ASTEROID SIZES FROM GAIA OBSERVATIONS

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ABSTRACT

Gaia will resolve the apparent angular sizes of a very large number of asteroids. In this way, it will produce for the first time a statistically significant and homogeneous data base of reliable and directly measured asteroid sizes. As a by-product, a new calibration of sizes obtained by means of indirect techniques, like polarimetry and radiometry, will also be obtained. We are analyzing the expected performances of Gaia in measuring asteroid sizes by means of a numerical algorithm to simulate asteroid signals produced on the Gaia astrometric focal plane. Assuming the most up-to-date instrumental parameters, we conclude that nearly all main-belt asteroids having diameters larger than 30 km will be measured with an accuracy equal to or better than 10% at least once during the mission operational lifetime.

Key words: Gaia, astrometric focal plane, asteroids, size measurement.

1. INTRODUCTION

The measurement of sizes is one of the major challenges for modern asteroid science. For a long time these objects have been point-like sources except in a very few cases, but today asteroids can be resolved in several cases using direct techniques of high-resolution optical observations based on adaptive optics and/or speckle interferometry, and also by means of radar experiments. ESA's Gaia mission (De Boer et al. 2000) will be an ideal tool to obtain direct measurements of asteroid sizes, due to its unprecedented resolving power. During its five-years operational lifetime, Gaia will observe some hundred thousand main-belt asteroids, in addition to a substantial number of near Earth asteroids, including probably a number of newly discovered objects, mostly belonging to the orbital classes of the Atens and IEOs, which orbit mostly or totally inside the Earth's orbit.

The purpose of the present analysis is to accurately assess the expected performances of Gaia in measuring asteroid sizes. The apparent angular size is derived in principle from the differences between the signal collected on the Gaia focal plane from an extended object, and the signal produced by a point-like source (star or quasar). Our analysis is based on a numerical code of simulation of asteroid signals recorded by Gaia (Dell'Oro & Cellino 2004), which takes into account all the different effects that determine the final signal recorded by the Gaia astrometric detectors, including the motion of the objects on the focal plane, the Time Delayed Integration (TDI) strategy, the role of sky brightness, etc. A major point is that, since we deal with signals consisting of limited numbers of photons (typically from 10^3 to 10^6), photon statistics play a role of primary importance, and must be taken into account. The noise due to photon statistics is such that the signal received from a given object in two identical observing conditions is not constant. Conversely, objects having different sizes and shapes may produce signals which are essentially identical. This is the primary factor which will limit the final accuracy in angular size measurements for the asteroids observed by Gaia. The number of collected photoelectrons depends essentially on the magnitude of the object, the entrance pupil and the integration time. Our capability to distinguish the signals of two asteroids having different sizes (assuming for the sake of simplicity that they are in all other respects identical) depends primarily on the level of noise produced by photon statistics. In this work we present the results of our investigation of this problem.

2. SIZE MEASUREMENTS WITH GAIA

When a celestial body enters the astrometric focal plane of Gaia, its signal is collected by a series of CCDs. The along-scan motion of the source is compensated by the TDI mode of CCD read-out, while the incoming signal is integrated in the across-scan direction. The final signal is the along-scan distribution of collected photoelectrons binned according to the along-scan width of the CCD pixels. In other words, the final signal is a series of n numbers representing the photoelectrons collected by a line of pixels aligned along the across-scan direction.

Depending on its apparent angular size at the epoch of observation, which in turns depends on the distance, absolute size, shape and rotational phase of the object, the signal generated by an asteroid can be more or less different from the signal coming from a point-like source. In particular, the signal from a point-like source is essentially the Point Spread Function (PSF) of the optical system. The signal produced by an asteroid will be instead

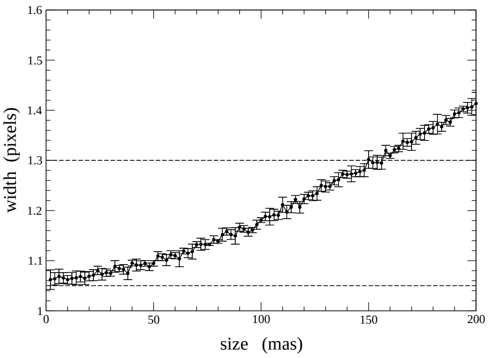

Figure 1. Example of relationship between asteroid signal width and asteroid size.

the convolution of the PSF with the angular distribution of asteroid photons incident on the focal plane.

The basic approach to the problem of the size measurement of an asteroid is based on an analysis of the recorded signal, and in particular on the determination of the standard deviation of the along-scan photoelectron distribution, a quantity that hereinafter we will simply call the *width* of the signal.

Our numerical simulations of asteroid signals are based on a ray-tracing procedure and a Monte Carlo approach. The final position of any given photoelectron in the produced image can be written down as:

$$X = \theta(x' + \delta x + \dot{x}t + x_0, t) \qquad (1)$$

$$Y = y' + \delta y + \dot{y}t + y_0 \qquad (2)$$

where X and Y are the along-scan and the across-scan coordinates, respectively. The coordinates x' and y' correspond to the angular direction of the incoming photon. The terms δx and δy represent the diffraction pattern due to the PSF. The proper motion is taken into account by the two terms $\dot{x}t + x_0$ and $\dot{y}t + y_0$, in which t is the time of arrival of the photon, $\dot{x}$, $\dot{y}$ the apparent angular velocities, and (x_0, y_0) an initial position at $t = 0$. Finally, the function θ accounts for the TDI shift of the photoelectron along the X direction. The final distribution of X, Y coordinates of N collected photoelectrons is determined by the above formula, assuming x', y', δx, δy and t to be random variables. In turn, the parent distributions of these random quantities are different, corresponding to different physical processes. In particular, x' and y' depend on the shape and the reflection properties of the object's surface, and they are simulated by means of a Monte Carlo ray-tracing mapping; δx and δy are computed according to the assumed PSF function; t is randomly (and uniformly) generated within an interval corresponding to the CCD integration time.

It is obvious that, to be measurable, an asteroid's apparent angular size must be such that the corresponding signal width can be distinguished from the signal of a point-like source (PSF). When this is the case, we are then interested in assessing how the signal width changes as a function of the apparent angular size. If we can establish a relation between signal width and apparent angular size, the problem of size measurement reduces then to the accurate determination of the collected signal width.

In this respect, the most important source of error is photon statistics. In particular, the noise in the collected signals due to photon statistics, strongly affects the capability of distinguishing the widths of signals from asteroids having different angular sizes. Simple statistical considerations suggest that the stochastic fluctuations of the signal width are of the order of $\sigma/\sqrt{N}$, where N is the number of collected photoelectrons, and σ is the 'true' signal width, i.e., the signal for $N \to \infty$. In practice, therefore, the final accuracy in measuring asteroid sizes will depend on the stochastic fluctuations of the signals and on the relation between signal width and size. Obviously, additional terms have also to be taken into account in the computations: sky brightness, read-out noise, number of the actually read-out pixels (windowing), all play a role in the final error budget. On the other hand, all the mentioned sources of uncertainty can be accurately reproduced by our numerical simulator. In Figure 1 an example of the relationship between size and width is shown. The signals from a series of simulated spherical objects observed in the same observing conditions, but having different sizes, have been computed, and the corresponding signal widths are plotted.

The resulting width-size relationship depends on the instrumental characteristics, and in particular the telescope PSF and the CCD windowing operation, on the observation conditions like phase angle and proper motion of the object, and on the physical properties of the asteroid, namely shape and surface light scattering properties. The fluctuations with respect to the general trend of the curve visible in this Figure are due to photon statistics. The error bars shown in the plot represent the standard deviations of the signal width fluctuations at different angular sizes. In this particular example, the mean number of collected photoelectrons is supposed to be 4000, corresponding to a visual magnitude around 18.5 for the entrance pupil (0.70 m^2) and CCD integration time (3.3 s) which characterize the nominal configuration of Gaia. The two values of signal width marked with two long dashed horizontal lines in the same plot have no special meaning, but they represent a couple of possible measurements. They are used as an example to introduce the following considerations. Having in mind Figure 1, the problem of asteroid size determination can be converted in practical terms into the following: which angular sizes are compatible with width values like the two ones marked in the Figure?

In principle, the range of sizes compatible with any given width measurement depends on the general shape of the width–angular size curve shown in Figure 1, that is variation of the standard deviation of the signal at different angular sizes. In particular, it is possible to build a probability distribution of angular sizes compatible with any given value of signal width. Figure 2 shows the result of this exercise for the two values of width marked by horizontal lines in Figure 1.

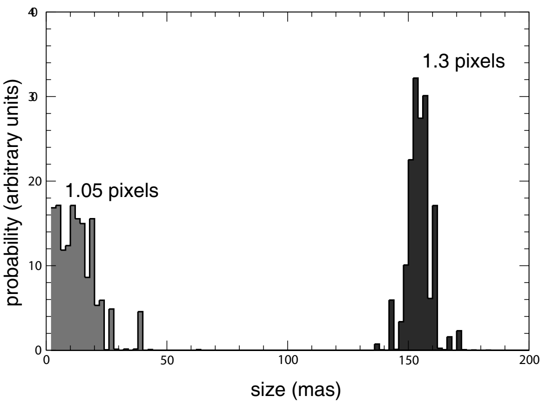

Figure 2. Probability distribution for the compatible sizes corresponding to the two marked values of the width in Figure 1.

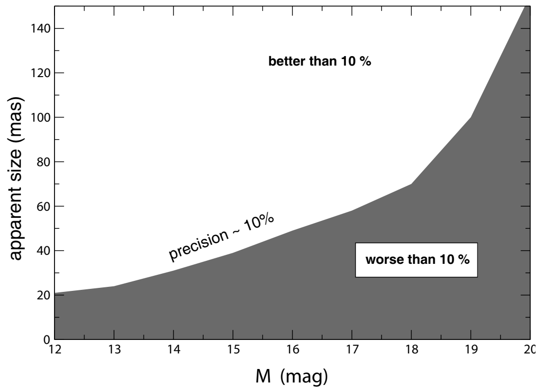

Figure 3. Precision measurement of extended object sizes expected by Gaia.

Histograms like the one shown in Figure 2 allow us to infer the expected accuracy in asteroid size determination. The nominal error in size determination can be defined as the standard deviation of the distribution of compatible sizes for an observed signal width. By performing simulations corresponding to a wide variety of possible observing circumstances, we can finally assess the expected performances of Gaia in measuring asteroid sizes.

The accuracy in size measurement depends essentially on the actual angular size and the magnitude, which determines the number of photons which are collected by the Gaia detectors. It is interesting to assess what is the practical limit in angular size d_0 that can be measured with a given precision. We focus here on a value of 10%, for a given apparent magnitude M. For a given magnitude, larger sizes can be measured at the same value of accuracy or better, whereas smaller sizes cannot. In this way, we can divide the magnitude *versus* apparent angular size plane in two regions, one corresponding to the 'good' conditions of observation, those for which a size measurement with an accuracy of 10% or better is possible, and one including the 'bad' observing conditions. The result of this exercise is shown in Figure 3. The two domains in the plane are divided by a border line corresponding to the accuracy of 10%. This diagram is characteristic of the particular instrument used. Of course, we assume the nominal configuration of the astrometric focal field of Gaia. In particular, we adopt the PSF delivered by the GIBIS facility[1], computed for the astrometric field CCD plates (Lindegren 1998), the sampling scheme proposed by Hoeg & Fabricius (2004), for which 12×12 ($n \times m$, where n is the number of pixels in the along-scan direction and m the number of pixels in the across-scan direction) pixel windows will be used for stars between magnitude 12 and 16 (in the G-band of Gaia), and 6×12 windows for stars with magnitudes between 16 and 20.

The minimum size that can be measured with a precision of 10% at magnitude 12 turns out to be about 20 mas, while at magnitude 20 only asteroids with sizes larger than 150 mas can be measured. We assumed a sky brightness at high ecliptic latitude around 23.3 mag arcsec^{-2}, and around 22.1 mag arcsec^{-2} on the ecliptic, corresponding to current estimates of the zodiacal light (HST/WFPC2 Instrument Handbook, De Boer et al. 2000). A read-out noise of 10 e^{-} has been also assumed.

A primary factor that will affect the value of the size threshold for which we can do a useful measure is the number of pixels used in the CCD windowing scheme. The best performance of the instrument is the result of a trade-off between adequate signal sampling and corresponding noise. An exceedingly small number of window pixels can entail an insufficient sampling of the signal, and an artificial cut-off of a part of it. In particular, extended objects having large angular sizes can be difficult to measure in this case. On the other hand, too many pixels can lead to the collection of unnecessary information. In particular, if the window size is much larger than the angular size of the object, the collected signal is contaminated by the noise produced by unnecessary pixels. These pixels contribute to the computation of the width with high momentum terms, so introducing a 'high momentum noise', which can prevail on the 'true' noise due to the source, if the number of pixels is too large. Even in this case the accuracy decreases significantly. The best choice for all the magnitudes turns out to be, according to our simulations, between 6 and 8 pixels.

3. ASTEROID OBSERVATION STATISTICS

In order to assess the capability of Gaia in measuring the sizes of main-belt asteroids, we can also take advantage of detailed simulations of asteroid detections performed by Gaia during its planned operational lifetime of five years. (Mignard 2001). These simulations provide the list of all expected transits of main-belt objects on the focal plane, and the corresponding observing circumstances, including the distances r from the satellite and the apparent magnitudes M. Using our simulator of asteroid signals, we have seen that, at least for main-belt

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[1] at http://gibispc.obspm.fr/∼gibis/

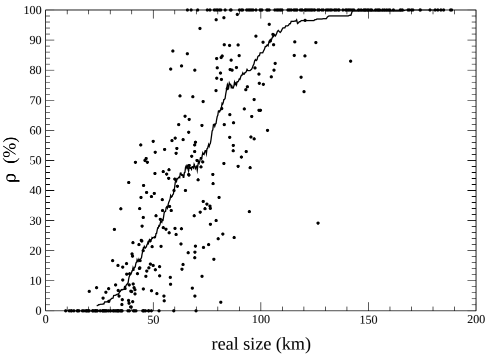

Figure 4. Expected Gaia efficiency in measuring the sizes of the main-belt asteroids.

objects, phase and apparent motion do not degrade significantly the size measurement accuracy (Dell'Oro et al. in preparation), then we can ignore them. Therefore, we selected among the whole list of simulated asteroid detections those for which the observed objects have a known IRAS diameter D. We assumed as a first approximation at this stage, that the objects have spherical shapes and we limited our analysis to the detections of objects having an apparent magnitude between 12 and 20. The corresponding apparent sizes are obviously given by $d = D/r$. Being given the apparent magnitude and the apparent size of an object at the epoch of each detection, we can assess if we can obtain in these circumstances a measurement of its size with a precision better (*good observation*) or worst (*bad observation*) than 10% on the basis of the diagram in Figure 3.

For each single object, in general some observations will be good and others will be bad, correspondingly with its magnitude and angular size. Let S be the total number of observations involving a single asteroid, while let s be the total number of *good* observations, in the sense explained above, of the same object. We will call *efficiency of Gaia* in measuring asteroid sizes the ratio $\rho = s/S$ for each object.

We show in Figure 4 the result of this exercise. Each point represents the efficiency and real diameter of each single asteroid. The solid line is the average value of the efficiency *versus* real diameter obtained by means of a running-box technique. For asteroids with sizes larger than 100 km the measurement efficiency is well above 50%, and almost all Gaia observations will be good. Below 20 km no good observation will be possible, because the objects are too small or too faint or both (note the group of crosses with $\rho = 0\%$). Around 20–30 km, the measurement efficiency is typically a few per cent.

4. CONCLUSIONS

Our simulations show that it will be possible, based on Gaia observations, to measure main-belt asteroid sizes with an accuracy of 10% or better if their angular size is larger than 20 mas at a magnitude around 12, or 150 mas at magnitude 20. In terms of physical sizes, this means that asteroids larger than ≈ 20 km will be measured with the same accuracy, whereas below this threshold no accurate size determination will be done. The expected number of measurable objects should be correspondingly of the order of 1000.

The results presented in this paper have been obtained assuming that asteroids have spherical shapes. At this stage, this simplification is not critical for our purposes. Dropping this assumption, the results of the measurements will be related to the angular size of the instant projection of the body shape at the epochs of detection. This will be used to derive information on the overall shape and spin axis orientation of the objects, since they will be typically observed by Gaia many times and in a large variety of observing geometries (see also Cellino et al. 2004).

Direct size measurements of such a large number of main-belt asteroids will constitute a decisive improvement of our knowledge of the asteroid size distribution. Gaia diameters will be important for a new calibration of the data base of asteroid sizes and albedos obtained by means of indirect techniques, primarily thermal IR observations (the IRAS Minor Planet Survey, see Tedesco et al. 2002), and polarimetry. The resulting knowledge of the size distribution, shapes and rotational state of main belt asteroids, will open a new era for the studies of the collisional evolution of minor planets. At the same time, the determination of sizes and overall shapes for asteroids for which Gaia will also determine the mass, will produce knowledge of the average densities of these objects.

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