ASTROMETRIC BINARIES WITH A VARIABLE COMPONENT

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ABSTRACT

We present two models dedicated to unresolved binary systems hosting a photometric variable: the first is the 'VIM' (for Variability Induced Movers) model, which deals with couples of fixed stars. The second consists of the calculation of the astrometric orbit of the so-called 'orbital VIM' systems; in this case, the photometric variability of the system is used to derive the semi-major axis of the orbit of the variable component around the barycentre, instead of the usual photocentric orbit. Additional information about the physical properties of the system are thus obtained.

Key words: Astrometry; Stars: variables: general; Binaries: general.

1. INTRODUCTION

In the course of the reduction of the Hipparcos data, several models were dedicated to various categories of binary stars, but only one concerned unresolved couples of stars including a photometric variable: the so-called VIM model (VIM for Variability Induced Movers) was used to detect double stars with fixed relative positions, but with moving photocentres due to the photometric variability of one component (Wielen 1996). The number of VIMs in the Hipparcos catalogue (ESA 1997) is rather small (Pourbaix et al. 2003). Still, Gaia will observe many more stars than Hipparcos, and it will also be more accurate in astrometry and in photometry. Therefore, a lot of VIMs will be detected in the Gaia mission, and their treatment must be prepared. We present in Section 2 an example of a VIM system which could be observed with Gaia, and we derive its parameters using a software based on Wielen's approach.

In addition to binaries with fixed components, we are preparing a treatment dedicated to orbital astrometric binaries with a variable component, i.e., to binaries with orbital periods sufficiently close to the duration of the mission to require the calculation of the orbital elements. Such systems are not expected to be very numerous, and they were ignored in the preparation of the Hipparcos catalogue. However, we will see hereafter that these 'orbital VIMs' deserve our attention since, in comparison to the classical astrometric binaries, the presence of a variable component is a source of additional information about the masses and luminosities of the components of the system. Without the appropriate model, such an effect would pop up as additional noise.

2. A VIM MODEL FOR GAIA

2.1. The Variability–Induced Movers

As explained above, a VIM is an unresolved binary with fixed components including a photometric variable. When the brightness of the variable changes, the photocentre moves along the radius vector joining both components. This displacement is related to the total magnitude of the system, m_T, and it is added to the motion of the stars due to their common parallax and their common proper motion. According to Wielen's calculation, for each observation, the abscissa of the photocentre along the scanning axis may be written as:

$$\begin{aligned} x = \; & f_{ss}(\Delta\alpha_*(\bar{m_T}), \Delta\delta(\bar{m_T}), \varpi, \mu_{\alpha*}, \mu_\delta, \mu_\tau | \\ & \alpha, \delta, \varphi, t) \; + \; (D_{\alpha*}\cos\varphi - D_\delta \sin\varphi) \\ & \times \left(10^{0.4(m_T - \bar{m_T})} - 1\right) \end{aligned} \quad (1)$$

where f_{ss} is the single-star model. It includes a reference position (α, δ) which must be close to the actual one, the position angle of the scanning axis, φ, and the observation epoch, t. The parameters to be determined are the rectangular coordinates of the photocentre when the magnitude is equal to the average magnitude $\bar{m_T}$, $(\Delta\alpha_*(\bar{m_T}), \Delta\delta(\bar{m_T}))$, the parallax ϖ, and the 3-dimensional proper motion $(\mu_{\alpha*}, \mu_\delta, \mu_\tau)$. The specific VIM parameters are $D_{\alpha*}$ and D_δ. For consistency with Wielen's definition, the rectangular coordinates of the variable component measured from the photocentre defined above are $(-D_{\alpha*}, -D_\delta)$. Since the single-star model is a linear function, the calculation of the 8 parameters of the VIM model consists of solving a system numbering as many linear equations as we get observations.

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The actual position of the variable star could be preferred to the position of the photocentre, which refers to a magnitude arbitrarily chosen. However, Wielen noted that it would be far to be as accurate as the position of the photocentre for magnitude $\bar{m_T}$, since $D_{\alpha*}$ and D_δ are extrapolated from the observed displacement of the photocentre.

2.2. The VIMs: Application to Simulated Data

In order to give an example of the application of the VIM process to Gaia observations, we consider a Cepheid variable with a 7-day period. The instantaneous magnitude is simulated as the radial velocity of a spectroscopic binary with the eccentricity $e = 0.6$ and the periastron longitude $\omega = \pi/2$. The magnitude is varying with a 0.5 mag semi-amplitude around 13 mag. The equatorial coordinates are $\alpha = 15$ h, $\delta = -30\mathrm{deg}$. The distance is 4 kpc. The companion is a 13 mag supergiant, 200 au away. All the astrometric parameters are summarized in Table 1. The proper motion along the τ axis corresponds to a 20 km s^{-1} radial velocity.

Table 1. The VIM solution compared to the input parameters of the simulation in the equatorial system of coordinates. The position refers to the photocentre when the total magnitude is m_T =12.2049 mag

Parameter	Simulation	Solution
$\Delta\alpha_*$ (μas)	12 020	12 012 $\pm$ 16
$\Delta\delta$ (μas)	20 819	20 799 $\pm$ 17
Parallax (μas)	250	233 $\pm$ 17
$\mu_{\alpha*}$ (μas/yr)	500	503 $\pm$ 6
μ_δ (μas/yr)	100	107 $\pm$ 7
μ_τ (μas/yr)	-1.3×10^{-6}	10 $\pm$ 7
$D_{\alpha*}$ (μas)	12 020	12 075 $\pm$ 65
D_δ (μas)	20 819	20 902 $\pm$ 65

The Gaia scanning law was simulated with a simple algorithm, which is quite sufficient for our purpose: we generated sequences of 4 to 6 scans with the same scanning direction. The scans were in pairs separated by 106 minutes of time, and the first scans of two consecutive pairs were spaced out 6 hours apart. The scanning directions of the sequences were randomly generated. The epoch of each sequence was randomly taken in a 32-day window, each window beginning 100 days after the preceding one. Assuming a 5-year mission for the satellite, we get a set of 78 observation epochs.

An error randomly generated with a 40 μas standard deviation was added to the abscissas along the scanning direction. The magnitude of the system was 'measured' with a 0.001 mag accuracy. The positions of the Cepheid star and of the photocentre of the system are plotted in Figure 1.

Our VIM reduction software was applied to the simulated observations, and the resulting parameters are shown in

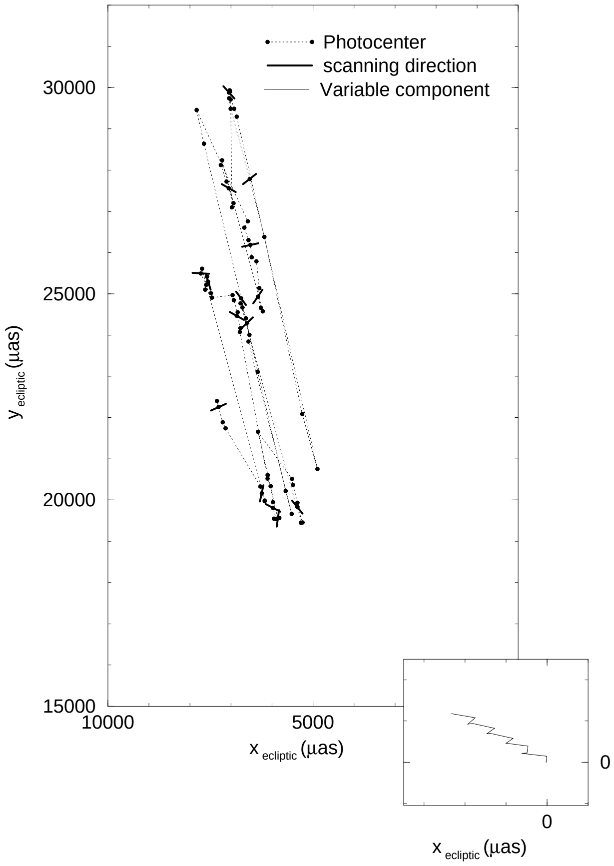

Figure 1. Simulation of a VIM: Gaia astrometric observations of a Cepheid star with a fixed unresolved companion, in ecliptic rectangular coordinates. The measurements are done only along the abscissa axis, whose orientation is indicated for each sequence of scans. The actual position of the Cepheid is on a separate graphic, since it is rather far from the photocentre.

Table 1 for comparison with their input values. The uncertainties were derived assuming the 40 μas error of the abscissas along the scanning axis, but the 0.001 mag error was ignored. The goodness–of–fit of the solution is 0.97.

3. ORBITAL ASTROMETRIC BINARIES WITH A VARIABLE COMPONENT

3.1. The Orbital VIMs: the Computation Method

As mentioned in the introduction, the orbital astrometric binaries with a variable component did not receive a dedicated treatment in the Hipparcos data reduction. However, the excellent accuracy of the Gaia photometric and astrometric measurements justifies a special effort to treat these kinds of binaries, which we call 'orbital VIMs' hereafter.

Even with constant magnitude components, the calculation of an astrometric orbit is much more complicated than the derivation of an astrometric solution for a single star. The reason is that three parameters (the period, P, the eccentricity, e, and the time of the periastron passage, T_0) contribute to the equations of the problem with non-linear terms. Moreover, due to the rather large number of objects which will probably deserve the orbital VIM process in the course of Gaia reduction, a reasonably fast algorithm is required. Pourbaix & Jancart (2003) have proposed a fast computation technique for classical astrometric binaries. They have shown that a rough estimate of the period and of the other elements of a classical astrometric binary - apart from e and T_0 - may be derived assuming a circular orbit. That initial guess makes it possible to derive a definitive solution with a few iterations.

When one component is variable, the problem is a bit more complicated, but it is also more interesting. The semi–major axis of the astrometric orbit of the photocentre, a_0, can be expressed as a function of a_1, the semi–major axis of the orbit of the variable component around the centre of mass of the system:

$$a_0 = a_1 \left[1 - \frac{L_2}{L_1 + L_2} . \frac{\mathcal{M}_1 + \mathcal{M}_2}{\mathcal{M}_2} \right] \qquad (2)$$

In this equation, L_1 and L_2 are the luminosities of the components, and $\mathcal{M}_1$ and $\mathcal{M}_2$ are their masses. A negative a_0 would mean that the photocentre is not between component ‘1’ and the barycentre, but between the barycentre and component ‘2’.

Introducing the mass ratio, $q = \mathcal{M}_2/\mathcal{M}_1$, and the total magnitude of the system at the observation epoch, m_T, Equation 2 becomes:

$$a_0 = a_1 \left(1 - g.10^{0.4 m_T} \right) \qquad (3)$$

where g is:

$$g = 10^{-0.4 m_2} \times (1 + q)/q \qquad (4)$$

Since the magnitude of component ‘2’ is assumed to be constant, g is constant. The abscissa of the photocentre along the scanning axis is:

$$\begin{aligned} x = \quad & f_{ss} \; + \; \left(1 - g.10^{0.4 m_T}\right) \times \; [(\cos E - e) \\ & \times(-A \sin\varphi + B \cos\varphi) + \sqrt{1-e^2}\sin E \\ & \times(-F \sin\varphi + G\cos\varphi)] \end{aligned} \qquad (5)$$

where f_{ss} and φ are the same as in Section 2. A, B, F and G are the Thiele-Innes elements of the orbit of the variable star around the barycentre, and e the eccentricity of the orbit. E is the eccentric anomaly for the observation epoch, which may be calculated when e, P and T_0 are known. Therefore, we deal again with a system of linear equations as soon as g, e, P and T_0 are determined.

An iterative method is used to derive the complete solution:

- The orbit is assumed to be circular, and several values for P and g are tested. P ranges from 10 to 10 000 days, and g is around $10^{-0.4\bar{m}_T}$. The other parameters are computed for every value of P and g, and those giving the smallest χ^2 are temporarily selected.
- Several values for e and T_0 are tested, using P and g found above. The other parameters are calculated for every values of e and T_0, and, again, those giving the smallest χ^2 are temporarily selected.
- The search for P and g is performed again, but assuming e and T_0. The eccentricity and the time of periastron are also improved afterwards.
- After 2 iterations, it is more efficient to search first P and T_0, and next g and e.

The 14 parameters of the system are thus derived. The A B, F, G Thiele–Innes elements are transformed in the Campbell elements: a_1, Ω, ω and i (Binnendijk 1960). It is worth noticing that a_1 may be derived only thanks to the variability of component ‘1’ and that it is an extrapolation for $m_T = -\infty$ of the values of a_0 corresponding to the actual measurements. As a consequence, we expect for a_1 a rather poor accuracy, similar to that of $D_{\alpha *}$ and D_δ when the components are fixed.

3.2. The Orbital VIMs: Application to Simulated Data

In order to give an example of orbital VIM, we consider a F8 V star orbiting a W UMa eclipsing binary. Both components of the W UMa are one-solar-mass stars, and their orbit is oriented edge-on. The light curve is simulated using the hybrid model of Halbwachs (1982). At the maximum of brightness, the magnitude of the W UMa is 12.25. The secondary magnitude is 12.6.

The reference position is the same as in Section 2.2, but the distance of the system is 667 pc, and the period of the wide pair is 1600 days (Figure 2). The model for Gaia observations is the same as above.

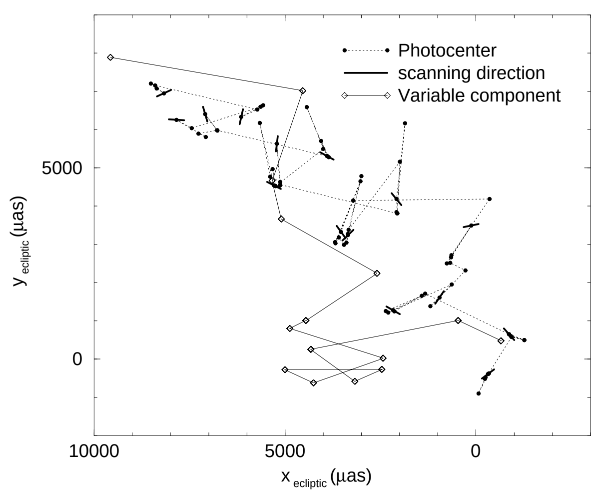

Figure 2. Simulation of an orbital VIM: an astrometric binary with a W UMa component observed with Gaia. The symbols are the same as in Figure 1.

Table 2. An example of orbital VIM: the astrometric solution compared to the input parameters of the simulation. (Radial velocity = 30 km s^{-1})

Parameter	Simulation	Solution
Parallax (μas)	1500	1457
$\mu_{\alpha *}$ (μas/yr)	2000	1957
μ_{δ} (μas/yr)	1000	1003
μ_{τ} (μas/yr)	-7×10^{-5}	25
g (10^{-5})	2.432	2.510
P (d)	1600	1633
e	0.6	0.577
T_0 (d)	80	55
a_1 (μas)	2219	2151
i (deg)	20	25
ω (deg)	130	132
Ω (deg)	60	57

The parameters derived by our software are compared to the input values in Table 2. The agreement is rather satisfactory, although the search for the minimum χ^2 could still be improved.

Combining the parameters in Table 2 with additional data would provide the masses of the components. It comes from the third Kepler law that:

$$\mathcal{M}_1 = \frac{{a_1}^3}{\varpi^3.P^2}.\frac{(1+q)^2}{q^3} \tag{6}$$

where ϖ is the trigonometric parallax of the system. The mass of the variable component, $\mathcal{M}_1$ (and consequently $\mathcal{M}_2$) may be derived from Equation 6 if the mass ratio q is obtained through a spectroscopic analysis. Assuming the actual value $q = 0.6$ in the example above, the mass of the WUMa pair would be $\mathcal{M}_1 = 1.91\mathcal{M}_{\odot}$ instead of $2\mathcal{M}_{\odot}$. Measuring the magnitude of the constant star would also permit the calculation of the masses. In our example, the actual value $m_2 = 12.6$ mag would lead to $q = 0.57$ and $\mathcal{M}_1 = 2.14\mathcal{M}_{\odot}$.

4. CONCLUSION

We have demonstrated with two examples that two processes dedicated to unresolved binaries must be included in the Gaia data reduction. They should be applied to the objects with variable magnitudes, at least when the goodness-of-fit of the single star solution is bad:

- The VIM model would be used to derive the astrometric parameters of very distant objects such as Cepheids. The position angle of the companions would then be calculated.
- The orbital VIM process would provide astrometric and orbital parameters for systems with a variable component. The semi-major axis of the orbit of the variable component around the barycentre would be derived, as well as a function of the mass ratio and of the secondary magnitude. A combination of the Gaia astrometric data with complementary observations would then provide the masses of the components.

Additionally, we note that it could be relevant to search orbital VIMs for all the planetary system candidates found by astrometry around a variable star. A negative result would then confirm that the photocentre of the system exactly coincides with the variable star, and that the small size of the astrometric wobble is not due to a shift of the photocentre induced by the combination of the light of two stars. The false alarms due to double stars with components having nearly similar luminosities would then be discarded.

In the same way, we could wonder why we need two distinct orbital models, whether it hosts a variable component or not. There are computational reasons not to do so and to keep distinct models. As mentioned in Section 2, the basic VIM model is linear and fitting it to the observations is straightforward. Furthermore, there is no guarantee that the most general model would converge to a sound solution. For instance, a basic VIM is an orbital VIM with a very long period, thus leading to a fixed configuration of the components during the whole mission. As shown by Pourbaix (2002), at low signal to noise ratio, fitting the observation with a too general model can lead to very spurious solutions.

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