GAIA-ASSISTED ON-BOARD DETECTION OF MOVING OBJECTS

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ABSTRACT

Objects that are moving relative to the stars (e.g., NEOs) are potentially detectable in the Gaia field of view. This work investigates, from simulated data, how this detection can be performed in the astrometric field of view on the ground. The objective of this work is twofold:

- to provide a 'flag' (Boolean) for each astrometric field crossing of each object, indicating whether or not an object is moving.
- to provide a first estimate of the velocity of the object at the time of the field crossing.

This information is determined from data available on-ground from the crossing of one astrometric field of view. The work includes a 'miniature comparative survey' of an array of applicable statistical methods from the fields of regression and trend testing, and an assessment of the velocity determination as a function of the magnitude and the kind of Solar System object, tested on a population of Main Belt asteroids, and on a simulated NEO population (Bottke et al. 2002). Due to the limited across-scan accuracy owing to the binning, the paper deals solely with along-scan data.

Key words: Gaia; Motion detection; Velocity estimation; Trend testing; NEO; Asteroid.

1. MOTION DETECTION

Figure 1 shows, as a function of actual object velocity, the probability that the object is flagged as stationary. The 'ideal' curve thus is a δ-function at $v = 0$. Four separate methods are compared. For a stationary object ($v = 0\sigma$), all methods have had their thresholds adjusted such that 91.7% of the objects are correctly categorized as stationary objects. As an example, looking at $v = 0.25\sigma$, i.e., a velocity of one fourth of the measurement error, the 'Chi squared' method erroneously characterizes the object in question as stationary in about 22% of the measurement series, whereas the 'Mann-Kendall' method makes this mistake in about 36% of the series. For velocities larger than one third of the measurement error, the 'Chi squared' method correctly flags a moving object in 95% of the cases.

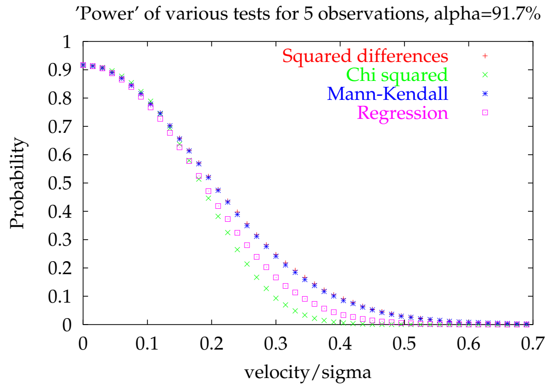

Figure 1. Probability, as a function of actual object velocity, that an object is flagged as stationary. One field crossing, five measurements.

Sets of twelve measurements (ASM and AF1 to AF11) yield a different result, as shown in Figure 2. Here, the 'regression' method appears the best, offering 95% correct decisions, even for velocities slightly less than one tenth of the measurement error, whereas the 'Chi squared' method yields a less admirable performance.

In Figures 1 and 2, the horizontal axes have been normalized by dividing the actual object velocity (in mas s^{-1}) by the standard deviation of simulated measurement error (from the centroiding), σ, in mas. The unit of the horizontal axes is thus 'sigmas per second', which makes the plots independent of the actual value of σ.

2. THE APPLIED TESTS

2.1. Successive Squared Differences

For a series of n values $x_1, x_2, \ldots, x_n$ taken from a Gaussian (or normal) distribution, the test statistic is calculated

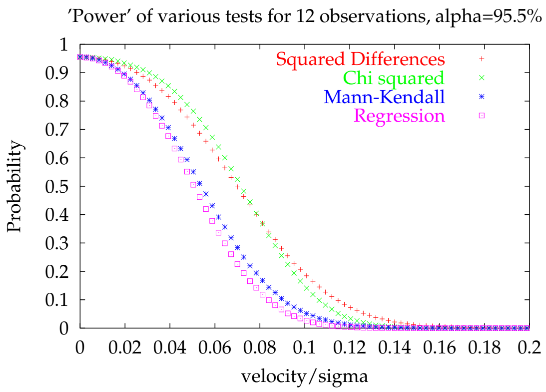

Figure 2. Probability, as a function of actual object velocity, that an object is flagged as stationary. One field crossing, twelve measurements.

as follows (see Aïvazian (1978)):

$$\gamma_{\mathrm{SD}}(n) = \frac{1}{2(n-1)\bar{s}^2(n)} \sum_{i=1}^{n-1} (x_{i+1} - x_i)^2 \qquad (1)$$

where $\bar{s}^2$ is the unbiased sample variance estimate.

For a series of completely random observations (i.e., the case of zero velocity), we expect $\gamma_{\mathrm{SD}}(n) = 1$. In case of a trend in the position measurements, i.e., in the case of non-zero velocity, we expect $\gamma_{\mathrm{SD}}(n) < 1$.

2.2. Mann-Kendall

Under the null hypothesis, i.e., when the observations are independent and identically distributed, the sign (plus or minus) of the difference between any pair of observations is expected to be completely random. This observation forms the basis of the Mann-Kendall test (Mann 1945). A skewed sign census indicates a trend in the series of measurements. The test statistic of this test is the sum of the *sign*-function taken over all measurement differences.

$$\gamma_{\mathrm{MK}}(n) = \sum_{i=2}^{n} \sum_{j=1}^{i-1} sign(x_i - x_j) \qquad (2)$$

The fact that the test statistic is an integer imposes a limitation on the freedom to choose detection thresholds, cf., the seemingly arbitrary thresholds of 91.7% and 95.5% in Figures 1 and 2, respectively.

2.3. Chi Squared (χ^2)

This test is based on the ratio of the empirical variance, $\bar{s}^2$, to the expected variance, i.e., the expected position standard deviation squared (denoted by σ^2_{AL}):

$$\gamma_{\chi^2} = \frac{\bar{s}^2}{\sigma^2_{AL}} \qquad (3)$$

If the observed variance (the empirical variance) can be explained solely by the expected variance (which is a function of object brightness), no trend is observed. Conversely, the case of the observed variance exceeding the expected variance can be explained by a trend in the series of measurements. Under the null hypothesis, i.e., when the observed variance is fully explained by the expected variance, σ^2_{AL}, it holds that

$$(n-1)\gamma_{\chi^2} = \frac{(n-1)\bar{s}^2}{\sigma^2_{AL}} = \frac{n-1}{\sigma^2_{AL}} \sum_{i=1}^{n} (x_i - \bar{x})^2 \qquad (4)$$

is distributed according to a $\chi^2(n-1)$ law. Thus, testing for a trend is done by performing a one-sided test of the above statistic against a $\chi^2(n-1)$ distribution.

2.4. Tests Based on Regression/Data Fitting

This method is based on performing a best fit of the observed data onto a straight line, and subsequently testing the significance of the linear coefficient. This of course assumes that the alternative to the null hypothesis ('no motion') is a linear motion, and not, e.g., a quadratic motion.

The linear coefficient is estimated as follows:

$$b_{\mathrm{est}} = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \qquad (5)$$

Using this, the test statistic may be written as

$$\gamma_R = \frac{b_{est}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2 - b^2_{\mathrm{est}} \sum_{i=1}^{n}(t_i-\bar{t})^2}{(n-2)\sum_{i=1}^{n}(t_i-\bar{t})^2}}} \qquad (6)$$

Under the null hypothesis, γ_R can be shown to be distributed according to a Student's t-distribution with $n-2$ degrees of freedom.

We assume the measurements can be described by a straight line, i.e., that the velocity vector of the observed object is constant during the field transit (which lasts less than one minute). Making this assumption, the null hypothesis, i.e., the case of a zero slope, is tested for by performing a two-sided test in a $t(n-2)$ distribution.

3. ERRORS IN THE VELOCITY ESTIMATION

An estimate of the object's instantaneous velocity in the along-scan direction is derived as a step inherent in the regression-based approach, i.e. the computation of the linear coefficient estimate b_{est} in Equation 5. Therefore, it is only natural to explore the quality of b_{est} as a velocity estimator. Deviating slightly from motion detection as such, the rest of this paper deals with the velocity estimation arising naturally from the regression-based motion detection method earlier.

Simulating Gaia's observations of Main Belt asteroids and a simulated population of NEOs for the full mission duration and estimating the along-scan velocity for each field crossing, we compare the velocity estimate with the actual (simulated) object velocity and plot the corresponding velocity estimate errors as a function of actual velocity. This can be seen in Figures 3 and 4.

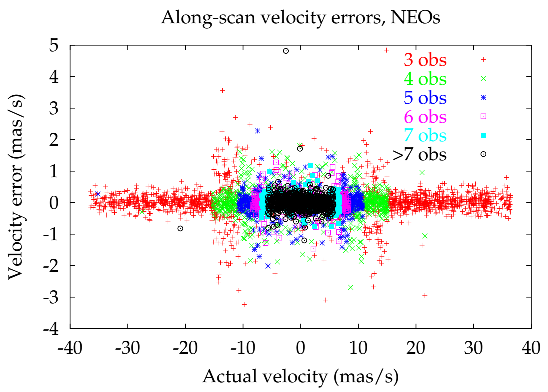

Figure 3. Errors in NEO velocity estimation, based on simulated observations.

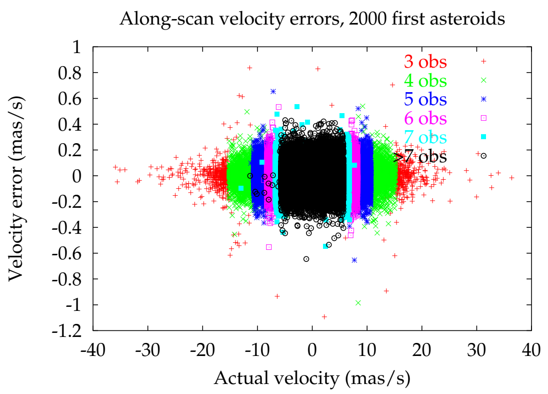

Figure 4. Errors in velocity estimation for the first 2000 Main Belt asteroids.

The accuracy of the velocity estimate from one field crossing (the RMS of the errors) of the simulated NEO population and that of the first 2000 Main Belt asteroids are:

$$\begin{aligned} \sigma_{V,\mathrm{NEO}} &= 0.6 \text{ mas s}^{-1} \\ \sigma_{V,\mathrm{MB}} &= 0.08 \text{ mas s}^{-1} \end{aligned}$$

This predicts that the velocity of about 95% of the NEOs and Main Belt asteroids can be determined with an absolute error of less than 1.2 mas s^{-1} and 0.16 mas s^{-1}, respectively, based on measurements from a single astrometric field crossing.

Furthermore, these simulations indicate that the relative error on the velocity estimation based on a single crossing is less than 10% for about 80% of the simulated NEO population. For 95% of this population, the relative error on the velocity is less than 30%. As to the velocity error of the 2000 Main Belt asteroids, 95% of these have a relative error better than 10%, and 80% better than 2.5%.

4. SUMMARY

Four different motion detection methods, based on statistical testing and regression, were employed and subsequently compared in their abilities to detect motion in noisy data arising from a single astrometric field crossing. The relative performance of each of the four methods varied depending on the number of measurements (the number of CCDs in which the object was observed).

When accepting that 8.3% of the stationary objects are erroneously characterized as moving, based on five measurements, the best method correctly characterizes an object as moving in 95% of the cases when the actual velocity of the object is greater than one third of the measurement error.

For twelve measurements (ASM, and AF1-AF11), accepting 4.5% erroneous characterizations of stationary objects, the similar 95% level velocity threshold is at slightly less than one tenth of the measurement error.

Working on a simulated population of NEOs and on the first 2000 Main Belt asteroids, respectively, an investigation of the error in the velocity estimation, derived from fitting measurements from one astrometric field crossing onto a straight line, shows that the standard deviation is $\sigma_{V,\mathrm{NEO}} = 0.6$ mas s^{-1} and $\sigma_{V,\mathrm{MB}} = 0.08$ mas s^{-1}, respectively.

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