OPTICAL SYSTEM FOR JASMINE AND CCD CENTROIDING EXPERIMENT

T. Yano[1], N. Gouda[1], Y. Kobayashi[1], T. Tsujimoto[1], T. Nakajima[1], H. Hanada[3], Y. Yamada[2], H. Araki[3], S. Tazawa[3], K. Asari[3], S. Tsuruta[3], N. Kawano[1], N. Takato[4]

[1]National Astronomical Observatory of Japan, Mitaka, Tokyo 181-8588, Japan
[2]Graduate School of Science, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan
[3]National Astronomical Observatory of Japan, Mizusawa, Iwate 023-0861, Japan
[4]Subaru Telescope, National Astronomical Observatory of Japan, 650 Noth A'ohoku Place, Hilo, HI 96720, Japan

ABSTRACT

We have investigated the optical design for the Japanese astrometry satellite mission (JASMINE). In order to accomplish measurements of astrometric parameters with high accuracy, optics with a long focal length and a wide focal plane for astrometry is required. In 1977, Korsch proposed a three mirror system with a long focal length and a wide focal plane. The Korsch system is one of the convincing models. However, the centre of the field is totally vignetted because of the fold mirror. Therefore we consider the improved Korsch system in which the centre of the field is not vignetted. Finally we obtain the diffraction limited optical design with small distortion.

Our project needs a common astrometric technique to obtain precise positions of star images on solid state detectors to accomplish the objectives. In order to determine the centres of stars, an image of the point source must be focused onto the CCD array with a spread of a few pixels. The distribution of photons (photoelectrons) over a set of pixels enables us to estimate positions of stars with subpixel accuracy. We modify the algorithm to estimate the real positions of stars from the photon weighted mean, which is originally developed by the FAME (Full-Sky Astrometric Mapping Explorer) group. Finally, we obtain the results from the experiment that the accuracy of estimation of distance between two stars is about a variance of 1/300 pixel, that is, the error for one measurement is about 1/300 pixel, which is almost an ideal result given by Poisson noise of photons. We also investigate the accuracy of estimation of positions with a different size of PSF. In this case also, we obtain that the accuracy of estimation is about a variance of 1/300 pixel.

Key words: JASMINE; Optics; Astrometry.

1. OPTICS FOR JASMINE

We have investigated the optical design for the Japanese astrometry satellite mission (JASMINE). JASMINE will measure parallaxes, positions and proper motions of stars in our Galaxy with the precision of 10 microarcsec in order to study the fundamental structure and evolution of the disc and the bulge components of the Milky Way Galaxy. In order to accomplish such measurements with high accuracy, optics with a long focal length and a wide focal plane is required. Numerous modern telescope objectives are of Ritchy–Chretien form, e.g., the Hubble space telescope, Subaru, and so on. Ritchey–Chretien is a two mirror system, and both mirrors are hyperboloids. This system is corrected for spherical aberration and coma, leaving astigmatism and strong field curvature uncorrected. This strong field curvature is hard to correct in such optics. In 1977, Korsch proposed a three mirror system with a long focal length and a wide focal plane. The Korsch system is one of the convincing models. However, the centre of the field is totally vignetted because of the fold mirror. Therefore we consider the improved Korsch system in which the centre of the field is not vignetted. The schematic of the optics is shown in Figure 1.

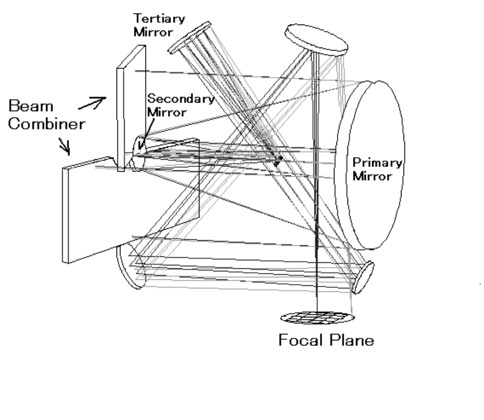

Figure 1. A schematic of the JASMINE optics.

The aperture size of the optics is 1.5 m, and its focal length, f,is 50 m in order to accomplish $f\lambda/Dw = 2$, where λ, D, and w is the wavelength, the aperture size, and pixel size, respectively. The size of the detector for z-band is 6 cm $\times$ 3 cm with 4096 $\times$ 2048 pixels. The pixel size is 15 μm which corresponds to 61.9 milliarcsec

Table 1. Summary of the instrument parameters.

Optics design	Korsch System (3mirrors)
Aperture size	1.5 m
Focal length	50 m
pixel size	15 μm
pixel on sky	61.9 mas
Array size	6 cm $\times$ 3 cm
Pixels per detector	4096 $\times$ 2048
Number of detectors	98 (7 $\times$ 14)
Basic Angle	99.5 $^\circ$

(mas) on the sky. These parameters are summarized in Table 1. We also show the summary of the surface data in JASMINE optics in Table 2.

The image quality of the field of view has been analyzed. The spot diagram is shown in Figure 2. This result shows that the field of view with diffraction limited image is achieved. Furthermore the field distortion is also investigated. The value of distortion is 0.02% in the maximum. This is small enough for our astrometry mission.

Finally we obtain the diffraction limited optical design with small distortion.

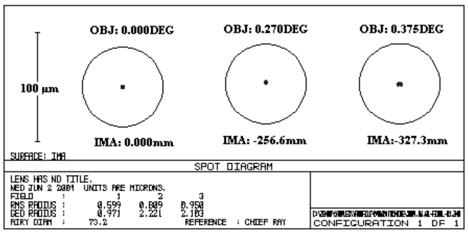

Figure 2. Spot diagram for JASMINE optics.

2. CCD CENTROIDING EXPERIMENT

Measuring the centroiding of stars is one of the most important problems for astrometry. We examined the accuracy of the centroid of stars (Yano et al. 2004). We obtain the relative distance of stars by an accuracy of 1/300 pixel, that is almost the ideal one. Our experimental method is shown below.

2.1. ALGORITHM

In order to estimate the precise distance of two point sources in image frames to sub-pixel accuracy, the following algorithm is proposed. Here, we show the algorithm used in this experiment. Before the analysis, each image frame is bias subtracted and flat fielded. First of all, we pick up two stars to measure the distance. Next we seek the pixel in which a number of photons is maximum in each star. Then we pick up a square subset of 5 $\times$ 5

Table 2. Surface data summary. The unit of the value is mm. In the 'Glass' column, 'M' means a mirror, and numerical value means a conic constant of the mirror. Surf, Thckns, Diamtr, OBJ, STO, IMA, STND, and CBRK in this table mean surface, thickness, diameter, object surface, stop surface, image surface, standard surface, and coordinate break, respectively.

Surf	Type	Radius	Thckns	Glass	Diamtr
OBJ	STND	Inf	Inf		0
1	STND	Inf	2325		1530.4
STO	STND	-5400	-2175	-0.98	1500.6
3	STND	-1750	1312	-5.17	322.9
4	CBRK	-	0		-
5	STND	Inf	0	M	107.0
6	CBRK	-	-1350		-
7	STND	2379	1350	-0.71	450.1
8	STND	Inf	1275		302.0
9	CBRK	-	0		-
10	STND	Inf	0	M	318.6
11	CBRK	-	-2175		-
12	CBRK	-	0		-
13	STND	Inf	0	M	438.1
14	CBRK	-	2725		-
15	CBRK	-	0		-
16	STND	Inf	0	M	568.9
17	CBRK	-	-2475		-
IMA	STND	$-14\,003$			654.6

pixels around the peak pixel of each star image. Accordingly, the number of photons is the maximum value at the centre of pixels in both two stars. Only the pixel values of the two subsets are used to measure the distance of the two stars. We calculate the photon weighted mean of each star by the following equation:

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \frac{1}{\sum_i \sum_j N_{ij}} \begin{pmatrix} \sum_i \sum_j N_{ij} i \\ \sum_i \sum_j N_{ij} j \end{pmatrix},$$

where N_{ij} is the number of photons at the position (i,j). The photon weighted means (x_c, y_c) derived by the above equation are different from the real positions (x_a, y_a). Here, we assume that the difference between the photon weighted mean and the real position is proportional to the deviation of the photon weighted mean from the centre of the pixel:

$$x_a - x_c = kx_c, \quad (1)$$

where k is a coefficient for the correction of the position of a star. This assumption was originally adopted by the FAME group for their laboratory experiment (Triebes et al. 2000). However our treatment of the coefficient k is different from that by the FAME group. Triebes et al. regarded k as a single parameter across the image frame, but we allow k to be specific to each star. This way, we take into account the variation of the shape of the PSF. We calculate parameters, k, by using the least squares method. Then we obtain the real position x_a from the estimated parameter, k. The algorithm, used in this experiment, is very useful. The reason is as follows. First, it is easy to calculate the photon weighted mean from the data. Second, we need not assume the shape of the PSF.

We note that the shape of the PSF is assumed from the estimated parameter, k, implicitly.

Below we show the above algorithm explicitly. We define the positions of star1 and star2 as x_{a1} and x_{a2}, respectively.

$$x_{a1} = x_{c1} + k_1 x_{c1} \tag{2}$$
$$x_{a2} = x_{c2} + k_2 x_{c2} \tag{3}$$

where x_c is the photon weighting mean of a star.

Here we define a function I as

$$\begin{aligned} I &= 0\ (x_{c2} > x_{c1}) \\ I &= 1\ (x_{c2} < x_{c1}). \end{aligned} \tag{4}$$

The relative distance of the two stars $|\delta x_a|$ is

$$\begin{aligned} |\delta x_a| &= x_{a2} - x_{a1} + I \\ &= x_{c2} - x_{c1} + k_2 x_{c2} - k_1 x_{c1} + I \\ &= (1 + k_2)(x_{c2} - x_{c1}) + (k_2 - k_1) x_{c1} + I \\ &\equiv \alpha\Delta + \beta x_{c1} + I \end{aligned} \tag{5}$$

where $\alpha = (1+k_2)$, $\beta = (k_2 - k_1)$, and $\Delta = (x_{c2} - x_{c1})$.

We wish to derive the values of the parameters k_1 and k_2, with which the above equation is satisfied with the smallest error. In other words, we use a least squares method. So, we define S as

$$S = \sum (\alpha\Delta + \beta x_{c1} + \gamma + I)^2, \tag{6}$$

where $\gamma = -|\delta x_a|$. The derivative of S by each parameter is equal to zero, i.e., $\frac{\partial S}{\partial \alpha} = 0$, $\frac{\partial S}{\partial \beta} = 0$, and $\frac{\partial S}{\partial \gamma} = 0$. Then the following relations are satisfied.

$$\begin{aligned} &\alpha \sum \Delta^2 + \beta \sum \Delta x_{c1} + \gamma \sum \Delta + \sum \Delta I = 0, \\ &\alpha \sum \Delta x_{c1} + \beta \sum x_{c1}^2 + \gamma \sum x_{c1} + \sum x_{c1} I = 0, \\ &\alpha \sum \Delta + \beta \sum x_{c1} + \gamma \sum 1 + \sum I = 0. \end{aligned} \tag{7}$$

From the above relations, we obtain the positions of two stars, x_{a1} and x_{a2} from the estimated parameters. Finally, we note again that the above least squares method is easy to calculate without explicit assumption of the PSF. Furthermore, it is an advantage for calculating thousands of stars because of its simplicity.

2.2. EXPERIMENTAL RESULTS

We have taken twenty image frames by sliding the CCD array. The interval of sliding is 1μm, that is, twenty steps correspond to 1 pixel. From these twenty image frames, we estimate the distance of two stars, using the algorithm shown in the previous section. An image of the point spread function (PSF) of a star is focused onto the CCD array with a spread of about three pixels, using a lens with focal length of 200 mm. The distance between the image field and the lens is about 570 mm, and the length between the lens and the CCD camera is about 310 mm. For a light source of the simulated stars, white light is used. The results for our experiment are shown in Figure 3. The abscissa indicates the photon-weighted mean of star1. The ordinate indicates the distance between two stars. Here, we note that the integer part of the distance is eliminated. Accordingly, the distance has a value between 0 and 1. The squares show only the separations between photon-weighted means of two stars, that is, no correction is performed. On the other hand, the diamonds are the estimated distances between two stars by using the algorithm. Distances of two stars with no correction, that is, difference of photon-weighted means of two stars, are distributed to two groups, one is a value around 0.24, and the other, around 0.68. On the other hand, estimated distances using the linear correction, all the measurements are the values of around 0.43. Then the accuracy of estimation becomes exceedingly high. These estimated distances (the diamonds) are shown again in Figure 4. As we see from Figure 4, the variance of the estimated distances of two stars is about 1/300 pixel, that is, the error of the estimation is 1/300 pixel for one measurement, which is almost the ideal one given by the Poisson noise of photons.

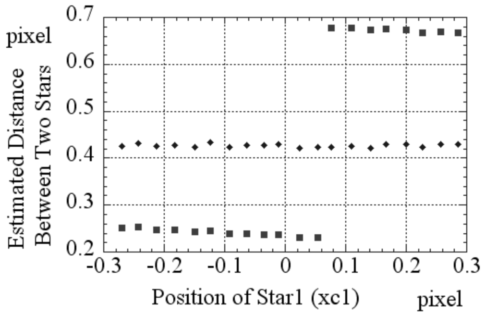

Figure 3. Relative distance between two stars against the photon weighted mean of star1. The squares indicate only the distances between photon weighted means of two stars, that is, no correction is performed. On the other hand, the diamonds indicate the estimated distances by linear correction of the photon weighted mean described in Section 3. In this experiment, $f\lambda/Dw$ is equal to about 3.

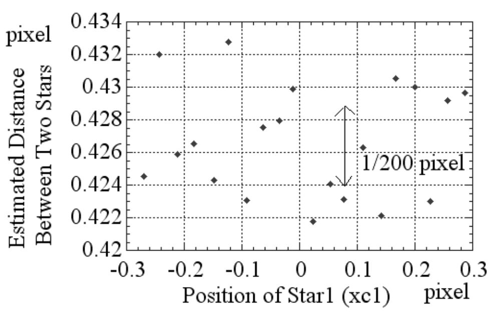

Figure 4. Same with the diamonds in Figure 3.

Next, we investigate the accuracy of estimation with a different size of PSF, using a lens with focal length of 100 mm. The distance between the image field and the lens is about 730 mm, and the length between the lens and the CCD camera is about 110 mm. In this case, an image size of the PSF is about 1 pixel. The results are shown in Figure 5. The squares show only the separations between photon-weighted means of two stars. On the other hand, the diamonds are the estimated distances between two stars, which are shown again in Figure 6. In this case also, the accuracy of estimation is about 1/300 pixel.

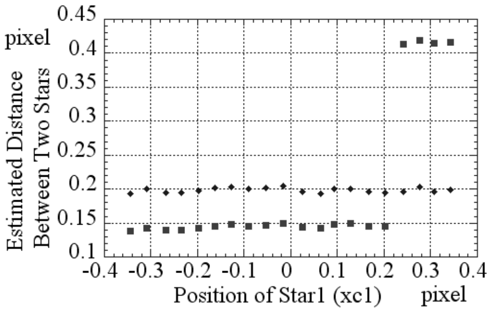

Figure 5. Same as Figure 3, but $f\lambda/Dw = 1$.

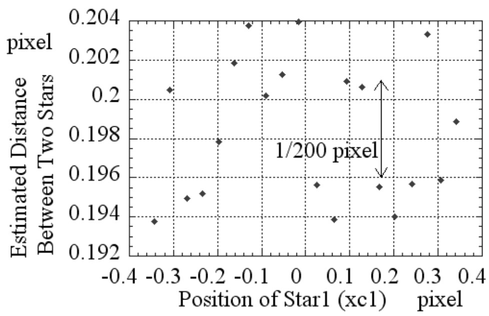

Figure 6. Same with the diamonds in Figure 5.

Here we consider the reason that the photon weighted mean differences between star1 and star2 are discontinuous. We pick up a square subset around the peak pixel of the star image in this experiment. When the peak pixel moves to the adjoining pixel by sliding the CCD array, the region of a square subset changes. Accordingly, the calculated photon weighted mean jumps discontinuously. The analytical value of this discontinuity is $\frac{k_2}{k_2+1}$. This analytical form shows that the small discontinuity represents the small k_2. The value of the discontinuity in the first case of experiments is about 0.68 – 0.23 $\simeq$ 0.45. In this experiment, the estimated parameters α, β, γ, k_1, and k_2 are 1.815, 0.112, -0.427, 0.703, and 0.815, respectively. Error in the experiment is about 3.3 $\times$ 10^{-3}. The value of k_2 is consistent with the discontinuity of 0.45. In the second case, the discontinuity in Figure 5 is about 0.41 – 0.14 $\simeq$ 0.27. In this case, these parameters, α, β, γ, k_1, and k_2 are 1.363, -1.80×10^{-2}, -0.198, 0.381, and 0.363, respectively. This discontinuity is consistent with the value of k_2. Comparing these two cases, the discontinuity in Figure 3 is larger than that in Figure 5. This is because the value of k_2 in the first case is larger than that in the second case.

For comparison, we estimate the distance of the two stars with the algorithm in which we use a common parameter k for the two stars, that is, $k_1 = k_2$ is satisfied. This algorithm is essentially the same with that in FAME. In this case we obtain that the accuracy of estimation is about 1/100 pixel, that is worse than the above results obtained from our algorithm.

3. CONCLUDING REMARKS

The three mirror optical design for the Japanese astrometry satellite mission, JASMINE, is studied. In order to accomplish measurements of astrometric parameters with high accuracy, optics with a long focal length and a wide focal plane for astrometry is required. Korsch proposed a three mirror system with a long focal length and a wide focal plane in 1977. However, the centre of the field is totally vignetted because of the fold mirror. Therefore we consider the improved Korsch system in which the centre of the field is not vignetted. Finally, we have obtained the diffraction limited optical design with small distortion (0.02%).

We have also experimented with the measurement of centres of star images on a CCD for investigating the accuracy of finding the positions of stars, using the algorithm for estimating the positions of stars from the photon weighted means of stars. Then we obtain the results from the experiment that the accuracy of estimation of distance between two stars is about a variance of 1/300 pixel, that is, the error for one measurement is about 1/300 pixel, which is almost an ideal result given by Poisson noise of photons. We also investigated the accuracy of estimation of positions with a different size of PSF. In this case also, we obtain that the accuracy of estimation is about a variance of 1/300 pixel.

In our experiment, the separation of two stars is measured by using the image of the field on the CCD array. Specifically, the lens system distorts an image of a star field, and then, the estimated distance of stars includes the error by the distortion. Therefore we must correct the distortion in order to estimate the real separations of stars. Developing the algorithm which corrects the distortion of the image by the lens system is needed. This is the future work of this experiment.

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