On a multi-resonant origin of high frequency QPOs in the atoll source 4U 1636-53

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The observational data and its possible interpretation

General belief dominating in the astrophysical community links the observed neutron star kHz QPOs to the orbital motion near the inner edge of an accretion disc.

The ratio between frequencies of the upper and lower observed QPOs mode cluster close to ratios of small natural numbers, most often close to the $3/2$ value, but the other rational ratios occur in some sources as well. The class of QPOs models considers a resonance between Keplerian and epicyclic frequencies of the geodesic motion.

The results of several studies [2, 11, 9, 10, 6, 22, 10] indicate that *for a given source the upper and lower QPO frequency can be traced through the whole observed range of frequencies but the probability to detect both QPOs simultaneously increases when the frequency ratio is close to the ratio of small natural numbers, namely 3/2, 4/3 and 5/4 in the case of six atoll sources* [27].

In Figure I we show correlation corresponding to the occurences of twin peaks for the atoll source 4U 1636-53 taken from [6], method A in the paper. This correlation was obtained by the shift-add [21] fitting of continuos segments of observations from all the at the time available RXTE data, see [9, 10, 6] for details.

We stress that in difference to the studies considering separated single QPO distributions, e.g., the recent paper of Belloni et al. [13], *the twin peak QPO distribution examined in such a way consider only simultaneous significant detections of both QPO frequencies* (i.e., the detections of both the peaks above 2.5σ significance having quality factor higher than 3). These two approaches in counting the number of occurences are different, but both legal being dependend on the reason (and assumptions) of the counting.

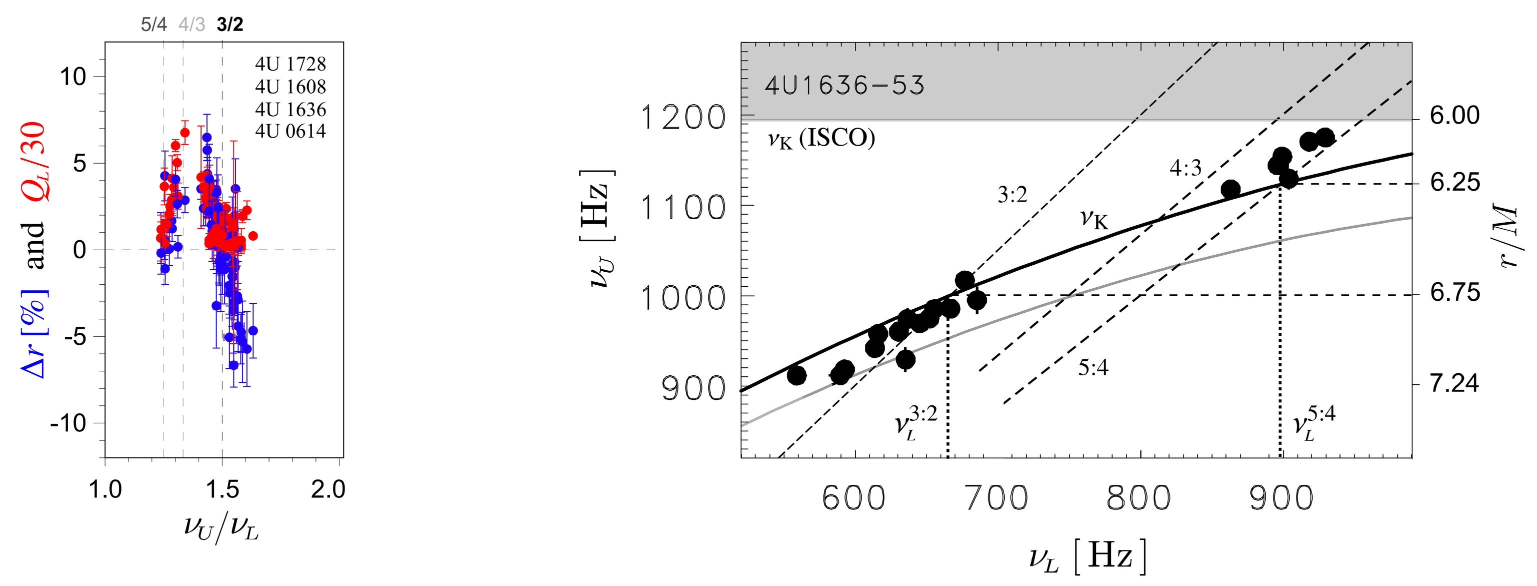

Fig. I Left, from [27, 17]: The twin QPO rms amplitude difference together with the lower frequency quality factor as a function of the frequency ratio. It has been recently showm [17] that such a behaviour may result from the resonant energy exchange between the two time dependent eigenfrequency modes. Right, from [27, 17]: The frequency correlation in the atoll source 4U 1636-53. Curve $\nu_{\rm K}$ determines the upper QPO frequency following from the relativistic precession model [25] under the consideration of the gravitational field described by the Schwarzschild metric with the central mass $M = 1.84 M_{\odot}$, the grey curve denotes the same relation but for $M = 2M_{\odot}$, i.e., the trend reported by [12]. Note that the actual (observed) frequencies of the resonance are allowed to differ from given resonant eigenfrequencies [19, 6].

In the sense described above, the atoll source 4U 1636-53 shows twin peak clustering around two distinct values ($3/2$ and $5/4$) of the frequency ratio. The same frequency ratios correspond to the change in the sign of the twin peak QPO amplitude difference, suggesting existence of a resonant energy overflow [27, 17].

We explore the idea [28] that the two clusters may follow from different instances of one orbital resonance.

Orbital frequencies of geodesic motion close to rotating neutron stars

The correct treatment of an orbital motion close to (rotating) neutron stars requires the general relativistic approach. For a given axially symetric spacetime the angular velocities of the azimuthal, radial and vertical "quasielliptic" orbital motion reads,e.g., [3],

$$\Omega_{\rm K} = u^{\phi}/u^{t}, \qquad \omega_{\rm i}^2 = \frac{(g_{tt} + \Omega_{\pm}\, g_{t\phi})^2}{2 g_{\rm ii}} \left(\frac{\partial^2 U}{\partial {\rm i}^2}\right)_{\ell}, \tag{1}$$

where $g_{\mu\nu}$ are components of the line element, ${\rm i} \in ({\rm r}, \theta)$ and U is an *efffective potential* $U(r,\theta,\ell) := g^{tt} - 2\ell\, g^{t\phi} + \ell^2\, g^{\phi\phi}$, with ℓ denoting the specific angular momentum of the orbiting test particle $\ell = -u_{\phi}/u_t$. In next we consider Keplerian motion and $l = l_K(r,\theta)$.

Due to the inequality between the azimuthal and radial frequency, the eccentric orbits waltz at the *periastron precession frequency* ν_P and in addition the orbits tilted relative to the equatorial plane of the spinning central mass wobble at the *nodal (often called Lense–Thirring) precession frequency*,e.g., [23]

$$\nu_{LT} = \nu_K - \nu_{\theta}, \qquad \nu_P = \nu_K - \nu_{\rm r}. \tag{2}$$

Both the declination of the quasiellipse plane and position of the periastron then reach the initial state simultaneously in the period characterized by the *total precession frequency*

$$\nu_T = \nu_P - \nu_{LT} = \nu_{\theta} - \nu_{\rm r}. \tag{3}$$

We consider the external neutron star spacetime described by the Hartle-Thorne metric [16], which represents the solution of vacuum Einstein field equations for the exterior of rigidly and relatively slowly rotating, stationary and axially symmetric body, and the explicit form of formulae (1) derived by [3].

Testing the hypothesis of a resonance between two time-dependent eigen-frequency modes

Frequency identification

Usually the $n:m$ orbital resonant models considering a non-linear resonance between Keplerian and/or epicyclic frequenciesm, see e.g. [5], identify the resonant eigenfrequencies $\nu_L^0,\ \nu_U^0$ as

$$\nu_L^0 = \nu_{\rm r}(r_{n:m}),\ \nu_U^0 = \nu_{\rm v}(r_{n:m}),\ \ \nu_{\rm v} \in [\nu_{\theta},\ \nu_K], \qquad \frac{\nu_{\rm v}(r_{n:m})}{\nu_{\rm r}(r_{n:m})} = \frac{n}{m}. \tag{4}$$

where n, m are small natural numbers and $r_{n:m}$ is the generic resonant radius.

In the case of a considerably weak forced or parametric non-linear resonance [19], the upper and lower observed QPO frequencies ν_L and ν_U are related to the resonant eigenfrequencies either directly $\nu_L \doteq \nu_L^0,\quad \nu_U \doteq \nu_U^0,$ or as their linear combinations $\nu_L \doteq \alpha\nu_L^0,\quad \nu_U \doteq \beta\nu_U^0,$ where α and β are small integral numbers.

In general case of a system in a non-linear resonance, the observed frequencies differ from resonance eigenfrequencies by a frequency corrections proportional to the square of small dimensionless amplitudes [19]. It was shown [4, 24] that a resonance characterized by one pair of eigenfrequencies may reproduce the whole range of frequencies observed in a neutron star source. Later [6] considered the idea of one eigenfrequency pair (so called resonant point in the frequency-frequency plane) common for a set of neutron star sources. They found that the coefficients of linear fits well approximating individual sources are anticorrelated which was in a good accord to the theory they presented and justified the hypothesis of one eigenfrequency-pair. On the other hand this approach, incorporating certain difficulties (e.g., the extremely large extension of the observed frequency range), is not proved yet, and some observational facts like the multipeaked ratio distribution suggest that *more then one resonant points may be responsible for the almost linear observed frequency correlation*.

In next *we focus on the hypothesis of more resonant points corresponding to different instances of one orbital resonance and suppose that the observed frequencies are close to the resonance eigenfrequencies, i.e. that the observed frequency correlation follows the generic relation between resonant eigenfrequencies*,

$$\nu_L \sim \nu_L^0, \quad \nu_U \sim \nu_U^0. \tag{5}$$

We checked in the frame of Hartle-Thorne spacetimes that the ratio between the Keplerian (or vertical epicyclic) frequency and radial epicyclic frequency monotonically increases with decreasing radius r whereas the Keplerian (vertical epicyclic) frequency increases. In other words, *for the models (4) considering resonance between Keplerian (vertical epicyclic) frequency and radial epicyclic frequency satisfying relation (5), the ratio of observed frequencies should increase with increasing QPO frequency, but that is opposite to what is observed.*

However, the above relations are not the only possible in the framework of resonance models. [14] discussed so called vertical precession resonance introduced in order to match the spin estimated from fits of the X-ray spectral continua for the microquasar GRO J1655-40. The resonance should occur between the vertical epicyclic frequency and the periastron precession frequency fulfilling the relation

$$\nu_L^0(r) = \nu_P(r) = \nu_K(r) - \nu_{\rm r}(r), \quad \nu_U^0(r) = \nu_{\theta}(r), \quad \text{"Bursa"} \tag{6}$$

for a particular choice of the resonant radius r defined by the condition $\nu_{\rm u} = 3/2\nu_{\rm l}$.

As noticed in [28], for the Schwarzschild spacetime the relations (6) coincide with those following from the relativistic precession model:

$$\nu_L^0(r) = \nu_P(r) = \nu_K(r) - \nu_{\rm r}(r), \quad \nu_U^0(r) = \nu_K(r). \quad \text{"Stella"} \tag{7}$$

Opposite to the relations (4) the two relationships (6,7) as well as the other two relationships

$$\nu_L^0(r) = \nu_{\theta}(r) - \nu_{\rm r}(r), \quad \nu_U^0(r) = \nu_{\theta}(r); \qquad \nu_L^0(r) = \nu_T(r) = \nu_{\theta}(r) - \nu_{\rm r}, \quad \nu_U^0(r) = \nu_K(r) \qquad \text{Total precession I; II} \tag{8}$$

imply the increase of ν_U^0 for increasing ν_L^0. *We fit the QPO frequencies observed in 4U 1636-53 by the four different frequency relationships above testing the hypothesis that an appropriate resonance may be responsible for all the observed datapoints.*

Matching the data

In order to obtain a rough scan we calculated the above frequency relations in the Hartle–Thorne metric for the range of the mass $M \in 1$–$4M_{\odot}$, the internal angular momentum $j \in 0$–0.5 and a physically meaningful quadrupole momentum q with a step equivalent to the thousand points in all three quantities, i.e., four 3-dimensional maps each having 10^9 points. Then, for each pair (M, j), we keep the value of the quadrupole momentum q which gives the lowest χ^2 with respect to the observed datapoints. For the Schwarzschild spacetime ($q = j = 0$), when relations the considered relations merge, the best fit is reached for the mass $M \doteq 1.77 M_{\odot}$, with a $\chi^2 \doteq 400 \sim 20\, d.o.f.$

Having a rough clue given by these maps we searched for local χ^2 minima using the Marquardt–Levenberg non-linear least squares method [20]. The map for the relation "Stella" coinciding with the prediction given by model of Stella & Vietri is shown in Figure II together with its representative fit.

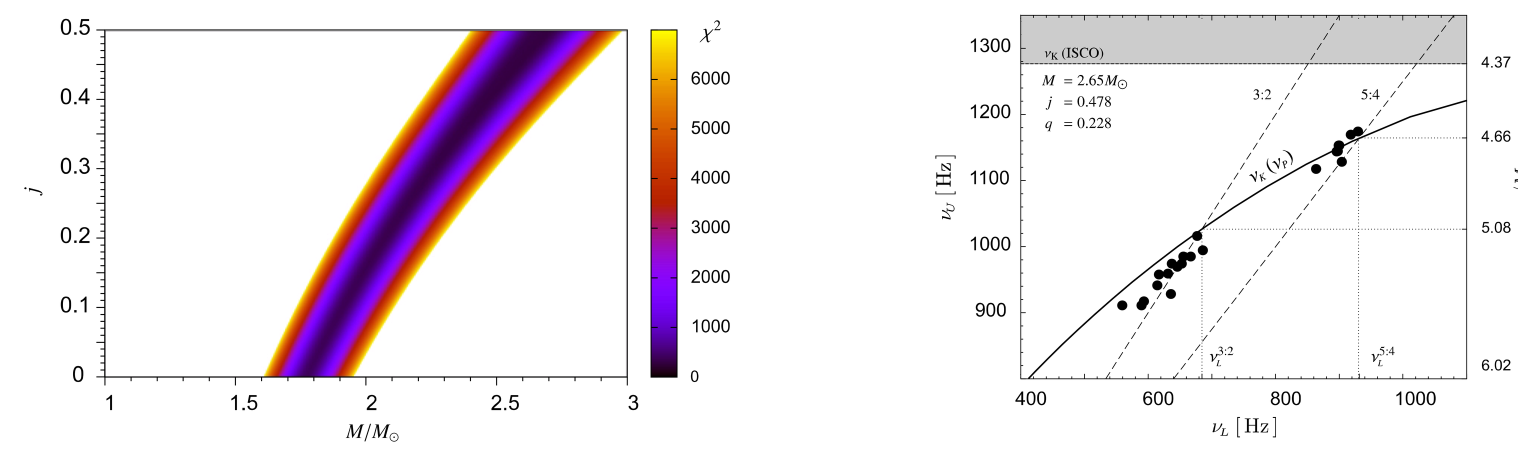

Fig. II Left: The chi-squared (inverse quality measure) of the fits by relation "Stella". Right: The best fit reached by this relation.

As previously realized, the relation "Stella" coinciding with the model of Stella & Vietri match the observational data most likely for relatively high angular momentum close to $j \sim 0.5$ and the central mass $M \sim 2.4$–$2.8 M_{\odot}$, reaching (not very satisfactory) $\chi^2 \sim 15 d.o.f.$. Unfortunately, the detailed analysis shown that the other three relations do not provide better results.

Discussion

The discussed geodesic relations provide fits which are in good qualitative agreement with general trend observed in the neutron star kHz QPO data (see, [12]). Nevertheless no one of this relations provides really good fits (we checked for the other five atoll sources, that trends are same as fo 4U 1636-53). In addition the best fits requires rather unrealistic values of mass and angular momentum with respect to the present knowledge of the neutron star equation of state [15]. This is of course problem for any models considering this geodesic relations, and not only for their resonant interpretation.

To check whether some non geodesic influence can resolve the problem above we consider the assumption that the effective frequency of radial oscillations may be lowered, e.g., by the hotspots interaction with the accretion disk or with the neutron star magnetic field (of course, in such a case all the frequencies would be modified by dependent corrections, nevertheless, e.g., in the case of the magnetic field it was shown by [8] that the corrections to radial epicyclic frequency should be the strongest one).

Then, in the lowest order approximation, the effective frequency of radial oscillations may be written as

$$\tilde{\nu}_{\rm r} = \nu_{\rm r}(1-k), \quad \text{where } k \text{ is a small constant.} \tag{9}$$

In Figure III we show the qualitative behaviour of frequency-frequency plot implied by the relation "Stella" vs. those implied by the total precesion relation II. They differ in the predicted frequency which is emphasized in the case when $\tilde{\nu}_{\rm r}$ is used instead of $\nu_{\rm r}$.

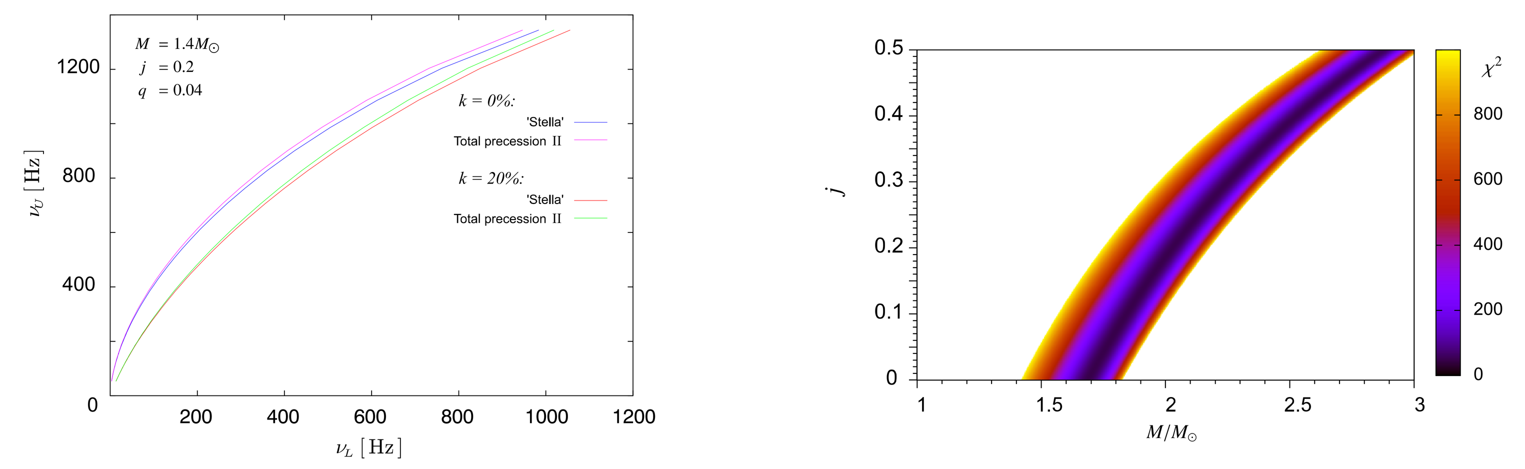

Fig. III Behaviour of frequency-frequency plot implied by the relation "Stella" vs.those implied by the total precesion relation II for the case of $\tilde{\nu}_{\rm r} = \nu_{\rm r}$ and $\tilde{\nu}_{\rm r} = 0.8\nu_{\rm r}$. Right: the χ^2 reached by the relation total precesion II for $k \in (0, 0.2)$.

Because the discussed frequencies of orbital motion scales roughly as $1/M$, the relations given by the totall precession implying lower frequencies in terms of frequency-frequency plot should require lower mass in order to fit the same datapoints. In general, the mass is lower when the vertical epicyclic frequency is included instead of Keplerian in the frequency relation. The same holds for substitution of $\nu_{\rm r}$ by $\tilde{\nu}_{\rm r}$.

Having this motivation, including lowering of both the χ^2 of fits and related mass, we repeated the described fitting procedure using the frequency $\tilde{\nu}_{\rm r}$ instead of $\nu_{\rm r}$ for all four discussed relationships. The resulting quality of fits is shown in the Figure IV for the three relations together with a representative fit.

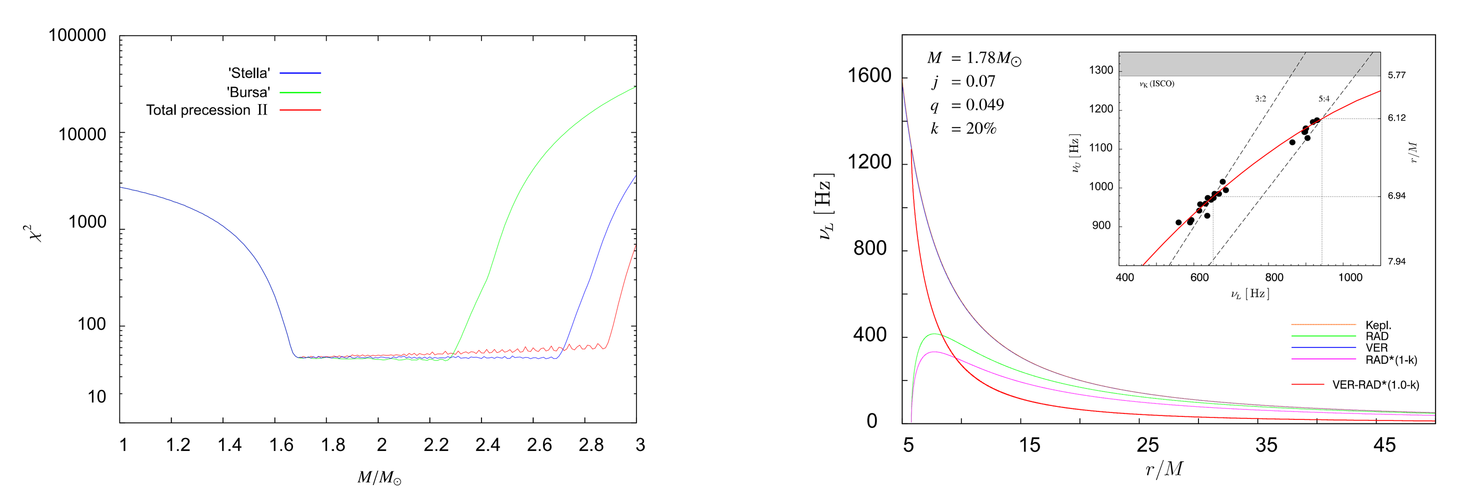

Fig. IV Left: The chi-squared (inverse quality measure) of the fits by relation coinciding with the model of Stella & Vietri under the consideration $\tilde{\nu}_{\rm r}$ instead of $\nu_{\rm r}$. For given combination M, j the coeficient k is chosen as the best one from the interval 0–20%. Right: The represenative low angular momentum fit for the total precession relation II. These results represent a coorection of those presented in [26].

Conclusions

In accord to the discussion above the modified relations provide the best fits having $\chi^2 \sim 2$–5d.o.f. when the coefficient k is in the interval 5–20%. The mass required for a reasonable χ^2 value is then in the interval 1.6–$1.8\ M_{\odot}$ and the relevant angular momentum $j \sim 0.05 - 0.2$. For the fixed angular momentum, the frequency relations including total precession implies slightly lower mass then those including relativistic precession.

We stress that the total precession frequency corresponds to the similar effect as the relativistic precession frequency but when considering a resonance, this may naturaly include all the three fundamental precessions: Keplerian, periastron, and Lense-Thirring. For the perfect free particle motion, if the Keplerian and total precession frequency form rational fractions, the trajectory is selfrepeating (i.e., closed) [26].

The debate above touching the hotspot QPO interpretation requires further future research including realistic consideration of the frequency corrections. In addition the proposed multi-resonance may also occur not between the considered hot spot modes but between similar disc oscillations modes as well which deserves attention too.

Nevertheless, the mentioned observational facts like the ratio clustering and rms amplitudes difference behaviour together with the fact that the discussed frequency relations can provide good fits conditioned by reasonable values of the neutron star mass and angular momentum indicates that the hypothesis of more instances of one orbital resonance has the potential to explain the neutron star kHz QPO nature.

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