Magnetic Field Instabilities in Neutron Stars

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INTRODUCTION

magnetic fields play a crucial role in the physics of NSs

- polar strengths up to $10^{13}$ G, magnetars $10^{15}$ G
- dipole radiation, magnetar flares, structure deformations and GWs, thermal evolution, ...

what is the internal magnetic field configuration?

- equilibrium models for magnetized neutron stars
- twisted-torus MF geometry (Braithwaite & Nordlund 2006)
- GR \{ Ciolfi et al. 2009, 2010 \}

which equilibrium configurations are stable?
INTRODUCTION

stability of magnetic field configurations studied in simple cases

- **perturbative analytic work**: purely poloidal or purely toroidal fields in non-rotating stars suffer the kink or ‘Tayler’ instability
  
  Tayler 1973; Markey & Tayler 1973; Wright 1973

- **most recently, fully 3D GRMHD simulations** to study the case of purely poloidal fields:
  (i) confirm the instability
  (ii) follow the subsequent nonlinear evolution
  (iii) electromagnetic and GW emission from field rearrangement → ‘test case’ for giant flares

  Lasky et al. 2011; Ciolfi et al. 2011
  Zink et al. 2012; Lasky et al. 2012; Ciolfi & Rezzolla 2012
PHYSICAL SYSTEM and NUMERICAL SETUP

isolated non-rotating magnetized neutron star → fully 3D GRMHD simulations, Cowling approximation

LORENE initial data:
- axisymmetric equilibrium
- purely poloidal magnetic field $10^{16} \text{ G} < B_0 < 10^{17} \text{ G}$
- polytropic EOS $p = K \rho^\Gamma$
  $\Gamma = 2 \quad M \sim 1.4 \ M_\odot$
  $K \sim 100 \quad R^* \sim 12 \ \text{km}$

80 km of computational box, flat BCs
3 refinement levels, up to a resolution of $h = 0.17 \ M_\odot$
MHD evolution performed with the WhiskyMHD code

**ATMOSPHERE**

- densities are reset to a minimum value and fluid velocity is set to zero
- **non-ideal MHD term** added in the induction equation
  \[
  \frac{\partial \vec{A}}{\partial t} = \vec{v} \times \vec{B} + \eta \Delta \vec{A}
  \]
- with the chosen profile we retain ideal MHD inside the star and allow for magnetic field evolution outside
- we avoid errors at the stellar surface and simulations are long-lived
INSTABILITY OF POLOIDAL FIELDS IN NEUTRON STARS: OUR SIMULATIONS

\[ B_D = 6.5 \times 10^{16} \text{ G} \]
INSTABILITY OF POLOIDAL FIELDS IN NEUTRON STARS: OUR SIMULATIONS

\[ B_p = 1.5 \times 10^{16} \text{ G} \]
MAGNETIC ENERGIES AND TIMESCALES

- 90% of magnetic energy is lost in few Alfvén timescales!
- toroidal-to-poloidal energy ratio tends to equipartition
no evidence for a stable equilibrium

- system tends to equipartition of toroidal and poloidal magnetic energies
- significant magnetic helicity produced (initially zero)
HYDROMAGNETIC INSTABILITIES AND MAGNETAR GIANT FLARES

- the violent, global rearrangement of magnetic fields induced by hydromagnetic instability proposed as trigger mechanism for magnetar Giant Flares
  
  Thompson & Duncan 1995, 2001

electromagnetic emission

- our system represents a test case for this scenario: we can estimate electromagnetic luminosity and duration of the process and compare with the observed Giant Flares (up to now, only simple analytic estimates)

gravitational wave emission

- in this scenario, magnetar flares are likely accompanied by f-mode oscillations and GW emission; this motivated recent LIGO/Virgo search for GWs in coincidence with SGR flares
  
  e.g. Abadie et al. 2011

- we can provide a realistic estimate of the amplitude of such GW signal, and establish its level of detectability with next future detectors
ELECTROMAGNETIC EMISSION

- most of the magnetic energy is lost, mainly due to em radiation
- initial $\sim$90% drop of magnetic energy corresponds to a spike in luminosity
- scaling laws: $\tau_{em} \propto 1/B$
  $E_m \propto B^2$
  $L_{em} \propto B^3$
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ELECTROMAGNETIC EMISSION

- most of the magnetic energy is lost, mainly due to em radiation
- initial ~90% drop of magnetic energy corresponds to a spike in luminosity

\[ \tau_{em} \ [s] \quad \langle L_{em} \rangle \ [\text{erg/s}] \]

<table>
<thead>
<tr>
<th>Source</th>
<th>( \tau_{em} )</th>
<th>( \langle L_{em} \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>our result @ ( 10^{15} ) G</td>
<td>( \sim 0.7 )</td>
<td>( 1.9 \times 10^{48} )</td>
</tr>
<tr>
<td>1806-20 (2004)</td>
<td>( \sim 0.5 )</td>
<td>( (0.3 - 1) \times 10^{47} )</td>
</tr>
<tr>
<td>1900+14 (1998)</td>
<td>( \sim 0.35 )</td>
<td>( &gt; 4.3 \times 10^{44} )</td>
</tr>
<tr>
<td>0526-66 (1979)</td>
<td>( \sim 0.25 )</td>
<td>( 6.4 \times 10^{44} )</td>
</tr>
</tbody>
</table>

Mereghetti 2008
SUMMARY

- GRMHD simulations of a non-rotating magnetized NS with purely poloidal magnetic field → evolution triggered by the poloidal field instability

- we confirm all the expectations from earlier perturbative work on the onset of the instability, and follow the subsequent field rearrangement

- at the final stages, most of MF energy is lost; toroidal-to-poloidal energy ratio tends to unity and significant amount of magnetic helicity is produced

- our system represents a test case for the internal rearrangement scenario of magnetar giant flares:
  I - we estimate the electromagnetic luminosity and find compatibility with the observations
  II - we establish that GW emission in giant flares is hardly detectable
FUTURE WORK

understanding the Giant Flare trigger mechanism

• repeat the evolutions with a fully resistive MHD version of the Whisky code
  Dionysopoulou et al. 2012

• explore the electromagnetic emission in more detail, provide full support to the internal rearrangement scenario of giant flares

looking for stable equilibria in magnetized NSs

• test the stability of different magnetic field configurations, in particular twisted-torus geometries

• known twisted-torus solutions are poloidal dominated and likely unstable
  Braithwaite 2009, Lander & Jones 2012

• we recently obtained higher energy in toroidal fields (up to 90%), these configurations could be stable!
  Ciolfi & Rezzolla, in preparation
BACKUP SLIDES
GW SIGNALS
SIGNAL-TO-NOISE RATIO AND DETECTABILITY

\[ f_0 \sim 1.55 \text{ kHz} \quad \text{(Cowling correction)} \]

\[ \frac{S}{N} = \frac{h_{rss}}{\sqrt{S_h(f_0)}} \]

\[ h_{rss} = \left[ \int_{-\infty}^{+\infty} h(t)^2 \, dt \right]^{1/2} \]

\[ h^2(t) = h_x^2(t) + h_+^2(t) \]

what is the expected scaling law for \( h_{rss} \)?

\[ h_{rss} \propto B^2 \]

Levin & van Hoven 2011

\[ \tau_{gw}=100 \text{ ms} \]

\[ \text{adv LIGO–Virgo} \]

ET

\[ h_{rss} \text{ [s/Hz]} \@ 10 \text{ kpc} \]

\[ |B_0| \quad [10^{16} \text{ G}] \]
SIGNAL-TO-NOISE RATIO AND DETECTABILITY

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\]

what is the expected scaling law for $h_{\text{rss}}$?

$\Rightarrow h_{\text{rss}} \propto B^2$

Levin & van Hoven 2011

in agreement with our results!
SIGNAL-TO-NOISE RATIO AND DETECTABILITY

..quadratic extrapolation gives

\[ \frac{S}{N} (\text{AdvLIGO-Virgo}) \sim 1.6 \times 10^{-4} \times \left( \frac{B_0}{10^{15} \text{ G}} \right)^2 \]

\[ \frac{S}{N} (\text{ET}) \sim 1.2 \times 10^{-3} \times \left( \frac{B_0}{10^{15} \text{ G}} \right)^2 \]

hardly detectable by present and near future GW detectors

\[ E_{GW} = \frac{2\pi^2 f_0^2 c^3}{G} r^2 h_{r,s,s} \sim 4.5 \times 10^{37} \text{ erg} \times \left( \frac{B_0}{10^{15} \text{ G}} \right)^4 \times \left( \frac{\tau_{GW}}{100 \text{ ms}} \right) \]

\[ E_{GW} \sim E_m \quad \text{would give} \]

\[ \frac{S}{N} (\text{advLIGO}) \sim 50 ! \quad \rightarrow \]

but only a very small fraction of available energy is pumped into f-mode, confirming expectation of Levin & Van Hoven 2011

Corsi & Owen 2011
TREATMENT OF THE ATMOSPHERE

Graphs showing the variation of $\eta/\eta_0$ with $r$ [km] and $E_{m,\text{tot}}/E_{m,\text{tot}}(t=0)$ with $t$ [ms]. The graphs illustrate the differences between new (thin layer) and old (thick layer) models, with parameters $\eta_0 = 0.02$, $0.10$, and $0.14$. The $E_{m,\text{tor}}/E_{m,\text{pol}}$ ratio is also shown with different $\eta_0$ values.
CENTRAL DENSITY

Graph showing the central density over time with different magnetic field strengths.
MAGNETIC HELICITY

\[ H_m = \int_{\Sigma_t} H_m^0 \sqrt{-g} \, d^3x \]

\[ H_m^\alpha = *F^{\alpha\beta}A_\beta \]

\[ |\tilde{H}_m| \sim R_N \times \sqrt{E_{pol}E_{tor}} \]

\[ = R_N \times E_m/2 \]

\[ |H_m/\tilde{H}_m| \ll 1 \quad \rightarrow \quad \text{small magnetic helicity} \]

\[ |H_m/\tilde{H}_m| \sim 1 \quad \rightarrow \quad \text{significant amount of magnetic helicity} \]
EQUILIBRIUM MODELS OF MAGNETARS

A theoretical explanation of magnetar observations relies on a good description of magnetar equilibrium configurations.

- Braithwaite et al. (2004, 2006): stable magnetic field equilibrium configuration found in 3D newtonian magnetohydrodynamical simulations

\[
\text{Twisted-torus configuration}
\]
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