A DESIDERATUM OF ASTROMETRY FOR GRAVITATIONAL MICROLENSING EVENTS

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ABSTRACT

We show that astrometric observations of proper motion and annual parallax of MACHOs during the visible event are indispensable for determining the MACHO mass and trajectory on the celestial sphere. Furthermore, we demonstrate that the annual parallax, besides the caustic formed by a binary lens, plays an important role in producing the spiky feature in the microlensing light curve. To perform the astrometry within the Einstein ring, we need an optical interferometric-technique in space of sub-milliarcsec to ten-microarcsec level.

Key words: microlensing, MACHOs, parallax, proper motion, galactic halo, GAIA

1. INTRODUCTION

There is a prevailing view that the main part of the gravitating matter in the galactic dark halo is composed of massive compact halo objects (MACHOs), which have not resulted in the nuclear burnning due to insufficient mass (less than 0.1 $M_\odot$). On the other hand, the initial mass function of stars has not yet been definitively established for the mass less than 1 $M_\odot$ (Richer & Fahlman 1992). Therefore, it is important to determine the MACHO mass and its mass function, without relying upon the statistical method.

The possibility of using gravitational microlensing to detect MACHOs in the galactic halo has been first pointed out by Paczyński (1986). Since then, several collaborations such as MACHO (Alcock et al. 1993), EROS (Aubourg et al. 1993), and OGLE (Udalski et al. 1993) have started to monitor the amplification of brightness for several million stars and discovered in total more than 50 candidates of the microlensing event toward both the Large Magellanic Cloud and the Galactic Bulge. The majority of the events detected so far by MACHO, EROS, and OGLE shows light curves with symmetry around a single maximum of brightness, as Paczyński (1986) has predicted. Such light curves are characteristic of events caused by a single lens and source, when observed by a heliocentric observer.

Meanwhile, Miyamoto & Yoshii (1995a, hereafter referred to as Paper I) have pointed out that both the annual parallax and the proper motion of the lens are equally influential in the microlensing events, *irrespective* of the lensing duration and lens mass, and moreover that even the light curve reported by Alcock et al. (1993) shows a deviation from the perfect symmetry due to the annual parallax. It has been believed so far that the effect of the annual parallax appears as a perturbation superposed on the symmetric light curve only for the event on annual timescales. Recently, MACHO collaboration (Alcock et al. 1995, Bennett et al. 1995) reported a long timescale event lasted almost one year, which showed indeed a slight asymmetry in the light curve, and they explained the asymmetry by the parallactic effect of annual timescales.

The most spectacular event reported so far is the one (OGLE No. 7) discovered by OGLE collaboration (Udalski et al. 1994) toward the galactic bulge. Later, MACHO collaboration (Bennett et al. 1995) confirmed the identical event and supplemented the portion of the light curve missed by OGLE. The light curve of this exotic event shows a characteristic U-shape (a sharp double-maximum shape) with a plateau between two spikes. Mao and Paczyński (1991) have already predicted the possibility that the light curve with a variety of the spiky feature will occur, whenever a source crosses the caustic formed by a binary lens. On the basis of the binary lens model, Udalski et al (1994) have reproduced the U-shape light curve of OGLE No. 7, introducing an additional lensed source for explaining the excess plateau level. In addition to the role of the caustic in the microlensing, we demonstrate here the role of the annual parallax as well, which distorts remarkably the symmetry of the light curve, and even produces the spiky feature.

2. COMBINED EFFECTS OF PARALLAX AND PROPER MOTION

We give here briefly the astrometric and photometric descriptions of the microlensing, and propose a scheme how to determine the MACHO mass and trajectory (see Paper I for details).

Consider the microlensing of a star (source) at the heliocentric distance D_S due to a single point mass (lens) with mass M at a distance D_L. Then, the most essential parameter at issue, the Einstein ring radius R_0 in radian on the celestial sphere with the radius D_L is given by

$$R_0^2 = \frac{4GM}{c^2}(\frac{1}{D_L} - \frac{1}{D_S}) \qquad (1)$$

Introducing the mutually orthogonal unit vectors (vector base) $\boldsymbol{\alpha}$ and $\boldsymbol{\delta}$ at (α, δ) on the celestial sphere, we have the position $\boldsymbol{r}(\mathrm{t})$ of the source relative to the lens for the terrestrial observer:

$$\boldsymbol{r}(t)=\boldsymbol{\mu}\cdot(t-t^{*})+\boldsymbol{d}+\pi\boldsymbol{p}(t) \quad , \qquad (2)$$

where $\boldsymbol{\mu}$ denotes the relative proper motion of the source, $\boldsymbol{d}$ the position of the source, seen from a heliocentric observer, at the closest approach ($t=t^{*}$) to the lens, $\pi = \pi_S$ - π_L the relative trigonometric parallax of the source, and $\boldsymbol{p}(\mathrm{t})$ the annual parallactic factor given by the Astronomical Almanac. Eq. (2) can be expressed as

$$\boldsymbol{r}(t)=R_0[\boldsymbol{\alpha}\ \boldsymbol{\delta}]\left[\begin{array}{c}\frac{1}{t_0}(t-t^{*})\sin\phi+d_0\cos\phi+\pi_0 p_\alpha(t)\\ \frac{1}{t_0}(t-t^{*})\cos\phi-d_0\sin\phi+\pi_0 p_\delta(t)\end{array}\right] \qquad (3)$$

where ϕ denotes the position angle of $\boldsymbol{\mu}$, and t_0^{-1}=μ/R_0, $d_0 = d/R_0$, and $\pi_0 = \pi/R_0$. The fully astrometric expression of Eq. (3) is given by

$$\boldsymbol{r}(t)=R_0[\boldsymbol{\alpha}\ \boldsymbol{\delta}]\left[\begin{array}{c}A+\pi p_\alpha(t)+\mu\sin\phi\cdot(t-T)\\ D+\pi p_\delta(t)+\mu\cos\phi\cdot(t-T)\end{array}\right] \quad , \qquad (4)$$

where $A = [\alpha_S(T)-\alpha_L(T)]\cos\delta$ and $D = \delta_S(T)-\delta_L(T)$ with the heliocentric positions $(\alpha_S(T),\delta_S(T))$ and $(\alpha_L(T),\delta_L(T))$ of the source and lens at the initial epoch T, respectively.

Next, defining the nondimensional distance u between the source and lens by

$$u=|\boldsymbol{r}(t)|/R_0 \quad , \qquad (5)$$

we have the combined amplification $A(t)$ of two images of the source, and the separation vector $\boldsymbol{\rho}(\mathrm{t})$ between the two images:

$$A(t)=\frac{u^2+2}{u\sqrt{u^2+4}} \qquad (6)$$

and

$$\boldsymbol{\rho}(t)=\frac{\sqrt{u^2+4}}{u}\boldsymbol{r}(t) \quad , \qquad (7)$$

which constitute the photometric and astrometric observations to be carried out during the microlensing event.

Now, noting that a time sequence of the observed amplifications $A(t)$ supplies the corresponding sequence of $u(t)$ on the basis of Eq. (6), we can solve the conditional equations constructed by Eq. (5) for the five parameters t_0, t^*, d_0, π_0, and ϕ. On the other hand, we can solve for the vector $\boldsymbol{r}(\mathrm{t})$ pointing from the unseen lens to the source on the basis of Eq. (7), since $u(t)$ is given by the photometric observation (cf. Eq. (6)). Then, the conditional equations constructed by Eq. (4) determine the five astrometric parameters A, D, π, μ, and ϕ.

Next, given the relative proper motion $\boldsymbol{\mu}$ and the time scale t_0, we can obtain the Einstein ring radius R_0 from the relation $R_0 = \mu t_0$. Then, replacing $1/D_S - 1/D_L$ by the relative parallax π, we can derive the lens mass M from Eq. (1). Furthermore, the astrometric parameters of the source are usually known or can be determined, so that if the five astrometric parameters are given, we can obtain all five astrometric parameters $(\alpha_L(T),\ \delta_L(T))$, $(\mu_{\alpha L},\ \mu_{\delta L})$, and π_L of the unseen lens, and thereafter we can track the lens, until it may cause the next microlensing.

3. AN ALTERNATIVE MODEL FOR THE SPIKY FEATURE IN THE MICROLENSING LIGHT CURVE

To demonstrate further the importance of the annual parallax in the microlensing, we reproduce, as an example, a spiky feature in the light curve, which is similar to the U-shape one of OGLE No. 7 (Udalski et al. 1994).

The importance of the parallax in the microlensing dominates that of the proper motion, when $|\pi\dot{\boldsymbol{p}}(t)| \gg |\boldsymbol{\mu}|$, while for $|\pi\dot{\boldsymbol{p}}(t)| \ll |\boldsymbol{\mu}|$ the parallactic effect can be considered as a perturbation (Gould 1992, Alcock et al. 1995). The conspiracy of the annual parallax with the proper motion of the lens in its trajectory is schematically illustrated in ecliptic coordinates (λ, β) in Fig. 1, where the origin of the coordinates is chosen at the position of a lens L viewed from a heliocentric observer. If the lens viewed from a terrestrial observer lacks the proper motion $\boldsymbol{\mu}$, the lens at a finite distance describes, taking one year, a closed parallactic ellipse $\pi\boldsymbol{p}(t)$ with the long axis π along the λ-axis (see the ellipse around the origin in Fig. 1). Furthermore, if the proper motion is superposed on the parallactic motion, there appears a variety of trajectories of the lens, depending on the magnitude and direction of the proper motion vector $\boldsymbol{\mu}$. It is noticed here that we can put the source anywhere at our option in the (λ, β)-plane of Fig. 1, since considered is the relative motion of the lens with reference to the source.

In Fig. 1 two families of the trajectories of the lens, which start from the phases A and B of the annual parallax, are demonstrated for the proper motion vectors $\boldsymbol{\mu}$ aligned to the line $\beta = \lambda\cos\beta_0$ with β_0 the latitude of the origin and to the λ-axis, respectively. According to the conditions $|\pi\dot{\boldsymbol{p}}(t)| \gg |\boldsymbol{\mu}|$ and $|\pi\dot{\boldsymbol{p}}(t)| \ll |\boldsymbol{\mu}|$, the trajectories of both families are helical (Type I) and sinusoidal (Type III), respectively. But, in certain circumstances, we encounter the condition $|\pi\dot{\boldsymbol{p}}(t)| \approx |\boldsymbol{\mu}|$ between the above extreme cases. Then, the trajectory describes a small cuspate loop (Type II), so that the lens turns back to a nearly identical position on the celestial sphere within a few months and even within a few days. The very fact gives an alternative basis for modelling the light curve with double spikes during a period shorter than one year, in a similar fashion to the caustic formed by the binary lens (Mao & Paczyński 1991, Udalski et al. 1994).

Fig. 2 shows an example of the light curve with double spikes (see the upper panel), which mimics the light curve of OGLE No. 7 (Udalski et al. 1994, Bennett et al. 1995), especially the observed double spikes with interval of about two months. But, on the basis of the present combination of the annual parallax with the proper motion, we cannot reproduce the plateau level between the two spikes. The lower panel of Fig. 2 shows the corresponding trajectory of the lens, where the small circle near the cusp indicats the source position. It is noticed here that the source position in the lower panel indicates only one of the four possibilities to reproduce two spikes. That is because there are four corners at the crossing point of the trajectory, so that the source can be put at any corner to reproduce two spikes, in practice. Therefore, the present model degenerates into fourfold. In the forthcoming paper (Miyamoto & Yoshii 1995b), we will present other exotic examples.

4. DISCUSSION

As is shown above, the astrometric determination of the relative proper motion μ and the relative trigonometric parallax π of the lens is indispensable for determining the Einstein ring radius, the lens mass, and its trajectory. To determine definitively the trigonometric parallax, it usually takes half a year at least, in principle. Turning to the general view of the microlensing, the possible mass range of MACHOs is considered to be $0.01M_{\odot}$–$1.0M_{\odot}$, whose event duration $2t_0$ is estimated to be one to five months in the standard case of the source in the LMC at $D_S = 50$ kpc ($\pi_S = 0.02$ mas), the MACHO distance $D_L = 10$ kpc ($\pi_L = 0.1$ mas), and its random spatial motion 200 km/s ($\mu = 4$ mas/yr).

Judging from the conventional astrometric method (van Altena 1983), it seems not easy to perform the parallax determination within such a short period. But, it is true anyway that exclusively within the short period of the visible microlensing we have to complete the astrometric measurements of MACHOs.

Table 1: Einstein ring radius, R_0

The vector $\pi \boldsymbol{p}(t)$ describes a parallactic ellipse on the celestial sphere. Therefore, the determination of π within a period shorter than six months is equivalent to determining the curvature of the ellipse. In the standard case considered here $\pi \sim 0.1$ mas and $\mu \sim 4$ mas/yr, the displacement due to the proper motion would amount to about 0.3 mas/month, while the one due to the annual parallax would be about 0.02 mas/month. Thus, the determinability of μ is one order of magnitude larger than that of π. In case that only the proper motion is determined reliably by the astrometric observation, we can still derive the MACHO mass, since the combination of μ with the photometric determination of t_0 and π_0 gives R_0 and π.

On the other hand, in order to determine $\boldsymbol{\mu}$ and π, we have to perform accurate observations of the separation vector $\boldsymbol{\rho}(t)$ of two images. The separation $|\boldsymbol{\rho}(t)|$ is always the order of magnitude of $2R_0$. To see the astrometric accuracy to be required, the Einstein ring radius R_0 as the function of lens mass $M/M_{\odot}$ and distance D_L for a fixed source distance $D_S = 50$ kpc at the LMC is given in Table 1. Table 1 shows that for determining definitively the proper motion and the parallax we need the next generation astrometric satellite (optical interferometric-technique) of sub-milliarcsec to ten microarcsec level, which should be considered seriously as the next goal of modern astrometry.

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Figure 1: Three types (I, II, III) of trajectories of the lens on the celestial sphere.

Figure 2: Upper panel: a microlensing light curve with double spikes, caused by a combination of annual parallax and proper motion. Lower panel: the corresponding trajectory of the lens on the celestial sphere, where the small circle indicates the source.