Equilibrium Models for Magnetized Neutron Stars in General Relativity

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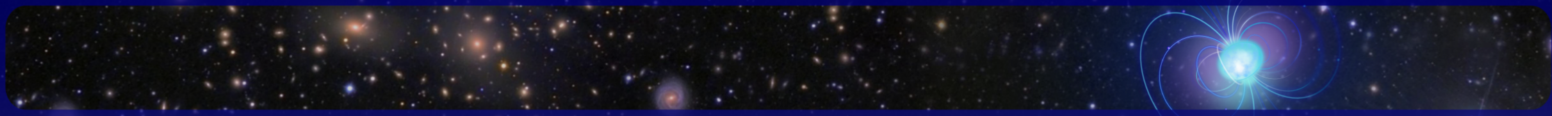


Abstract

Neutron stars can be characterized by very strong magnetic fields. The magnetar paradigm for AXPs and SGRs, requires fields in excess of 10^{14}G, and the recently proposed millisecond-magnetar model for GRBs advocates the possibility of even higher magnetic fields $\sim 10^{16}$ G. A strong magnetic field is supposed to induce deformations in a NS that in principle might give rise to gravitational wave emission in the case of rapid rotators. Modeling the role and effects of strong magnetic fields is thus necessary to properly constrain the properties of NSs. We present here numerical models of magnetized NS derived in the so called XCFC approximation. Results are shown for purely toroidal, purely poloidal configurations as well as for mixed cases, usually referred to as "twisted torus".

Governing Equations

- Axisymmetric and static equilibria
- Ideal GRMHD regime
- Polytropic EoS

} **Hypotheses**

XCFC approximation:

In the context of (3+1) formalism [3] the metric line is written as:

$$ds^2 = -\alpha^2 dt^2 + \psi^4 f_{ij} dx^i dx^j$$

where α is the lapse function, ψ is the conformal factor and finally f_{ij} is the flat 3-metric.

Einstein's equations are then written as a system of eight elliptic PDEs that can be solved in a hierarchical way [1,2]. For the purpose of modelling axisymmetric static polytropic NSs the only equations that must be solved are the two Poisson-like equations:

$$\Delta\psi = -2\pi \hat{E}\psi^{-1}$$

$$\Delta(\alpha\psi) = [2\pi(\hat{E} + 2\hat{S})\psi^{-2}]\alpha\psi$$

$\hat{E}$ and $\hat{S}$ are the energy density and the trace of the stress-energy tensor measured by the Eulerian observer rescaled by a factor ψ^{-6} (this ensure local uniqueness).

Bernoulli-like integral:

The equilibrium condition is derived from GRMHD equations and reads:

$$\log\left(\frac{h}{h_c}\right) + \log\left(\frac{\alpha}{\alpha_c}\right) - \mathcal{M} = 0$$

where h is the enthalpy and $\mathcal{M}$ is the electromagnetic term induced by the Lorentz force.

Relativistic Grad-Shafranov equation:

For mixed field configurations the ϕ-component of the magnetic vector potential is related to $\mathcal{M}$ through:

$$\Delta_* A_\phi = -\rho h \psi^8 r^2 \sin^2\theta \frac{d\mathcal{M}}{dA_\phi} - \partial_r A_\phi \partial_r \log\frac{\alpha^2}{\psi^2} - \frac{1}{r^2}\partial_\theta A_\phi \partial_\theta \log\frac{\alpha^2}{\psi^2} - \frac{\psi^4}{\alpha^2}\frac{d\mathcal{F}}{dA_\phi}$$

The new function $\mathcal{F}$ is related to the ϕ-component of magnetic field while Δ_* is the vector Laplacian.

Numerical Scheme

To produce self-consistent equilibrium models we use **XNS** [1] [sites.google.com/site/niccolobucciantini/xns] a numerical code that solves the coupled Einstein-Maxwell equations for a stationary magnetized polytropic NS in spherical coordinates using an iterative method. While originally XNS could only handle purely toroidal configurations, we have now extended it to also manage configurations endowed with poloidal magnetic fields. Given the value of the free parameters plus a starting guess for the central density the main steps are:

- initialize metric functions and source terms from solutions provided at the previous step (TOV solution for a spherical star at first step);
- solve equation for conformal factor;
- solve equation for the lapse function;
- impose the equilibrium solving the Bernoulli-like equation together with the Grad-Shafranov equation;
- repeat steps until convergence to a desired tolerance is achieved.

Poisson-like equations and Grad-Shafranov equation are iteratively solved exploiting a semi-spectral method. Along the angular direction the equations are decomposed with spherical-harmonics. Then the obtained set of ODEs is solved, for each harmonic, with direct inversion.

References

[1] Niccolò Bucciantini & Luca Del Zanna - *General Relativistic Magnetohydrodynamics in Axisymmetric Dynamical Spacetimes:the X-ECHO Code* - A&A 528, A101, 2011

[2] Isabel Cordero-Carrion et al. - *Improved Constrained Scheme for the Einstein Equations: An Approach to the Uniqueness Issue* - Phys. Rev. D79, 2009

[3] E. Gourgoulhon - *3+1 Formalism in General Relativity: Bases of Numerical Relativity* - Springer, 2012

Related recent works by other authors:

- K. Kiuchi et al. - *Relativistic Stars with Purely Toroidal Magnetic Fields* - Phys.Rev. D78, 2008
- J.Frieben et al. - *Equilibrium Models of Relativistic Stars with a Toroidal Magnetic Field* - MNRAS 427, 2012
- R.Ciolfi et al. - *Structure and Deformations of Strongly Magnetized Neutron Stars with Twisted-Torus Configurations* - MNRAS, 406, 2010

Results

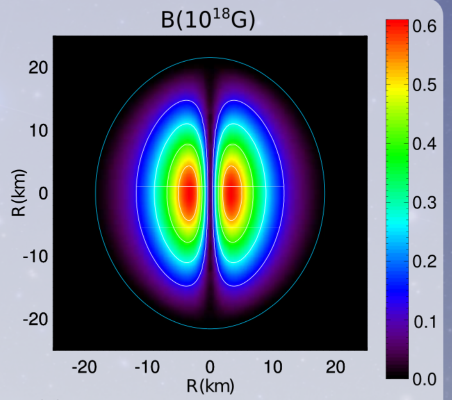
Purely Toroidal Configuration

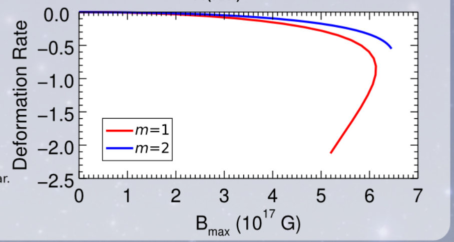
$$\mathcal{M} = -\frac{mK_m^2}{2m-1}(\alpha^2\psi^4\rho h r^2 \sin^2\theta)^{2m-1}$$

We have calculated models in a wide range of the parameter space (ρ_c, K_m) both in m=1 and m=2 cases.

Main results:

- models exhibit a prolate deformation that increases with K_m;
- the magnetic field remains buried under the star surface;
- the magnetic pressure causes an expansion of the star and a growth of its equatorial radius;
- in the cases of m=2 the stars are less concentrated around the magnetic axis;
- the maximum value of magnetic field strength inside the star is not a monotonic function of the magnetization parameter.





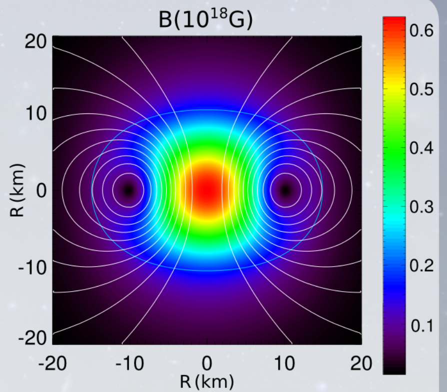
Up: NS with rest mass M_0=1.68 $M_\odot$ and gravitational mass M=1.60 $M_\odot$. The blue line is the surface of the star.
Down: deformation rate as a function of maximum value of the magnetic field inside star for a sequence of stars with constant M_0=1.68 $M_\odot$.

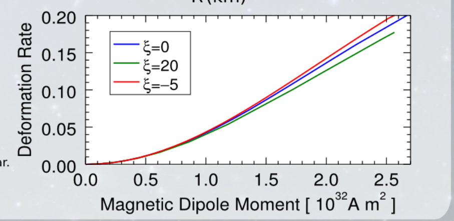
Purely Poloidal Configuration

$$\mathcal{M} = k_{pol}(A_\phi + \tfrac{1}{2}\xi A_\phi^2) \qquad \mathcal{F} = 0$$

Main results:

- purely poloidal magnetic field induces an oblate distortion of the star structure that increases as the magnetization parameter k_{pol} grows;
- the magnetic field extends over the stellar surface;
- the magnetic field causes a flattening of the density profile at the center of the star in the direction orthogonal to the magnetic axis;
- in the domain of convergence of the scheme the maximum magnetic field strength inside the star is a monotonic function of the magnetization parameter;
- configurations with $\xi \neq 0$ introduce new current terms but they do not affect significantly the structure of the star.





Up: NS with rest mass M_0=1.68 $M_\odot$ and gravitational mass M=1.58 $M_\odot$. The blue line is the surface of the star.
Down: deformation rate as a function of magnetic dipole moment for a sequence of stars with constant M=1.55 $M_\odot$.

Twisted-Torus Configuration

$$\mathcal{F} = a(A_\phi - A_\phi^{max})\theta(A_\phi - A_\phi^{max})$$

$$\mathcal{M} = k_{pol} A_\phi$$

where θ is the Heaviside step function.

Main Results:

- in our models stars are oblately distorted even if the toroidal magnetic energy contribution is dominant;
- distortion increases with a and k_{pol};
- the poloidal part of the magnetic field extends outside the star decaying at infinity as a dipole;
- the toroidal part of the magnetic field remains contained in a little region inside the star vanishing at the surface.