Gravitational waves from pulsars with measured braking index

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ABSTRACT

We study the putative emission of gravitational waves (GWs) in particular for pulsars with measured braking index. We show that the appropriate combination of both GW emission and magnetic dipole brakes can naturally explain the measured braking index, when the surface magnetic field and the angle between the magnetic dipole and rotation axes are time dependent. Then we discuss the detectability of these pulsars by aLIGO and the Einstein Telescope.

Outline

- Power by a rotating magnetic dipole (traditional mechanism of electromagnetic radiation in pulsars)
- Gravitational wave power by pulsars
- Modeling the spindown of the (9) pulsars with known “n” that can naturally explain the range of braking indices measured accurately
- GW amplitudes generate by these pulsares
- Detectability of these pulsares by aLIGO and ET

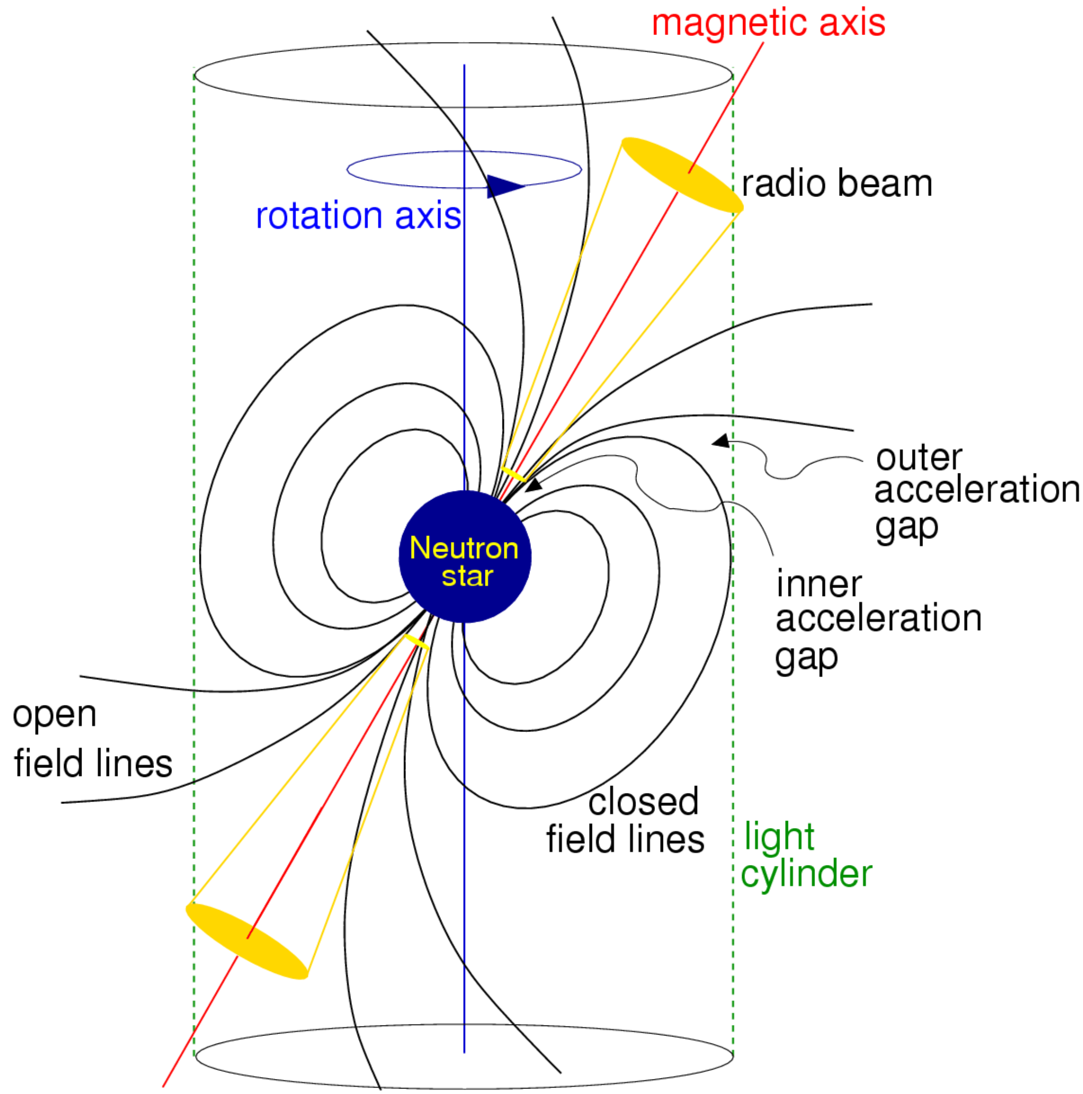
Recall that ...

The power of magnetic dipole is given by

$$\dot{E}_\mathrm{d} = \frac{16\pi^4}{3}\frac{B_0^2 R^6 \sin^2\phi}{P^4 c^3}$$

Where:

B_0 – magnetic field
R – radius
P – period of rotation
ϕ – angle between the rotating and the magnetic dipole axes



http://www.cv.nrao.edu/course/astr534/Pulsars.html

Recall that ...

Pulsars might generate continuous GWs whether they are not perfectly symmetric around their rotation axes.

The amplitude of GWs for pulsars : $$h = \frac{16\pi^2 G}{c^4} \frac{I \epsilon f_{\rm rot}^2}{r}$$

Where:

$I = I_{zz}$ with respect to the rotation axis (z)

and

$$\epsilon = \frac{|I_{xx} - I_{yy}|}{I_{zz}}$$ (Ellipticity)

The corresponding power in GWs [see Shapiro 1983]:

$$\dot{E}_{\rm GW} = \frac{2048\pi^6}{5} \frac{G}{c^5} \frac{I^2 \epsilon^2}{P^6}$$

What is the braking index (n)? What it is for?

Recall that "n" is given by

$$n \equiv \frac{\Omega_{rot}\,\ddot{\Omega}_{rot}}{\dot{\Omega}_{rot}^{2}}$$

As it is well known, in the context of pulsars, this quantity gives us important information about the spindown of pulsars.

It is also worth recalling that a pure magnetic brake leads to n=3, whereas a pure GW brake leads to n=5

What is n for a magnetic brake?

Starting from:

$$\dot{E}_{rot} = I\,\Omega_{rot}\,\dot{\Omega}_{rot}$$

$$\dot{E}_{rot} = \dot{E}_{d}$$

$$\dot{E}_{\mathrm{d}} = \frac{16\pi^4}{3}\,\frac{B_0^2 R^6 \sin^2\phi}{P^4 c^3}$$

One has

$$\dot{\Omega}_{rot} = \frac{1}{3}\,\frac{B_0^2 R^6 \sin^2\varphi}{I\,c}\,\Omega_{rot}^3$$

It is easy to show that:

$$\ddot{\Omega}_{rot} = \frac{3\,\dot{\Omega}_{rot}^2}{\Omega_{rot}}$$

$\Rightarrow$ n = 3

What is n for a GW brake?

Starting from:

$$\dot{E}_{rot} = I\,\Omega_{rot}\,\dot{\Omega}_{rot}$$

$$\dot{E}_{rot} = \dot{E}_{GW}$$

$$\dot{E}_{\mathrm{GW}} = \frac{2048\pi^6}{5}\frac{G}{c^5}\frac{I^2\epsilon^2}{P^6}$$

One has

$$\dot{\Omega}_{rot} = \frac{32}{5}\frac{G}{c^5}\,I\,\epsilon^2\,\Omega_{rot}^5$$

It is easy to show that:

$$\ddot{\Omega}_{rot} = \frac{5\,\dot{\Omega}_{rot}^2}{\Omega_{rot}}$$

$\Rightarrow$ n = 5

What about the observations?

Pulsar	P (s)	$\dot{P}$ (10^{-13} s/s)	n
PSR J1734-3333	1.17	22.8	0.9 ± 0.2
PSR B0833-45 (Vela)	0.089	1.25	1.4 ± 0.2
PSR J1833-1034	0.062	2.02	1.8569 ± 0.0006
PSR B0540-69	0.050	4.79	2.140 ± 0.009
PSR J1846-0258	0.324	71	2.19 ± 0.03
PSR B0531+21 (Crab)	0.033	4.21	2.51 ± 0.01
PSR J1119-6127	0.408	40.2	2.684 ± 0.002
PSR B1509-58	0.151	15.3	2.839 ± 0.001
PSR J1640-4631	0.207	9.72	3.15 ± 0.03

From the observational point of view, the literature shows that almost all pulsars with measured braking index have $n<3$.

What about the observations?

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Several interpretations for the observed braking indices have been put forward, like the ones that propose either accretion of fall-back material via a circumstellar disk (Chen & Li 2016), the so-called QVF effect (Coelho et al. 2016), relativistic particle winds (Xu & Qiao 2001; Wu et al. 2003), or modified canonical models to explain the observed braking index ranges (see, e.g., Allen & Horvath 1997; Eksi et al. 2016)

Modeling pulsars’ braking indices

Modeling the braking index of PSR J1640-4631 (n =3.15)

This value of n suggests that the spindown can be given by a combination of a magnetic dipole and a gravitational wave brakes

Starting from:

$$E_{rot} = \frac{1}{2} I \Omega_{rot}^2 \quad \Longrightarrow \quad \dot{E}_{rot} = I \Omega_{rot} \dot{\Omega}_{rot}$$

Assuming that $\dot{E}_{rot} = \dot{E}_{GW} + \dot{E}_d$

Recall that $\dot{E}_{\mathrm{d}} = \frac{16\pi^4}{3} \frac{B_0^2 R^6 \sin^2 \phi}{P^4 c^3}$ $\quad \dot{E}_{\mathrm{GW}} = \frac{2048\pi^6}{5} \frac{G}{c^5} \frac{I^2 \epsilon^2}{P^6}$

$$\Longrightarrow \quad \dot{\Omega}_{\mathrm{rot}} = \frac{32}{5} \frac{G}{c^5} I \epsilon^2 \Omega_{\mathrm{rot}}^5 + \frac{1}{3} \frac{B_0^2 R^6 \sin^2 \phi}{I c^3} \Omega_{\mathrm{rot}}^3$$

Since $n \equiv \frac{\Omega_{rot} \ddot{\Omega}_{rot}}{\dot{\Omega}_{rot}^2} \quad \Longrightarrow \quad n = \frac{5 \dot{\Omega}_{GW} + 3 \dot{\Omega}_d}{\dot{\Omega}_{GW} + \dot{\Omega}_d}$

Now, the fraction of power in GWs can be defined by:

$$\eta = \frac{\dot{E}_{GW}}{\dot{E}_{rot}} \qquad \left(\Rightarrow \quad \eta = \frac{\dot{\Omega}_{GW}}{\dot{\Omega}_{rot}} \right)$$

Substituting into

$$n = \frac{5\,\dot{\Omega}_{GW} + 3\,\dot{\Omega}_{d}}{\dot{\Omega}_{GW} + \dot{\Omega}_{d}}$$

One has:

$$\eta = \frac{n-3}{2} \qquad (\text{para } 3 \leqslant n \leqslant 5)$$

For $n = 3.15 \implies \eta = 0.075$

Thus, 7.5% of the power in GWs!

What about the amplitude of GWs?

Let us start from:

$$h^2 = \frac{5}{2}\frac{G}{c^3}\frac{I}{r^2}\frac{|\dot{f}_{\rm rot}|}{f_{\rm rot}}$$

This equation implicitly considers that the spindown is exclusively given by a GW brake, i.e., n = 5.

…. it should be modified to take into account that $3 \leq n \leq 5$.

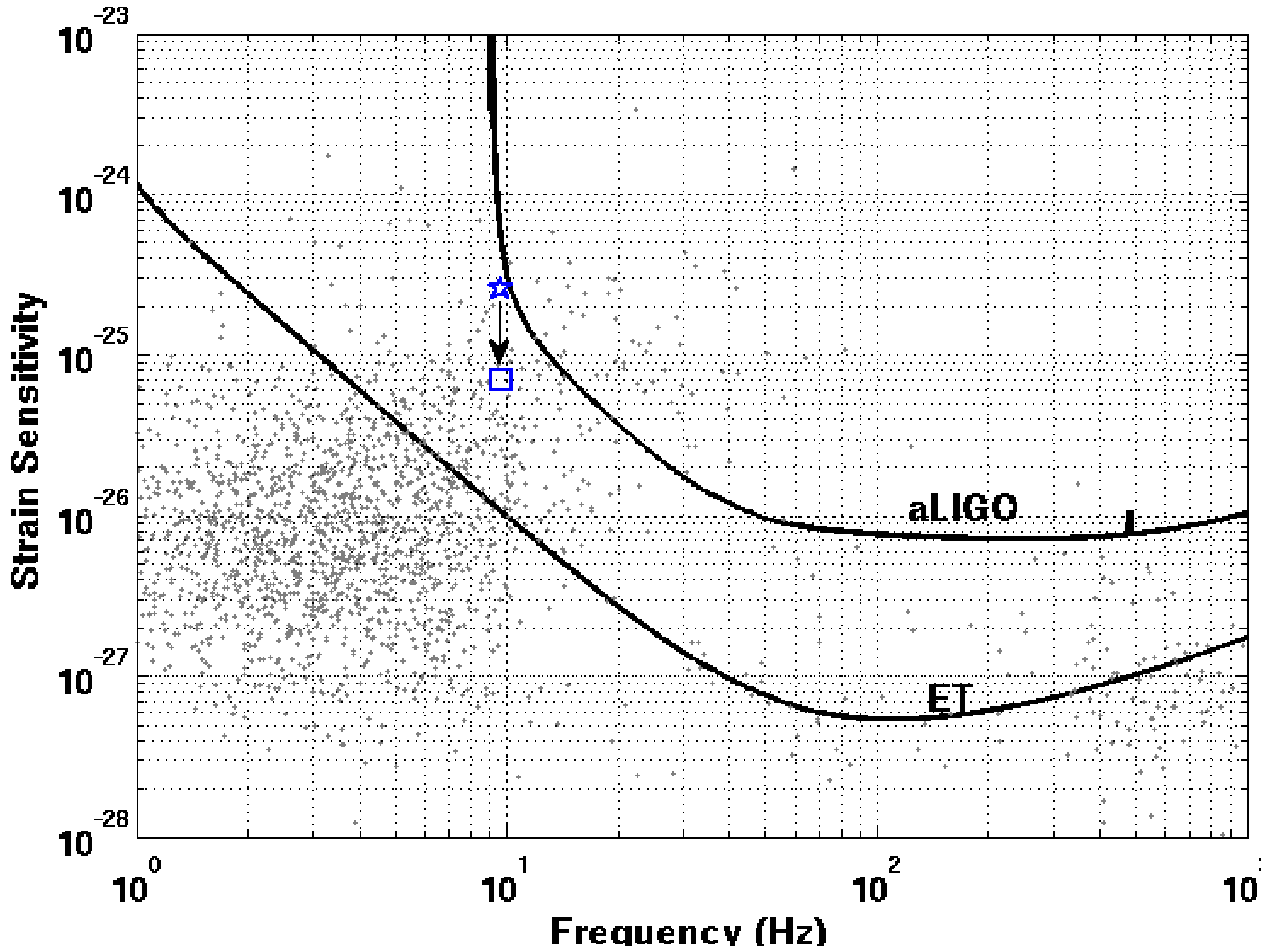
From the previous slide one sees that:

$$\dot{\Omega}_{GW} = \eta \dot{\Omega}_{rot} \quad \Rightarrow \quad \dot{\bar{f}}_{rot} = \eta \dot{f}_{rot}$$

Therefore, the amplitude of GWs can also be written as follows

$$\bar{h}^2 = \frac{5}{2}\frac{G}{c^3}\frac{I}{r^2}\frac{|\dot{\bar{f}}_{\rm rot}|}{f_{\rm rot}} = \frac{(n-3)}{2} h^2$$

For $I = 10^{38}$ kg m^2 (fiducial), r = 12 kpc (distance Earth-Pulsar). Almost a factor of four lower than the amplitude found when one assumes that the energy loss is completely given by GW emission.



Amplitudes of GW for n = 5 (star) and for n = 3.15 (square). The GW luminosity would be 7.5% of the total power lost. Strain sentivities for aLIGO and ET for one year integration time.

What would be the ellipticity?

Combining the equations for the amplitude of GWs:

$$\bar{h}^2 = \frac{5}{2}\frac{G}{c^3}\frac{I}{r^2}\frac{|\dot{f}_{\rm rot}|}{f_{\rm rot}} = \frac{(n-3)}{2}h^2$$

$$h = \frac{16\pi^2 G}{c^4}\frac{I\epsilon f_{\rm rot}^2}{r}$$

One has:

$$\epsilon = \sqrt{\frac{5}{1024\pi^4}\frac{c^5}{G}\frac{\dot{P}P^3}{I}(n-3)}.$$

Substituting the appropriate quantities:

$$\epsilon = 4.8\times 10^{-3}$$

… this is extremely large! This could be an indication that other mechanisms, apart of GW and dipole magnetic brakes must necessarily be considered.

Modeling the spindown of the other 8 pulsars and revisiting PSR J1640-4631 ($n = 3.15$)

In order to model pulsars with n < 3, one can consider, for example, that the magnetic field and the angle between the magnetic dipole and the rotating axes are time dependent.

…. after some algebra one obtains:

$$n = 3 + 2\eta - 2\frac{P}{\dot{P}}(1-\eta)\left[\frac{\dot{B}_0}{B_0} + \dot{\phi}\cot\phi\right]$$

Combining appropriately the ingredients of the above equation, it is possible to obtain n < 3.

The term in brackets can be written in the following form:

$$g = g(B_0, \dot{B}_0, \phi, \dot{\phi}) \equiv \left[\frac{\dot{B}_0}{B_0} + \dot{\phi}\cot\phi\right]$$

This term as a function of η for a given pulsar reads

$$g = -\frac{(n-3-2\eta)}{2(1-\eta)}\frac{\dot{P}}{P}$$

Now, it is appropriate to consider how the GW amplitudes for pulsars with n < 5 can be calculated. For the amplitude of GWs we follow the same prescription as for PSR J1640-4631.

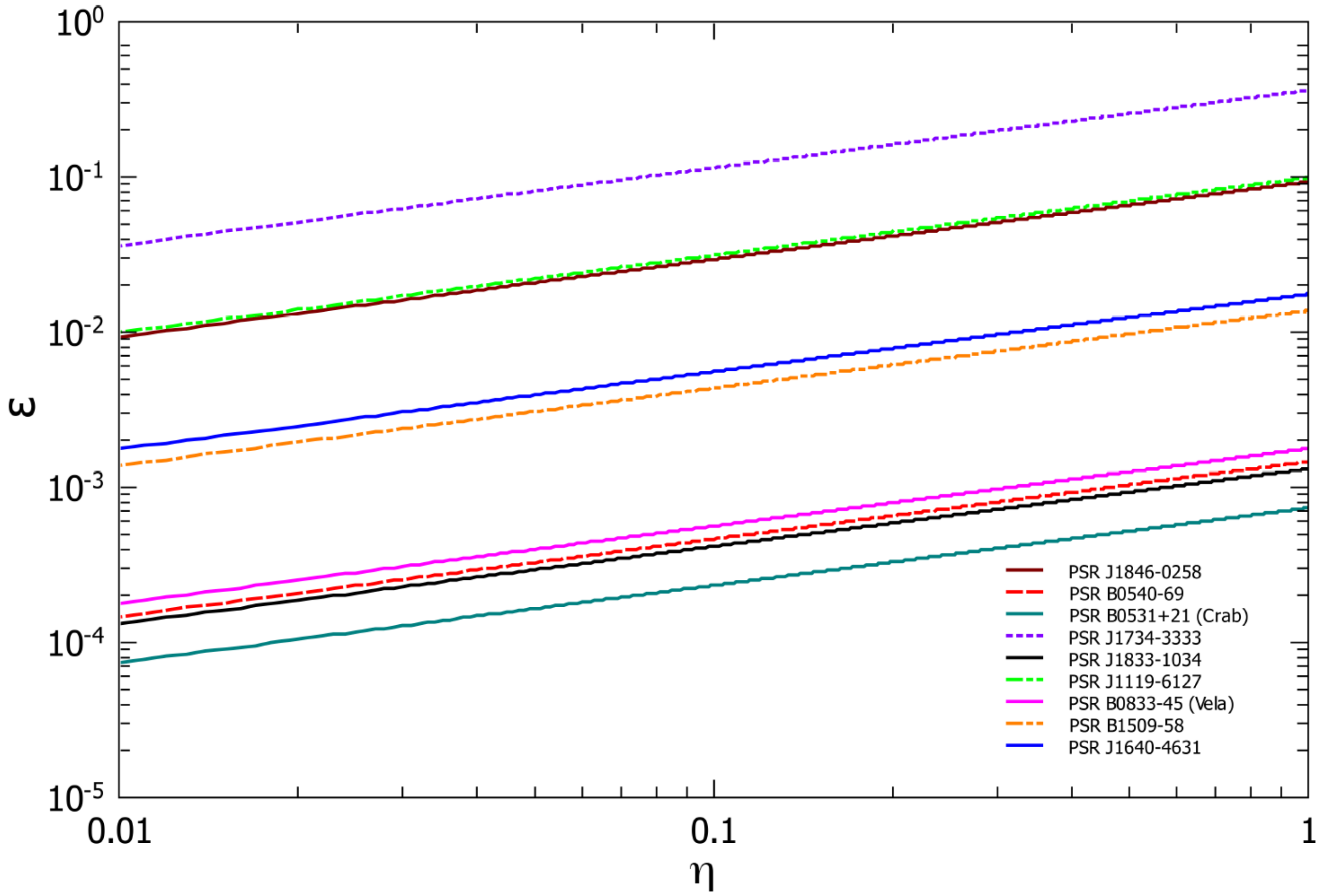
Then, we have:

$$\bar{h}^2 = \frac{5}{2}\frac{G}{c^3}\frac{I}{r^2}\frac{\dot{\bar{f}}_{rot}}{f_{rot}} = \eta\, h^2$$

….. and for the ellipticity we have

$$\epsilon = \sqrt{\frac{5}{512\pi^4}\frac{c^5}{G}\frac{\dot{P}P^3}{I}\eta}$$

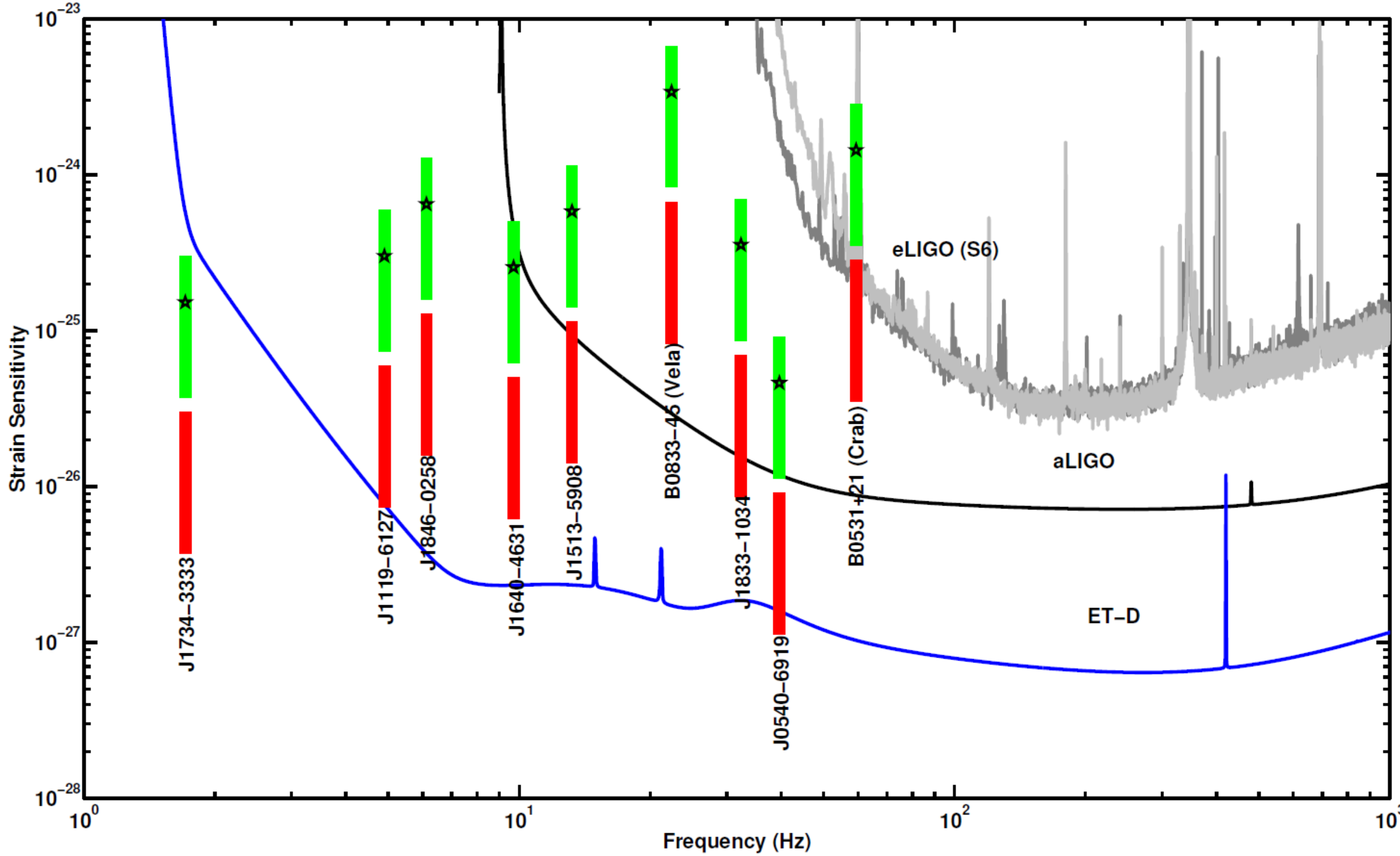
… we obtain



Note that, for example

$$\varepsilon \ll 10^{-4} \Rightarrow \eta \ll 0.01$$

Strain sensitivities for eLIGO S6, aLIGO e ET-D for one year integration time. A range of possible amplitudes (for two distinct efficiencies) calculated from the realistic moment of inertia.



GW amplitude for $\eta = 1$ (green bars) and $\eta = 0{,}01$ (red) for $7\times10^{36} < I < 1\times10^{38}$ kgm^2 (EoS GM1; Glendennig & Moszkowski 1991). "Black star": $I = 10^{38}$ kgm^2 and $\eta = 1$. aLIGO can well detect at least some of the pulsars considered here, in particular Vela within one year of observation.

Conclusions

It is possible to model the braking index of the Pulsar with $n = 3.15$ considering that the spindown is a combination of magnetic dipole and GW brakes. Although the ellipticity obtained is extremely large.

For the other 8 pulsares, which has $n < 3$, it is possible to model them by considering that the magnetic field and the angle between rotating and the magnetic dipole axes are time dependent. This model can be extended to the pulsar with $n = 3.15$. As a result, its predicted ellipticity can have more moderated values ($\ll 10^{-3}$).

Our results also show that aLIGO, and more probably ET, could well be able to detect GWs from some pulsars considered here.

Thank you!