GEOMETRIC OBSERVATION EQUATIONS FOR GAIA MICROARCSECOND ASTROMETRY

B. Bucciarelli, M.G. Lattanzi, A. Spagna

Osservatorio Astronomico di Torino, Strada Osservatorio 20, 10025 Pino Torinese TO, Italy

ABSTRACT

The geometric observation equations for an aberration-free, GAIA-like instrument are formulated in a flat space-time metric. Given the expected measurement accuracy of 10 microarcsec (μarcsec), the significance of perspective acceleration and foreshortening terms is addressed. Apparent place effects such as velocity aberration and relativistic light deflection are also discussed. The unprecedent potential astrometric accuracy of a GAIA-like mission calls for a re-formulation of the observing equations in a general relativistic framework, which is best suited to account for light aberration caused by the non-stationary gravitational field of the Solar system. This effort is underway.

Key words: space astrometry; GAIA; relativity.

1. INTRODUCTION

In the presently adopted mission-concept design, GAIA basic observation consists of measuring the displacement of a stellar object with respect to the centre of the field of view, at a certain mean time t, in the instantaneous scanning direction. Thanks to the use of a Fizeau-type interferometer, the accuracy of such measurement is expected to be of the order of 10 μarcsec (see Lindegren 1995). The telescope is meant to operate in continuous scanning mode, following a scanning law similar to the Hipparcos one, with the difference that GAIA measures all the objects which enter the field of view up to a given limiting magnitude, whereas Hipparcos could only observe stars from a list of pre-selected objects. Moreover, the capability of observing stars at large separations is ensured by using two interferometers rigidly connected.

GAIA will then produce an observation of the projection along the scanning direction of the angular distance between any couple of stars brigther than $V = 15 - 16$ mag that appear simultaneously in either one of the two fields of view. Similarly to Hipparcos, the characteristic scanning law permits to re-observe any object under very different viewing directions; therefore, its actual position on the sky can be reconstructed from solely uni-dimensional observations.

In the following, after briefly reviewing the satellite scanning law and the geometry of a single observation, we will formulate the differential observation equation in vectorial form for a flat space-time metric. We will also evaluate the significance of the different terms, stating the relations among vectors and the classical astrometric quantities λ, β, μ_λ, μ_β, π. Finally, we will discuss the treatment of apparent place effects, such as velocity aberration, and of relativistic light deflection, which play a very critical role at the μarcsec accuracy level.

2. MODELLING THE OBSERVATION

2.1. Scanning Law

The satellite spin-axis, positioned at a fixed angle $\xi \simeq 55°$ to the Sun, revolves around the Sun with velocity $\omega_1 \simeq 6.3$ rev/year. The optical axes of the two interferometers lie on the plane perpendicular to the spin axis, and rotate rigidly on such a plane with velocity $\omega_2 \simeq 8$ rev/day. We also call ω_0 the mean velocity of the Sun around the ecliptic. We define $\mathbf{E}(\mathrm{t})$ the unit vector perpendicular to the optical axis at time t, and directed along the instantaneous scanning velocity. Then, combining the three rotations ω_0, ω_1, ω_2, one obtains (Betti 1982):

$$\begin{aligned} E_1(t) &= -\sin\xi\cos\omega_0 t\cos\omega_2 t - \\ &\quad \cos\xi\sin\omega_0 t\cos\omega_1 t\cos\omega_2 t + \\ &\quad \sin\omega_0 t\sin\omega_1 t\sin\omega_2 t \qquad (1) \\ E_2(t) &= -\sin\xi\sin\omega_0 t\cos\omega_2 t + \\ &\quad \cos\xi\cos\omega_0 t\cos\omega_1 t\cos\omega_2 t - \\ &\quad \cos\omega_0 t\sin\omega_1 t\sin\omega_2 t \qquad (2) \\ E_3(t) &= \cos\xi\sin\omega_1 t\cos\omega_2 t + \\ &\quad \cos\omega_1 t\sin\omega_2 t \qquad (3) \end{aligned}$$

At the initial time $t = 0$, both the spin-axis and the optical axis lie on the ecliptic, with the spin-axis preceding the Sun in the direction of Sun's motion on the ecliptic. The above equations describe the nominal along-scan attitude of the satellite, allowing the determination of the direction onto which the angular measurement on the sky is projected. Deviations from such a law induced by, e.g., jet-firing actuators, mechanical instabilities, etc., have to be estimated along with the adjustments to the astrometric parameters.

2.2. Geometry of Observation

We consider the plane tangent to the celestial satellitocentric sphere in the direction of the instrumental optical axis. If $\mathbf{r}'$ is the unit vector representing the *apparent* direction to the star, then the observation is given by the quantity x, i.e., the component along the scanning direction of the projection of $\mathbf{r}'$ onto the defined tangent plane. We also call $\Delta\mathbf{r}$ the small vector correction including all the apparent place effects that modify the path of the light-ray from the source to the observer. Then, the quantity $\mathbf{r} \equiv \mathbf{r}' - \Delta\mathbf{r}$ represents the *geometric* direction to the stellar source. We introduce at this point the following quantities:

- π annual stellar parallax (rad)
- $\mathbf{u}_\mathrm{b}$ unit vector in the barycentric direction to the star;
- $\mathbf{V}_\mathrm{T}$ star barycentric tangential velocity (AU/year);
- $\mathbf{V}_\mathrm{R}$ star barycentric radial velocity (AU/year);
- $\dot{\mathbf{u}}_\mathrm{b}$ unit vector in the direction of $\mathbf{V}_\mathrm{T}$;
- $\mathbf{u}_\odot$ unit vector Satellite-Sun;
- ρ distance Satellite-Sun (AU).

The following vectorial expression holds:

$$\mathbf{r}(t) = \mathbf{u}_\mathrm{b}(T)[1 + \pi V_\mathrm{R}(t-T)] + \pi\dot{\mathbf{u}}_\mathrm{b} V_\mathrm{T}(t-T) + \pi\mathbf{u}_\odot \rho(t) \qquad (4)$$

where t is the epoch of observation and T the reference epoch. The right hand side of Eq. (4) is a function of the five classical astrometric parameters, whose improvement is the main goal of the mission, namely, λ, β, μ_λ, μ_β, and π. Spherical coordinates are trivially derived from $\mathbf{u}_\mathrm{b} = (\cos\lambda\cos\beta, \sin\lambda\cos\beta, \sin\beta)$; proper motion components μ_λ, μ_β can be easily computed as

$$\pi\mathbf{V}_\mathrm{T} = \mu_\lambda cos\beta\mathbf{e}_\lambda + \mu_\beta\mathbf{e}_\beta,$$

where the proper motion units are rad/year, and $\mathbf{e}_\lambda$ and $\mathbf{e}_\beta$ are unit vectors tangent to the celestial sphere at the star given by

$$\mathbf{e}_\lambda = \begin{vmatrix} -\sin\lambda \\ \cos\lambda \\ 0 \end{vmatrix}, \qquad \mathbf{e}_\beta = \begin{vmatrix} -\cos\lambda\sin\beta \\ -\sin\lambda\sin\beta \\ \cos\beta \end{vmatrix}.$$

3. THE LINEARIZED CONDITION EQUATION

Before formulating the condition equation, we recall that Eq. (4) is non linear in the parameters to be estimated. In addition, as previously mentioned, the observable, which we called x, is the projection of $\mathbf{r}'$ along $\mathbf{E}$. Therefore, we can write the observation equation in compact form as

$$x = \mathbf{r}' \cdot \mathbf{E}. \qquad (5)$$

We also note that $\mathbf{r}'$ is not a unit vector, and shall therefore be normalized before inserting it in (5). Eq. (5) must be linearized with respect to catalog values $\mathbf{r}_0$ and to some provisional attitude knowledge $\mathbf{E}_0$. Thus, linearization of Eq. (5) gives:

$$\Delta x = \delta\mathbf{r}\cdot\mathbf{E}_0 + \delta\Delta\mathbf{r}\cdot\mathbf{E}_0 + \delta\mathbf{E}\cdot\mathbf{r}_0' + \epsilon \qquad (6)$$

where ϵ includes both higher order terms and observational errors. Terms $\delta\mathbf{r}$ and $\delta\mathbf{E}$ contain the adjustments to the astrometric parameters and to the along-scan attitute respectively. We also explicitely added the term $\delta\Delta\mathbf{r}$, which represents possible additional adjustments due to imperfect knowledge of the parameters used for the computation of apparent place effects. Nonetheless, to the level of accuracy attainable, such effects are to be removed a priori from the observation x before entering it in Eq. (5). Δx is simply the difference between the actual observation and its *calculation* based on the best a priori knowledge of the quantities involved. Now, substituting Eqs (1–4) in Eq. (5), and writing Eq. (6) in explicit form one obtains the condition equation in a usable form. When differentiating Eq. (5), we must make sure that all the terms neglected (which are implicitely carried in ϵ), are of the order of a fraction of μarcsec or less; otherwise, the risk would be there to spoil the quality of GAIA observations by introducing systematic errors of magnitude comparable with those of the measurements themselves.

3.1. The Effect of Stellar Radial Velocity

The formulation presented above is quite general, and applies as well to the Hipparcos observations, as in Lattanzi et al. (1990). We now specialize it to the mission design under study by evaluating to which extent the change on a star position caused by its radial velocity can be detected by GAIA. First, we examine *perspective acceleration*, which induces a secular change in the barycentric coordinates of the star. We call γ the yearly angular change due to proper motion, and ϕ the total yearly angular change if one takes into account also radial velocity. Assuming for simplicity that the moduli of V_R and V_T are identical, from planar geometry one obtains, to first order in πV_R:

$$\gamma = \pi V_T(1 - 0.4\pi V_R); \qquad \phi = \pi V_T.$$

Hence, the perspective acceleration effect is given by the quantity $\gamma - \phi = 0.4\pi^2 V_R V_T$. This quantity becomes of the order of 10 μarcsec or larger for objects having $V_R \geq 100$ km/s, at a distance $D \leq 10$ pc, after a time span $\geq$1 year. In conclusion, GAIA will be able to detect *perspective acceleration* for some stars, for which one would therefore need accurate radial velocities in order not to bias their proper motion estimate.

We now want to evaluate the so-called *foreshortening* term, which is the variation of the star position, as seen by the observer, induced by the radial velocity of the source. In this case also, we call γ the angle defining the apparent direction of the source, calculated by taking into account its radial velocity; while ϕ indicates the analogue angle, this time computed diregarding V_R. Again, applying simple geometry, one obtains, to first order in π

$$\phi - \gamma = \frac{\pi^2 V_R \sin u}{1 + \frac{\pi}{2}(2V_R - \cos u)}$$

where u is the angle between the observer and the star as seen from the Sun. In this case, the effect is probably negligible for all accessible stars over a time span of several years.

3.2. Relativistic Velocity Aberration

In the framework of special relativity, the velocity aberration is computed by applying a Lorentzian transformation between the barycentric reference frame and the one coinciding with the reference frame of the observer at the time of observation t, which moves with constant velocity V, again coincident with the instantaneous velocity of the observer at time t. The second-order formula transforming the barycentric direction $\mathbf{s}$ into the observed one, i.e. $\mathbf{s}'$, is the following (see, for example, Green 1985):

$$\mathbf{s}' = \mathbf{s} - \frac{V}{c}\mathbf{s}\times(\mathbf{s}\times\mathbf{n}) + \frac{1}{2}\frac{V^2}{c^2}[2(\mathbf{s}\cdot\mathbf{n})^2 - 1][\mathbf{s} - (\mathbf{s}\cdot\mathbf{n})\mathbf{n}]$$

where $\mathbf{n}$ is the direction of the observer velocity V. Considering a geostationary orbit, the instantaneous satellitocentric velocity would be of the order of 3 km/s. The magnitude of the second-order term (V^2/c^2) is about 20 μarcsec, significantly larger than the budgeted error.

Very accurate knowledge of the satellite ephemerides is critical to the calculation of velocity aberration. In fact, an uncertainty of only 1 mm/s on V corresponds to an error of $V/c \simeq 1\mu$as on the stellar apparent displacement caused by velocity aberration.

3.3. Gravitational Light Deflection

A light ray passing near a perfectly spherical body is deflected, in accordance with the predictions of general relativity, by the full angle

$$\Gamma = \frac{4GM}{c^2 b}$$

where G is the constant of gravitation, M is the perturbing mass, and b the so called impact parameter, which is equal to the radius of the perturbing mass for a limb-grazing ray. For b in AU, M in solar masses, and c in AU/year, G is numerically equal to $4\pi^2$. Using the above formula, one can see for example that the maximum deflection caused by the Sun, i.e., for a limb-grazing ray ($b_\odot$= 0.00466 AU), is 1.75 arcsec, while the analogue value for Jupiter is only 17 mas. However, for observations made by an Earth satellite like GAIA one has to consider the entity of the deflection in case of sources at large angular separations from the perturbing mass, and its variation with observing conditions. A general expression for this general case (Ward 1970) is given by

$$\gamma = \frac{4GM}{c^2 b}\frac{(1+\cos\theta)}{2}$$

where θ is the angle between the light source and the perturbing mass as seen by the observer on a geostationary orbit. If we then calculate the effect of light bending due to Earth for a source, say, at $\theta = 90^\circ$, we obtain $\gamma \simeq 50$ μas, comfortably observable by GAIA. If we examine the gravitational perturbation by Saturn as seen by GAIA, the amount of light deflection becomes of the order of 10 μas for a light source at a 10° from it.

The above discussion deals with ideal bodies. In reality, since the masses of planets are not perfectly spherical, both their quadrupole moment and angular momentum can, in principle, affect the deflection, and therefore need to be carefully estimated (Schutz 1982). It is clear then, that in the context of a GAIA-like mission, gravitational light bending has to be carefully gauged for all Solar system bodies, or else constraints are to be put on the minimum angular distance between the planet under consideration and the observed stellar source.

4. GENERAL RELATIVITY

The problem of the reduction of high-precision astrometric observations in a General Relativistic framework has been addressed by several papers in the past years by Murray (1981), Brumberg et al. (1990) and Klioner & Kopejkin (1992). Such an approach permits to naturally take into account the effect of gravitational light deflection induced by a non-stationary local field. In this framework, the *observable* (or *natural*) direction is defined to be the direction of the tangent to the photon track (geodesic) at the point of observation as seen in a local flat space-time frame. Therefore, after computing the *natural* direction starting from the geodesic equation of the light ray, one can derive the *proper* direction, which is the direction as seen by a moving observer, by a simple Lorentzian transformation.

An attempt at rederiving these equations for the reduction of astrometric measurements involving angular separations on the sky is being made in collaboration with the University of Padova (Vecchiato 1996).

5. CONCLUSIONS

We have presented the condition equation for a GAIA-like mission in a flat space-time metric, neglecting instrumental perturbances. An analysis of the effect of stellar radial velocity on the star's angular displacement has shown that the perspective acceleration term is significant at the μarcsec level for very few stars, while the foreshortening term is negligible. Gravitational light deflection caused by the Earth and Saturn—and possibly by other Solar system bodies—besides that due to the Sun and Jupiter, has to be taken into account. A fully general relativistic approach to the formulation of GAIA observation equation is under study.

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