#### HIGH ACCURACY ASTROMETRY OF STAR CLUSTERS AND ASSOCIATIONS

# Imants Platais<sup>1</sup>, Floor van Leeuwen<sup>2</sup>, Jean-Claude Mermilliod<sup>3</sup>

Yale University Observatory, New Haven, USA
Royal Greenwich Observatory, Madingley Road, Cambridge, UK
Observatoire de Lausanne, Chavannes-des-Bois, Switzerland

#### ABSTRACT

The impacts on the research of star clusters and associations of 10 microarcsec accuracy astrometric data is discussed. The target magnitude range as presented in the GAIA concept is compared with the typical contents of these stellar systems.

Key words: space astrometry, GAIA, star clusters, associations

## 1. INTRODUCTION

Studies of star clusters and associations provide essential information on a wide range of astronomically very important aspects: (a) star formation in clusters and associations (luminosity and mass functions); (b) stellar evolution (HR diagrams of stars of the same age and chemical composition); (c) small-N stellar system kinematics and dynamics; (d) formation and evolution of double stars (statistics and orbital parameters of double stars in clusters of different age); (e) variable stars, blue stragglers and other odd stars in open clusters.

Perhaps, the basic intricacy of star cluster and association studies is to establish genuine memberships of those systems. The star's membership is defined through its three positional coordinates and three velocity components. Measured with respect to the mean position and velocity of the cluster, and compared with the potential energy distribution of the cluster, they show whether or not a star is likely to be bound to the cluster. In practice, however, it is nearly impossible to derive full spatial and velocity information of cluster stars.

Almost all open clusters, in total about 1 200 (Lyngå, 1987), have been discovered as local star-density enhancements against the general star background: which in principle provides two out of the three positional coordinates, with no information on the velocity components. The measurements of proper motions can add two velocity components, and for nearby clusters this is sufficient to distinguish members from non-members. The remaining positional and velocity component can be obtained from the parallax and radial velocity measurements, respectively.

The commonly used proper-motion technique for separating cluster members depends heavily upon the availability of photographic exposures having been made over

a long time interval, and with a reasonable plate scale. To date merely 90 open clusters (van Leeuwen 1985; van Altena et al. 1993) have at least one determination of relative proper motions. Proper motion surveys of star clusters are always confined by the limiting magnitude and area of sky covered by the old epoch plates: nothing can ever be changed there. As a result, in general, only the cluster centres have been studied, and the completeness limits usually do not advance much fainter than 13th to 15th magnitude.

If the cluster is situated near the galactic plane, which is usually the case as can be seen from Fig. 1, its proper motion is very close to that of the field stars. This makes the membership determination increasingly problematic, particularly, at the level of proper-motion accuracy of 1–2 mas/yr, commonly found in many early studies. The highest quoted proper-motion accuracies, 0.1–0.2 mas/yr, have been reached for very few, mostly nearby open, clusters: Pleiades, Praesepe, Orion cluster and M67, and for one globular cluster:  $\omega$  Cen (Reijns et al, in preparation). Such an accuracy already allows one to estimate the internal velocity dispersion and velocity anisotropy.

It seems that photographic astrometry has essentially reached its limitations set by the availability of the old epoch material and the presence of subtle systematic errors which depend upon the stability of the 'optical system and photographic emulsion', frequently over as much as a hundred years. It should be noted that the Hipparcos Input Catalogue density per square degree due to satellite observing constraints greatly limited the number of observed cluster stars with exception for nearby clusters. On average, for the 241 programme open clusters included in the Hipparcos Input Catalogue, 5 stars per cluster were measured, and in many cases only one star was included (Sellier et al. 1992).

In addition to the problems discussed above we would like to outline some more which cannot be solved properly with available ground-based instrumentation:

- the bulk of open clusters do not have any first epoch plates and/or wide-field CCD frames, so the number of clusters with material available for proper motion determinations is likely to increase very slowly;
- the need for plate modelling in proper motion determinations essentially precludes one from detecting any expansion, contraction or rotation of clusters and associations;

Figure 1. Distribution of 418 open clusters with known distances in the galactic coordinate system. The size of circles denotes the cluster's distance modulus. The nearest cluster, UMa, has a distance modulus of 2.0 mag, and the most remote ones, about 14.0 mag. Some well-known open clusters, like UMa, M67, NGC 188, etc are also marked. Figure 2. Distribution of 61 OB associations in the galactic coordinates. The meaning of the circle sizes as in Fig. 1. ever, they are useless in restoring the spatial distribution 3. it is extremely difficult to get absolute proper motions, (with the exception of the Hyades). It is hardly surprising that the existing rather idealized reduction techniques needed for galactic orbit calculations, since most of the open clusters are located near the galactic planeand cluster models provide rather uncertain results and, consequently are of limited use in studies of cluster dygalaxy avoidance zone; 4. the actual size of open clusters is essentially unknown. namics (King 1980). In associations the main problem is the large area on There is not a single cluster or association for which we the sky covered by the system and low spatial density (Blaauw 1991; de Zeeuw et al. 1994). For instance, know its dimensions and true density profile. Note that

the Orion association covers at least 200 square degrees,

which, putting aside the bright stars from meridian cat-

alogues, requires either large fields on single plates with

the cluster-star distribution and the full velocity vectors

are convolved along the line of sight. Radial velocities can

in principle provide the missing velocity component, how-

unfavourable plate scale, or many overlapping small fields which, in turn, pose specific field adjustment problems. In addition, the proper motions of young OB associations are very small, so that the membership analysis is not decidedly reliable. Summarizing, in the case of associations we have basically the same, but even more cumbersome problems as with open clusters. Although a significant improvement of our knowledge for some nearby associations, like Cas-Tau, RCr A, Sco OB2, Per OB3, etc, is expected from the Hipparcos mission results, still the majority of known associations (about 60 in total; see Fig. 2) are and will remain poorly studied. Clearly, accurate proper motions and more desirably the sub-mas accuracy parallaxes could radically improve our understanding of those elusive star systems.

The small class of so-called T associations, perhaps, is beyond the reach of the GAIA (Lindegren & Perryman 1994) concept. Even in the nearby T association, Tau-Aur dark cloud, apparent magnitudes of T Tauri stars tend to gravitate around the limiting  $B\approx 15$  mag (Jones & Herbig 1979).

The galactic halo dwellers—the globular clusters—are in many respects different from open clusters and associations. While the majority of open clusters have inconveniently low number of cluster members, the the globular clusters are severely crowded. This is further enhanced by their large distances. The parallaxes of the nearest globular clusters are at a level of 0.3 mas and are not detectable from the ground. There may be an exception if a strong point radio source is located in the cluster, then the VLBI technique could in principle provide a marginally significant parallax for the very nearest clusters, like M4 which does contain a pulsar PSR B1620-26 (Lyne et al. 1988). Proper-motion studies of globulars and, especially, an absolute proper-motion determination is presently quite a popular issue. However, the uncertainty of absolute proper motions is still somewhere at the 1 mas/yr level with little hope of significant improvement soon forthcoming. Due to the mentioned crowding problem and faint apparent magnitudes, a space mission like the GAIA concept can make only a limited contribution towards the globular astrometry, measuring the parallaxes and absolute proper motions of the nearest clusters using bright giant-branch stars. More details on the globular clusters can be found in the paper by Tucholke et al.

## 2. HIGH ACCURACY PARALLAXES

The possibility of obtaining parallaxes at a level of 10 microarcsec revolutionizes the research on star clusters and associations. Essentially for nearly all known open clusters the distances can be derived directly, avoiding the commonly used main-sequence fitting procedure. For the the first time it will be possible to deconvolve the apparent spatial distribution of the nearest clusters (for  $r<130~{\rm pc}$  if  $\sigma_{\rm T}<0.1~{\rm pc}$  is required), hence eliminating problems caused by mass segregation and low numbers of stars. In addition, it will yield highly accurate distance moduli and remove the apparent spread in absolute magnitudes due to the size of a cluster relative to its distance, provided the reddening effects are effectively minimized. The latter problem stresses the need for adequate on-board photometric follow-up measurements.

An individual cluster-member parallax with an accuracy of 10 microarcsec is, even at the distance of 30 kpc, still as good as  $(\sigma_{\pi}/\pi)=0.3$ . If at the same distance we can measure, for instance, ten cluster members, then  $(\sigma_{\pi}/\pi)$ can be reduced to 0.1, which means that within the Galaxy the distance determination itself is no longer a problem. However, at those distances the limiting factor is set by the intrinsic magnitudes of the brightest cluster-stars. This is further worsened by the interstellar reddening of clusters, especially, in the galactic plane  $(|b| < 10^{\circ})$ . In a study based upon distances and colour excesses of 462 open clusters Pandey & Mahra (1987) have derived the value of absorption in the galactic plane  $A_0 = 1.35 \text{ mag/kpc}$ . If we extrapolate this value beyond  $r \approx 3$  kpc where the cluster data are scarce and unreliable and ignore the clumpiness of the reddening material or its abundance as a function of galactic longitude, then at the distance of about 7 kpc the total absorption reaches  $A_v \approx 10$  mag! Perhaps, this  $A_v$  value is overestimated since between the spiral arms the absorption is negligible, therefore we adopt  $A_v \approx 5$  mag as more realistic for such a distance. Even so, with this value in hand at 7 kpc we will see only very few bright B0 or earlier main sequence stars as  $V \approx 15$  mag objects.

In the infrared K-bandpass ( $\lambda_o = 2.2 \mu m$ ) the interstellar absorption is ten times less than in V-bandpass. On the other hand, the absolute flux from an A0V star drops nearly 100 times (or 5 mag) in switching from Vto K-bandpass wavelengths (Johnson 1966). Thus, with an ideal detector, the same width of the bandpass and smooth change of absorption with distance, the gain of moving into intermediate infrared wavelengths is, probably, marginal. Even for the old open clusters the main sequence turnoff point will be essentially invisible in the K-bandpass wavelengths if it is not detectable in visual light. An exception might be the dense cores of giant molecular clouds where the absorption in V-bandpass can be enormous, completely hiding the population of young stellar objects in visual light (Lada et al. 1991). If those objects are of great concern then usage of infrared bandpass is a necessity.

Summarizing, the practical limit for distance determinations of the young open clusters and associations will most likely be at around 6–7 kpc. At this limit the parallaxes will be at least 0.14 mas, many times exceeding their errors. The distance limit for the old open clusters (approximately 3 kpc) can be estimated bearing in mind the luminosity of the giant-branch clump  $M_v=+1$  mag (Cannon 1970) and interstellar absorption. Virtually all open clusters and associations which can be observed by the proposed GAIA mission will get a very good distance estimate.

There is an interesting benefit from knowing the stellar parallaxes in cluster membership analyses. Usually, the membership probabilities are calculated from the propermotion distribution alone, or at best coupled with some model of the apparent distribution of cluster stars. If the distance of every single star is known we can effectively delete from consideration all obvious field stars as far as their distances are incompatible with the cluster location. Hence, we are left just with the space volume occupied by the cluster stars and random background, perhaps, at the density of 0.1 star/pc<sup>3</sup>. At this point the membership analysis based on a proper-motion distribution can be inverted, i.e. rather than looking for cluster stars we should find possible field stars which will be in solid minority.

### 3. HIGH ACCURACY PROPER MOTIONS

Most of all, the high accuracy proper motions will improve the membership segregation for all accessible gravitationally-bound star clusters. The principal limitation, though is the cluster internal velocity dispersion and binaries whose motion should be integrated over a sufficiently long time span. With this in mind, we will be able to establish clean and complete (down to  $V\approx 15$  mag) samples of cluster stars, finally solve the cluster halo (or corona) problem, check the reality of very loose clusters (Loden 1987) and find the boundary between the true clusters and unbound multiple star-systems.

In addition, there will be numerous discoveries of not yet known poorly-populated open clusters which do not show up as density enhancements but will manifest themselves in velocity space. It's very likely to get good statistics on the cluster disintegration process and discover the lost cluster members, presumably, dispersed along the cluster orbits in the Galaxy as stellar streams (Eggen 1994).

Before discussing the dynamic processes and kinematic implications we would like to mention some of the fundamental problems which are described in the excellent review by Mathieu (1985). There are a few problems which can be addressed largely, or sometimes only, by the study of open clusters:

- (a) the theoretical and observational aspects of small-N stellar systems;
- (b) the effects of a large stellar-mass spectrum and mass loss during the evolution;
- (c) a wide span in cluster ages and, hence a possibility to study fundamental dynamical time-scales: the crossing, relaxation and energy equipartition times;
- (d) testing of the external environment via variation of anisotropy in the cluster velocity distributions.

As indicated by Mathieu (1985) we have only just started to get a flavour of those effects and problems, whereas the GAIA mission would be an ideal tool to solve them.

Within a sphere of about 1 kpc around the Sun the open cluster proper motions of 10 microarcsec/yr accuracy will be dominated by internal velocity dispersion known to be at a level of 0.5-1.0 km/sec (Mathieu 1985). As it was shown in the previous section, only the open clusters at distances less than 6 kpc can be expected to be observable. For clusters within 2.5 kpc, however, the limiting magnitude is at  $M_v = +3$ , which in most cases ensures a fairly large number of cluster members being available. At 2.5 kpc a velocity dispersion of 1 km/sec translates into the proper motion dispersion of 0.09 mas/yr and, hence, will be easy to obtain. The proper motions combined with the parallaxes and preferably supplemented with radial velocities should give internal velocity dispersions in the centres of several hundred clusters, thus providing a picture of mass distribution in star clusters as a function of age and position in the Galaxy.

In a nearby cluster, like the Pleiades, the proper-motion accuracy of 10 microarcsec/year translates into an equivalent tangential velocity accuracy of 6 m/sec, which can be obtained for 400 to 500 cluster members ranging in mass from 3 to  $0.3M_{\odot}$ . At this level of accuracy we should be able to detect the halo internal motions. Velocity dispersion in the halo is crucial to the study of angular momentum distribution in the cluster. The halo

velocity dispersion contains information on the mass segregation process, tidal heating and dynamical evaporation. This also echoes in the spatial density distribution of stars which, for quasi-equilibrium states, is governed by the kinetic energy, potential energy and angular momentum distributions in all but the very centre of the open clusters. The potential energy distribution can also be estimated from the total mass of the cluster and the space density distribution. Discrepancies showing up when comparing the two estimates will indicate the differences between assumed and actual mass distribution (provided the cluster is old enough to have reached a state of semi equilibrium).

A spin-off of very high accuracy proper motions and parallaxes would be the possibility to detect the non-cluster stars accidentally crossing the cluster space (see previous section). In principle, interactions with the cluster stars as well as an occasional capture of the random passerby are possible in such a situation. The volume of the Pleiades cluster ( $\sim 20$  pc diameter) may contain a few dozens of such stars but, probably, very few of them will be associated with the cluster longer than the crossing time.

Using the high accuracy astrometric data along with radial velocities in studies of binaries can provide direct mass determinations, thus allowing to confront evolutionary masses against measured masses and improve the mass-luminosity relation. The favourable target, the short-period binaries ( $P < 10~\rm yrs$ ), are, however, not easy to resolve into visual pairs. On the other hand, the long-period binaries ( $P > 100~\rm yrs$ ) can be resolved with ease but do not have long enough time coverage to solve for the orbital parameters. In this regard the results from Hipparcos will be very instructive for planning GAIA.

Binaries are interesting for themselves but in clusters they are also the dominant factor in dynamical evolution and also possibly responsible for the blue straggler phenomenon. Probably, one of the most important contributions relative to the population of binary stars could be their reliable identification. It is believed that much of the noise and confusion seen in various datasets is just due to undetected binaries.

Between, say, 200 pc and 1000 pc the combined information of proper motions and parallaxes will provide a wealth of information on mass and luminosity functions of open clusters, as well as mass distributions. This information for clusters of different age and size should give an estimate of the star-loss rate for a typical open cluster, a value which can be compared with numerical simulation results which are strongly dependent upon the presence and character of binary stars in a cluster centre. Ultimately, we expect to re-assess the cluster mass function and its universality.

The all-sky astrometry can easily achieve the requirement for successful studies of associations: a uniform accuracy of the proper motions over a large area on the sky. Hence, large-scale streaming motions should become recognizable from a combination of distances and proper motions supplemented by radial velocities. Similarly to open clusters, it will be possible to create a 3-dimensional distribution of an association, in both position and velocity. This will most likely show the stars formed in the association but no longer bound to it, and will give a picture of what fraction of stars remains bound to the cluster after their formation.

Finally, we would like to mention the possibility to measure the radial acceleration in star clusters (van Leeuwen 1994). It was estimated that a 5-yr mission could lead to an acceleration accuracy of about 0.6 microarcsec/yr $^2$ . As a matter of fact the GAIA mission could be considered as the 'first epoch' since with repeated observations the accuracy can be improved by a factor of  $T^{-2.5}$ , where T is the time span between the epochs.

# 4. IMPLICATIONS FOR KINEMATICS AND ASTROPHYSICS

Of crucial importance in the study of clusters and associations is the combination of parallax, proper motion, radial velocity and photometric data. For open clusters within 6 kpc this should give a clear picture of both the distribution of clusters in the Galaxy as well as their kinematics, thus, to some extent describing the mass distribution in the Galaxy.

Beyond 6 kpc only some very young open clusters containing O and early B type stars (provided the interstellar absorption is low) would be observable because all main sequence stars later than A0 will be fainter than the limiting 15 mag. Distance determinations for clusters containing only one or two very bright objects (e.g. Cepheids) will be important mainly for those particular stars, as in most cases no further information is available to link that distance to a reliable selection of other cluster members.

Obtaining radial velocities seems to be the most difficult and time consuming part of the observable quantities required for the Galaxy kinematic studies. The total amount of radial velocity data lags well behind the acquired photometric and astrometric data due to well-known reasons. One of the most productive instruments, the radial velocity scanner CORAVEL, has in well over a decade produced radial velocity measurements of 1 200 red giants in 186 open clusters and of nearly 700 dwarfs in a handful of nearby clusters.

Note that a majority of those stars have multiple measurements and many suspected binaries have been observed over several years. The real problem in building a database of radial velocities is the binaries. It has been found, for instance, that the binaries among the cluster giants have a wide maximum between 150 to 1500 days in the distribution of periods. Evidently, one first of all has to identify such binaries and the follow-up observations must span at least over one orbital period.

A promising development is planned for the new 3.5-m WIYN telescope at Kitt Peak equipped with the multiobject spectrograph HYDRA. If accepted, one of the key projects for this instrument will be open cluster radial velocity determination (R. Mathieu, private communication). However, even this cannot fill the enormous gap in radial velocity data, unless some concerted effort is taken or new concepts/instruments developed.

The core of the GAIA mission surely is its astrometric part. However, in star-cluster studies information on a star's magnitude is also of paramount importance. A compilation of cluster photometry in the Database for Galactic Open Clusters (BDA) by Mermilliod (1993) shows discordant photometry for a number of clusters. Apparently, it is quite difficult to ensure uniform external photometric accuracy from the ground-based obser-

vations. Likewise, it is absolutely necessary to find the star's reddening and metallicity [Fe/H] via intermediate passband photometry. These effects can distort the true luminosity estimates and, in the case of metallicity, also influence the path and rate of stellar evolution. In addition, from some photometric indices it would be desirable to get the estimates of CNO element abundances which provide a test of stellar evolution theory.

A detailed study of open cluster main-sequence turnoff regions provides the needed check on the stellar structure and evolution theories. To illustrate this here we reproduce a new colour-magnitude diagram (CMD) of the old open cluster NGC 3680 and the theoretical isochrone fit to the CMD (Kozhurina-Platais et al. 1995). In Fig. 3 the data-points represent all possible cluster members with membership probability  $P \geq 5\%$  within an area of 13' in diameter around the cluster centre. The BV photometry is based on new CCD observations at CTIO with the 0.9-m telescope. The stellar tracks were calculated using the Yale Rotating Evolution Code in the range of masses from 0.7 to 2.5  $M_{\odot}$  and subsequently converted into isochrones of different ages.

As convincingly shown in Fig. 3 a clear preference should be given to the 1.6 Gyr isochrone with an overshoot of  $P_{\rm mix}{=}0.2$  at the edge of convective core. This shows that convective overshooting does exist and is an observable phenomenon near the main sequence turnoff or, in other words, how the fine astrometric data combined with excellent photometry can be crucial in testing the theoretical model.

Figure 3. Color-magnitude diagram of the open cluster NGC 3680. The dashed line shows the standard model isochrone fit (yielding the age of 1.3 Gyr) adjusted for the true distance modulus of 10.25 and color excess  $E_{\rm B-V}=0.04$  mag. The solid line represents the model with overshoot at the edge of convective core, and is giving a larger age of 1.6 Gyr. It is virtually impossible to fit the CMD features at the main sequence turnoff point by using the standard model.

Not many old clusters will be observable due to their intrinsic faintness. As indicated by Phelps et al. (1994) there are about 20 open clusters with age similar to M67 or older. However, only half a dozen of those clusters have the upper part of the color-magnitude diagram brighter than  $V \approx 15$  mag.

Additional information on stellar evolution is provided by the photometric observations of the pre-main sequence members of star clusters. Regular monitoring of G and later-type star brightness can directly provide rotational periods (see e.g. van Leeuwen et al. 1987). An understanding of the star's angular momentum evolution can shed light on magnetic fields and mixing processes in stars as well as provide clues on discs around the young stars. In addition, the rotational periods furnish information on physical conditions in star formation regions well after those regions were depleted of gas. It is important to realize that  $v \sin i$  estimates from stellar spectrum lines cannot substitute for direct rotational periods because of unknown angle of incidence i and uncertainty in the star's radius estimate.

#### 5. CONCLUSIONS

With the astrographic refractors getting out of regular use and a clear trend of replacing the photographic emulsions with CCDs it is becoming increasingly difficult to improve and enlarge upon the astrometric information on open clusters. Therefore, a high-accuracy astrometric space mission is a most welcome development for the studies of clusters and associations, in particular if this mission also provides reliable multi-colour photometric data. At the accuracies of 10 microarcsec a wealth of data becomes available for single stars and small stellar systems, like open clusters, making it possible to study the structure and evolution of stars and the Galaxy as a whole.

The increased length of the mission would affect directly the possibility to segregate proper motion and the orbital motion, which is important in two ways: reduces disturbance in the study of the kinematics, and facilitates the study of binary systems with well-determined ages. Including the possibility to measure fainter stars would probably mainly benefit the study of luminosity and mass functions, and the internal dynamics of the nearest clusters. For the open clusters and associations a selection of infrared detectors does not provide any expected gain in securing more distant objects unless they contain numerous red giants.

Within the current specifications for GAIA an amount of data will be obtained that will revolutionize through the study of star clusters and associations our knowledge of star formation, structure and evolution.

#### ACKNOWLEDGEMENTS

We are grateful to P. Demarque, R. Larson and S. Barnes for helpful discussions on various aspects of this paper.

#### REFERENCES

- Blaauw, A., 1991 in The Physics of Star Formation and Early Stellar Evolution, p. 125, eds. C.J. Lada and N.D. Kylafis, NATO ASI Series C.
- Cannon, R.D., 1970, MNRAS, 150, 111
- de Zeeuw, P.T., Brown, A.G.A., Verschueren, W., 1994, in Galactic and Solar System Optical Astrometry, p. 191, eds. L.V. Morrison and G.F. Gilmore (CUP)
- Eggen, O.J., 1994, in Galactic and Solar System Optical Astrometry, p. 191, eds. L.V. Morrison and G.F. Gilmore (Cambridge University Press)
- Johnson, H.L. 1966, Ann. Rev. Astron. Astrophys., 4,
- Jones, B.F., Herbig, G.H., 1979, AJ, 84, 1872
- King, I.R. 1980, in IAU Symp. 85 Star Clusters, p. 139, ed. J.E. Hesser (Reidel, Dordrecht)
- Kozhurina-Platais, V., Demarque, P., Platais, I., Orosz, J.A., Barnes, S., 1995, in preparation
- Lada, E.A., DePoy, D.L., Evans, N.J., Gatley, I., 1991, ApJ, 372, 171
- Lindegren, L., Perryman, M.A.C., 1994, GAIA Global Astrometric Interferometer for Astrophysics, Noordwijk, the Netherlands
- Loden, L.O., 1987, Irish Astron. J., 18, 95.
- Lyne, A.G., Biggs, J.D., Brinklow, A., Ashworth, M. and McKenna, J., 1988, Nature, 332, 45
- Lyngå, G., 1987, Lund Catalogue of Open Clusters, Fifth edition, CDS de Strasbourg
- Mathieu, R.D., 1985, in IAU Symp. 113 Dynamics of Star Clusters, p. 427, eds. J. Goodman & P. Hut (Reidel, Dordrecht)
- Mermilliod, J.-C., 1993, in Workshop on Databases for Galactic Structure, p. 27, eds. A.G.D. Philip, B. Hauck and A.R. Upgren (Davis Press, Schenectady)
- Pandey, A.K., Mahra, H.S., 1987, MNRAS, 226, 635
- Phelps, R.L., Janes, K.A., Montgomery, K.A., 1994, AJ, 107, 1079
- Sellier, A., Turon, C. and Mermilliod, J.-C., 1992, The Hipparcos Input Catalogue, vol. 7, Annex 3, ESA SP-1136
- van Altena, W.F., Cudworth, K.M., Johnston, K., Lasker, B., Platais. I., Russell, J.L. 1993, in Workshop on Databases for Galactic Structure, p. 250, eds. A.G.D. Philip, B. Hauck & A.R. Upgren (Davis Press, Schenectady)
- van Leeuwen, F., 1985, in IAU Symp. 113 Dynamics of Star Clusters, p. 579, eds. J. Goodman & P. Hut (Reidel, Dordrecht)
- van Leeuwen, F., Alphenaar, P., Meys, J.J.M, 1987, Astr. Astroph. Suppl. Ser., 67, p483
- van Leeuwen, F., 1994, IAU Symp. 166 Astronomical and Astrophysical Objectives of Sub-Milliarcsecond Optical Astrometry, 15-17 August 1994, The Hague