DETERMINATION OF THE PPN β AND SOLAR $J_2^{\odot}$ FROM ASTEROID ASTROMETRY

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ABSTRACT

The improvement of known asteroids' orbits from Gaia astrometric observations is discussed. Further we show how the PPN parameter $(2 + 2\gamma - \beta)/3$ and the solar quadrupole can be determined separately, providing hence a direct and independent determination of the solar $J_2^{\odot}$.

Key words: Gaia; Asteroid; Perihelion-precession; Relativistic PPN β; Solar quadrupole.

1. INTRODUCTION

The Gaia satellite will observe a large number of Solar System objects, mainly main-belt asteroids, with an un-precedented sub-milliarcsecond astrometric precision. Given the limiting magnitude of $V \leq 20$, one can expect that about 5×10^5 asteroids will be observed, most of them being already discovered at the time of the Gaia mission with present ground-based surveys such as e.g., LINEAR. The astrometric measurements will enable the improvement of the orbit of each known object taken alone, but also the derivation of global parameters that affect their motion around the Sun such as the precession of the perihelion that is due in one part to general relativity and in the other part to the oblateness of the Sun. We give here the results for the orbit improvement from a simulation of Gaia observations of selected asteroids that lie in the main-belt, the jovian Trojans, and the NEAs population. The present simulation does not consider the orbit of newly discovered asteroids, errors due to un-modelled photocentre offset from phase effect, or un-modelled mutual perturbations from massive asteroids.

2. ASTEROIDS ASTROMETRY

The measured astrometric position of an asteroid depends on its spectral type, size, velocity, shape, solar phase, etc, but the astrometric precision depends mainly on the photon noise due to its brightness. Making use of the Gaia Instrument and Basic Image Simulator (GIBIS, Babusiaux et al. 2001; Babusiaux 2005), one can simulate asteroid images on the focal plane that take into account the different instrumental design, CCD noises and gain, as well as the asteroid's magnitude, motion and size.

Estimation of the astrometric precision was next obtained with the Pyxis code (Arenou et al. 2004; Arenou 2005) and a centroiding based on the image barycentre which is known to be less precise than e.g., an LSF fitting. Further we will consider that one position corresponds to 1 crossing of the object in the field of view (FOV) keeping the same single CCD precision. Hence we are conservative with respect to the achievable astrometric precision, balancing also the fact that no special treatment was applied at this stage for fast moving objects that would not be observed throughout their whole FOV transit. Eventually the astrometric precision is in the range 0.2–3 mas for objects in the $V \sim$ 8–20 magnitude range, and generally below the mas level, with a noticeable increase when $V \gtrsim 14$ (see Figure 1). In the following we have adopted the simple relation:

$$\begin{aligned} \log(\sigma_\lambda) &\simeq - 0.411 \quad ; \quad \text{if } V < 14 \\ \log(\sigma_\lambda) &\simeq 0.147.V - 2.710 \quad ; \quad \text{if } V \geq 14 \end{aligned} \qquad (1)$$

for the estimation the astrometric precision and weightings of the equations of condition.

The simulation also considers a realistic time distribution for the asteroids observation. Considering the asteroids ephemerides and the satellite scanning law, the Gaia crossing dates of selected numbered asteroids have been simulated by F. Mignard. The number of crossings – varying from one asteroid to another – is on the average equal to 65, and drops to 40 for the NEAs.

3. ORBIT IMPROVEMENT

The orbit improvement is expressed in terms of small correction to its size $(da/a, de)$ and its orientation through $(dl_o + dr, dp, dq, e.dr)$, where the angles (dp, dq, dr) are related to the usual osculating elements by:

$$\begin{pmatrix} dp \\ dq \\ dr \end{pmatrix} = R_z(-\omega) \cdot \begin{pmatrix} di \\ \sin i \; d\Omega \\ d\omega + \cos i \; d\Omega \end{pmatrix} ; (\sin i \neq 0) \quad (2)$$

where $R_z(-\omega)$ is the rotation along the ecliptic pole of angle $-\omega$. For any planet i the partial derivatives

Table 1. Results for the (formal) precision of the orbit improvement of selected asteroids.

	$dl_o + dr$	dp	dq	$e.dr$	de	da/a
NEA	3.0×10^{-09}	1.4×10^{-09}	1.1×10^{-09}	1.4×10^{-09}	1.0×10^{-09}	4.7×10^{-10}
MBA	1.4×10^{-10}	1.8×10^{-10}	4.1×10^{-10}	1.2×10^{-10}	1.3×10^{-10}	7.7×10^{-11}
Trojan	2.9×10^{-09}	2.5×10^{-09}	9.5×10^{-09}	3.5×10^{-10}	4.4×10^{-10}	6.2×10^{-10}

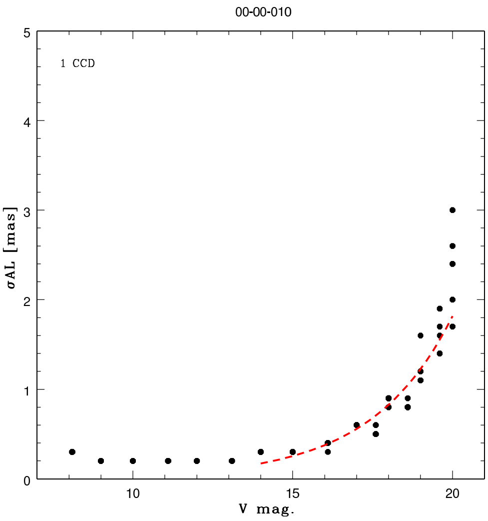

Figure 1. Astrometric precision $\sigma(\lambda)$ (in the along-scan direction) of the barycentre centroiding as a function of the V magnitude.

matrix $\mathbf{B_i} = [\frac{\partial \lambda}{\partial \mathbf{q_i}}]$ can be obtained from a numerical integration but also in close form from the two body approximation (Brouwer & Clemence 1961). The formal precision of the orbit improvement $\mathbf{dq_i} = (dl_o + dr, dp, dq, e.dr, da/a, de)$ is next obtained from the variance/covariance matrix in a – supposed linear – least-squares inversion:

$$\mathbf{B_i} \cdot \mathbf{dq_i} = \mathbf{d}\lambda \quad ; \quad \sigma^{\mathbf{2}}(\mathbf{dq_i}) = \mathbf{B_i^-} \cdot \sigma^{\mathbf{2}}(\mathbf{d}\lambda)$$

where $\mathbf{B_i'}$ is the transpose of matrix $\mathbf{B_i}$. Only the variance $\sigma^2(\mathbf{d}\lambda)$ of the one-dimensional along-scan astrometric observation is needed in this analysis.

We consider, as an example for illustrating the achievable precision, three bodies with different orbital period: one NEA (1862 Apollo), one MBA (39 Laetitia) and one Jupiter trojan (624 Hektor). The standard deviation of the orbital corrections are given in Table 1. We have also considered a single parameter correction for the perihelion precession, but for sake of brevity it will not be reproduced here.

4. PRECESSION OF THE PERIHELION

Note that, following Equation 2, we will consider the argument of the perihelion ω, not the longitude of the perihelion $\varpi = \omega + \Omega$. Putting $m_\odot = GM_\odot/c^2 \approx 1.48$ km, the secular drift of the argument of the perihelion $\dot{\omega}$, for a body of semi-major axis a and eccentricity e, is given by:

$$\dot{\omega} \;=\; \frac{3\pi m_\odot}{a\,(1-e^2)}\lambda_p \quad \text{[rad/cycle]} \qquad (3)$$

where λ_p should include both the relativistic effects and the effect due to the non-sphericity of the Sun's gravitational potential (the other effects are assumed to be negligible or assumed to be known). In the following we neglect the terms depending on the bodies masses' ratio[1], terms from moments larger than the solar J_2 (in particular $J_4 \sim 0$), terms violating the equivalence principle (hypothetical post-PPN $\alpha = 1$) and those arising from the cosmological constant[2] ($\Lambda = 0$).

Eventually, the correction to the (secular) perihelion drift is modelled by the PPN parameter $\Gamma = (2 + 2\gamma - \beta)/3$ and the solar quadrupole moment $dJ_2^\odot$:

$$\begin{aligned}\Delta\omega &= \Delta\omega_{|PPN} + \Delta\omega_{|J2} \\ &= \left[\frac{6\pi m_\odot}{a^{5/2}\,(1-e^2)}\,\Gamma + \frac{6\pi R_\odot^2}{4}\,\frac{5\cos^2 i - 1}{a^{7/2}\,(1-e^2)^2}\,J_2\right](t-t_0) \\ &= \frac{3m_\odot}{a\,(1-e^2)}\left[\Gamma + \frac{R_\odot^2}{4\,a\,m_\odot}\,\frac{(5\cos^2 i - 1)}{(1-e^2)}\,J_2\right] n\,(t-t_0)\end{aligned} \qquad (4)$$

The formal precision with which these two parameters are obtained depends on the magnitude of the object but also – as is well known – on its eccentricity and semi-major axis. We can see in Figure 2 that in addition to Mercury – that is not observable by Gaia – Near Earth asteroids are good candidates for such an experiment. Previous attempts by Lieske & Null (1969); Shapiro et al. (1971) with the observations of asteroid 1566 Icarus were unfortunately limited by the quality and precision of the available observations. The situation was slightly improved – yet insufficiently – in the work of Sitarski (1992), but with Gaia the number of observed bodies and the astrometric precision will be considerably improved. Moreover Gaia is an opportunity to separately determine the PPN and solar components of the perihelion drift (Will 1984; Hestroffer et al. 1999; Mignard 2002).

In addition to the matrix $\mathbf{B_i}$ and the correction $\mathbf{dq_i}$ to the orbit of asteroid i given in Section 3, we introduce the partial derivatives matrix $\mathbf{A_i} = [\frac{\partial \lambda}{\partial \mathbf{q}}]$ for the secular variation of the perihelion that depends on the global parameters $\mathbf{dq} = (\Gamma;\, J_2)$. After some manipulation, the

[1]We consider only small bodies in the Solar System with masses about less than $10^{-11}\,M_\odot$.

[2]The cosmological constant very likely enters the formulation with much too small a weight to be determined from such Solar System perihelion precession experiment.

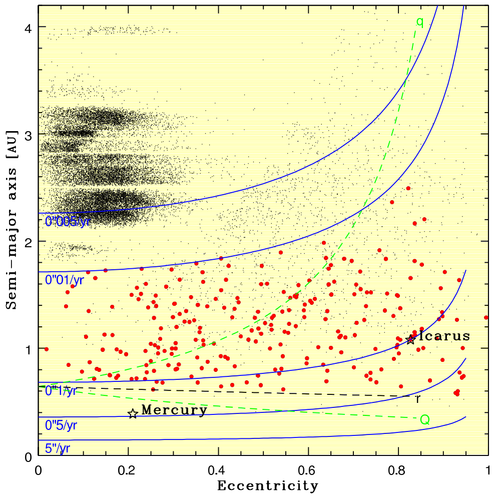

Figure 2. Relativistic precession for asteroids in the eccentricity and semi-major axis space. The main-belt is composed of presently known objects (limited to the first 20 000), while the Near Earth population is mainly based on simulated data from Bottke et al. (2002).

variance matrix of these global parameters is found to be given by:

$$\begin{aligned} \sigma^2(\mathbf{dq}) &= \mathbf{U^{-1}}\,\sigma^2(\mathbf{d}\lambda) \\ \mathbf{U} &= \sum_{\mathbf{i}} (\mathbf{A_i'A_i})^{-1} - \mathbf{A_i'}\,\mathbf{B_i B_i^-}\,\mathbf{A_i} \end{aligned} \quad (5)$$

where $\mathbf{B_i^-} = (\mathbf{B_i'B_i})^{-1}\mathbf{B_i'}$ is the pseudo-inverse of $\mathbf{B_i}$. The results for the formal precision of these global parameters are given in Table 2. It is stressed that the PPN parameter γ will be known with sufficient accuracy within Gaia itself from the light deflection experiment, and that hence the precision for the determination of β is $\sigma(\beta) \simeq 3\,\sigma(\Gamma)$. The correlation between β and J_2 is in any case very strong (larger than 0.9). Both the PPN parameter and solar quadrupole appear to be determined with a precision better than that estimated in Mignard (2002) ($\sigma(\beta) \sim 1.5 \times 10^{-3}$ and $\sigma(J_2) \sim 1.5 \times 10^{-7}$ for the most optimistic). These are encouraging results since they would put Gaia on a competitive level for the determination of β and mostly for an independent and direct measure of the solar J_2.

Table 2. Results for the (formal) precision of the simultaneous determination of β and $J_2^{\odot}$.

	$\sigma(\beta)$	$\sigma(J_2)$
$\sim$ 50 NEAs	3.0×10^{-4}	1.0×10^{-8}
$\sim$ 1100 MBAs	9.0×10^{-4}	1.0×10^{-8}
$\sim$ 1300 asteroids	1.5×10^{-4}	5.0×10^{-9}

Moreover the actual population of known NEAs is likely to be completed in the coming years. Among the $\sim$250 000 presently known asteroids in the `astorb.dat` data base, about 15 have a perihelion drift larger than $0''1$/cy at solar elongations and brightness observable by Gaia. This number would be increased to 50 if one considers the simulated debiased population of NEAs from Bottke et al. (2002) (see Figure 2).

5. CONCLUSION

Considering the simulated Gaia observation of selected known asteroids and their astrometric precision over a 5 years mission, we have analysed the orbit improvement that would be achievable by a simple linear least-squares procedure. It appears that all the osculating elements could be obtained – from the Gaia mission alone – with a relative precision of $\sim 10^{-9}$– 10^{-10}. By considering moreover a global analysis of the perihelion precession, the PPN parameter β and the solar quadrupole J_2 could be determined separately with a precision of the the order of $\sim 10^{-4}$, and $\sim 5 \times 10^{-9}$, respectively. Present values – mainly deduced from heliosismology and model dependent – are of the order of $J_2 \sim 2 \times 10^{-7}$ (e.g. Pijpers 1998; Mecheri et al. 2004), so that Gaia should provide a valuable independent and direct estimates of this quadrupole, and test the GR at different distances from the Sun.

Further simulation will be performed by considering a larger simulated-population of NEAs (Bottke et al. 2002). Finally external and/or systematic effects have been disregarded in the present analysis. Since the observations are preferably performed in a zone around the quadratures rather than around the opposition, the solar phase is not negligible; and it will be particularly large for the NEAs. Un-modelled photocentre offset could deteriorate this formal precision and moreover introduce systematic effects; this should also be analysed in a further study.

ACKNOWLEDGMENTS

Thanks to F. Mignard (OCA, France) who provided us with the Gaia scanning law and the asteroid crossings data (‘rendez-vous’). Thanks to C. Babusiaux (Brussels University, Belgium & GEPI, France) and F. Arenou (GEPI, France) for developing and providing us the GIBIS and Pyxis softwares, and for their assistance.

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