GRAVITATIONAL WAVES AS ASTROMETRIC TARGETS

V.V. Makarov

Copenhagen University Observatory
Østervoldgade 3, 1350 Copenhagen K, Denmark

ABSTRACT

It has been recently realized that sources of gravitational waves in the Galaxy are capable of producing astrometric effects on the apparent position of background stars. This effect can be considered as a way to discover and measure gravitational waves, supplementary to the on-ground gravitational detectors, provided one has a very accurate and high-time-resolution astrometric satellite. General requirements for such a satellite are specified for different continuous sources of gravitational waves, such as binary systems and neutron stars, as well as for burst emitters of strong radiation like the collapse of a supermassive star and neutron star–neutron star coalescence.

An astrometric precision of a 'single' observation of 10^{-8} to 10^{-7} arcsec is needed to make the gravitational radiation of known close binaries detectable, and there is no stringent requirements on time resolution of the instrument. On the other hand, for the cataclysmic short-term events mentioned above, a precision of 10^{-6} arcsec would probably do, but the time resolution must be as high as 10^{-4} to 10^{-3} sec. It is concluded that the first possibility seems to be more in the line of the GAIA proposal.

It is noted that the GAIA satellite could observe gravitational radiation of a 100 sec period binary star under some special circumstances. A special off-line treatment of individual GAIA observations over a wide sample of programme stars is proposed as a kind of blind search for periodic sources of gravitational waves.

Key words: GAIA, space astrometry, gravity waves

1. INTRODUCTION

Despite a few decades of effort, detection and measurement of gravitational radiation remains a challenge for experimental physics and observational astrophysics. The reason for this is related to the remoteness of strong sources and inherent weakness of gravitational radiation. Only relativistic short-living events can produce a signal strong enough to be detected on earth. Yet, even this signal should be disentangled from parasitic signals and noise of earth's origin. A few ambitious and rather sophisticated projects of gravitational astronomy, using state-of-the-art technology, are under development at present.

Fakir (1994a, 1994b, 1995) has recently demonstrated that gravitational radiation is capable of producing an astrometric effect on the light of background stars. The effect can be significant when a source of gravity waves (SGW) is situated very close to the line observer–source of light (SL). Then the light from the SL penetrates through a region of relatively strong waves and deflects on the ripples of space-time, to the effect that the apparent position of the SL oscillates with the frequency of the gravitational wave. Fakir has estimated the amplitude of this oscillation in the quadrupole approximation.

The advance of projects of optical interferometry in space, such as GAIA (Lindegren & Perryman, 1994), makes this *astrometric* way of detection and measurement of gravitational waves be not a mere dream. Time resolution and angular resolution are the two principal characteristics which determine the ability of an instrument to detect and measure gravitational waves in this way. In this paper, these characteristics are evaluated for surely existing sources (binary stars), as well as for some more hypothetical objects.

2. BINARY STARS

Binary stars are sources of steady monochromatic gravitational radiation of angular frequency $\Omega = 4\pi/P$, where P is the orbital period. According to Peters & Mathews (1963), the power output due to the radiation, averaged over one period is

$$L_{\rm GW} = \frac{32}{5}\,\frac{\mu^2 M^3}{a^5} L_0, \qquad (1)$$

for a circular orbit, where a is the semimajor axis, M is the total mass of the system, $\mu = m_1 m_2/M$ and $L_0 = 3.63 \times 10^{59}$ erg/sec. Masses and distances are expressed in the geometrized units of [cm]. Fakir (1994) derived an approximate expression for the maximum deflection of light from the SL:

$$|\Delta\phi|_{\max} \approx \frac{3}{2}\,\pi^2 h_{\max}(d = \Lambda), \qquad (2)$$

where $h_{\max}$ is the dimensionless amplitude of gravitational wave at a distance of one wavelength Λ. Combining the above two formulae, we derive

$$|\Delta\phi|_{\max} \approx 1.54 \times 10^6 \sqrt{\frac{L_{\rm GW}}{L_0}}\ \text{arcsec}, \qquad (3)$$

which holds if the impact parameter $b \le \Lambda$. In order to detect the astrometric effect with a signal-to-noise ratio

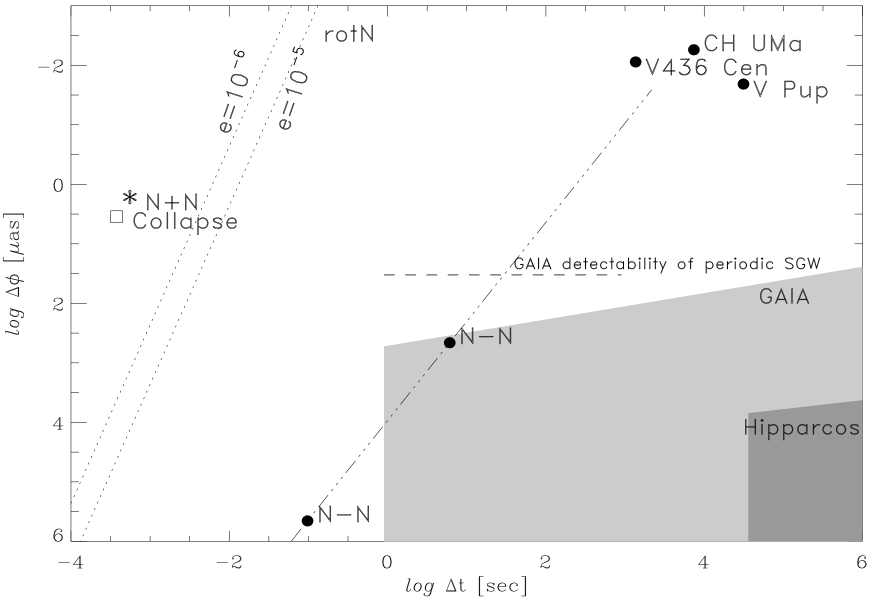

Figure 1: Accuracy of angular measurement $\Delta\phi$ versus time resolution Δt, required to detect astrometric effects of gravitational radiation, emitted by different sources. For comparison, performances of the Hipparcos astrometric satellite for a star of $V = 12$ mag and GAIA interferometer for a star of $V = 15$ mag are shown with shaded areas. Three known binaries are represented in the upper part of the diagram. The dashed line indicates the position of one-solar-mass binary stars of different periods. The horizontal thick dashed line indicates the theoretical level of detectability of periodic SGWs with GAIA for a background star of $V = 15$ mag. N–N denotes neutron star–neutron star binaries. rotN and the two dotted lines show the expected characteristics of single rotating neutron stars of two different degrees of asymmetry. The asterisk, marked with N+N quantifies approximately a neutron star–neutron star coalescence event (see assumed parameters in the text), and the small square indicates the collapse of a supermassive star.

about 1, having only two observations, the required precision of angular measurements must be approximately equal to $|\Delta\phi|_{\max}$. The required time resolution of observations, or duration of a 'single' observation is $\Delta t \approx P/4$ for the periodic GWS.

Fig. 1 represents these characteristics, computed for a few binary stars, in comparison with performances of the Hipparcos and GAIA satellites. Estimations by Høg (1995) have been used to evaluate the GAIA performance. The three large dots in the upper part of the diagram correspond to some known binaries. We point out that such sources are out of reach of GAIA by 3–4 orders of magnitude on precision, while the time characteristics are quite suitable.

It is difficult to find better candidates than V Pup among known binaries because the shortest period systems have too low masses. Binaries of different periods consisting of two solar mass stars lie on the straight dashed line. The dot which happens to be within the ranges of GAIA correspond to a rather hypothetical neutron star–neutron star binary with $P = 25$ sec. A possibility to discover such a system could be in itself a strong motivation to accomplish the GAIA project but it seems to be meagre for the following reasons.

The angular radius of the near zone for such a source at a distance 1 kpc from the Earth would be as small as 8×10^{-5} arcsec. It is clear, that the probability for an observable background star to occur by chance within such a tiny spot is negligible. Yet, the probability becomes much higher if the N–N binary itself is locked into a binary system with an 'ordinary' luminous star. Moreover, eclipsing binaries SL – N–N yield an excellent opportunity to discover gravitational waves in this way, but none of the kind is known, of course.

Another difficulty is that a 25 sec binary should be short-lived. Its spiral time (see, e.g., Misner, Thorne & Wheeler, 1973) is only some 20 years:

$$\tau_0 = \frac{5}{256}\,\frac{a_0^4}{\mu M^2}. \tag{4}$$

In this time the two stars spiral into each other due to the loss of energy by the gravitational radiation, coalesce and end up with a strong pulse of gravity waves. Although this seems to be a normal way for a tight binary star to evolve, the chance to find such a system in the Galaxy at a given moment is small.

3. ROTATING NEUTRON STARS

A single neutron star can be a source of gravitational waves due to different plausible mechanisms. At present the strength and other characteristics of this radiation can only be vaguely guessed. Let us consider as a periodic SGW perhaps the least unlikely case in our Galaxy: a rotating neutron star, deviating in shape from symmetry around the rotation axis. The degree of this deviation is the most critical and poorly known parameter.

More specifically, the amplitude of periodic waves is proportional to a so-called 'gravitational ellipticity' e of the star in the equatorial plane (Thorne 1987). The observed small slow-down rates of the Crab, Vela and PSR 1937+21 pulsars imply typical values of e as small as 10^{-5} to 10^{-6}. For a realistic value of the moment of inertia 10^{45} [g cm^2] the maximum deflection of light penetrating through the near zone of the SGW is

$$|\Delta\phi|_{\max} \approx 2.3\times 10^{-7}\, e \cdot \left(\frac{\nu}{1\ \mathrm{Hz}}\right)^3 \ \mathrm{arcsec.}$$

Here again the angular size of the near zone is so small that only SGWs locked into eclipsing binaries could be of interest for the optical interferometry.

4. BURST SOURCES

Little is known about sources of strong pulses of gravitational waves. Even less is known about the characteristics of the waves themselves. Our aim is therefore to make an order-of-magnitude estimation of possible astrometric effect.

In doing so, we make use again of the Fakir's formula (2), substituting the $h_{\max}(d=\Lambda)$ with $h(b) = h_{\max}/b$, since it has been shown (Fakir, 1993) that the astrometric deflection decreases only slightly faster with distance than $1/b$. Now we have to consider the case $b \gg \Lambda$, for the size of the near zone is so small that the angular size of the SL can well be larger than the former.

A characteristic amplitude of gravitational wave (Thorne, 1987) as a function of distance d and total energy output ΔE_{GW} is

$$h_c \approx 2.7\times 10^{-17}\left(\frac{\Delta E_{\mathrm{GW}}}{M_\odot c^2}\right)^{\frac{1}{2}} \cdot \left(\frac{1\ \mathrm{kHz}}{\nu_c}\right)^{\frac{1}{2}}\left(\frac{10\ \mathrm{kpc}}{d}\right),$$

where ν_c is the characteristic frequency of the wave. If an SL is situated at an angular distance r_a from the GWS, as seen from the Earth, the expected amplitude of astrometric displacement is roughly

$$|\Delta\phi| \approx 1.8\times 10^{-5}\left(\frac{\Delta E_{\mathrm{GW}}}{M_\odot c^2}\right)^{\frac{1}{2}} \left(\frac{1\ \mathrm{kHz}}{\nu_c}\right)^{\frac{1}{2}}\left(\frac{1''}{r_a}\right)\left(\frac{10\ \mathrm{kpc}}{d}\right) \ \mathrm{arcsec.}$$

4.1. Neutron star–neutron star coalescence

It is widely believed that a large number of binaries is close enough together that their spiral time (4) is less than the age of the universe. The well-known binary pulsar PSR 1913+16, for example, will coalesce 3.5×10^8 years from now. The typical duration of the final pulse of gravitational waves is about 0.1 sec, the peak frequency $\nu_c \approx 900$ Hz and the efficiency $\epsilon \approx 0.02$ (Clark 1979). Then for two stars of one solar mass each

$$|\Delta\phi| \approx 3.4\times 10^{-6} \cdot \left(\frac{1''}{r_a}\right)\left(\frac{10\ \mathrm{kpc}}{d}\right) \ \mathrm{arcsec.}$$

This case is presented in Fig. 1 with the asterisk.

4.2. Collapse of a star to form a black hole

The expected birth rate of black holes of a few solar masses is probably not larger than $\frac{1}{3}$ the birth rate of neutron stars, hence formation of a black hole through collapse of a massive star should be a very rare event in our Galaxy. Still, it can generate a strong pulse of gravitation waves with the following characteristics (Thorne 1987):

$$\nu_c \approx (1.3\times 10^4\ \mathrm{Hz})\left(\frac{M_\odot}{M}\right), \qquad (5)$$

$$h_c \approx 7\times 10^{-19} \cdot \left(\frac{\epsilon}{0.01}\right)^{\frac{1}{2}}\left(\frac{M}{M_\odot}\right)\left(\frac{10\ \mathrm{kpc}}{d}\right).$$

The expected astrometric effect is then

$$|\Delta\phi| \approx 4.4\times 10^{-7}\left(\frac{\epsilon}{0.01}\right)^{\frac{1}{2}}\left(\frac{M}{M_\odot}\right) \cdot \left(\frac{1''}{r_a}\right)\left(\frac{10\ \mathrm{kpc}}{d}\right) \ \mathrm{arcsec.}$$

The efficiency ϵ can probably range from 7×10^{-4} for an axisymmetric collapse to 0.1 for a non-axisymmetric one. Observational requirements for $M = 10M_\odot$ and $\epsilon = 0.01$ are shown in Fig. 1 with the small square.

5. CONCLUSIONS

The GAIA instrument will be a scanning survey astrometric interferometer rather than a pointing telescope. It will be specially aimed at construction of a rigid reference frame based on a dense network of stars. During a five year mission, each star will get a few hundred observations, unevenly distributed in time. At a given moment GAIA will observe several small patches on the sky of a few square degrees altogether, the location of which will be strictly determined by a scanning law.

Thus, detection of burst sources of gravity waves is absolutely out of reach of the planned astrometric mission. At the same time, steady sources of gravity waves, and binary stars in particular, look more promising as astrometric targets. There are lots of them, and it is not unlikely that a suitable configuration SL–SGW can be found by GAIA. Such a system, if any exists, would be observed not once but a few hundred times during a five year period.

Taking into account the periodic character of the apparent displacements the whole series of observations for a given SL can be folded up with the expected period and averaged, to derive a phase curve. This would lead to further improvement of precision by a factor of 10 compared to the performance shown in Fig. 1. Hence, a gravity wave from a binary star of $P = 100$ sec (for an SL of $V = 15$ mag) can in principle be discovered by GAIA. This falls close to the shortest known orbital periods.

It is clear from the above consideration how modest are the chances to find an SGW through the GAIA mission. Nonetheless, the payoff in case of success would be so great that it is worth to plan a special off-line treatment of single astrometric observations for that purpose.

The model for the treatment can in fact be fairly complicated because of possible orbital motion and interfering micro-lensing effects. A set of suspected stars can first be selected due to for instance increased astrometric residuals. Then periodic oscillations in the range of frequencies 1 to 0.01 Hz should be looked for for each selected object individually. In the frame of proposal by Fabricius & Høg (1995) on the organization of data processing for GAIA, the sorted Transit Catalogue can be used for the search.

An alternative way is proposed by Fakir (1995) that implies a prior selection of the best candidates SL–SGW among known binaries and then concentrating efforts only on these stars. In my opinion it would hardly be possible to find any much better examples of SGWs than those shown in Fig. 1 among the binaries known up to now, because the shortest periods are always accompanied by the lower masses. The total mass and the frequency contribute to the strength of the wave with the same power $5/3$, therefore relatively slow but very massive V Pup remains one of the best targets. With GAIA, one has perhaps to hope more for unknown and typically invisible neutron star binaries, close to the coalescence.

ACKNOWLEDGEMENTS

This work was supported by the Danish Space Board. The author is grateful to Dr. Fakir for useful discussions and comments.

REFERENCES

Clark, J.P.A., 1979, in *Sources of Gravitational Radiation*, ed. L.Smarr (Cambridge: Cambridge Univ. Press), 447–459

Fabricius, C., Høg, E., 1995, Scientific analysis of data from a scanning satellite, ESA SP–379, this volume

Fakir, R., 1993, Univ. of British Columbia preprint UBCTP–93–016, 10 pp.

Fakir, R., 1994a, ApJ, 426, 74–78.

Fakir, R., 1994b, Physical Review D50, 6, p. 3795–3800.

Fakir, R., 1995, A first direct detection of gravity waves through high precision astrometry, ESA SP–379, this volume

Høg, E., 1995, Some designs of the GAIA detector system, ESA SP–379, this volume

Lindegren, L., Perryman, M.A.C., 1994, Global Astrometric Interferometer for Astrophysics, *Supplementary Information Submitted to the Horizon 2000+ Survey Committee*, 52 pp.

Misner, C.W., Thorne, K.S., Wheeler, J.A., 1973, Gravitation (Freeman: San Francisco)

Peters, P.C., Mathews, J., 1963, Phys. Rev., 131, 435-440.

Thorne, K.S., 1987, in *300 Years of Gravitation*, ed. S.W. Hawking & W. Israel, (Cambridge: Cambridge Univ. Press), 330–448.