STATISTICAL INVERSION OF GAIA PHOTOMETRY FOR ASTEROID SPINS AND SHAPES

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ABSTRACT

The Gaia satellite will provide us with a large amount of photometric asteroid data, sparse in time. We have used two methods to analyse simulated Gaia data to obtain the spin and shape solution for an asteroid-like object. A convex inversion method was used to obtain the spin state and convex shape for the target, and a spherical harmonics technique to obtain a set of non-convex shapes that reproduce the observed data. Our results show that the period of asteroids may be determined accurately using Gaia data alone, while the pole solution and the related convex shape are constrained to specific regions. Non-convex features cannot uniquely be extracted from the data, but a set of non-convex solutions characterizes the limits where the correct shape lies.

Key words: Gaia; Asteroid; Photometry; Physical properties.

1. INTRODUCTION

Methods for deriving asteroids' spins and shapes from their photometric brightness data have recently progressed significantly. Currently the most complex shape models are expressed either as arbitrary convex polyhedrons or spherical harmonics series. In comparison to triaxial ellipsoids, these novel methods result in more realistic shapes and in higher accuracy of the rotational parameters. Sometimes information about the non-convex features can be derived, uncertainties of which, however, being hard to be estimated. Here we use two methods to estimate the spin and shape of an asteroid from sparse Gaia-like data. The theory of the first, convex inversion method, is described in Kaasalainen & Torppa (2001) and Kaasalainen et al. (2001), and will not be repeated here (see also Kaasalainen et al. 1992a,b). Until now, it has been applied only to data consisting of more or less complete lightcurves (e.g., Torppa et al. 2003), not sparse Gaia-like data. Since in the Gaia case we do not have complete lightcurves, we cannot use relative magnitudes. Thus, we have to use a selected phase function and consider magnitudes as absolute. This is no problem, due to the high accuracy of the data. The initial steps towards what we call the spherical harmonics method (Muinonen & Torppa, in preparation) were taken by Muinonen (1998) (the direct problem) and Muinonen & Lagerros (1998) (the inverse problem). It has not yet been actively applied to any real asteroid data, and the main features are briefly described in Section 4.

Gaia data of asteroids consists of single brightness values ranging over five years of time, the expected lifetime of the satellite. Mignard (2003) states that the time span between individual observations will be less than one month for main-belt asteroids and more for near-Earth targets, providing us with a maximum of about 100 brightness values at varying observing geometries. In particular, small solar elongations and large ecliptic distances will be better covered than with Earth-based observations. The fact that makes the data particularly useful for physical studies of asteroids is that the derived magnitudes are standard absolute magnitudes with accuracy of 0.01 on average (for bright objects the accuracy is higher than for faint ones). This error is of the same order as the errors in the best methods used in asteroid data analysis. Also, since Gaia will do multi-colour photometry, it will provide us with the colours and possible colour variations of its targets after five years. This enables classifying the asteroids by their taxonomy, and is also a significant aid when selecting suitable standard stars for the future ground-based photometry.

Mignard (2003) has estimated that the total amount of asteroids to be observed is 500 000. The limiting magnitude is about 20. We show here that Gaia data really is abundant for obtaining valuable information about the spin and shape properties of its target asteroids. Also, while making it possible to physically characterize a huge number of completely unknown asteroids, Gaia data also provides a remarkable addition to the existing photometric data, and enables making more accurate determination of the spin and shape parameters for the previously observed asteroids. In Section 2 we introduce the simulated data used in this paper. Section 3 gives an overview of the convex inversion method and in Section 4 application of the spherical harmonics technique to this problem is presented.

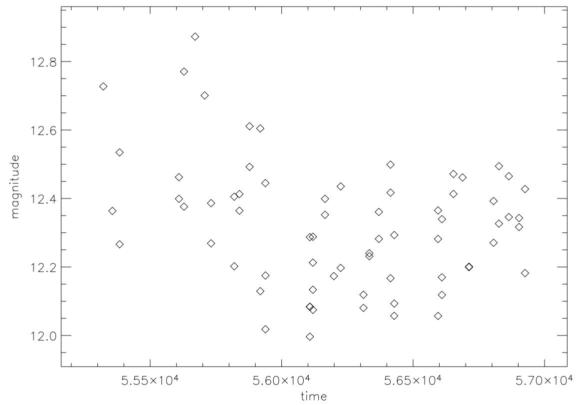

Figure 1. Simulated Gaia data.

2. SIMULATED GAIA DATA

Here we use simulated Gaia-like data that was generated for a Gaussian sphere mimicking an asteroid (Figure 4). The observing geometries were those computed for asteroid Vesta for Gaia's five-year observing time, using the orbital elements of Gaia and Vesta. The data is plotted in Figure 1. The amount of data for Vesta, 69 brightness values, represents the average of what Gaia will provide us for asteroids. The accuracy of the data was the same as what will be typical for Gaia, i.e., 0.01 mag.

As the scattering law we used the H,G magnitude system with parameter G = 0.1102. Geometric albedo was 0.05032, and no Lambertian scattering was included. The future Gaia data will not reach very small phase angles of asteroids, and thus the opposition effect is not needed to be modelled in the data analysis. The rotational parameters of the simulated target are tabulated in Table 1.

3. CONVEX INVERSION

In the convex inversion described in the following subsection, we have transformed the magnitudes to brightnesses as $I = 10^{(5+0.4m)}$, and use a linear-exponential phase function, that expresses the relative brightness of the target at each phase angle. The parameters describing the opposition effect (the exponential part) are not constrained, since the phase-angle range of the data does not reach small values. The adopted value for the slope of the linear part in units of brightness change per degree was −0.009.

3.1. Period Determimation

We determined the period using a technique described in Kaasalainen et al. (2001), where we take fixed pole values distributed evenly in space, and for each pole we compute *rms* values for a set of fixed periods while fitting a shape which is expressed as a very low order functional series. This is plausible, since the period is not sensitive to the exact pole and shape.

Table 1. Spin parameters of the original and convex model shapes.

Shape	period	pole latitude	pole longitude
Original	10.17395622	62.8942754	25.01849912
Model 1	10.173961	62.5	26.0
Model 2	10.173996	67.8	215.4

Here, the period space was first roughly scanned over 1-30 hours since most of the asteroid periods lie within this range. The most clear peaks were found near the period of 10 hours for pole directions $\beta = 45°$, $\lambda = 45°$ or 225°. Clear peaks were also evident at twice the 10 hour period for $\beta = 135°$, $\lambda = 45°$ or 225°. Since also the *rms* value for these four pole-period combinations was smaller than others, they were chosen as starting values for the more accurate spin and shape inversion. If there had been no such clear and consistent peaks in four of the eight plots, it would have suggested that the period sampling density is too small, or that the correct period is not within 1–30 hours. The period plots are shown in Figure 2.

The second step in period analysis was to generate more detailed *rms*-period plots near the starting spin values. This is because for the data ranging five years in time, the distance between the *rms* minima in the period space for periods 10-20 hours is about 0.002 hours, and setting the starting value for the period a few thousands of hours wrong in final convex inversion would lead to a false period solution. An example or these plots is shown in Figure 3 for $\beta = 45°$ and λ =45°. It can be seen, that there is a clear narrow peak also within this range near 10.17 hours. Near 20 hours there are several peaks with equal *rms* one of which is twice the 10.17 hour period.

3.2. Spin Properties and Convex Shape Determimation

Once the period-pole candidates were determined to the accuracy described above, we used each of the spin states as starting values for the final spin and convex shape inversion. For the models, we tested two degrees for the series representation of the shape: $l_{\max} = 4$ and $l_{\max} = 6$ with 15 and 28 shape parameters, respectively. It became clear, that only two solutions produced realistic shapes and rotational states and *rms* values considerably smaller than the other spins: period P=10.173961 and pole β =62.5°, λ =26.0° (Model 1) and P = 10.173996 and pole $\beta = 67.8°$, $\lambda = 215.4°$ (Model 2), both producing *rms*=0.01. The two shapes were mirror images of one another. The shape of Model 1 is shown in Figure 5 and Table 1 includes information on the spin parameters of the results.

Comparing the model shape to the original one shows

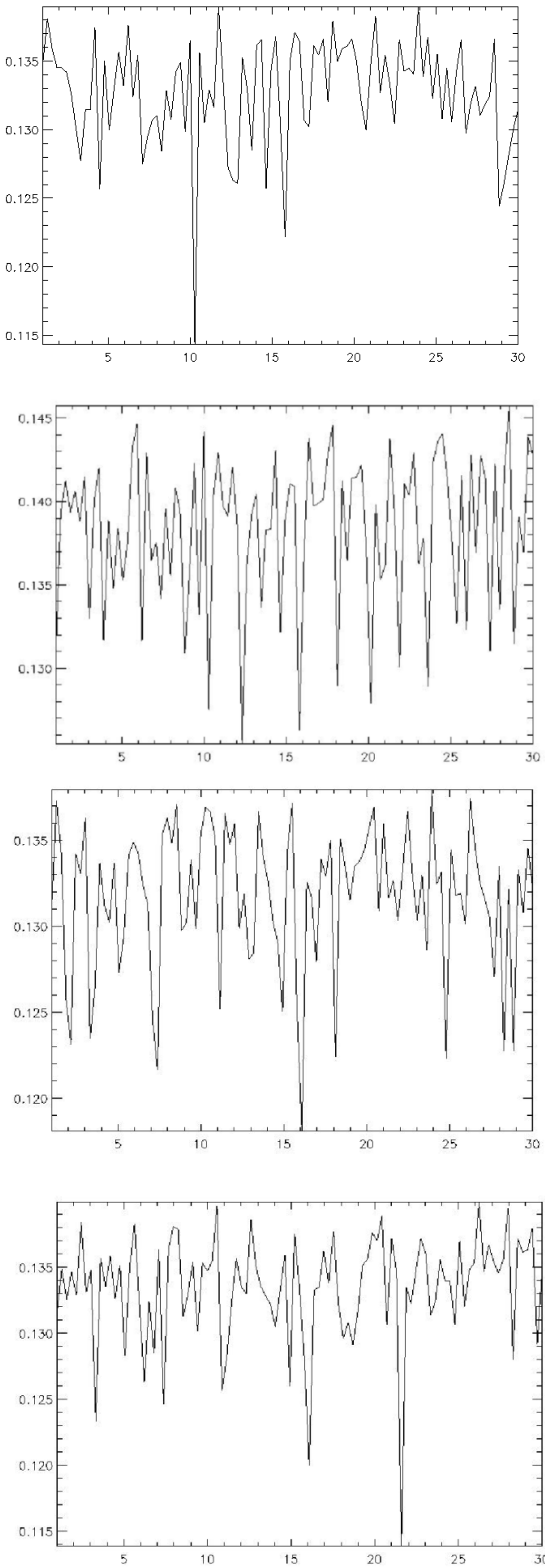

Figure 2. Period plots for observing geometries: $\beta = 45°$, $\lambda = 45°$; $\beta = 45°$, $\lambda = 135°$; $\beta = 135°$, $\lambda = 45°$; $\beta = 135°$, $\lambda = 135°$

striking similarities in the overall shape. For each pole, the only difference in the shapes with 15 or 28 parameters is, that the 28 parameter shape has a bit sharper edges, making it somewhat unrealistic. An interesting fact is, that the pole solution obtained with this shape, is more accurate than the one obtained with the smoother shape, though the fit to the data was ‘too’ good, *rms* being less than the error of the data.

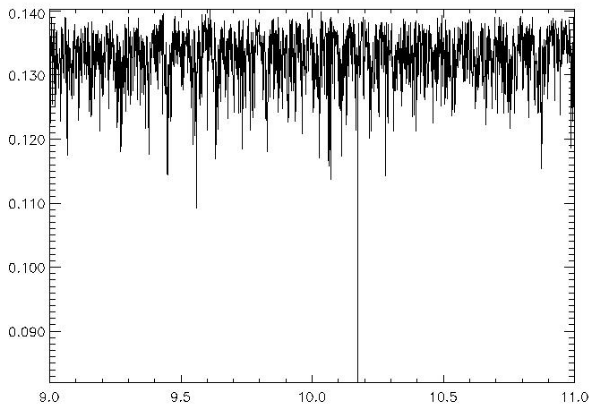

Figure 3. Plot of rms vs. period for period range 9 to 11 hours.

4. NON-CONVEX INVERSION

The spherical harmonics technique (Muinonen & Torppa, in preparation) divides into three main parts. First, as a starting point, it is convenient to use a preliminary shape model, which can be an octant ellipsoid (Cellino et al. 1989), low-degree spherical harmonics series solution (above), or a convex polyhedron (above), the second one of which is also used in the determination of the rotational parameters. Second, in the vicinity of the initial spin and shape solution, incorporating higher-degree spherical harmonics, the parameter distributions are mapped using a systematic study through the spin parameters. For a finite number of such parameters, best-fit spherical harmonics shape solutions are obtained using the Levenberg-Marquardt nonlinear least-squares procedure or the Nelder-Mead downhill simplex method. The resulting shape solutions characterize the regime of solutions in the phase space of the spin and shape parameters. Third, sample spin and shape solutions are generated via Monte Carlo sampling with the help of the spin and shape grid obtained in the second phase. A trial solution qualifies for a sample solution if and only if it produces an acceptable fit to the observational data.

In the inverse spherical-harmonics technique, it is crucial to regularize the possible outcomes of the inversion. The regularization is here carried out using the correlation function of Gaussian asteroid models derived by Muinonen & Lagerros (1998): during the inversion, the spherical-harmonics shape parameters are allowed to vary in the proximity of solutions suggested by the Gaussian modelling. It is worth noting that the regularization procedure does not prevent us from obtaining excellent fits to the data at hand.

The range of variability of the resulting sample shapes tells us about the information content of the asteroid's photometric data. If, on one hand, the sample shapes vary at large scales, the data are not informative enough for accurately pinpointing the shape. On the other hand, if the shapes turn out to be similar, it encourages the idea that the errors are not large.

The first results for simulated Gaia data are shown in Figure 2. They are in agreement with the results from convex inversion above: the overall shape is well determined with the Gaia data, whereas local hills and valleys suffer from large uncertainties.

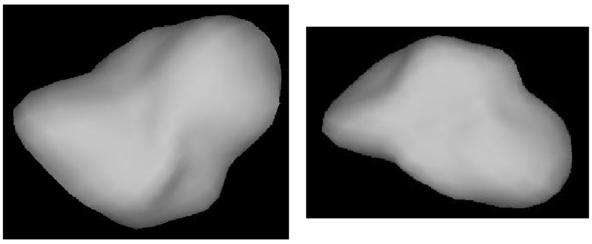

Figure 4. Original shape viewed from two directions.

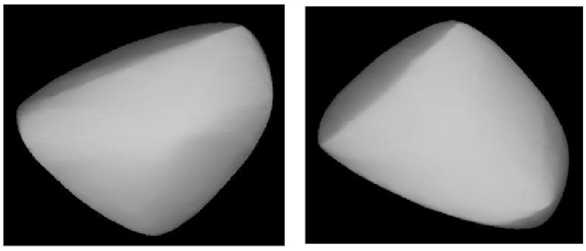

Figure 5. Convex shape solution viewed from two directions.

5. CONCLUSIONS AND FUTURE WORK

The main outcome of this study is that we should use the future Gaia data for analysis of asteroids' physical characteristics such as spin state and shape. Gaia will provide us with enough information to derive periods of high accuracy for hundreds of thousands of asteroids, and to restrict the possible pole solutions to only a few, most often two, since the longitude of the pole may be hard to be defined uniquely using sparse data, and two mirror solutions fit the data equally well. Having the preliminary pole solutions for the Gaia target asteroids, we can later, with a small observing effort, obtain a more accurate unique solution for desired objects. A unique convex shape is connected with each pole solution, but a larger number of non-convex shape solutions is always obtained. The variety of these give us an idea of the maximum and minimum scale of non-convex features at the concave areas.

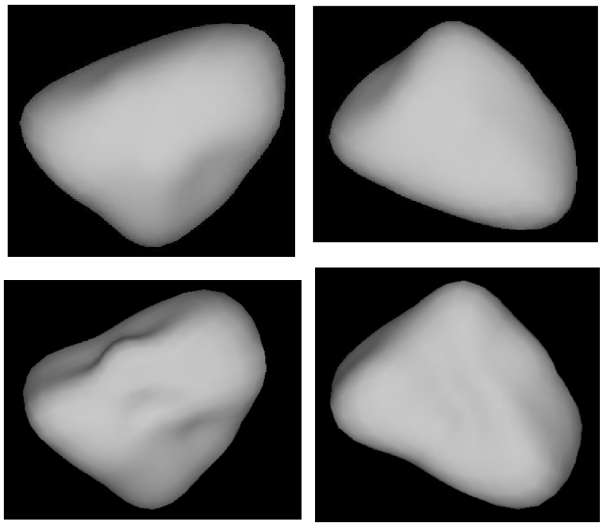

Figure 6. Two non-convex shape solutions viewed from two directions.

In the near future, we will complete the development of the spherical harmonics technique. We will also participate in continuing studies about the potential of Gaia data (Cellino et al. 2005).

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