The Equation of State of Neutron Stars:

Neutron-star masses, radii and internal composition.

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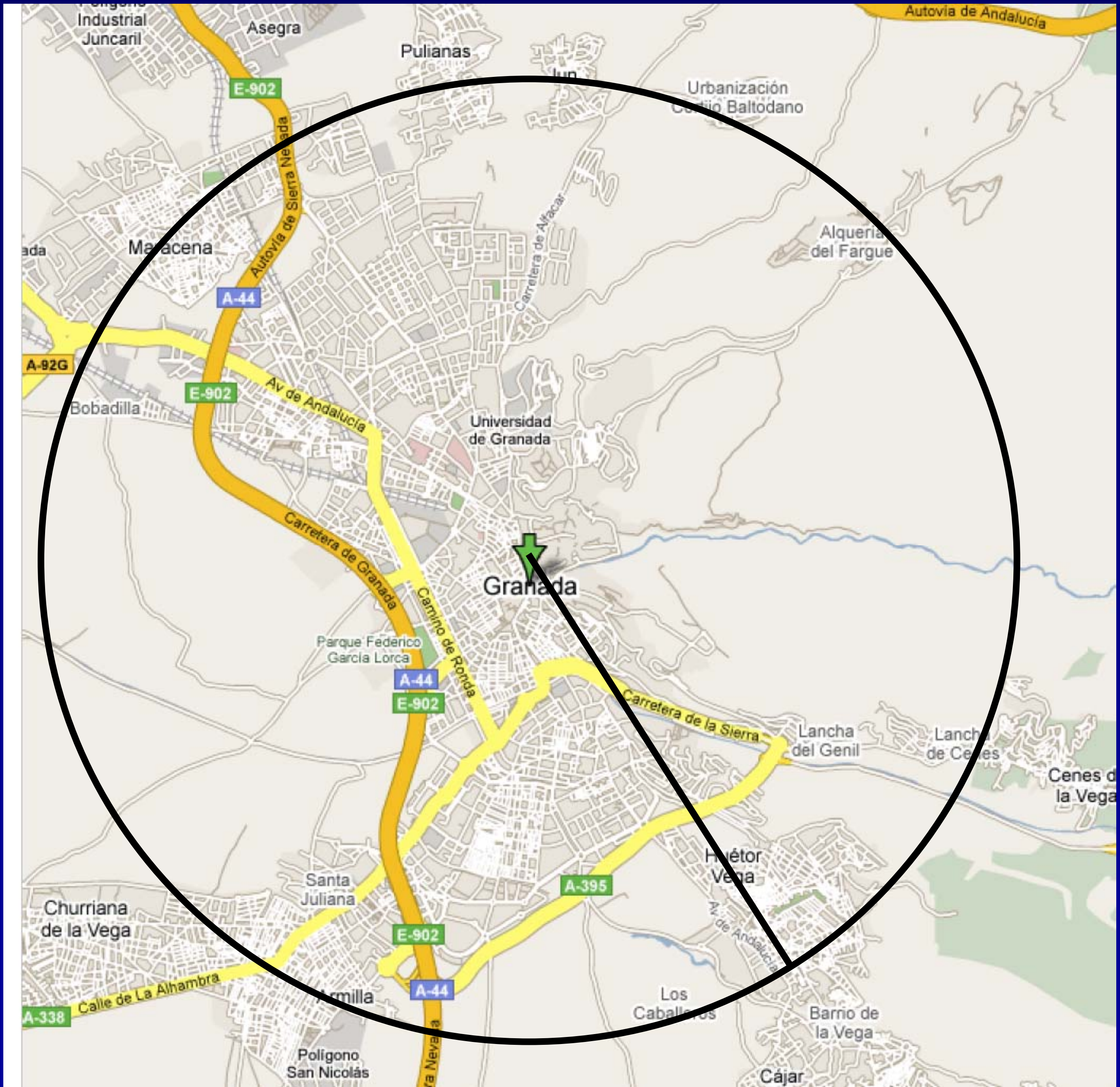
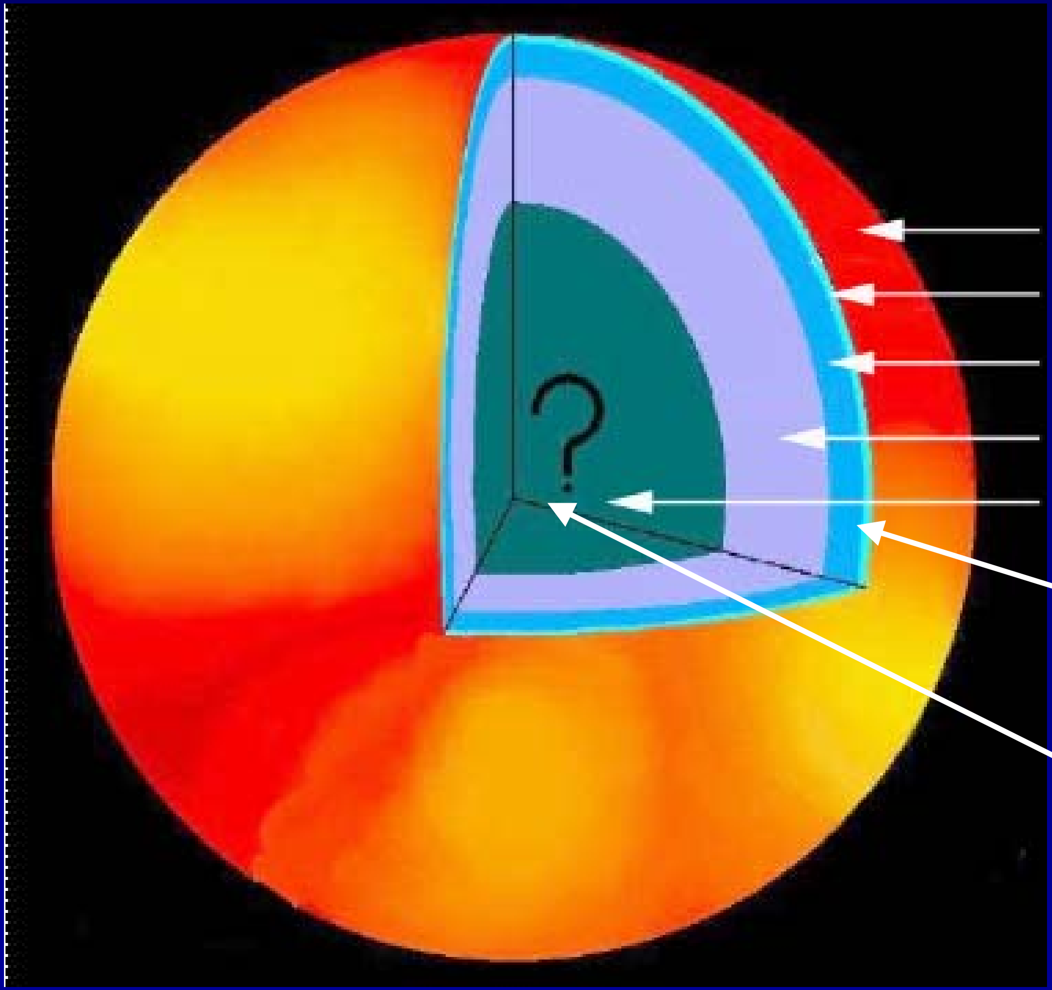
Neutron Star



Radii ~ 10 km

Masses ~ 1 – 2 $M_\odot$

Equation of State of nuclear matter.



Atmosphere
Envelope
Crust
Outer core
Inner core

$\rho \sim 10^{14}$ gr cm^{-3}

$\rho \sim 10^{15} - 5 \times 10^{15}$ gr cm^{-3}

Figure courtesy of D. Page

Equation of State of nuclear matter.

The interactions between the particles that constitute stars determines the equation of state (EOS), a relation between pressure and density, $P = P(\rho)$, that can be translated into a mass-radius relation, $M = M(R)$. For neutron stars:

$$\frac{dP}{dr} = -\frac{G\rho m}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{mc^2}\right)\left(1-\frac{2GM}{c^2 r}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

plus a prescription for $P = P(\rho)$

e.g., Shapiro & Teukolsky

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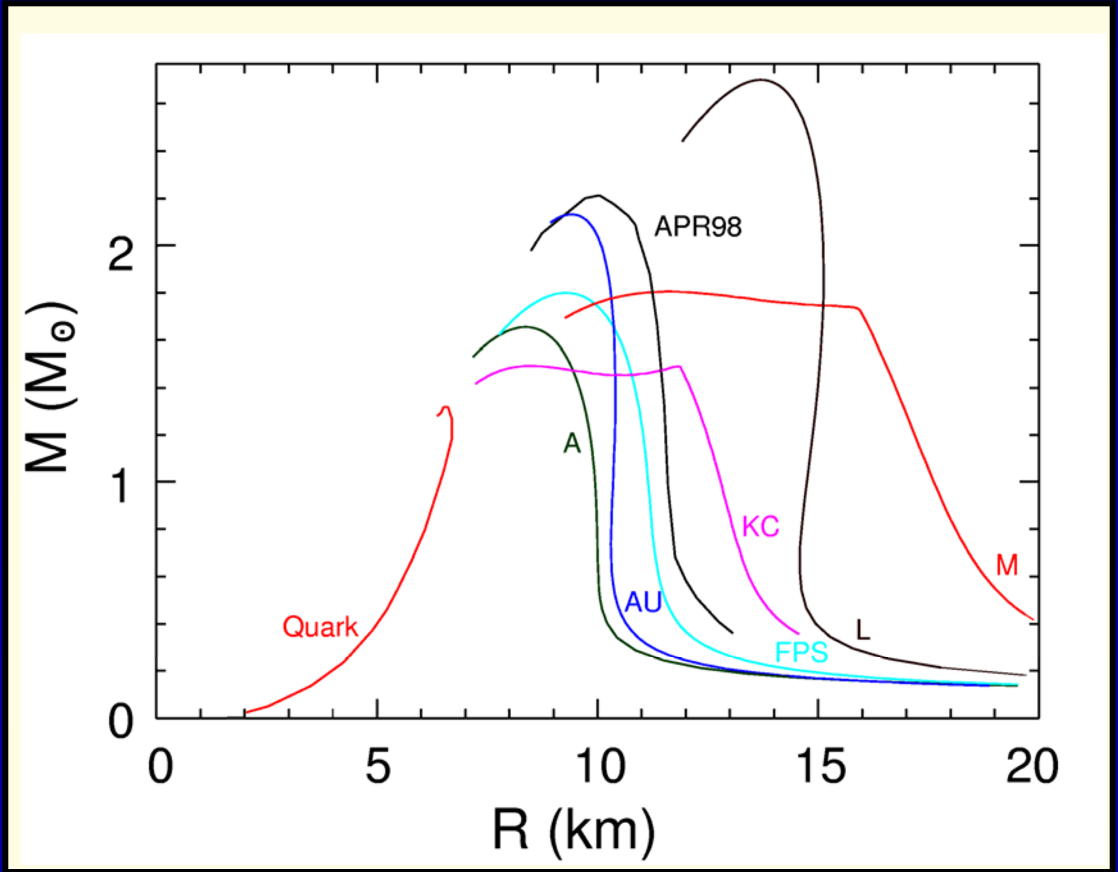
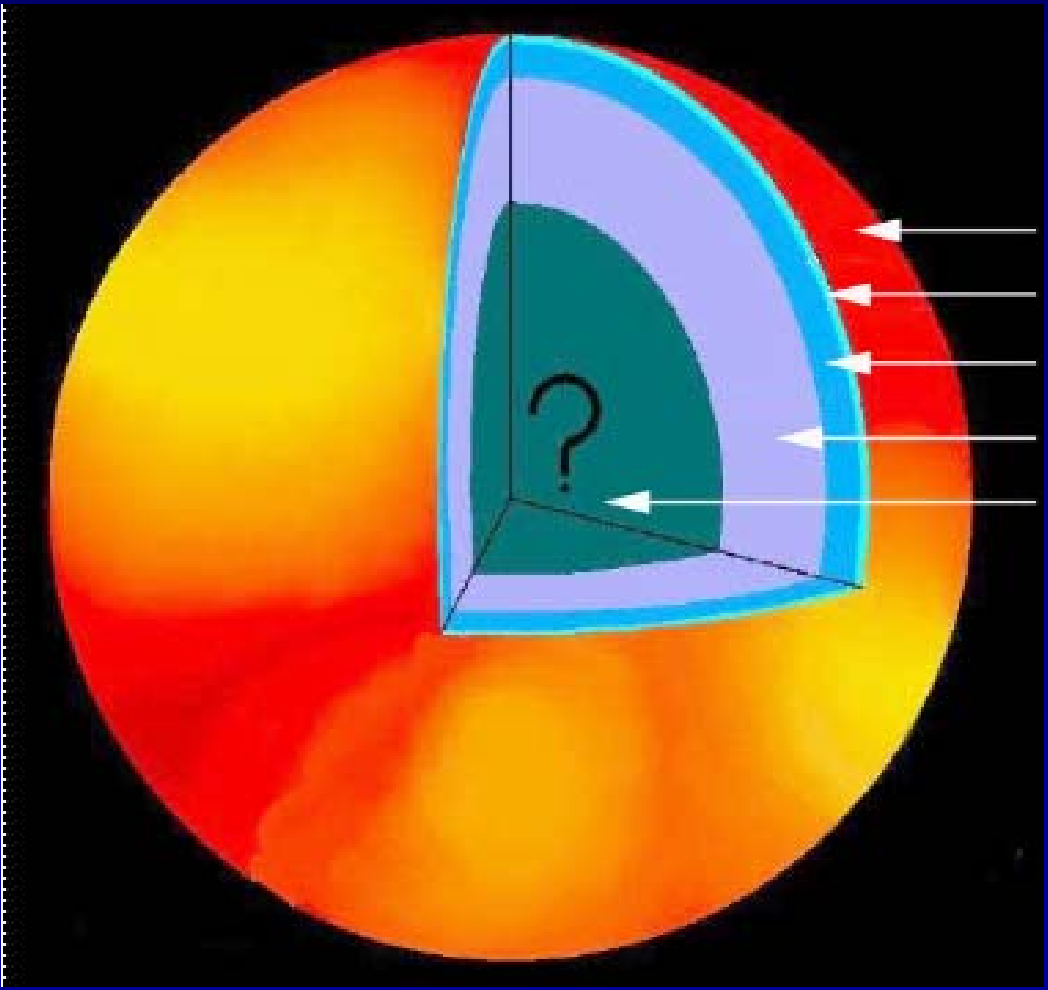


Cook, Shapiro & Teukolsky; Akmal,Pandharipande, Ravenhall; Heiselberg

Equation of State of nuclear matter.



Neutron star equation of state is known for the outer parts of the star, but is unconstrained for the high-density inner core.

This uncertainty arises from an inability to extrapolate our knowledge of normal nuclei (with 50% proton fraction) to the high-density regime of nearly 0% proton fraction.

Consequently, equation of state models depend upon assumptions about the matter phase of the inner core:

- Hadronic matter
- Bose-Einstein condensates (pion, kaon)
- Quark matter

Each new phase of matter increases the compressibility of the star, and for the same mass, the radius of the neutron star can be smaller.

Measurements of the NS mass and radius are the only way to constrain the models.

Mass and Radius in physical units

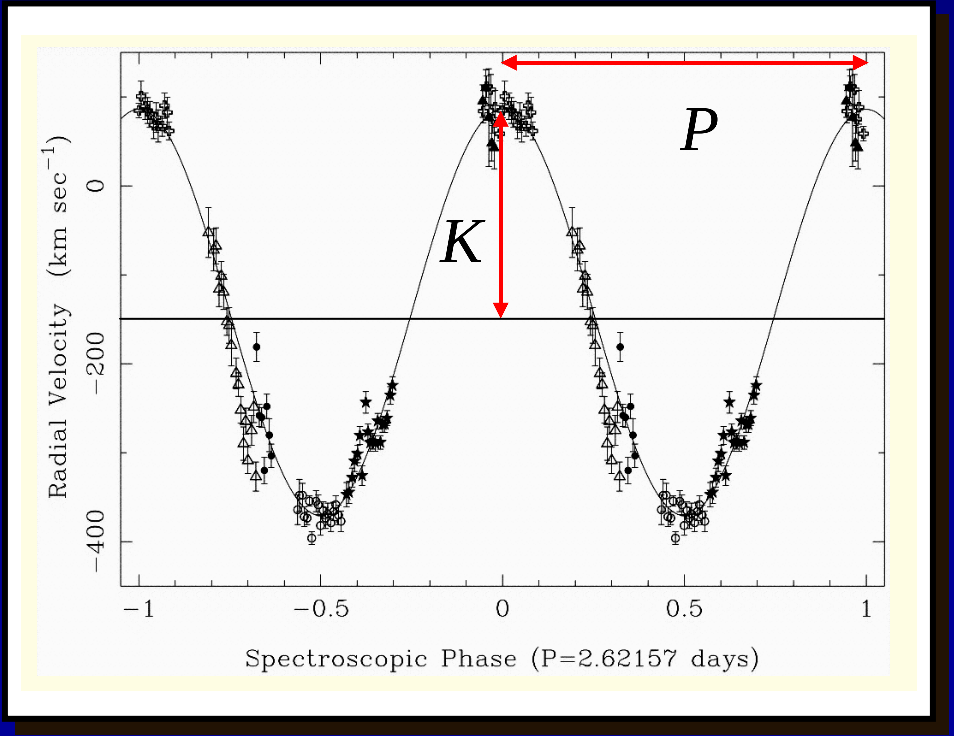
MASS:

From mass function (binary systems):

$$f_1 = \frac{PK^3}{2\pi G} = \frac{M_1 \sin^3 i}{(1 + M_2/M_1)^2}$$

If both spectra are measured, exchange M_1 and M_2 to get f_2.

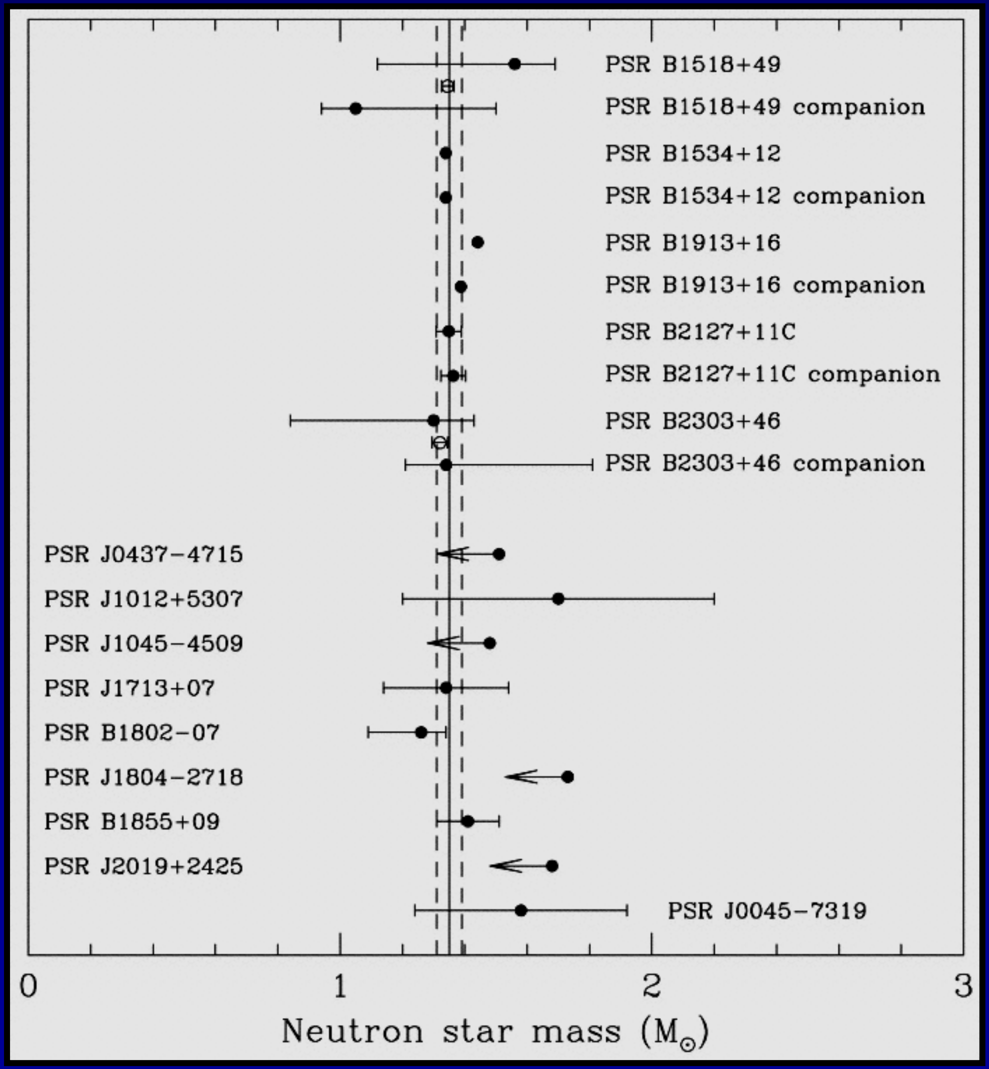
If the *inclination i* is known (e.g., eclipsing systems), the two equations can be solved for M_1 and M_2.



Dynamical constraints of the masses.

Masses of NS obtained from pulsars in binary systems.

$$\langle M \rangle = 1.35 \pm 0.04 M_{\odot}$$



Thorsett & Chakrabarty

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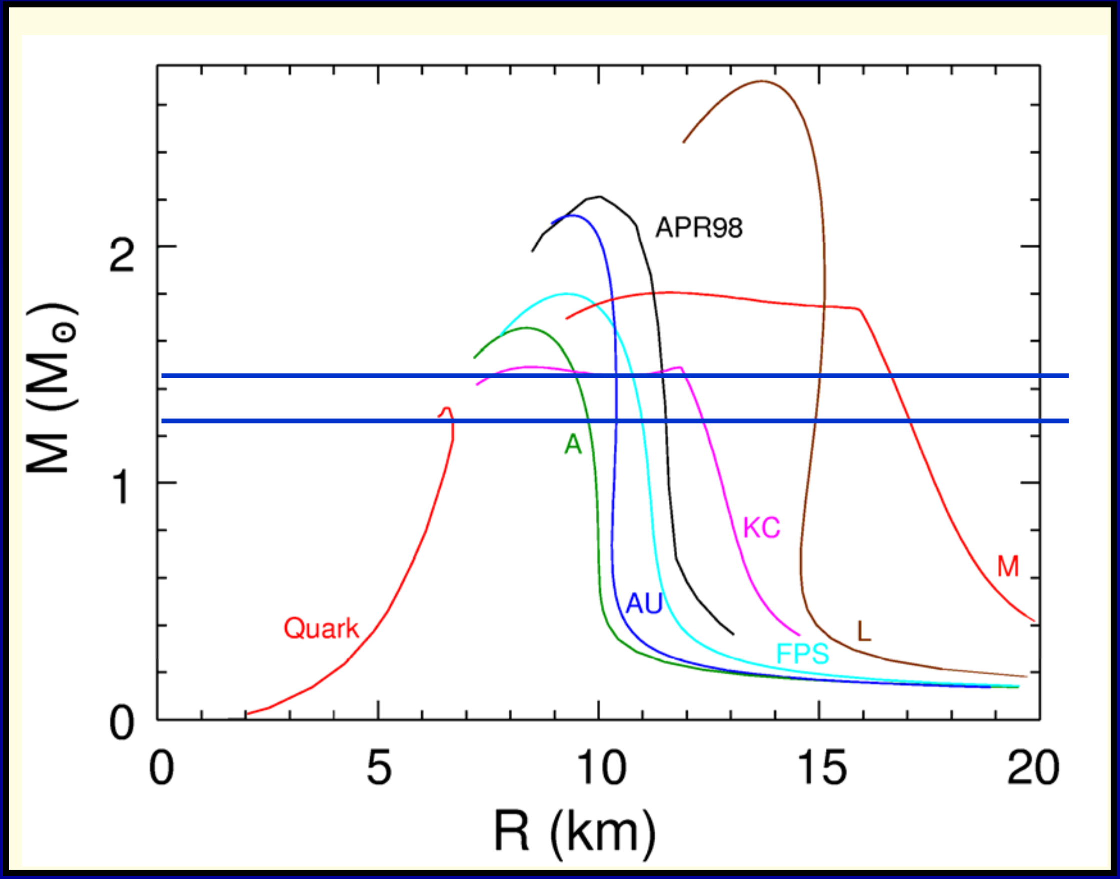
Some NS masses are 0.1% accurate!



Neutron star EoS measurements and constraints

Spectroscopy:

- Solid angle from flux and effective temperature → R_∞/d.
- Cooling → Internal structure.
- Redshifted photospheric lines → M/R, potentially M/R^2.
- Spectral line profile → M–R.

Timing:

- Kilohertz quasi-periodic oscillations → M–R.
- Pulse-profile fitting (persistent/bursts pulsations) → M–R.

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Radius from solid angle measurement

Classical astronomy: Use continuum spectrum to measure flux and effective temperature, hence determine solid angle of emitting source:

$$\left(\frac{R_\infty}{d}\right)^2 = \frac{f_\infty}{\sigma T^4_{\mathrm{eff},\infty}}.$$

For known distance, this yields radius. Temperature, flux, and radius are subject to significant gravitational redshift:

$$T_{\mathrm{eff},\infty} = \frac{T_{\mathrm{eff}}}{1+z}, \quad f_\infty = \frac{f}{(1+z)^2}, \quad R_\infty = R(1+z).$$

Radius from solid angle measurement. Complications

- Strong neutron-star magnetic fields complicate atmosphere models.
- Magnetospheric activity in young neutron stars can generate strong non-thermal emission that outshines the thermal emission.
- Pulsar activity in old neutron stars can lead to highly anisotropic temperature distribution, with poles hotter than rest of the star.
- For neutron stars in accreting binaries, accretion disk emission generally much brighter than thermal emission from neutron star surface (except in transient systems in quiescence).
- Distance measurements usually of limited precision. Exceptions: globular cluster sources, parallax (eventually with *Gaia*), radius expansion bursts.

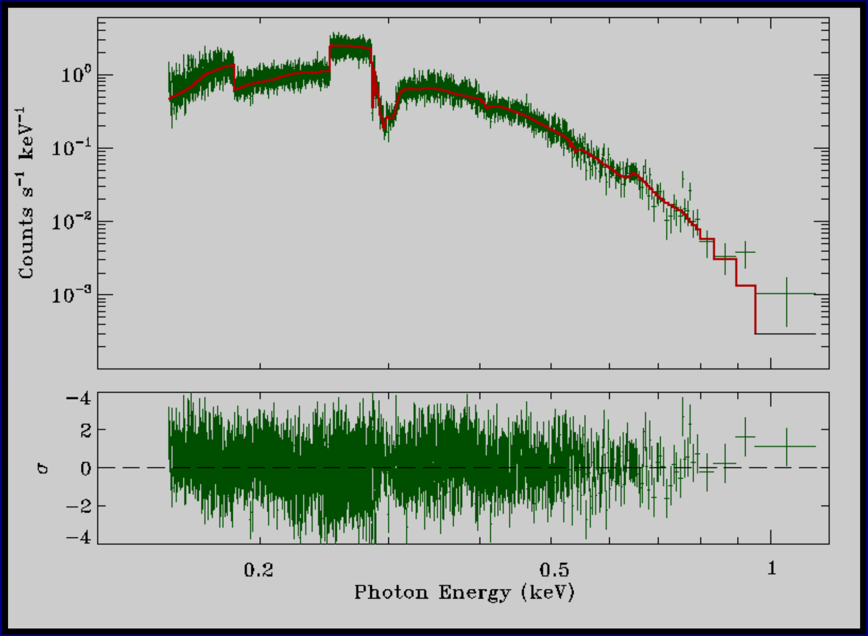
RX J1856.5–3754

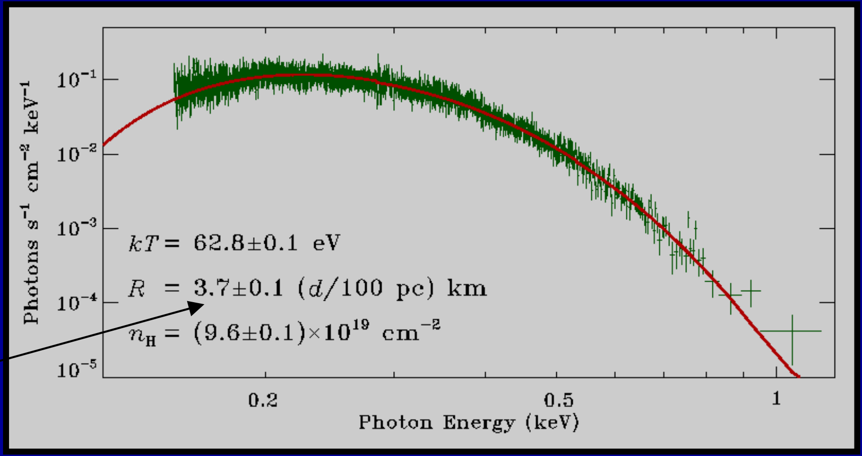
Chandra/LETGS observations of the nearby, isolated NS, RX J1856.5–3754.

$\mu = 0.3' / \text{yr}$

$d = 117 \pm 12$ pc

Perfect blackbody fit





Quark star?

Burwitz et al.; Pons et al.; Walter et al.; Drake et al.

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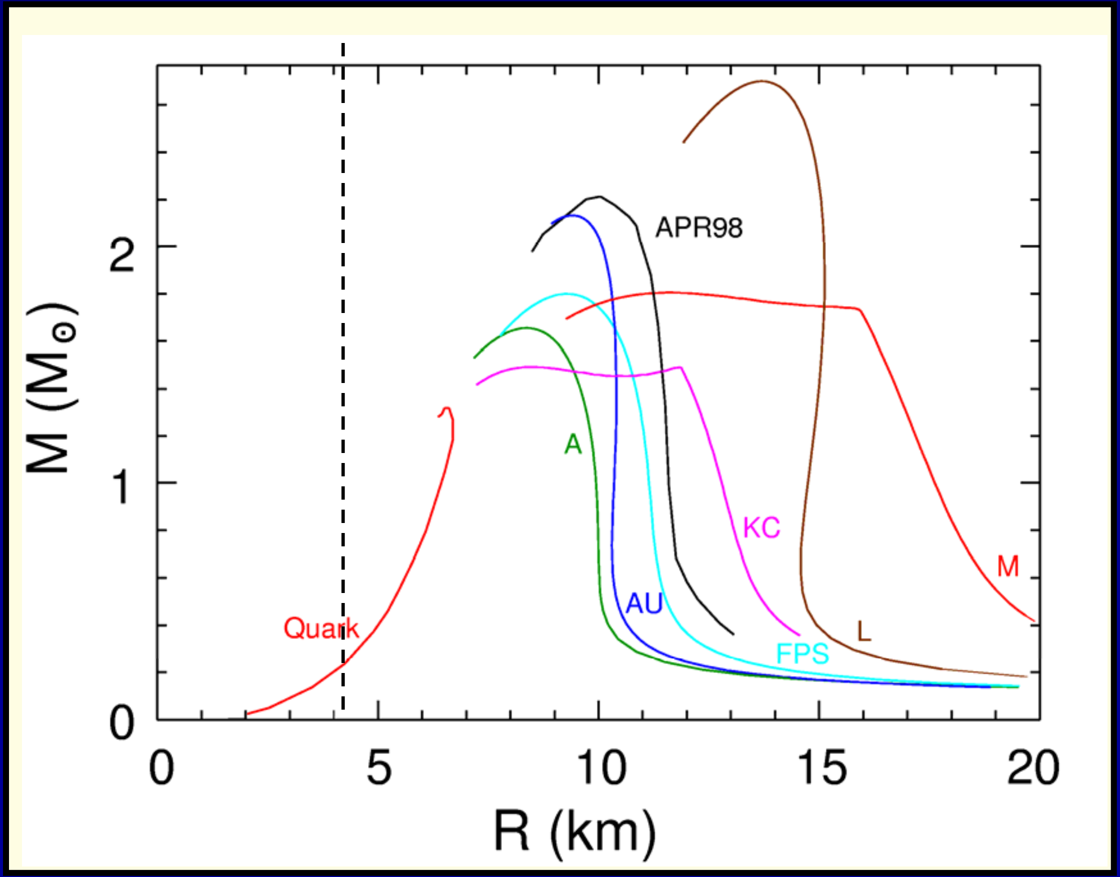
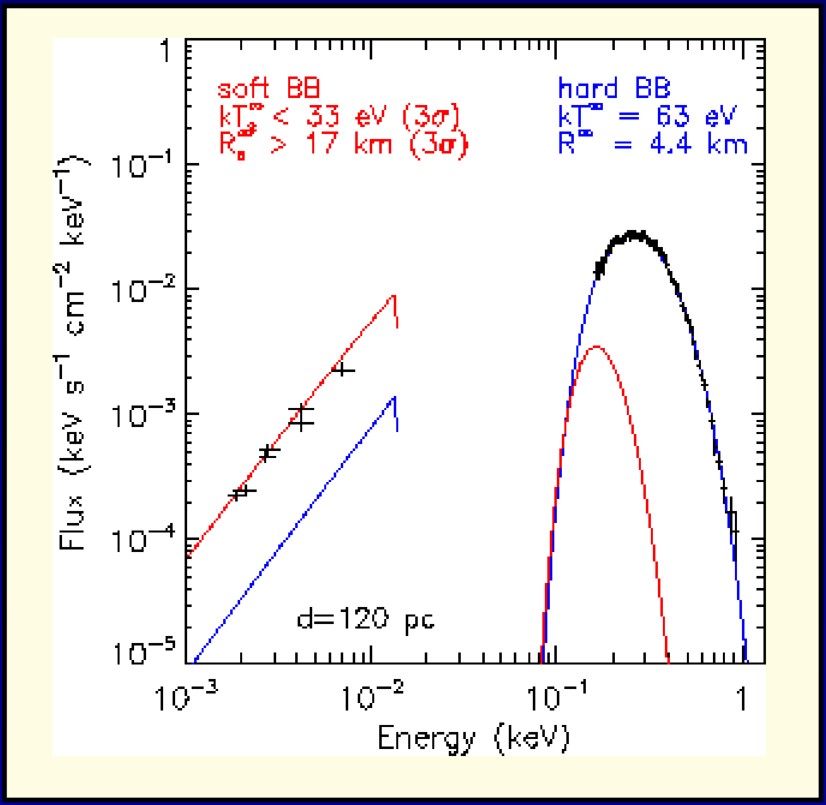


Neutron star radius?



However, the model that fits the X-ray data under-predicts the HST and EUVE data by a factor of ~ 10.

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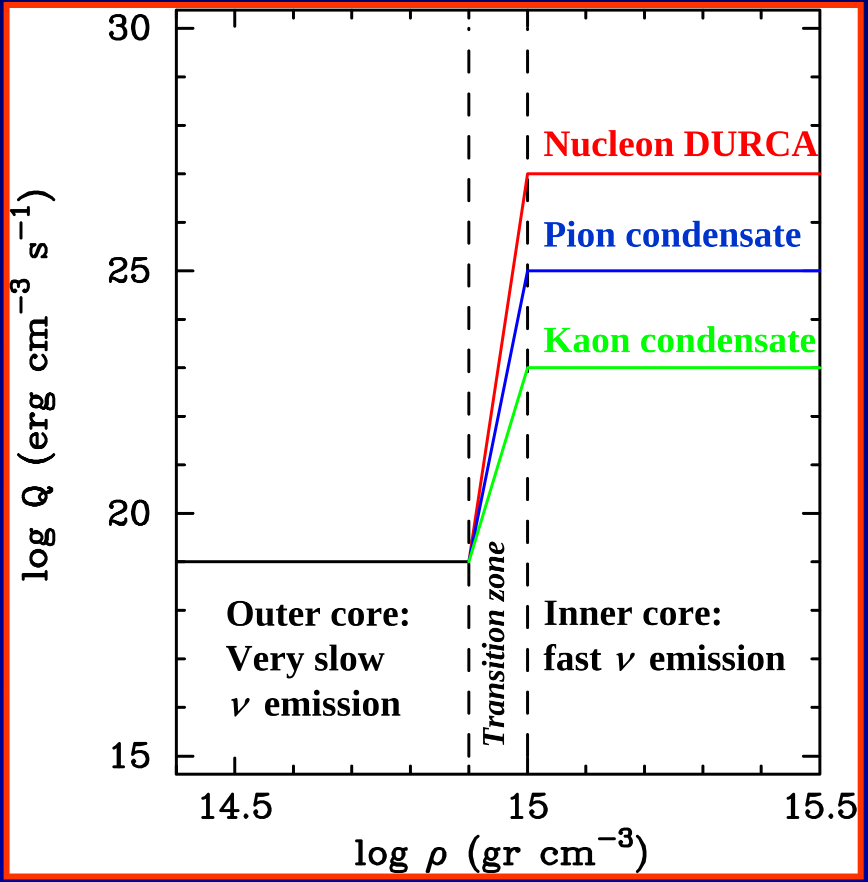
Cooling neutron stars



<u>Direct URCA</u>

Nucleonic matter

$$n \rightarrow p + e + \bar{\nu}$$
$$p + e \rightarrow n + \nu$$

Quark matter

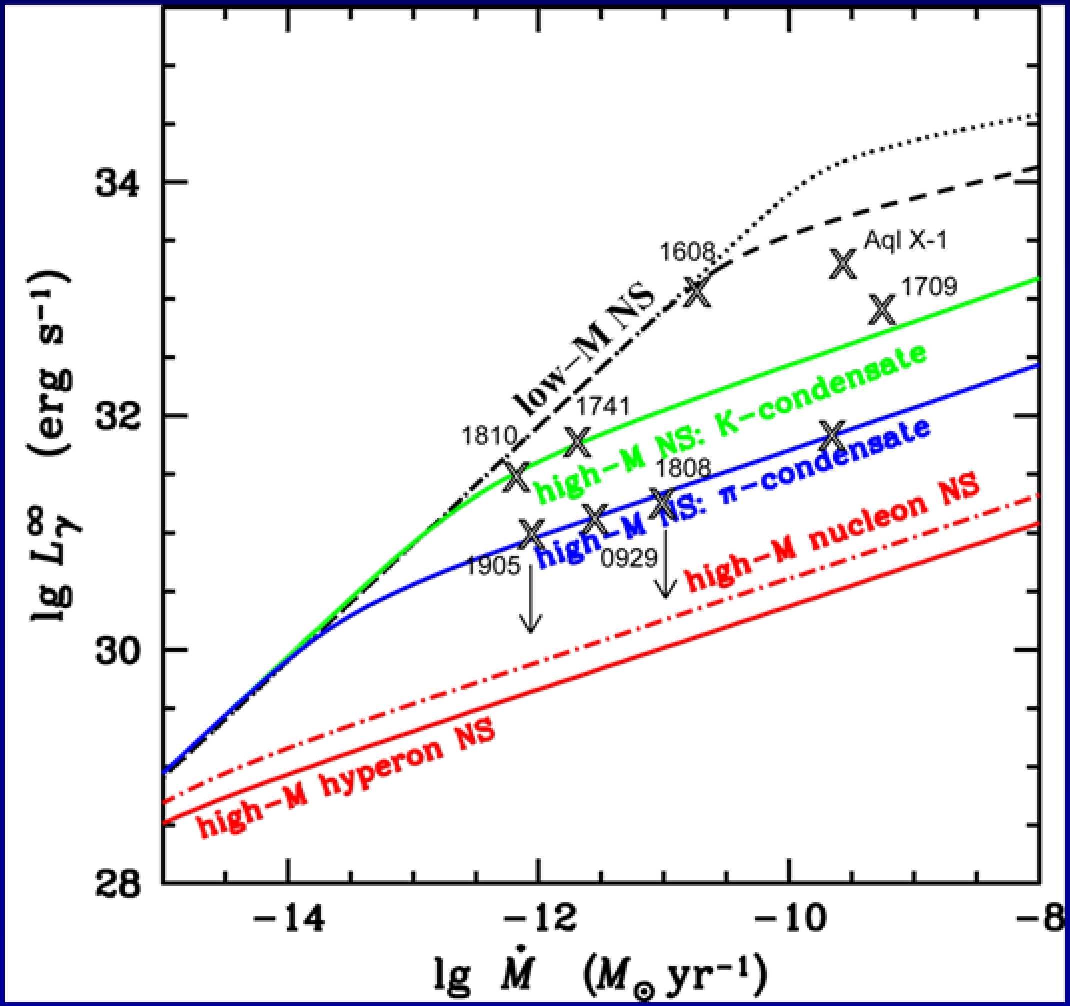
$$d \rightarrow ue\bar{\nu}$$
$$ue \rightarrow d\nu$$

<u>Modified URCA</u>

$$n + n \rightarrow n + p + e + \bar{\nu}$$
$$n + p + e \rightarrow n + n + \nu$$

Yakovlev & Pethick

Cooling neutron stars



Due to evolutionary arguments, average mass-accretion rate cannot be less than ~ 10^{-12} $M_\odot$/yr.

For NS in quiescent soft-Xray transients, bolometric surface emission only depends on cooling in the core.

Observing very low bolometric luminosities would discard Bose-Einstein condensates EoS.

Jonker et al.; Heinke et al.; Wijnands et al.; Rutledge et al.

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Gravitational redshift

- The time dilation: $dt = (-g_{00})^{-1/2} \Delta t$

  g_{00} : time-like component of the metric tensor,

  Δt : the proper time in the gravitational field,

  dt : time measured by an observer at infinity,

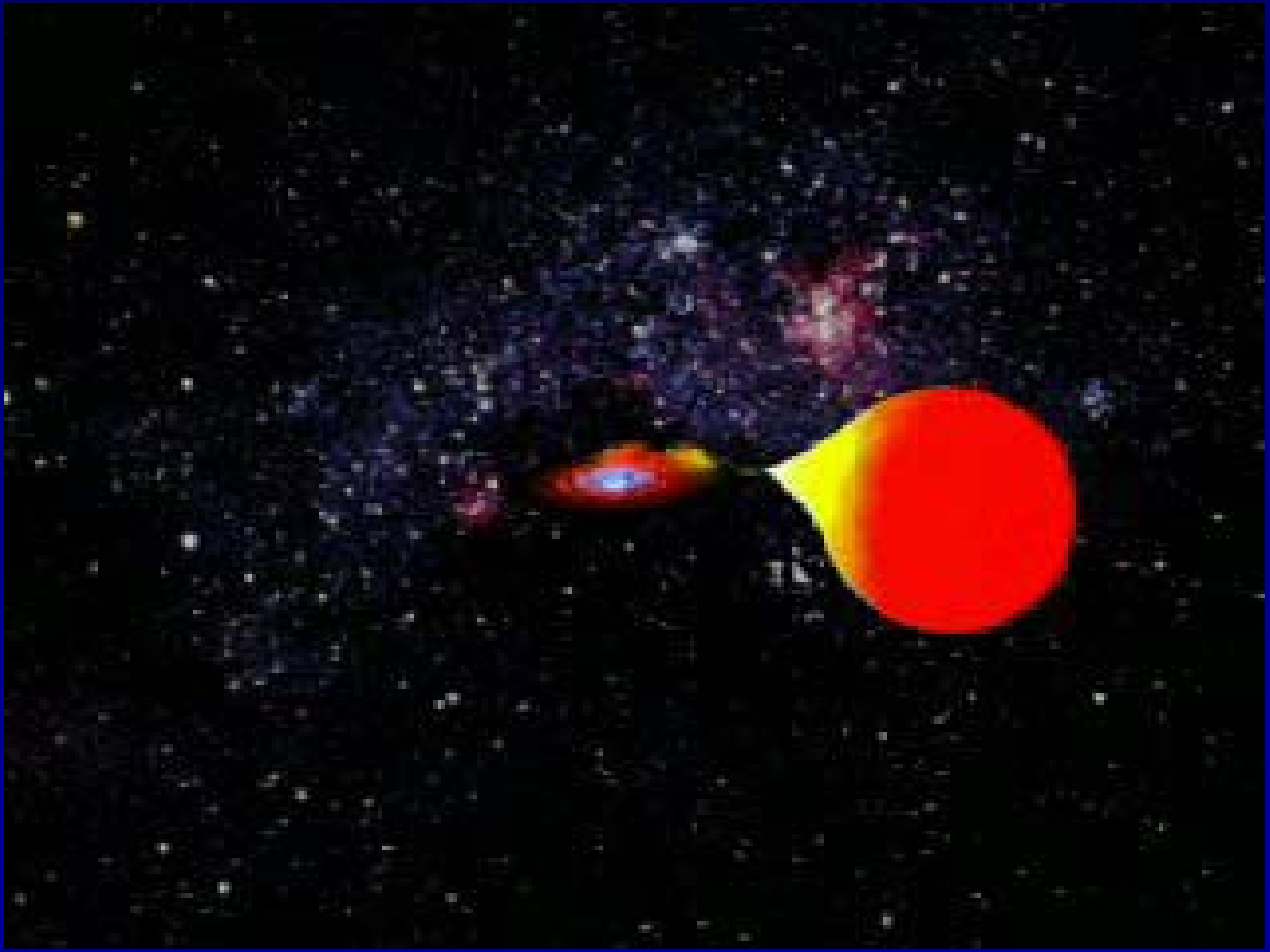
which in the Schwarzschild's metric yields:

$$\frac{\Delta t}{dt} = \left(1 - \frac{2MG}{c^2 R}\right)^{1/2} = \frac{\nu}{\nu_0} = \frac{\lambda_0}{\lambda}.$$

From a measurement of the gravitational redshift, $z = \Delta\lambda/\lambda$, we can infer the compactness (M/R) of the neutron star:

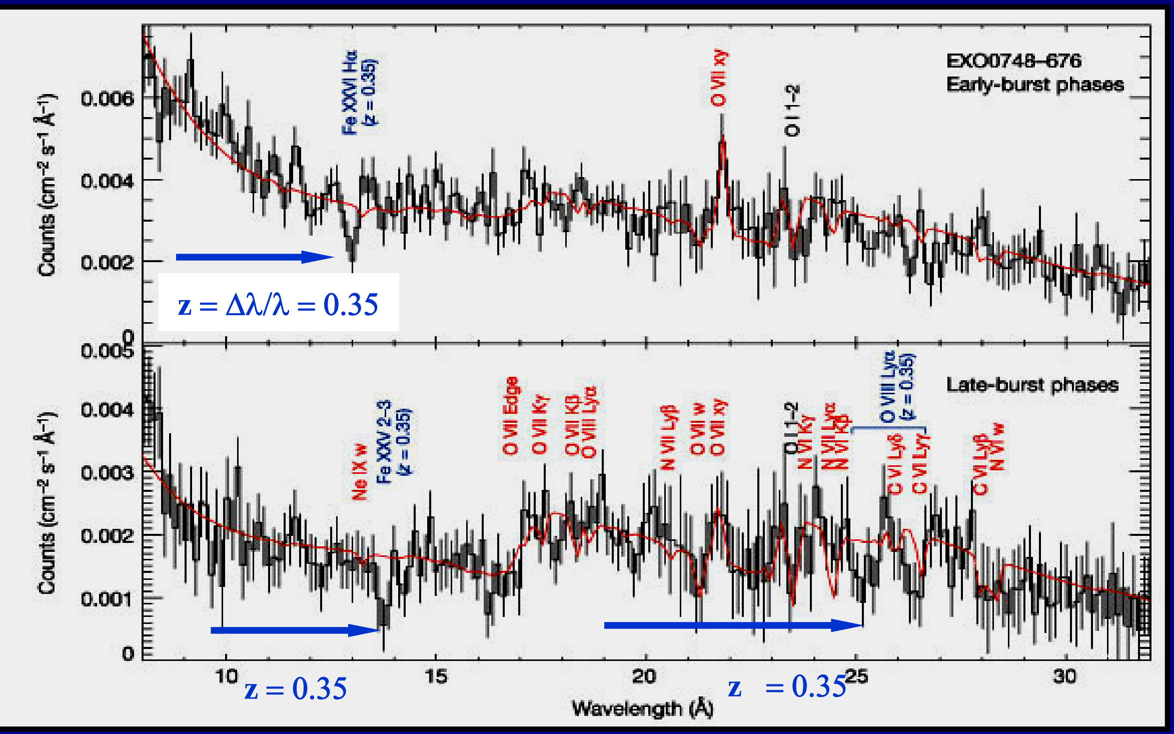
$$\boxed{\frac{M}{R} = \frac{c^2}{2G}\left[1 - (1+z)^{-2}\right]}$$

X-ray Bursts: Thermonuclear explosions on the surface of neutron stars



Photospheric absorption during X-ray bursts

EXO 0748–676, a known X-ray burster
XMM-Newton observed it as a calibration target:
~ 335 ks with RGS cameras; 28 X-ray bursts.



Absorption lines at λ 13.0Å and λ13.7Å in the combined early- and late-burst spectra, respectively.

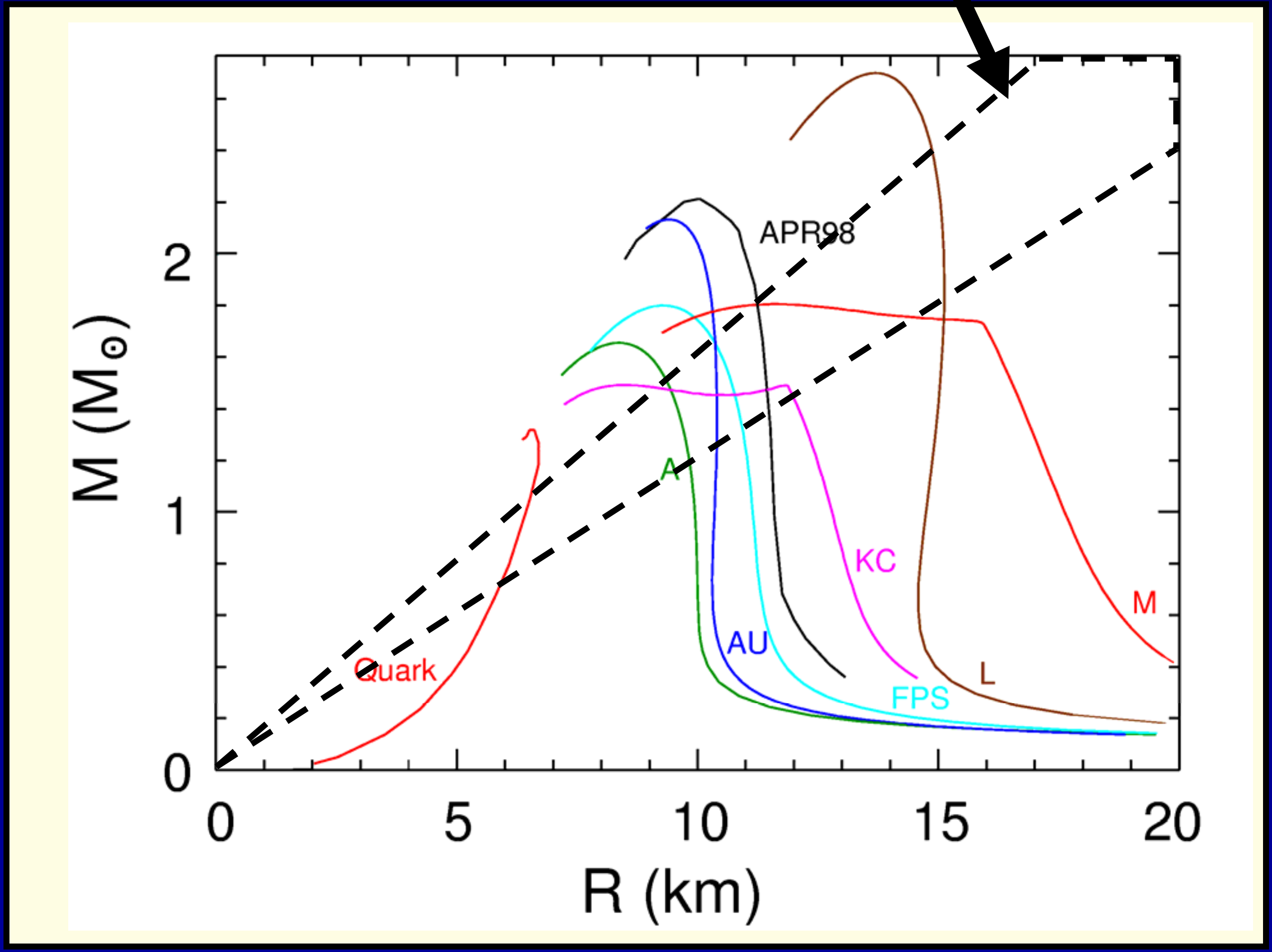
FeXXVI Hα (n = 2–3) and FeXXV Heα (n = 2–3), respectively, at the same redshift z = 0.350 ± 0.005.

The feature at λ25.3 Å in the late-burst spectrum would then be consistent with OVIII Lyα.

Cottam, Paerels & Méndez

EOS – Constraining mass and radius

$$M/R = 0.15 \pm 0.01 M_{\odot}/\mathrm{km}$$



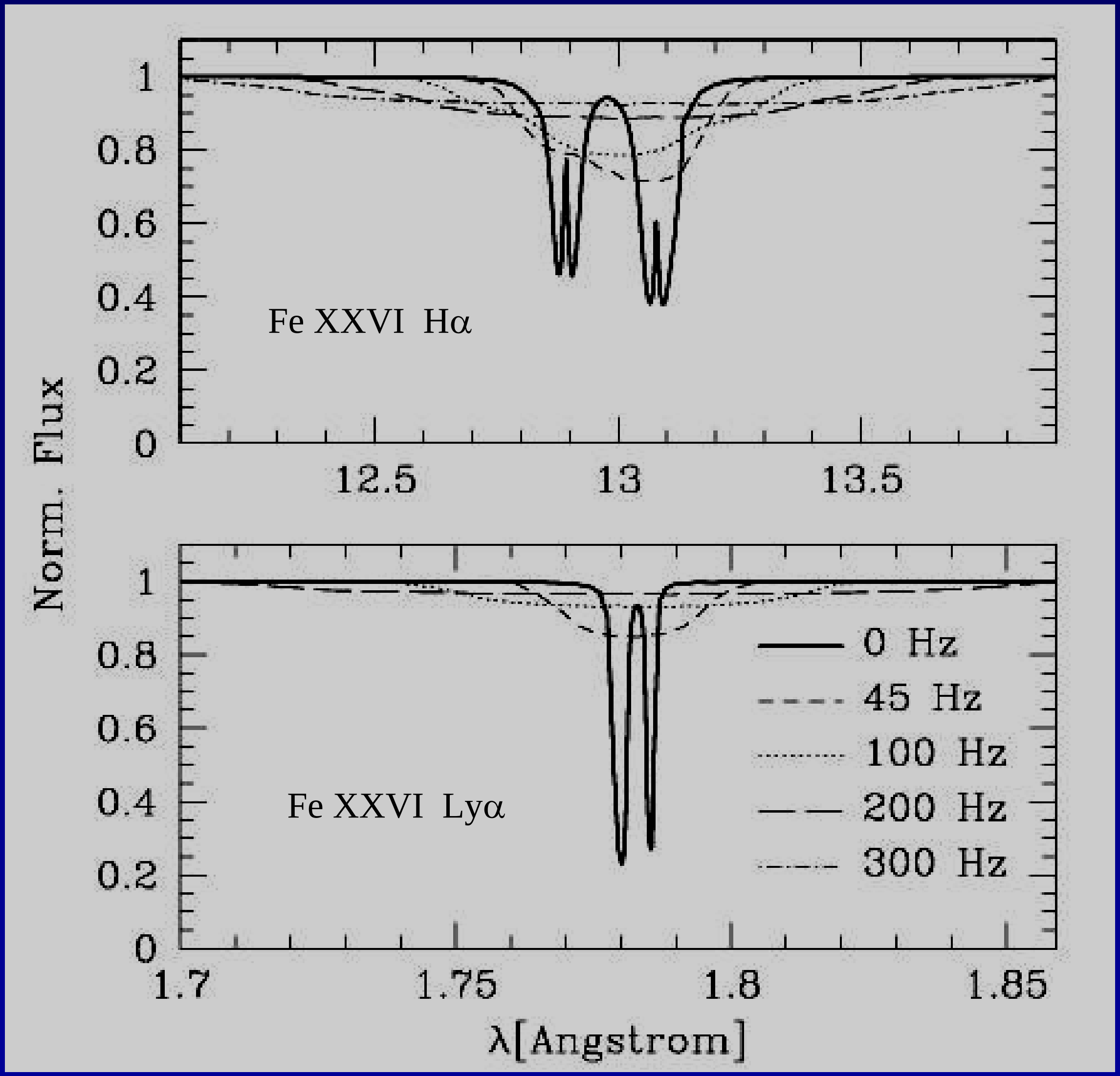
Issues

- Rotational broadening very serious for >100 Hz; complex line profile.

- Follow-up observations of EXO 0748–676 with *XMM-Newton* do not detect similar line features. Entire source spectrum is very different.

- Are the lines real? Reanalysis of previous data verifies significance of original detection. Interesting *Chandra*/HETGS data burst spectrum.

Rotational broadening of “structured” lines



- Even for slow rotators, rotational broadening will be significant if the intrinsic line is not narrow (e.g. fine structure, pressure broadening, magnetic splitting.)
- Redshift measurements are definitely feasible. Extracting parameters from line profiles may be possible, but detailed calculation of line physics required.

Chang et al.

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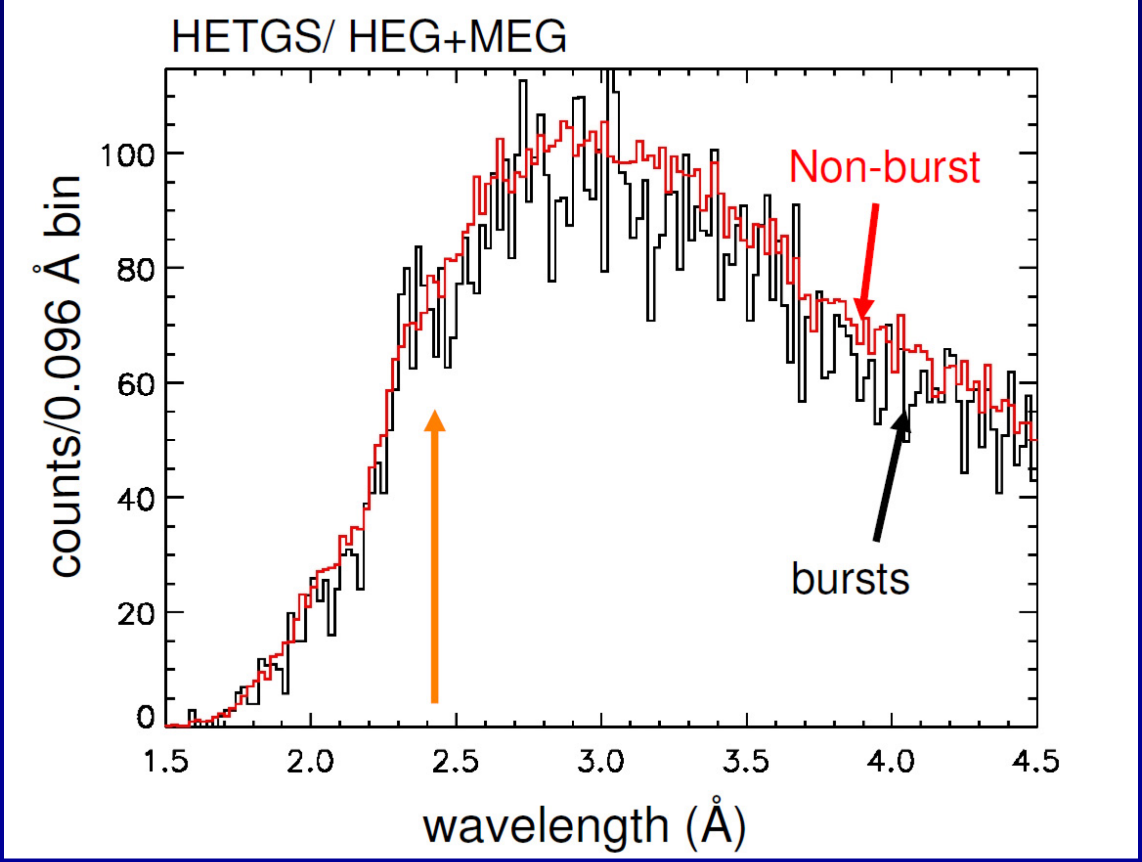
Photospheric absorption during X-ray bursts

Fe Lyman spectroscopy with Chandra/HETGS;
~300 ks; 30 X-ray bursts;
~3.4 ks effective exposure



H-like Fe Lyα

$\lambda = 1.778\ \text{Å} \times 1.35 = 2.400\ \text{Å}$

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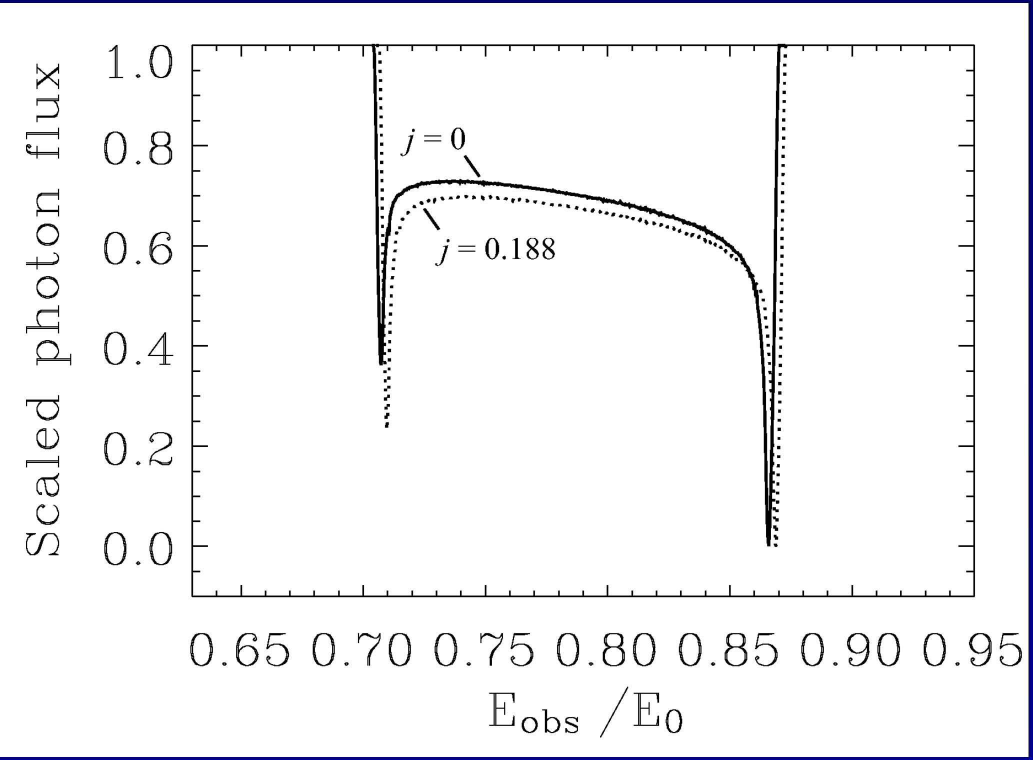
Spectral line profile

Mechanisms that affect the shape of spectral lines:

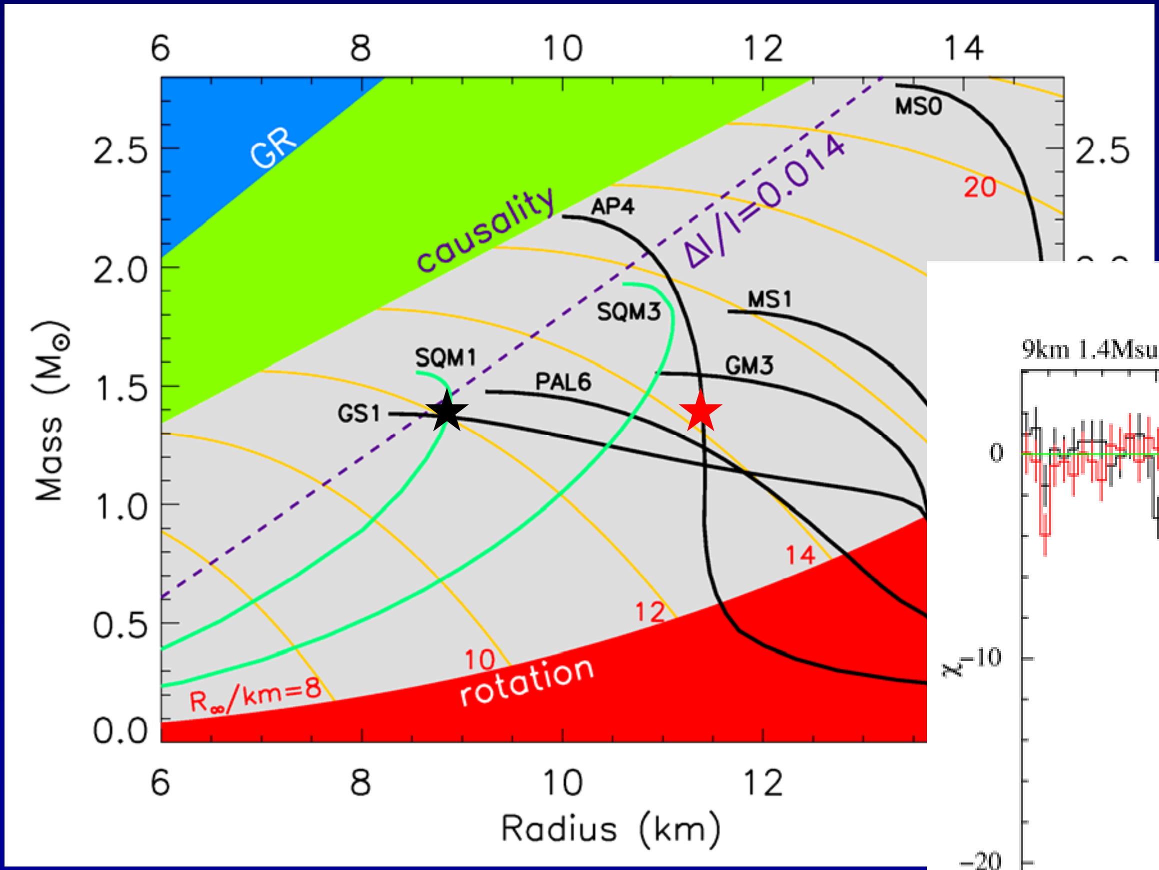
- longitudinal and transverse Doppler shifts,
- special relativistic beaming,
- gravitational redshifts,
- light-bending,
- frame-dragging.

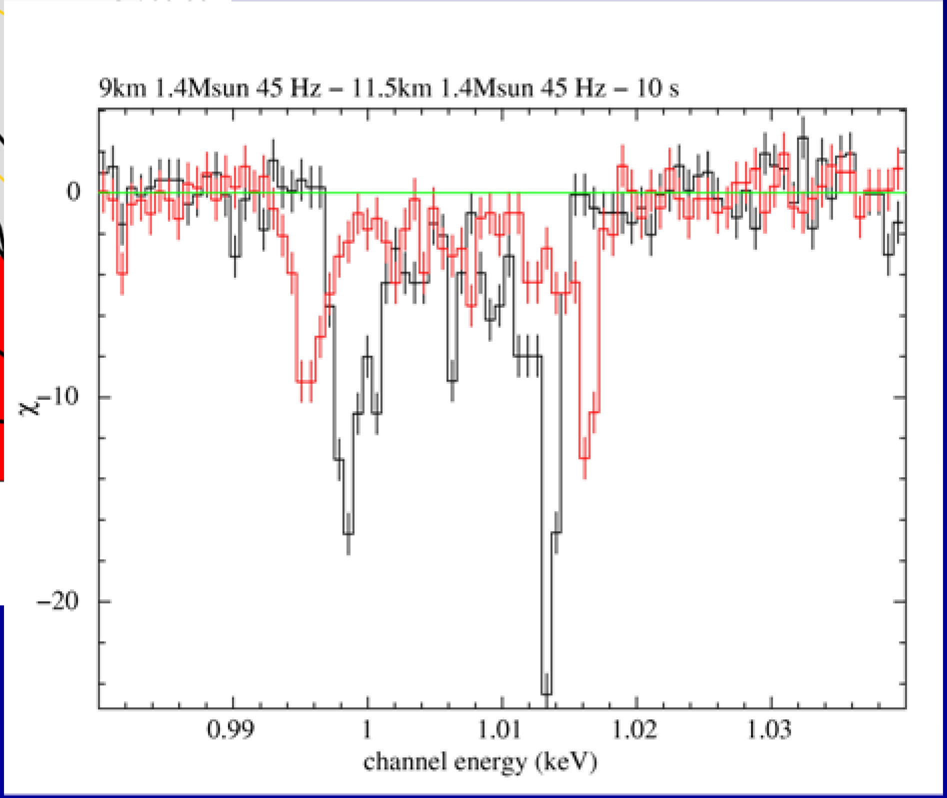


Assumption: Line initially narrow

Bhattacharyya et al.

Spectral line profile: Xeus simulation

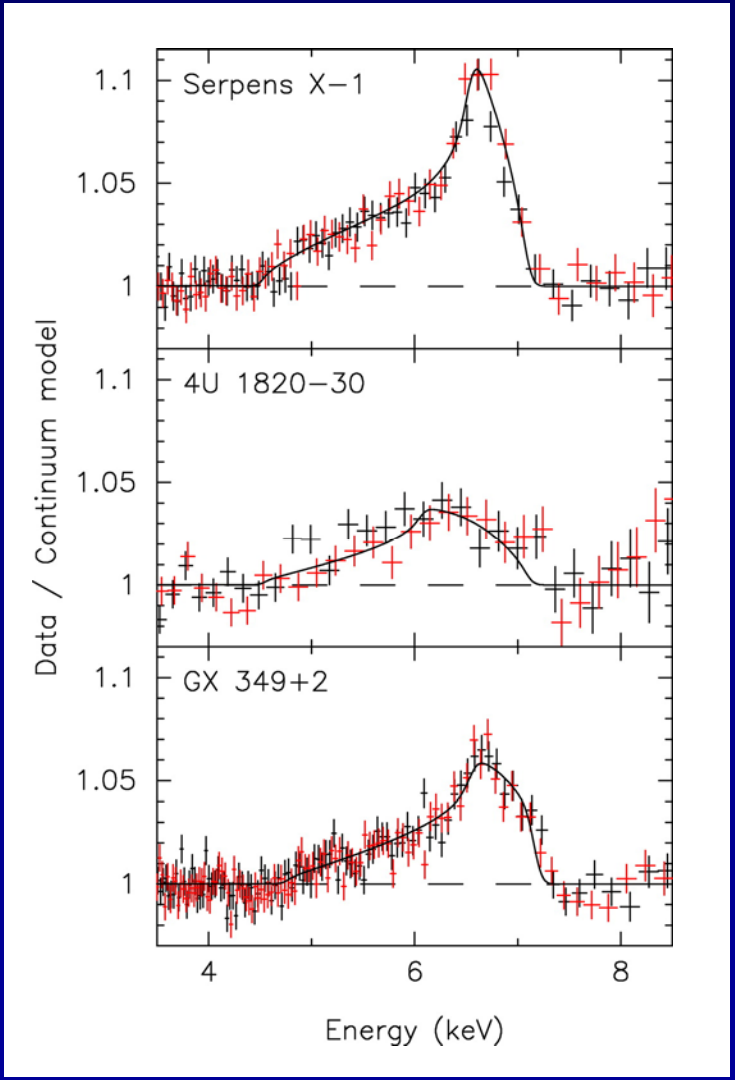




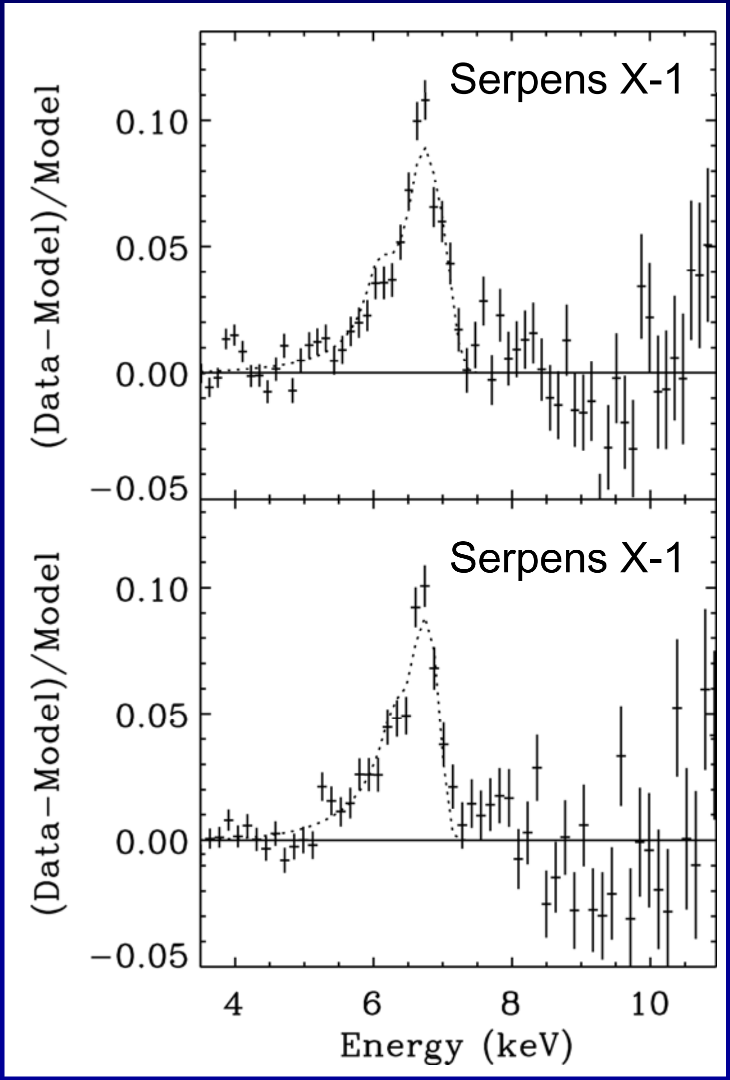
Line profile calculations courtesy of
S. Bhattacharyya

Emission lines from the inner disc

Suzaku



XMM-Newton



Cackett et al.; Bhattacharyya & Strohmayer

$$M/R = 0.03 - 0.17 M_{\odot}/\text{km}$$

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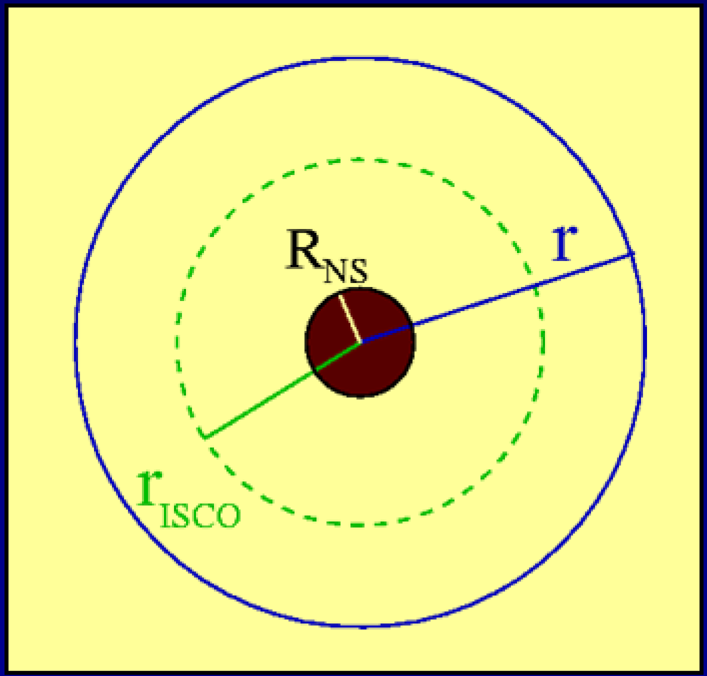
Rossi X-ray Timing Explorer

Proportional Counter Array

Effective area: ~ 6500 cm^2 @10 keV
Time resolution: ~ 1μs
Telemetry rate: ~ 18 Kbps (48 – 1024)
Energy range: 2 – 60 keV

Launched: Dec. 1995

Mass and radius constraints from timing



$$\nu = \frac{1}{2\pi}\sqrt{\frac{GM_{\rm NS}}{r^3}}$$

$$r_{\rm isco} \leq r$$

$$R_{\rm NS} \leq r$$

$$r_{\rm isco} = \frac{6GM_{\rm NS}}{c^2}$$

$$\boxed{M_{\rm NS} \leq 2.2(\nu/1000{\rm Hz})^{-1} M_\odot}$$

$$\boxed{R_{\rm NS} \leq 14.6(M_{\rm NS}/M_\odot)^{1/3}(\nu/1000{\rm Hz})^{-2/3}\ {\rm km}}$$

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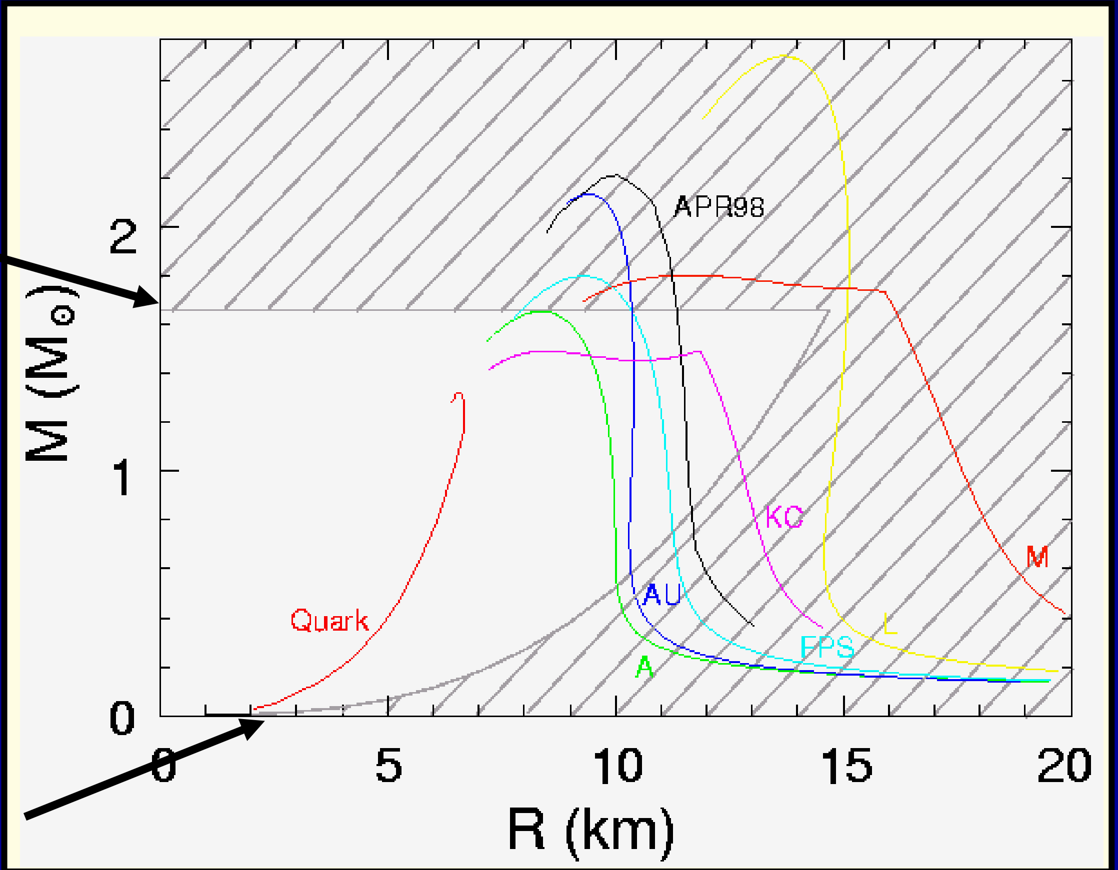
$\nu = 1329$ Hz

$M \leq 2.2(\nu/1000 \text{ Hz})^{-1} M_\odot$

$R \leq 14.6(M/M_\odot)^{1/3}(\nu/1000 \text{ Hz})^{-2/3}$ km



XEUS/CON-X