STATISTICAL METHODS FOR CALIBRATING TRIGONOMETRIC PARALLAXES

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ABSTRACT

We examined statistical methods for calibrating trigonometric parallaxes to retrieve the absolute magnitude of stars, using Monte Carlo simulations. Here we consider the case of the zero-point of the period-luminosity relation for Cepheid variables. The method originally proposed by Ratnatunga & Casertano was revisited by introducing the *realistic* density distribution of sample stars belonging to the catalogue through prior calculations of the photometric distance for each star. It is found that our method gives an unbiased estimate, regardless of any dispersions in their absolute magnitude. We further investigate the reliability of results that depends on an accuracy of parallax. Our finding is that the accuracy ($\sim$1 mas) of Hipparcos parallaxes is not enough to obtain a reliable result due to a large variation among different ensembles of stars. More precise determination of parallaxes by an accuracy of 200 μas at least, which will be easily realized by the ongoing astrometric space satellites, will give a precise zero-point together with a dispersion in absolute magnitude.

Key words: Cepheids; Distance scale; Stars: distances; Hipparcos; Jasmine.

1. INTRODUCTION

The Hipparcos satellite has brought us a new era for the distance determination using the trigonometric parallaxes π of stars. Before Hipparcos, the ground-based observations offered π only for stars within a few tens of pc. We have now reached the stage where we can obtain π for stars with their distances extending to $\sim$1–2 kpc. As a result, the Hipparcos catalogue contains many valuable stars, in which Cepheid variables and RR Lyrae stars which serve as primary distance indicators are included.

It is well known that Cepheids obey the period-luminosity (PL) relation. The zero-point for the PL relation gives a distance modulus $\mu_{\rm LMC}$ to the Large Magellanic Cloud (LMC), which is no doubt an important step for determining the distances to distant galaxies and thus for the Hubble constant (H_0). Instead of the indirect calibrations of the zero-points so far, Hipparcos has enabled us to derive them directly from π of Galactic Cepheids for the first time. However, in fact, direct calibrations have confronted a serious problem which comes from the fact that almost all of π for these stars are measured with very large errors including negative parallaxes. In such cases, the distance d for each star cannot be calculated from $d({\rm pc}) = 1/\pi$. We therefore need to deduce the physical quantities statistically from an ensemble of Hipparcos parallaxes (e.g., Smith 1987, 1988).

To retrieve statistically an *unbiased* estimate is indeed a difficult task (e.g., Smith 2003). A lot of work has been done since Roman (1952) and Jung (1971) (see the review by Arenou & Luri 1999). Essentially two different approaches have been investigated since then, and both were applied to the calibrations with Hipparcos parallaxes. Ratnatunga & Casertano (1991) have proposed a maximum likelihood method (hereafter, ML method) for correcting the notable Lutz-Kelker bias (Lutz & Kelker 1973), allowing full use of low-accuracy and negative parallaxes. Tsujimoto et al. (1998) applied their method to Hipparcos RR Lyraes, and found the absolute magnitude M_V(RR) of RR Lyraes at [Fe/H]$= -1.6$ is 0.59, which corresponds to $\mu_{\rm LMC}$ =18.41 mag with the data of LMC RR Lyraes by Walker (1992), given the slope 0.20 of the M_V(RR)–[Fe/H] relation. Arenou et al. (1995) also investigated extensively the statistical properties of the errors of Hipparcos parallaxes by a similar algorithm to the ML method.

Feast & Catchpole (1997) have proposed another method, the so-called ‘reduced parallax method’ (hereafter, FC method), to estimate the zero-point for Cepheid PL relation from the weighted mean of the formula free from biases such as the Lutz-Kelker one. Using Hipparcos Cepheids, they found 0.2 mag brighter zero-point than the previous value (Laney & Stobie 1994), which results in $\mu_{\rm LMC}$ =18.70 mag. This value gives a upper bound for $\mu_{\rm LMC}$ among numerous determinations of the distance to the LMC (Gibson 2000), and it seems a bit too large (e.g., Freedman et al. 2001). However, Pont (1999) concluded this method to be the most rigorous one, and Lanoix et al. (1999) further confirmed it by Monte Carlo simulations. In any case, even confined to the determinations of $\mu_{\rm LMC}$ from Hipparcos parallaxes, there exists a large uncertainty in $\mu_{\rm LMC}$, which is one of the largest remaining uncertainties in the overall error budget for the determination of H_0.

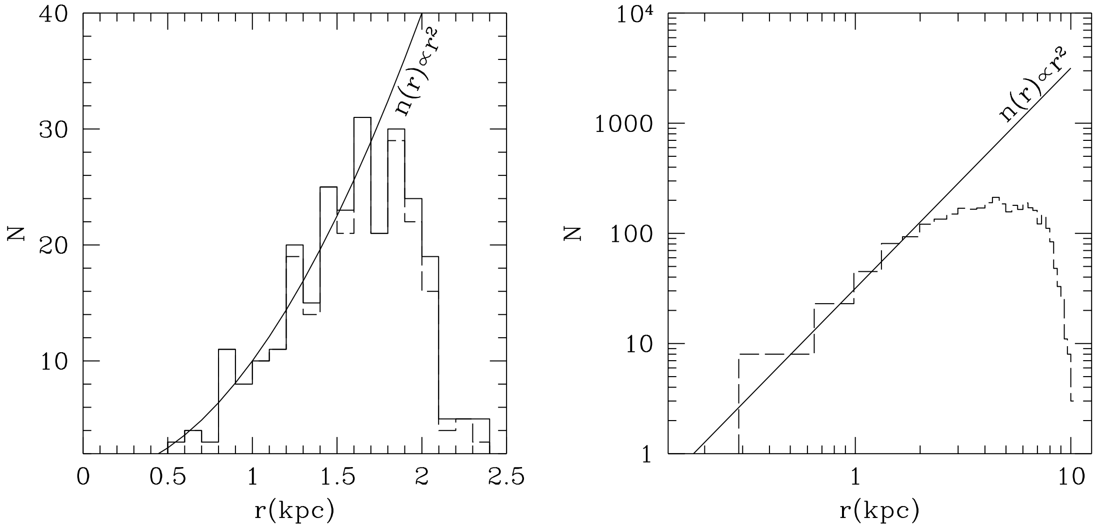

Figure 1. Left panel: The approximate distribution of Hipparcos Cepheids deduced by giving the photometric distance to each star with the help of the PL relation (solid histogram) together with one in a pseudo catalogue that we made (dashed histogram). The solid line denotes a uniform distribution. Right panel: The stellar distribution in a pseudo catalogue with an accuracy of parallax $\sim$ 200μas (dashed histogram) compared with a uniform distribution (solid line).

Here we perform Monte Carlo simulations as done by Lanoix et al. (1999) to analyze the zero-point for the Cepheid PL relation with the ML method. The ML method allows us to incorporate the density distribution of stars into the model arbitrarily. Without knowing the zero-point of the Cepheid luminosity in advance, *relative* approximate distances of individual Cepheids can be deduced from their photometric information assuming an arbitrary zero-point. Then from these relative distances, a realistic density distribution of stars belonging to the catalogue can be obtained. We show that the ML method combined with a density distribution of stars thus obtained leads to an unbiased estimate of the zero-point ρ for Cepheid PL relation, regardless of any values for the dispersion σ_m of the absolute magnitude. In fact, the intrinsic dispersion for Cepheids is estimated to be $\sim$0.2 mag (e.g., Ngeow & Kanbur 2004). However, it could be possibly broadened by a large reddening (Luri et al. 1998).

Furthermore, the dependence of reliability of results on the accuracy of parallaxes is investigated. We will show how not only the absolute magnitude precision but also its dispersion precision will improve according to the parallax accuracy. These results are also compared with those obtained by the FC method. The precise determination of biases and variances involved in the final results by the statistical calibrations is certainly demanded for not only the further study using Hipparcos data but also the future work using the highly-precise astrometric data brought by the ongoing space satellite projects such as Gaia (Perryman 2002) and JASMINE (Japan Astrometry Satellite Mission for INfrared Exploration, Gouda et al. 2003).

2. STELLAR DISTRIBUTION

We make the pseudo catalogues based on the Hipparcos data for Cepheids. We prepare two kinds of pseudo catalogues. One is totally based on the Hipparcos data, which is identical to thesimulated catalogues made by Lanoix et al. (1999). Another is constructed with some changes to be done in order to be compatible with forthcoming catalogues with a high accuracy of parallax determination. Here we follow the way of DIVA, which was an astrometric satellite project once planned to be launched by the German Space Agency.

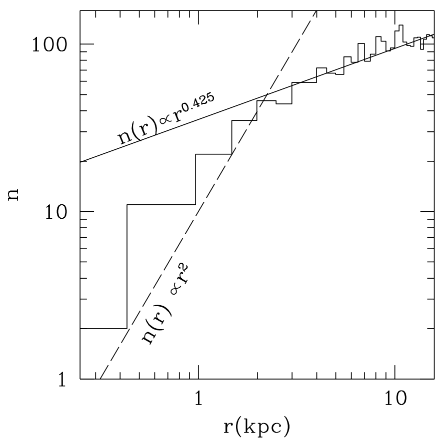

Figure 2. The stellar distribution of one sample in the pseudo catalogues with an accuracy of parallax $\sim$200 μas. The solid line is obtained by the power-law fitting at larger distance. The dashed line denotes a uniform distribution.

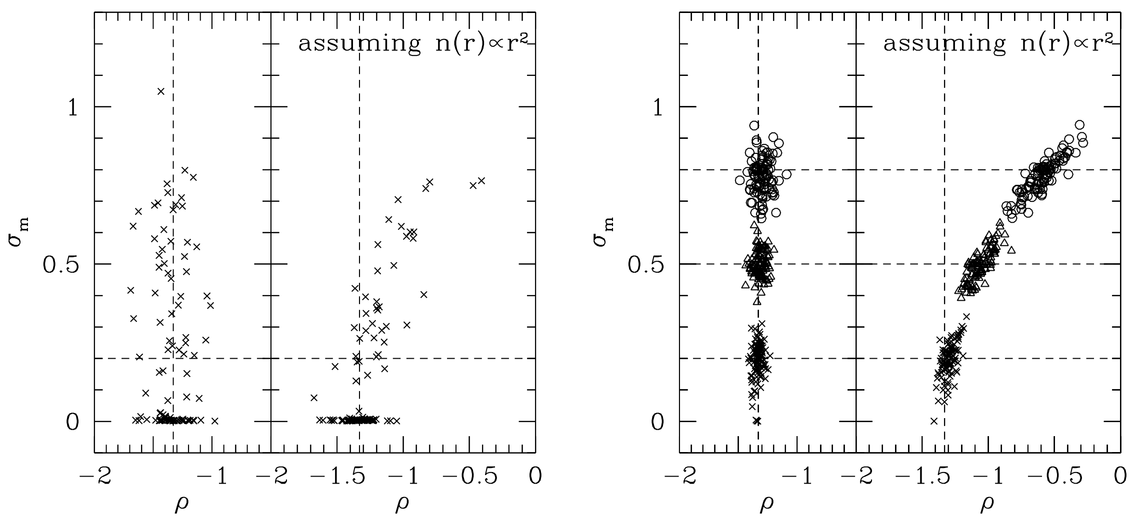

Figure 3. Left panel: The left figure shows a distribution map of one hundred trial results for the case of the accuracy equivalent to Hipparcos parallaxes. The dashed lines correspond to the input values of ρ and σ_m. The figure on the right of this panel shows the results obtained by the original ML method in which a uniform stellar distribution is assumed. Right panel: The distribution maps for one hundred pseudo catalogues with an accuracy of parallax ~200 μas for three cases of σ_m = 0.2, 0.5 and 0.7.

The actual stellar distribution in the catalogue should be deviated from a uniform one mainly due to the Malmquist bias. The approximate distribution of Hipparcos Cepheids can be deduced by giving the photometric distance to each star with the help of the Cepheid PL relation as shown in the left panel of Figure 1 (solid histogram), which is indeed different from the uniform distribution (solid line) at larger distance. According to the derived distribution, we should reconstruct the stellar distribution for the pseudo catalogues. The resultant distributions are shown by dashed histograms in Figure 1 for two kinds of pseudo catalogues. As shown in the left panel of Figure 1, the final distribution is in good accord with the one for Hipparcos Cepheids (solid histogram). It is noted that in the pseudo catalogues with higher accuracies of parallaxes, the deviation from a uniform distribution is significant (the right panel of Figure 1).

According to these results, we should determine the number density $\nu(r)$ in the ML method, which was originally assumed to be constant (Ratnatunga & Casertano 1991). The power-law form of $\nu(r) \propto r^{-\alpha}$ is applied to realize the stellar distribution close to the actual one. We perform the power-law fitting for each catalogue, and obtain different values of the power index. Figure 2 shows one sample with $n \propto r^{0.425}$, which gives $\nu(r) \propto r^{-1.575}$ from the relation $n(r) = \nu(r)r^2 = r^{2-\alpha}$. As in this case, we tried to fit the distribution at larger distances such as $r >$ 2 kpc because most of stars reside in this area.

3. RESULTS

For one hundred pseudo catalogues with an accuracy of parallax corresponding to Hipparcos, the resultant distribution of ρ and σ_m is shown in Figure 3 (in the left figure of the left panel). The dashed lines denote the input values of them, i.e., $\rho = -1.32$ and $\sigma_m = 0.2$. It is found that there is no bias for the returned values of ρ, regardless of the returned values of σ_m, though there exists a large scatter in the values of ρ, ranging over $-1.7 < \rho < -1$. It means that the accuracy (~1 mas) of Hipparcos parallaxes is not enough to obtain a reliable result for the zero-point for Cepheid PL relation. Besides, we cannot expect to obtain the right σ_m. For comparison, the results calculated with the ML method in which a stellar distribution is assumed to be uniform are shown in the right figure on the left panel. For the cases which return higher values of σ_m than the true (input) value, biases are apparently seen in the returned values of ρ. These biases appear in the opposite sense to the Malmquist bias which yields a bias toward a brighter magnitude. It is caused by an overcorrection of the Malmquist bias for the assumed uniform density distribution of stars.

The results of one hundred pseudo catalogues with an accuracy of parallax corresponding to DIVA, i.e., 200 μas are shown on the right panel of Figure 3. Here we perform calculations for three input values of σ_m, i.e., σ_m = 0.2, 0.5, and 0.7 denoted by three dashed lines. Although the values of σ_m = 0.5 and 0.7 are much larger than an intrinsic dispersion of M_V, they could be possible due to the error in the value of reddening. Furthermore it should be remarked that a large value of $\sigma_m \sim 0.7$ is suggested by the statistical calibration of parallaxes and proper motions for Hipparcos Cepheids (Luri et al. 1998). For all cases, our modified ML method gives unbiased estimates of ρ with a small scatter (Figure 3, left figure on the left panel). However this is not the case for the ML method with a assumption of a uniform stellar distribution (Figure 3, right figure on the left panel). The biases become larger according to larger input σ_m.

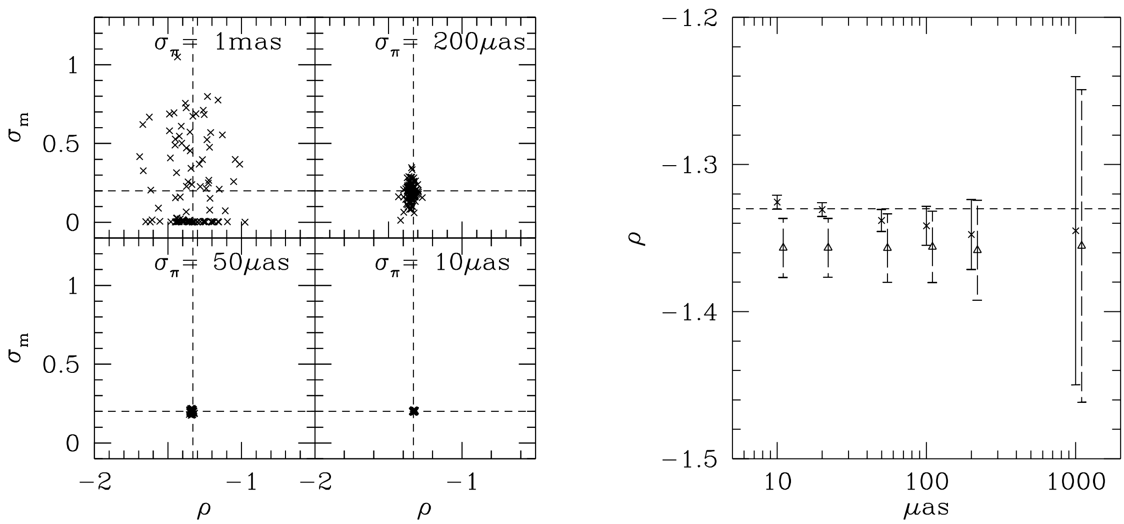

Figure 4. Left panel: Distribution maps of one hundred trial results in the ρ-σ_m diagram for four cases with accuracies of 1 mas, 200 μas, 50 μas, and 10 μas for parallax. Right panel: The returned values of ρ as a function of an accuracy of parallax calculated by two methods, i.e., the modified ML method (crosses with solid bars) and FC method (open triangles with dashed bars).

The left panel of Figure 4 clearly demonstrates that the area of the distribution of (ρ, σ_m) becomes smaller according to higher accuracy of parallaxes. Providing a parallax accuracy of more than $\sim 50\mu$as is gained, we will be able to obtain the precise combination of ρ and σ_m. The right panel of Figure 4 shows the returned values of ρ with 1σ errors as a function of an accuracy of parallax (crosses with solid bars) together with the results obtained by the FC method (open triangles with dashed bars). Interestingly in the case of the FC method, the bias always remains regardless of an accuracy of parallax, though the application of the FC method to the cases with high accuracies such as 10 μas is not appropriate. This bias might be ascribed to an effective intrinsic dispersion $\sigma_{\rm eff}$ of the absolute magnitude. In fact, the FC method gives a reliable value of the zero-point as long as $\sigma_{\rm eff}$ could be reduced to a small value such as 0 – 0.1. It was then proposed by Feast & Catchpole (1997) that such a small $\sigma_{\rm eff}$ could be realized if we use a colour-period relation to estimate a reddening, and combine it with the luminosity-colour-period relation. However, we obtain a relatively large value of $\sigma_{\rm eff} \sim 0.176$.

4. CONCLUSIONS

Here we present a maximum likelihood method in which the *realistic* density distribution of stars to be analyzed is introduced. The prior information on the stellar density is obtained through the calculations of the photometric distance for each star instead of the use of the Galactic model as done by Arenou et al. (1995). Our method gives an unbiased estimate of the absolute magnitude of stars, regardless of any dispersions in their absolute magnitude. However an accuracy of 200 μas at least for parallaxes is required to obtain a reliable result from only one ensemble of stars. If not, as in the case of Hipparcos, we cannot get an assertive conclusion due to a large variation in the estimate. Furthermore, an accuracy more than $\sim 50\mu$as promises to get a precise value of the dispersion in absolute magnitude together with its mean value.

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