due to its gravitational field. For Jupiter its magnitude for a grazing ray is ~ 16 mas, to which a component from the quadrupole moment of Jupiter is superimposed with an amplitude of ~ 240 μas. This secondary deflection has a very specific pattern as a function of the position of the star with respect to Jupiter and according to the orientation of its spin axis. We establish in the following section the relevant formulas to express the astrometric observable effect and explore the favourable circumstances in 2011-2018 to detect this effect predicted by General Relativity, but never observed.

2. THE LIGHT DEFLECTION AMOUNT OF AN AXIS-SYMMETRIC PLANET

Several works (Epstein & Shapiro 1980; Kopeikin & Mashhoon 2002; Klioner 2003; de Felice et al. 2004) tackle the relativistic problem of the light path reconstruction, which, at the μas accuracy, implies taking into account all the contributions due to the bulk mass of the Solar System bodies and their quadrupole moment. In this work, having in mind the new capabilities of Gaia to observe close to the Jupiter limb, we compute the light deflection as a lens effect acting on the grazing photons coming from distant stars. The validity of this approach relies on the fact that the stellar source, the observer and the planet have a small velocity compared to the velocity of light and that the Solar System generates a weak gravitational field. Specifically, we consider a local Minkowskian space-time perturbed by a spherically symmetric mass distribution plus its first relevant deviation from it. Then, the space-time curvature acts like a non-homogeneous medium with an index of refraction $n = 1 - 2U/c^2$, where U is the gravitational potential. From the theory (Narayan & Bartelmann 1995) we know that the deflection is the integral along the unperturbed light path λ of the gradient of n perpendicular to it, namely

$$\triangle \mathbf{\Phi} = -\int \nabla_{\perp} n d\lambda = \frac{2}{c^2} \int \nabla_{\perp} U d\lambda. \qquad (1)$$

As a first approximation, for a single lens and a distant star, let us consider the deflection angle $\triangle \mathbf{\Phi}$ small enough to integrate $\nabla_{\perp} U$ along an unperturbed light ray with the same impact parameter b. Assuming, then, the origin of coordinates at the centre of an axis-symmetric planet, i.e., z considered as its axis of maximum moment of inertia, located between the star and the observer, we get

$$\begin{aligned} \nabla_{\perp} U \quad = \quad & \left[-\frac{b}{r} \left(\frac{GM}{r^2} + \frac{3GM}{r^4} J_2 R^2 \frac{5\cos^2\theta - 1}{2} \right) \right. \\ & \left. + \left(-\frac{3GM}{r^4} J_2 R^2 \cos\theta \right) (\mathbf{z} \cdot \mathbf{n}) \right] \mathbf{n} \qquad (2) \\ & + \left(-\frac{3GM}{r^4} J_2 R^2 \cos\theta \right) (\mathbf{z} \cdot \mathbf{m}) \mathbf{m}, \end{aligned}$$

where r and θ are, respectively, the radial distance and the co-latitude (on the planet) of the field point stemming from the origin of the coordinates to the light trajectory, J_2 is the dimensionless coefficient of the second zonal harmonic, R the radius and M the mass of the planet, $\mathbf{n}$ the radial direction perpendicular to the unperturbed ray pointing towards the centre of gravity along b, and, finally, $\mathbf{m}$ represents the orthoradial component (see Figure 1 and Figure 2). Then, the positional vector of the photon with respect to the principal axes centred at the planet can be expressed as:

$$\mathbf{r} = \mathbf{t}\lambda + \mathbf{n}b. \tag{3}$$

where $\mathbf{t}$ is unit the tangential vector on the unperturbed light trajectory in the direction of the observer.

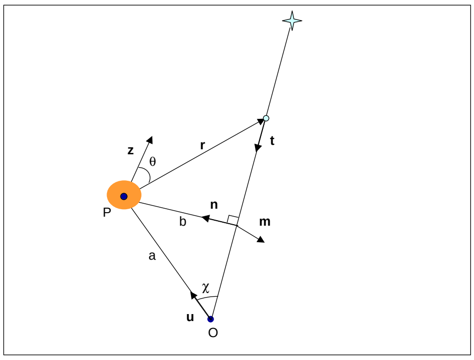

Figure 1. Geometry of light deflection due to a planet (P): the spin axis of the planet $\mathbf{z}$ is out of plane, $\mathbf{t}$ represents the unit tangential direction from a distant star to the observer (O) on the unperturbed light trajectory, $\mathbf{u}$ is the unit direction from O to P along their distance a, χ is the angle star/O/P, and b is the impact parameter.

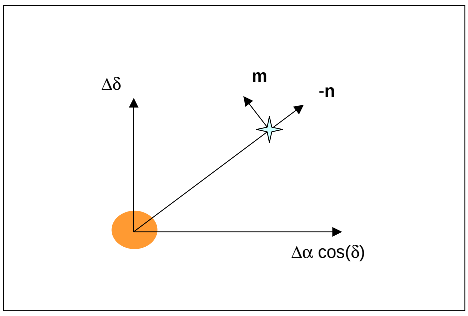

Figure 2. Light deflection by a planet: observer's view. The position of the star is displaced both in the radial (-$\mathbf{n}$) and orthoradial $\mathbf{m}$ directions. The spin axis of the planet $\mathbf{z}$ is somewhere out of the plane.

The length λ on the photon path can be scaled by the impact parameter as follows

$$d\lambda = b d\ell, \tag{4}$$

so the radial distance becomes

$$r = b(1 + \ell^2)^{1/2}. \tag{5}$$

Each integral in expression (2) must be computed with ℓ running positively in the same direction as the photon from $\ell = -\infty$ to $\ell = 1/\tan\chi$, assuming χ the angular separation between the directions star/observer and observer/planet (Figure 1). At the closest approach on the unperturbed ray one has $\ell = 0$. The evaluation of the integrals in (2), after some algebra, splits the light deflection vector into two components: the first one along $\mathbf{n}$ and the second one along $\mathbf{m}$, both including the quadrupole contribution of the planet in function of the angular separation χ:

$$\Delta\boldsymbol{\Phi} = \Delta\Phi_1\mathbf{n} + \Delta\Phi_2\mathbf{m}, \tag{6}$$

where, precisely,

$$\begin{aligned} \Delta\Phi_1 \quad = \quad & \frac{2GM}{c^2 b}\Big\{(1+\cos\chi) + J_2\frac{R^2}{b^2} \\ & \Big[(1+\cos\chi+\frac{1}{2}\cos\chi\sin^2\chi) \\ & -2(1+\cos\chi+\frac{1}{2}\cos\chi\sin^2\chi \\ & +\frac{3}{4}\cos\chi\sin^4\chi)(\mathbf{n}\cdot\mathbf{z})^2 \\ & +(\sin^3\chi - 3\sin^5\chi)(\mathbf{n}\cdot\mathbf{z})(\mathbf{t}\cdot\mathbf{z}) \\ & -\ (1+\cos\chi+\frac{1}{2}\cos\chi\sin^2\chi \\ & -\frac{3}{2}\cos\chi\sin^4\chi)(\mathbf{t}\cdot\mathbf{z})^2\Big]\Big\} \end{aligned} \tag{7}$$

and

$$\begin{aligned} \Delta\Phi_2 \quad = \quad & \frac{2GMJ_2R^2}{c^2b^3}\,[2(1+\cos\chi \\ & +\frac{1}{2}\cos\chi\sin^2\chi)(\mathbf{n}\cdot\mathbf{z})(\mathbf{m}\cdot\mathbf{z}) \\ & \sin^3\chi(\mathbf{m}\cdot\mathbf{z})(\mathbf{t}\cdot\mathbf{z})]\,. \end{aligned} \tag{8}$$

The first term in the radial component (that along $\mathbf{n}$) is the classical monopole deflection, widely used in astronomy. All the other terms are factored by J_2 as being due to the quadrupole of the planet. As said before, in general one has for the planet $\chi \ll 1$ and applied to Gaia this leads to the more convenient and accurate enough formulas,

$$\begin{aligned} \Delta\Phi_1 \quad = \quad & \frac{4GM}{c^2 b}\,[\,1 \\ & +J_2\frac{R^2}{b^2}\left(1 - 2(\mathbf{n}\cdot\mathbf{z})^2 - (\mathbf{t}\cdot\mathbf{z})^2\right)\Big] \end{aligned} \tag{9}$$

$$\Delta\Phi_2 = \frac{8GMJ_2R^2}{c^2b^3}(\mathbf{m}\cdot\mathbf{z})(\mathbf{n}\cdot\mathbf{z}). \tag{10}$$

In this final expression the deflection vector depends on the orientation of the spin axis of the planet and, moreover, on the direction of the star with respect to the planet as the terms proportional to $(\mathbf{n}\cdot\mathbf{z})$ and $(\mathbf{t}\cdot\mathbf{z})$ show. We have not yet introduced the full complexity of the physical context, i.e., considering the motion of Jupiter and the fact that a retarded effect on the light propagation exists. But we will refer to Equations 9 and 10 as test when we deal with the real experimental conditions. Here, we recover the same expression deduced in Epstein & Shapiro (1980) in the case of the Sun. In fact, our assumptions coincide with those of a locally flat, Minkowskian space-time which is weakly perturbed by the gravitational potential of the lens to first post-Newtonian order assuming the parameter γ equal to its GR value.

3. THE *GAIA* *R*ELATIVISTIC *EX*PERIMENTS

Gaia should carry out in the Solar System several tests of General Relativity: mainly the estimate of the parameter γ at the level of accuracy needed to reveal the scalar component from which the current tensorial formulation of the space-time is deduced (Damour et al. 2002a,b). For the first time, with Gaia it should be possible to disentangle among the deflection effects those due to Jupiter's gravitational presence. Because of the scanning law and the Jupiter background noise, the observation strategies at the partial illuminated edge need to be planned and the astrometric accuracies to be evaluated. These will be achieved by the so called GAREX (GAia Relativistic EXperiments).

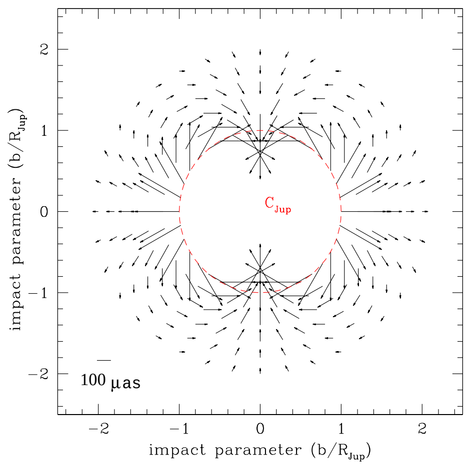

Figure 3. The quadrupole deflection vector field of the stellar background around the center of gravity (C_{Jup}) of Jupiter. The scale of 100 μas is shown on the lower left of the plot.

In Figure 3 we have plotted the deflection vector field due only to the contribution of the quadrupole of Jupiter. Each vector is a composition of the radial component plus the orthoradial one and measures the relative displacements of the stellar positions on the background field, i.e., the difference between the background position without the planet and the deflected position due to the axis-symmetric mass distribution. Table 1 give some values obtained using Equations 9 and 10; the second and third column refer, respectively, to the average magnitude of the radial and orthoradial quadrupole deflection component for stars evenly distributed around the planet.

Table 1. Magnitude of light deflection due to Jupiter deduced by Equations 9 and 10 at several radii.

	M_J mas	$< J_2 >_{\mathbf{n}}$ μas	$< J_2 >_{\mathbf{m}}$ μas
1.0 R_{Jup}	16	155	155
1.5 R_{Jup}	11	46	46
2.0 R_{Jup}	8	20	20

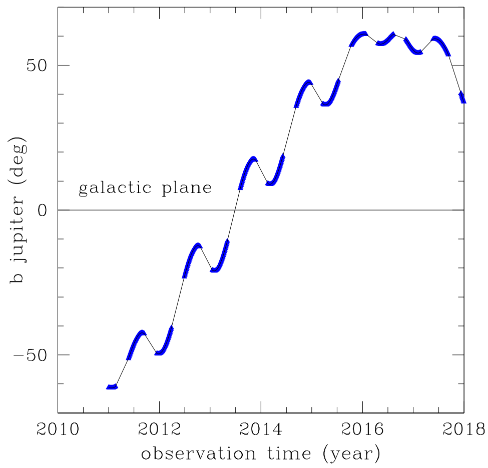

Figure 4. Galactic latitude (b, blue dots) of Jupiter versus the observing time (in years). The solid line represents the galactic latitude of Jupiter (as seen from Gaia) over the years 2011-2018 and the highlighted patches correspond to the visibility periods when the angular distance to the Sun makes the observation possible.

4. THE POSSIBLE SCENARIO IN 2011-2018

Before simulating GAREX, we have investigated what are the favorable events to perform experiments using Jupiter during the mission's lifetime. This depends on the number of times Jupiter will cross one of the astrometric fields and on the stellar density around Jupiter during these observations. The detail of the former can only be known statistically, as long as the precise initial conditions of the scanning law remain unknown. The latter condition can be assessed exactly as it depends only on the motion of Jupiter between 2011 and 2018. Figure 4 shows a typical sequence of observations of Jupiter by Gaia during the mission, together with the favorable visibility period when the angular distance to the Sun makes the observation feasible with Gaia. The ephemeris has been computed in galactic coordinates between 2011 and 2018, to indicate the period when the maximum of stars are expected around Jupiter. Figure 5, instead, illustrates Jupiter in a real starfield near the galactic plane where it will be in mid-2013 and, consequently, when we expect many favorable observations of the planet in front a high stellar density. The plate is from the Palomar digitized survey and the faintest stars are around V = 18. The red spots are stars from the UNSNO-B2 and the faintest

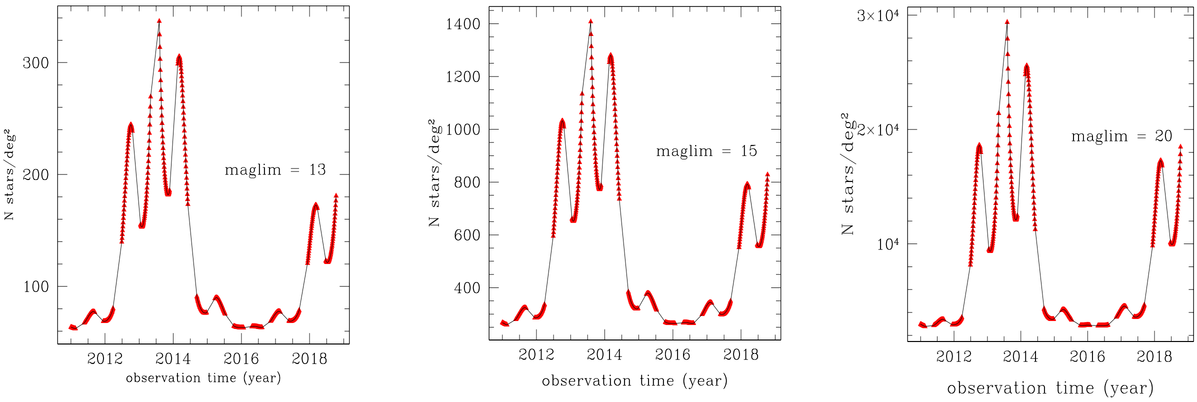

Figure 6. The triangles represent the number of stars per square degrees versus the observation time in years for limiting magnitudes $G < 13$, $G < 15$, $G < 20$. The large variations in density are just an amplification of the small bumps of Figure 4 when Jupiter is nearing the Galactic plane.

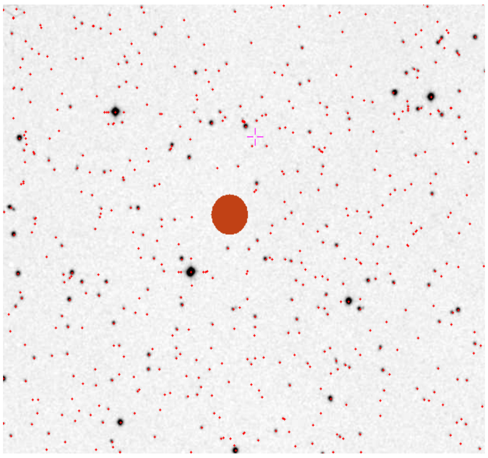

Figure 5. The real star field on the galactic plane around Jupiter in mid 2013.

are around V = 20, all visible with Gaia. Between the preceding and following field the motion of Jupiter is between 1 and 2 R_J. The comparison of the star mutual distances between these two fields will bring the information of the direction and magnitude of the deflection. The method needs only to compare small fields within a short interval of time and is largely independent on the attitude restitution of Gaia. Finally, we have computed the stellar densities up to various limiting magnitudes as they will be seen by Gaia in L2 from 2011 to 2018. Figure 6 shows the star counts per square degree obtained in the case of G = 13, G = 15, and G = 20. As said before, the maximum number of stellar observations occurs around mid 2013, when Jupiter crosses the Galactic plane.

5. CONCLUSION

High astrometric accuracy performed by Gaia will allow new effects predicted by different astrometric models to be tested in the context of General Relativity. The microarcsecond accuracy sets a goal for the observational models, which will have to include all the details and the implication of the Solar System gravitation and instrument development and modelling. In particular, the new design of Gaia's CCDs will allow the quadrupole deflection due a planet to be detected for the first time. Eddington-like experiments will be carried out by observing many times the same background field with and without the deflecting mass and will give the chance to determine accurately the parameter γ at the level of accuracy needed to test alternative theories of gravity. Then, as a final application, but no less important, the measurements of the relative displacements of the stars on the background field with and without planet will indirectly determine the position of the center of gravity of the planet.

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