Modal bispectrum estimation with Planck data

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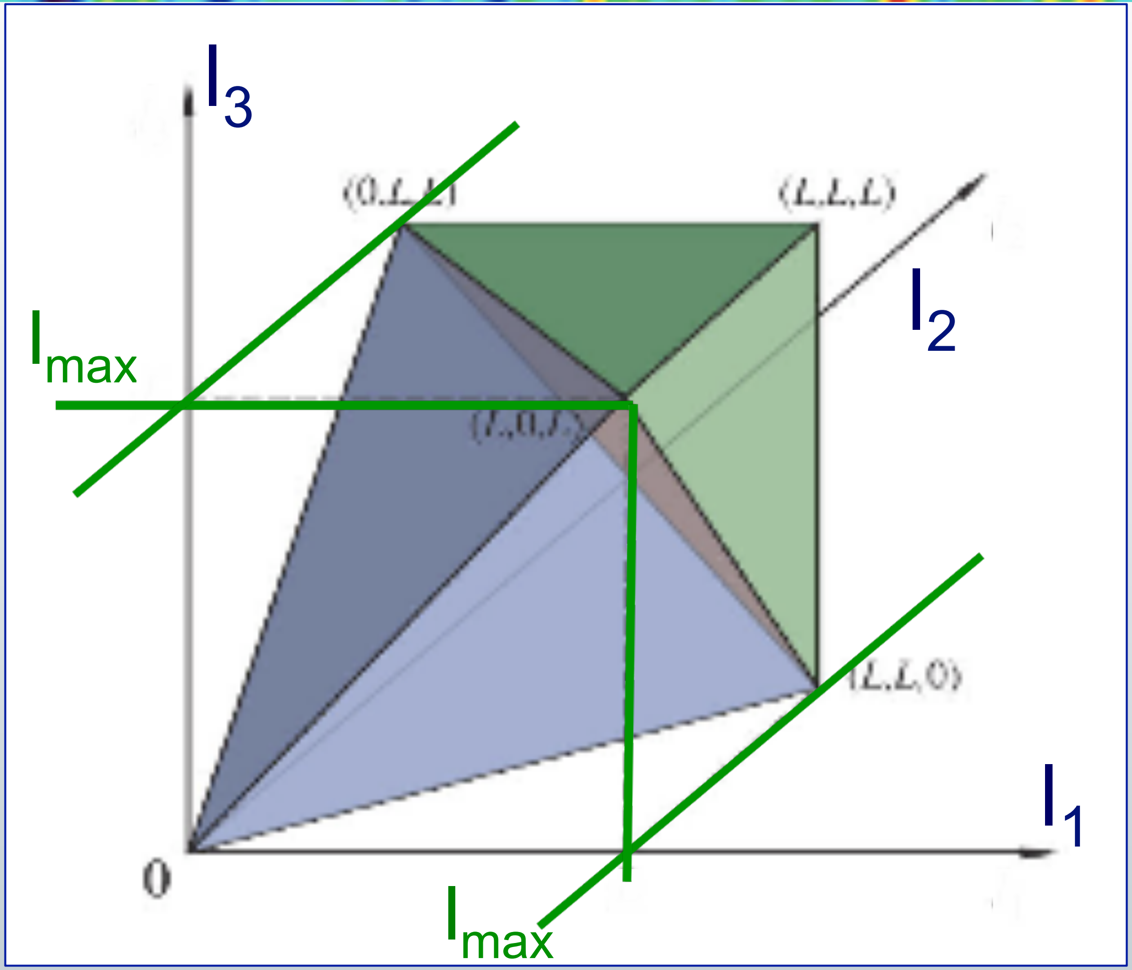
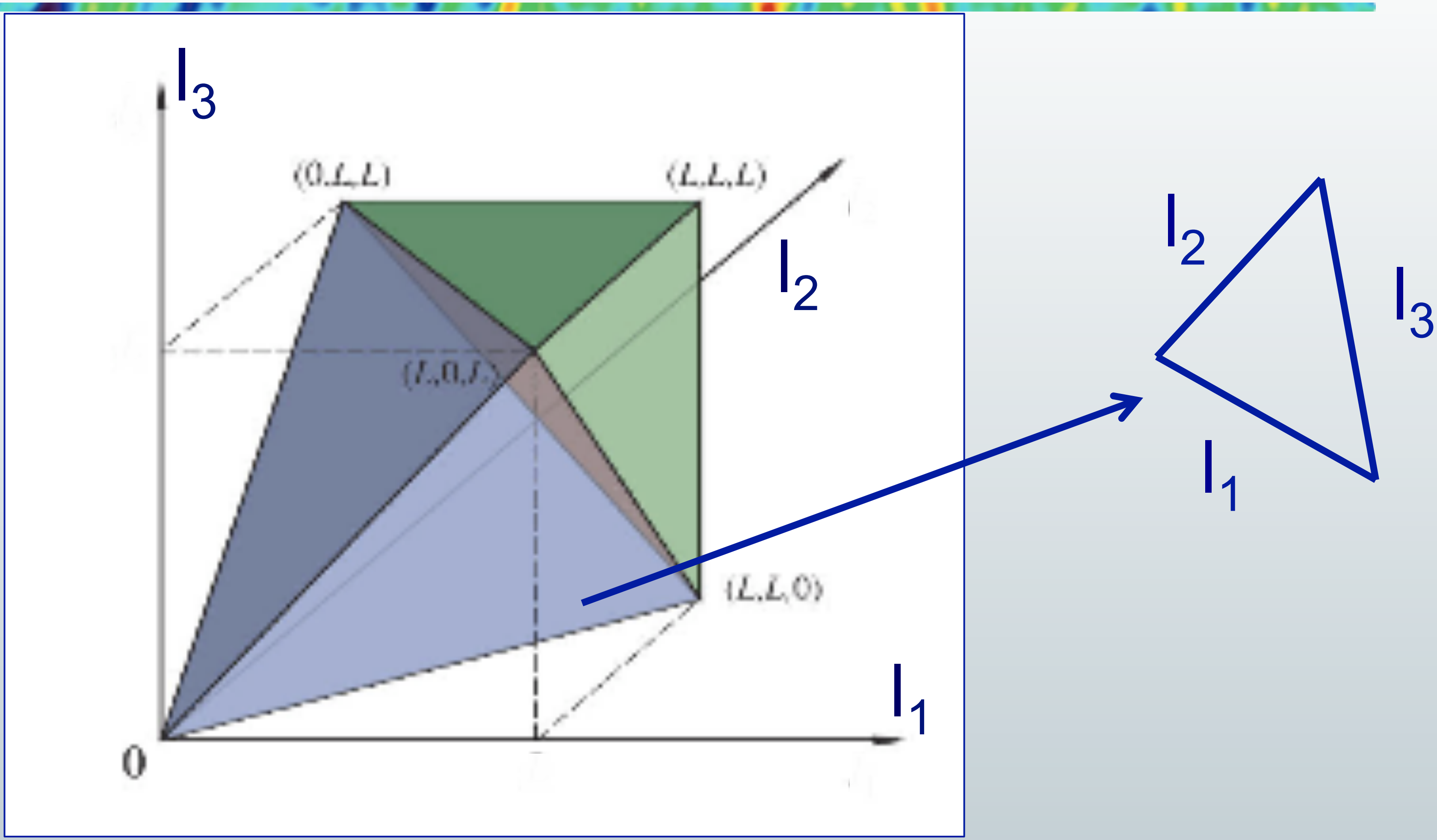
On behalf of the Planck collaboration

3D bispectrum



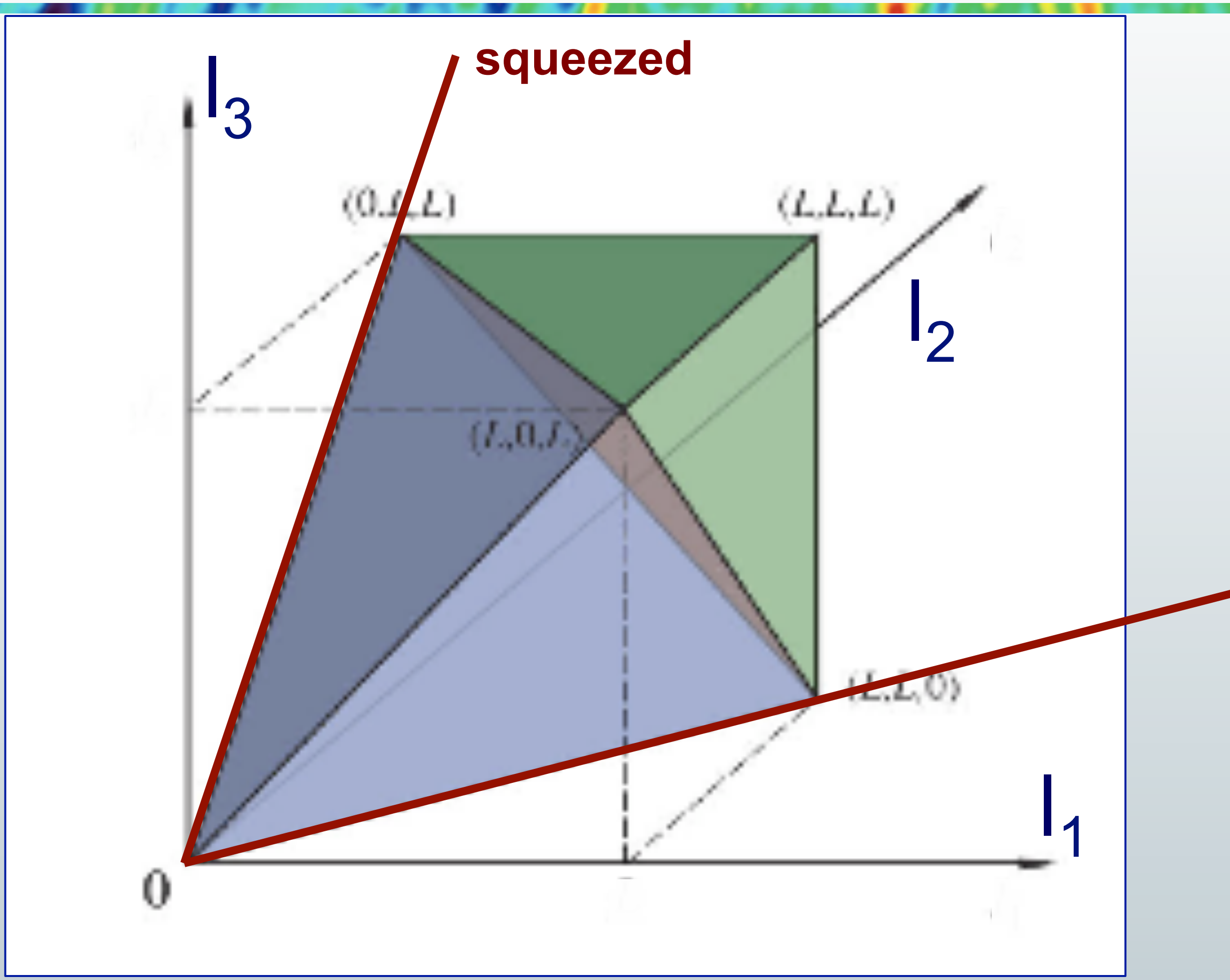
$$B_{\ell_1\ell_2\ell_3} = \sum \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} \right\rangle \;;\; B_{\ell_1\ell_2\ell_3} = h_{\ell_1\ell_2\ell_3} b_{\ell_1\ell_2\ell_3}$$

Bispectrum definition



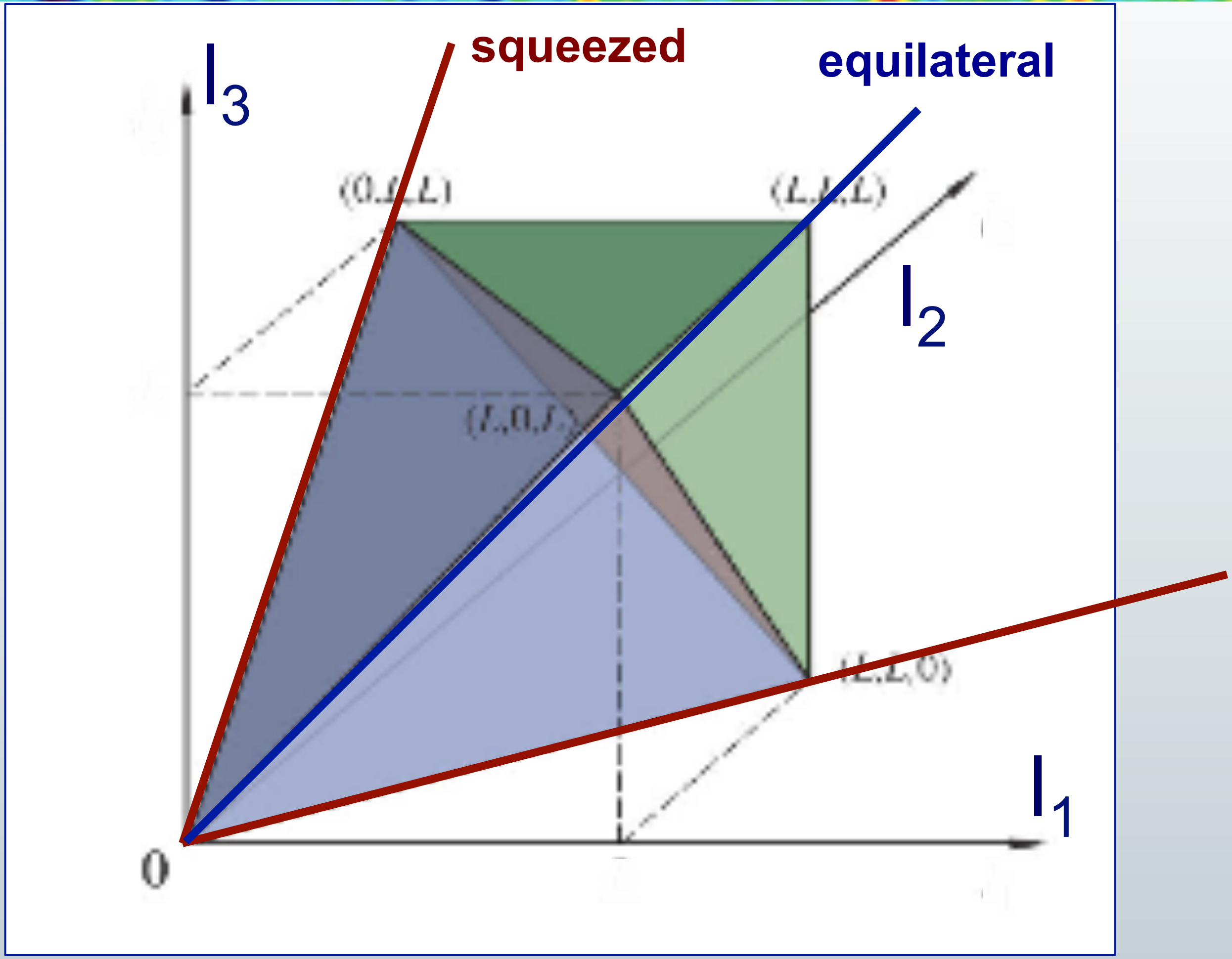
$$B_{\ell_1\ell_2\ell_3} = \sum \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} \right\rangle \; ; \; B_{\ell_1\ell_2\ell_3} = h_{\ell_1\ell_2\ell_3} b_{\ell_1\ell_2\ell_3}$$

Bispectrum definition



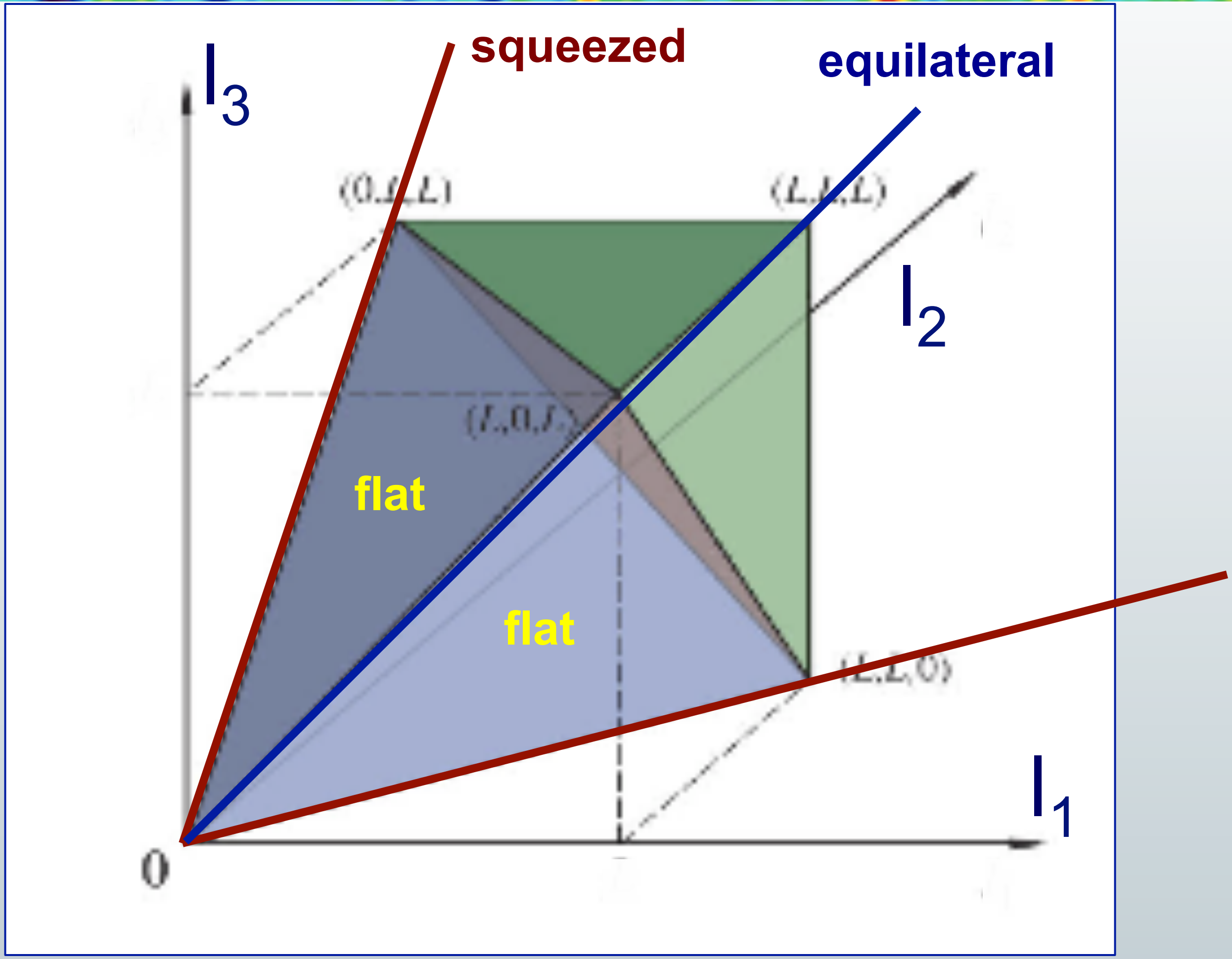
$$B_{\ell_1\ell_2\ell_3} = \sum \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} \right\rangle \; ; \; B_{\ell_1\ell_2\ell_3} = h_{\ell_1\ell_2\ell_3} b_{\ell_1\ell_2\ell_3}$$

Bispectrum definition



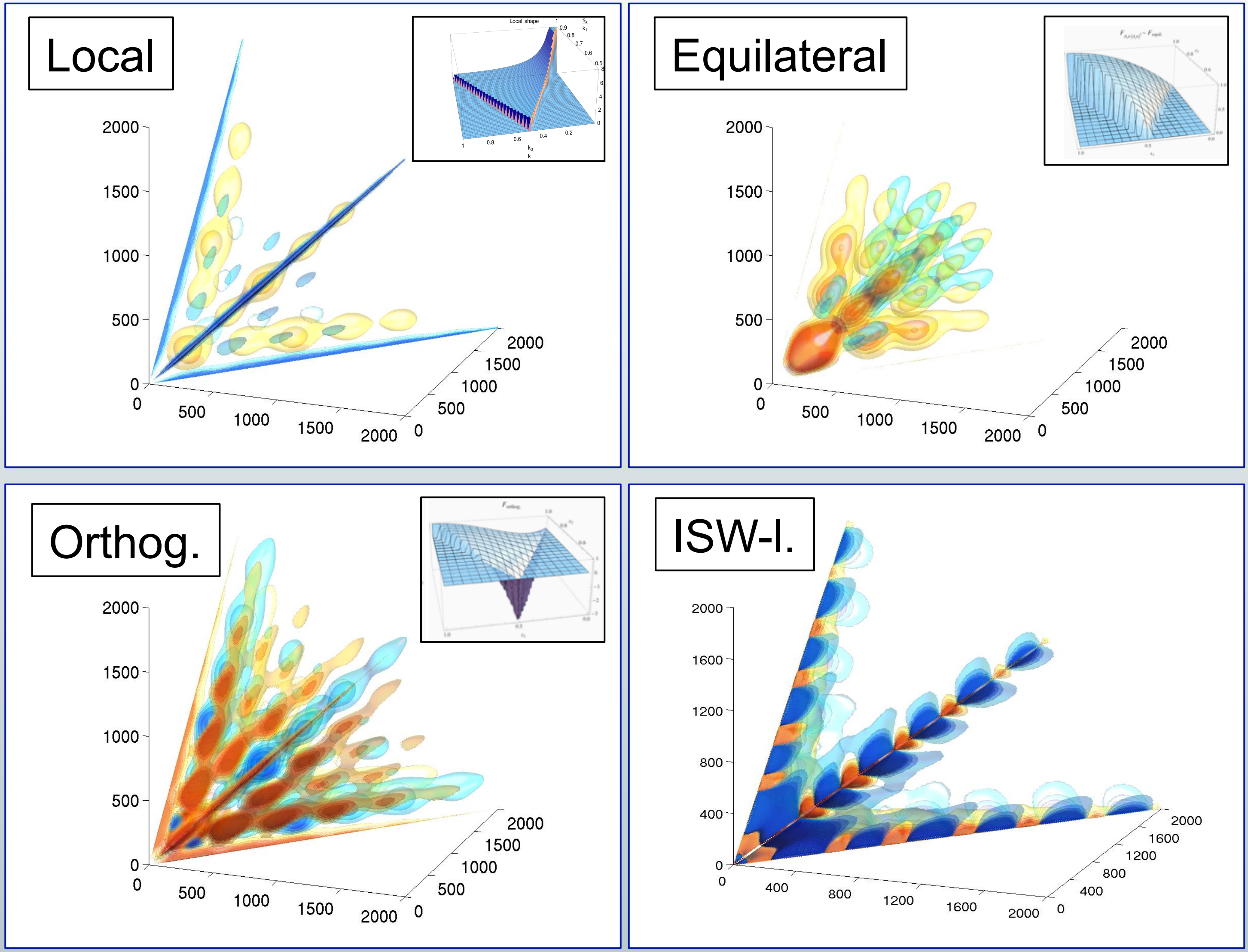
$$B_{\ell_1\ell_2\ell_3} = \sum \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} \right\rangle \; ; \; B_{\ell_1\ell_2\ell_3} = h_{\ell_1\ell_2\ell_3} b_{\ell_1\ell_2\ell_3}$$

Bispectrum definition



$$B_{\ell_1\ell_2\ell_3} = \sum \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} \right\rangle \; ; \; B_{\ell_1\ell_2\ell_3} = h_{\ell_1\ell_2\ell_3} b_{\ell_1\ell_2\ell_3}$$

Shapes


Local
Equilateral
Orthog.
ISW-l.
2000
1500
1000
500
0
1600
1200
800
400

Optimal f_{NL} bispectrum estimator

$$\hat{f}_{NL} = \frac{1}{N}\sum B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \left(C^{-1}a\right)^{m_1}_{\ell_1} \left(C^{-1}a\right)^{m_2}_{\ell_2} \left(C^{-1}a\right)^{m_3}_{\ell_3} - 3C^{-1}_{\ell_1 m_1 \ell_2 m_2} \left(C^{-1}a\right)^{m_3}_{\ell_3}$$

Leaving aside complications coming from breaking of statistical isotropy (sky-cut, noise…), one can see that we are extracting the three point Function from the data and fitting theoretical bispectrum templates to it

$$\hat{f}_{NL} = \frac{1}{N}\sum_{\ell_i m_i} B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \frac{a^{m_1}_{\ell_1}}{C_{\ell_1}} \frac{a^{m_2}_{\ell_2}}{C_{\ell_2}} \frac{a^{m_3}_{\ell_3}}{C_{\ell_3}}$$

A brute force implementation scales like $\ell^5_{\max}$. Unfeasible at Planck (or WMAP) resolution.

Can achieve massive speed improvement ($\ell^3_{\max}$ scaling) if the reduced bispectrum is *separable* (Komatsu, Spergel, Wandelt 2003)

$$b_{\ell_1\ell_2\ell_3} = \sum_{ijk} X^i_{\ell_1} Y^j_{\ell_2} Z^k_{\ell_3} \Rightarrow B^{m_1 m_2 m_3}_{\ell_1\ell_2\ell_3} = b_{\ell_1\ell_2\ell_3} \int Y^{m_1}_{\ell_1}(\Omega) Y^{m_2}_{\ell_2}(\Omega) Y^{m_3}_{\ell_3}(\Omega)$$

Modal expansion

We expand theoretical shapes as a series of *separable* bispectrum templates ("modes"), forming a complete orthonormal basis in bispectrum space, with the scalar product:

$$\langle f,g\rangle = \sum_{\{\ell\}} w_{\ell_1\ell_2\ell_3} \frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} f_{\ell_1\ell_2\ell_3} g_{\ell_1\ell_2\ell_3}$$

Define modes with $\Re_n(\ell_1,\ell_2,\ell_3)$ and expand:

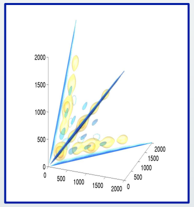
$$\frac{b^{theory}_{\ell_1\ell_2\ell_3}}{\sqrt{C_{\ell_1}C_{\ell_2}C_{\ell_3}}} = \sum_n \alpha_n \Re_n$$

Truncate when the expanded shape is highly correlated to the initial one (r > 0.95)

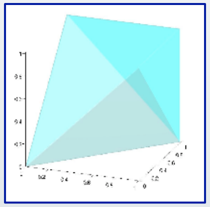
(Fergusson, ML, Shellard, 2009,2010)

Modal expansion in figures

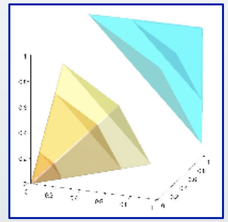
Expansion



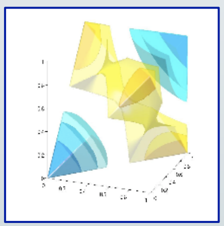
$= \alpha_0$



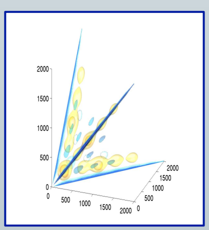
$+ \alpha_1$



$+ \alpha_2$

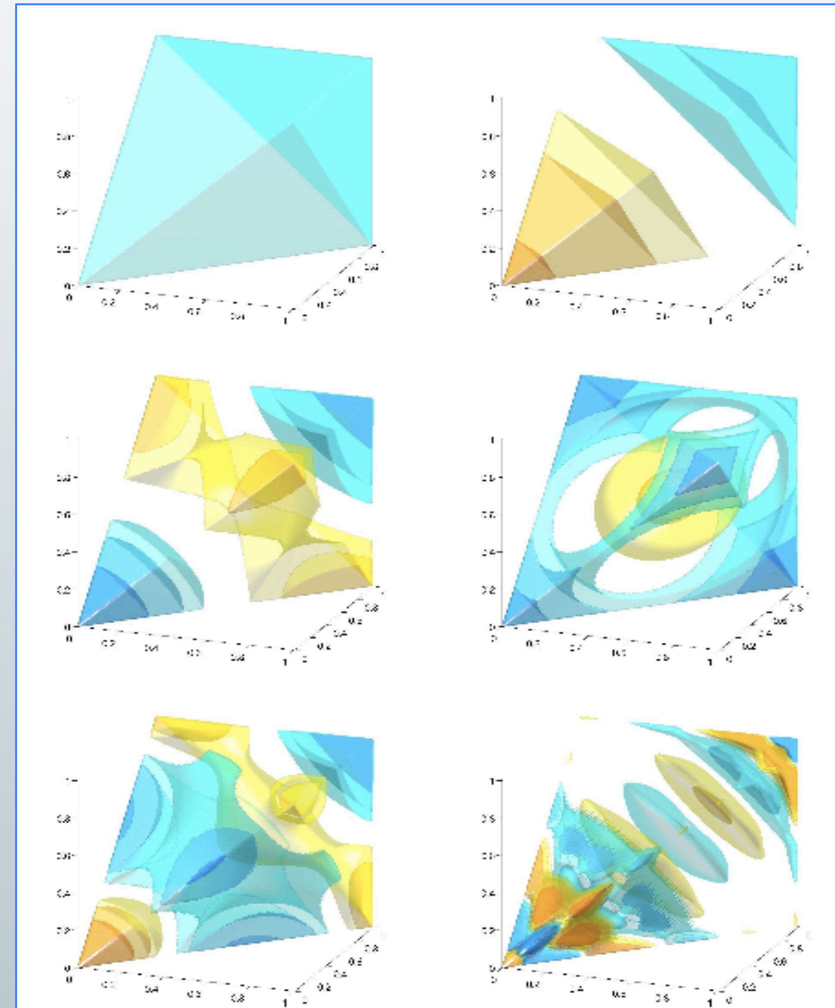


$+ \ldots$

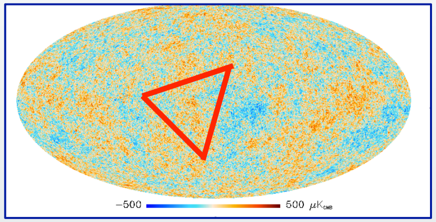


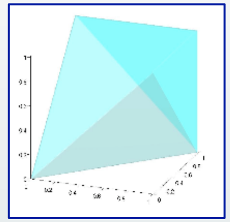
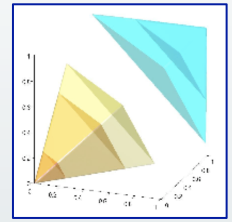
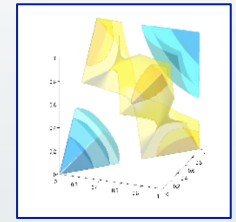
$\Rightarrow (\alpha_0, \alpha_1, \ldots, \alpha_n)$

Basis modes



Bispectrum estimation



$= \beta_0$

$+ \beta_1$

$+ \beta_2$

$+ \ldots$

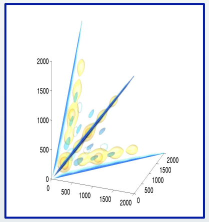
For a given dataset, extract best-fit b_i, i=1,…,n

- The basis elements pictured on the right *are by construction factorizable*

- Apply position space cubic statistics, “KSW”, to each separable template on the right to estimate the amplitudes β_i

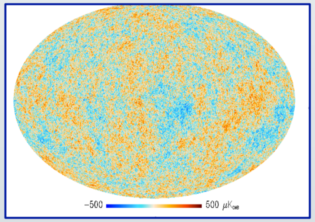
- Orthonormal basis ➔ β_i uncorrelated (in first approx.)

f_{NL} and bispectrum reconstruction



$$\Rightarrow (\alpha_0, \alpha_1, \ldots, \alpha_n)$$

Theory

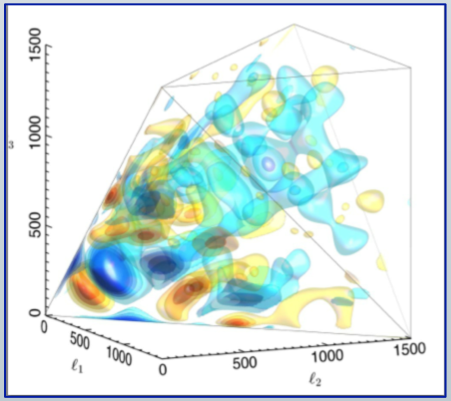


$$\Rightarrow (\beta_0, \beta_1, \ldots, \beta_n)$$

Data
("mode spectrum")

$$f_{NL} = \frac{1}{N} \sum_n \alpha_n \beta_n$$

$$N = \frac{1}{6} \sum_n \alpha_n^2$$



$$= \sum_n \beta_n \mathfrak{R}_n$$

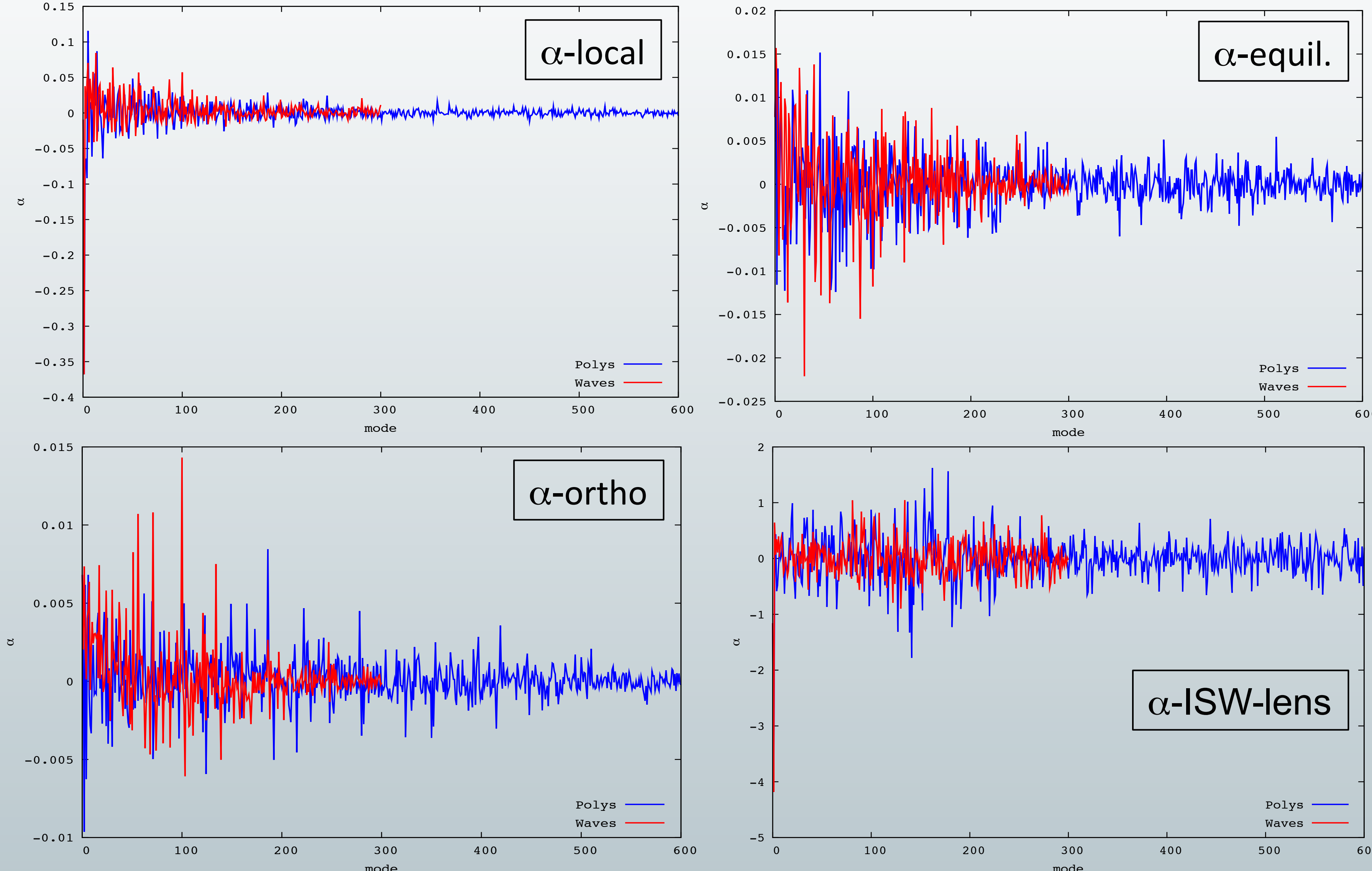
Planck modal expansion

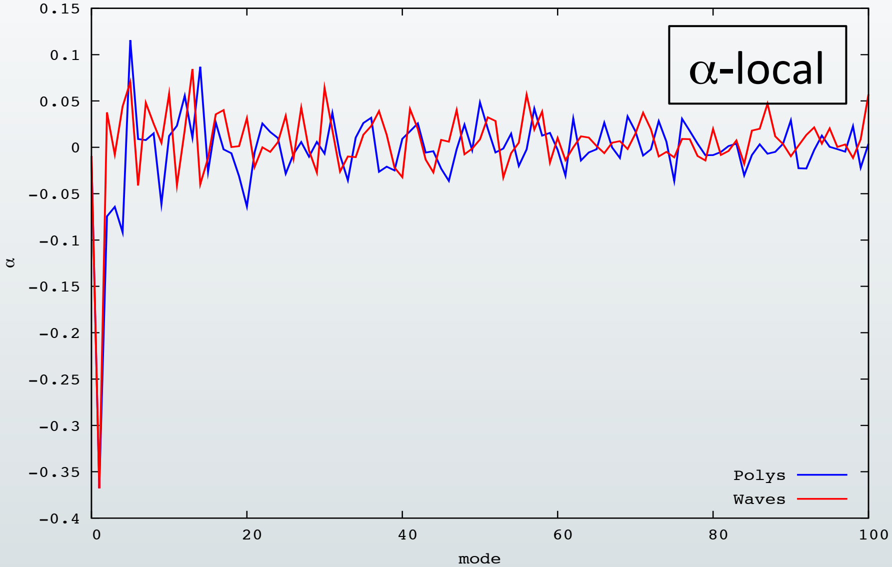
We work at l_{max} = 2000 and consider two sets of modes throughout the analysis

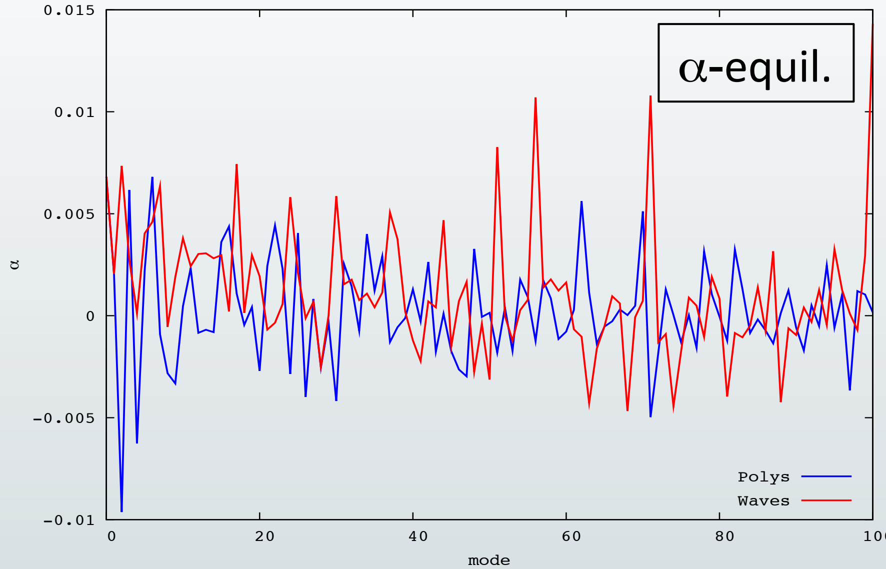
- Plane wave templates, n_{max} = 300 (faster, slightly less accurate)
- Polynomial modes, n_{max} = 600 (slower, slightly more accurate)

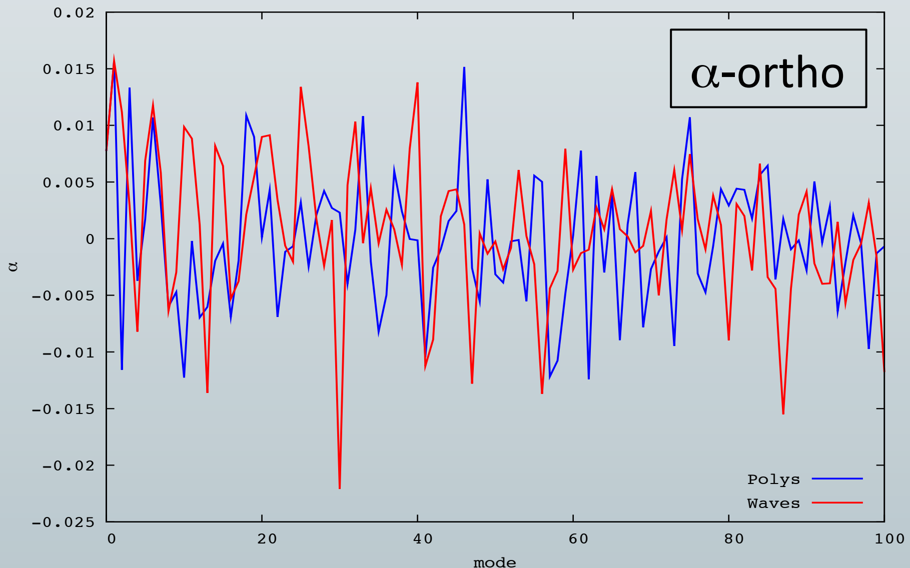
Both sets are augmented with a local Sachs Wolfe template (B~CC+perm.) to improve convergence in the squeezed limit

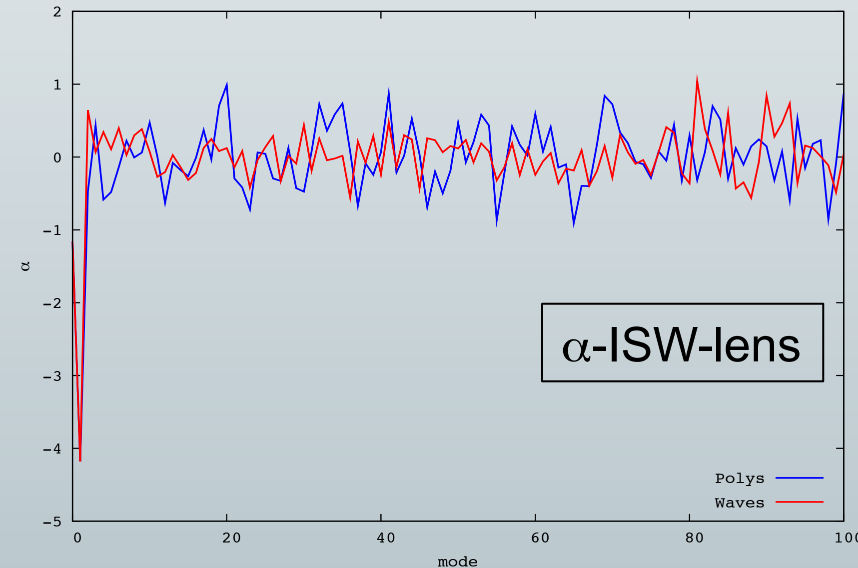
	Local	Equil.	Ortho.	ISW-lens
Waves	95%	99%	97%	70%
Polys	99%	99%	99%	90%


α-local
α-equil.
α-ortho
α-ISW-lens
α
mode
Polys
Waves
0
100
200
300
400
500
600


α-local
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
−0.3
−0.35
−0.4
α
0
20
40
60
80
100
mode
Polys
Waves


α-equil.
0.015
0.01
0.005
0
−0.005
−0.01
α
0
20
40
60
80
100
mode
Polys
Waves


α-ortho
0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
−0.025
α
0
20
40
60
80
100
mode
Polys
Waves


α-ISW-lens
2
1
0
−1
−2
−3
−4
−5
α
0
20
40
60
80
100
mode
Polys
Waves

Validation on simulations

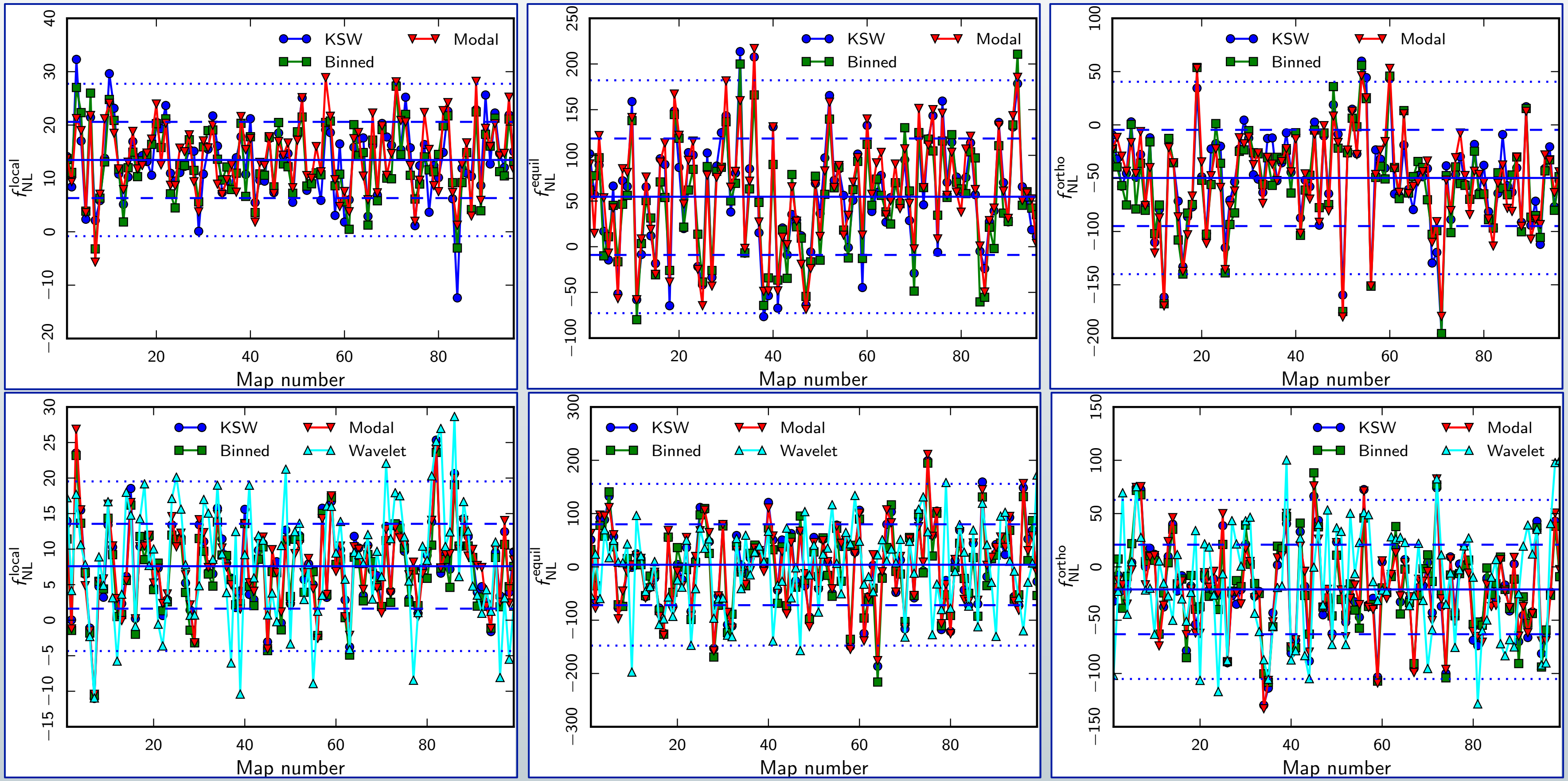
- Different pipelines (KSW, binned, modal) and expansions (waves,polys) have *highly correlated* but *not identical* weights.
- Expected scatter on average (Gaussian, full sky, homogenous noise):

$$\sigma_{\delta f_{NL}} = \Delta_{f_{NL}} \frac{\sqrt{1-r^2}}{r}$$

- We perform a large amount of tests on G and NG simulations (loc.+ equil.+ortho.)
- We start from full sky noiseless maps, and include several features in various steps: corr. noise, sky cut, NG, lensing, foreground residuals.
- We check:
  - ✓ unbiasedness,
  - ✓ optimality,
  - ✓ agreement on a map-by-map basis.

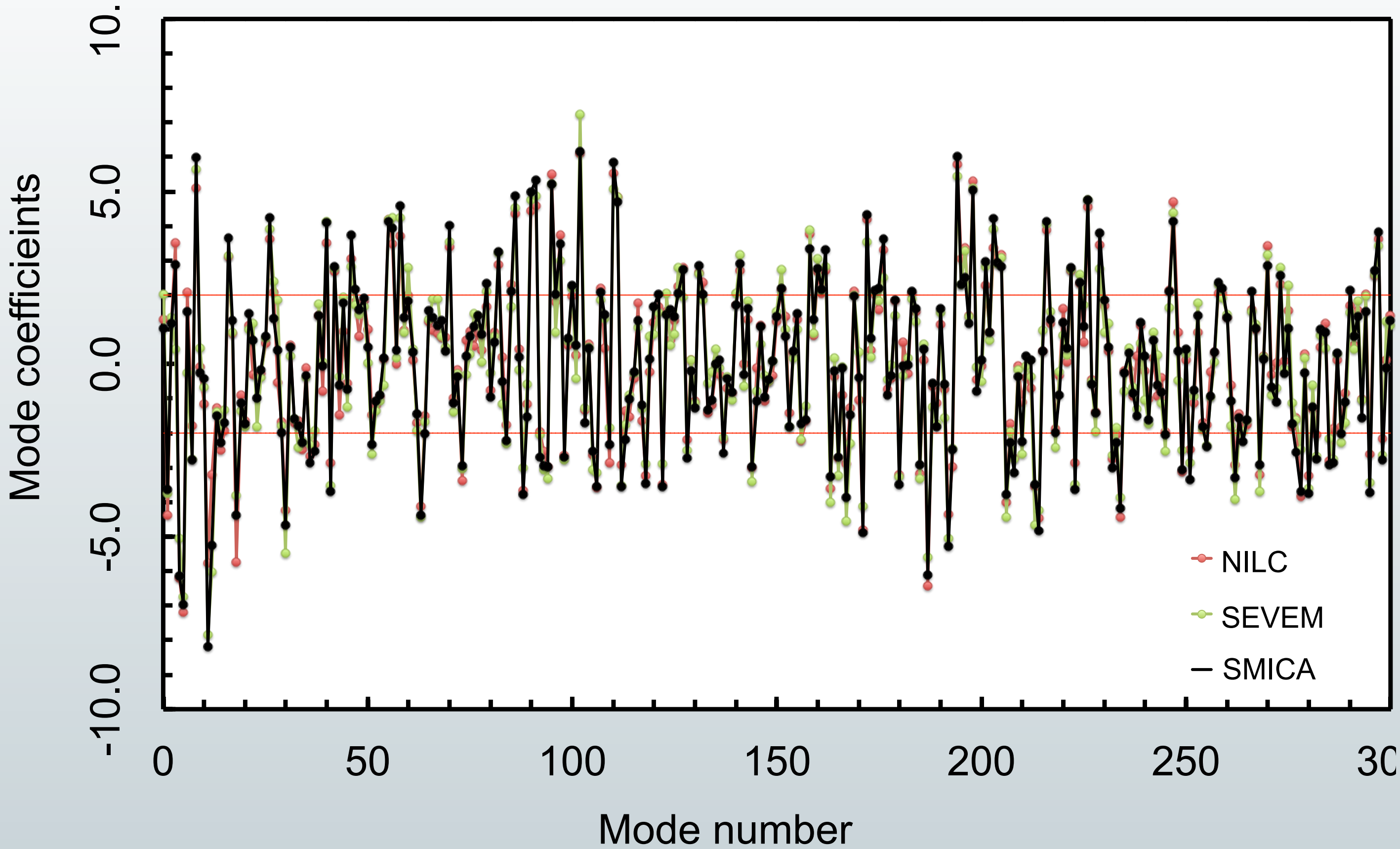
Comparison between pipelines

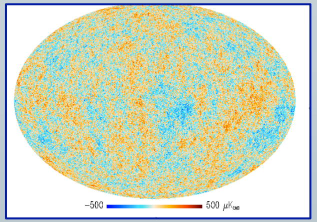
Local   Equilateral   Orthogonal



- Top row: wave expansion, NG sims (loc+eq+ortho, anis. noise, U73 mask)
- Bottom row: polynomial expansion, lensed FFP6 sims

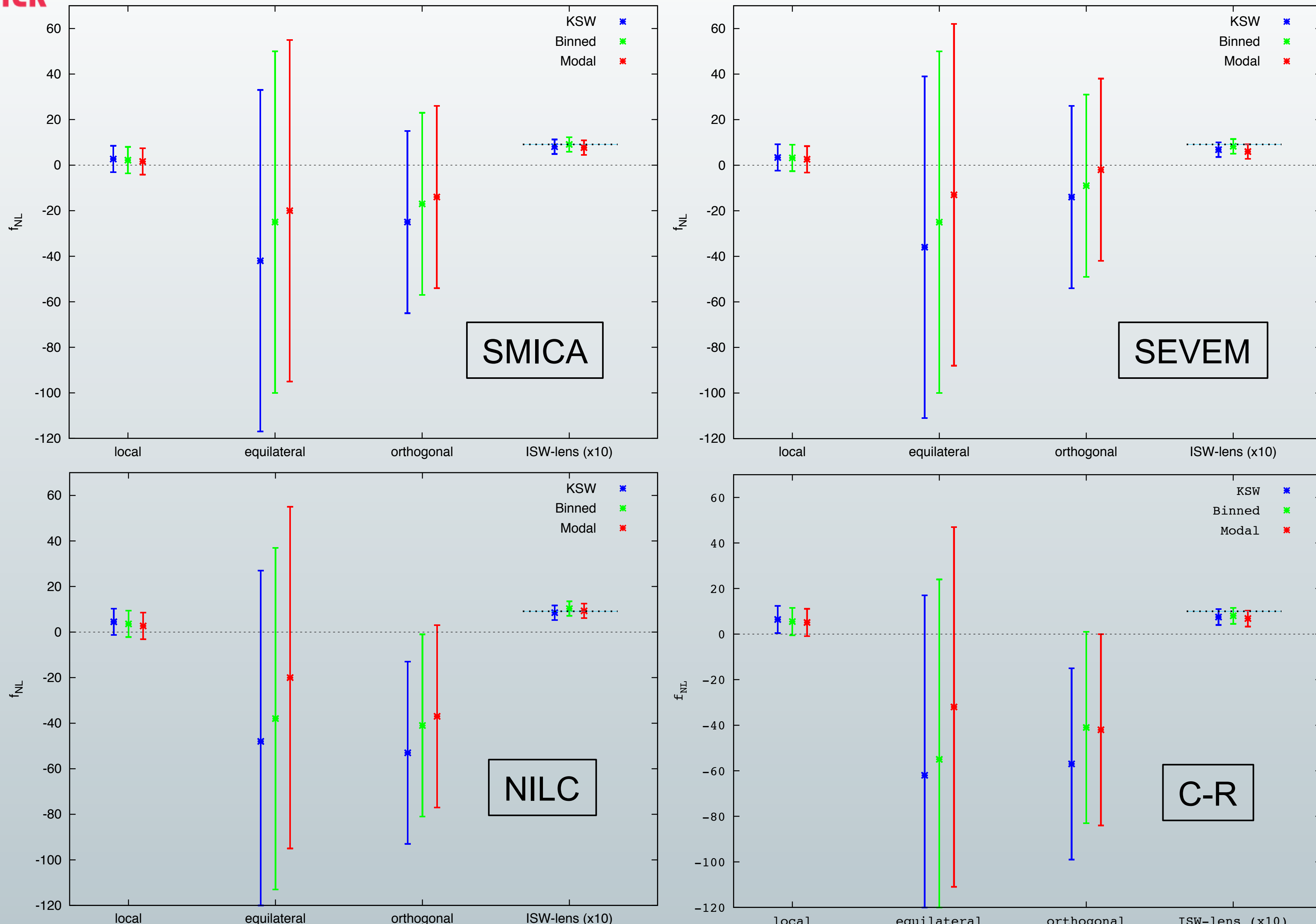
Modes from *Planck* data



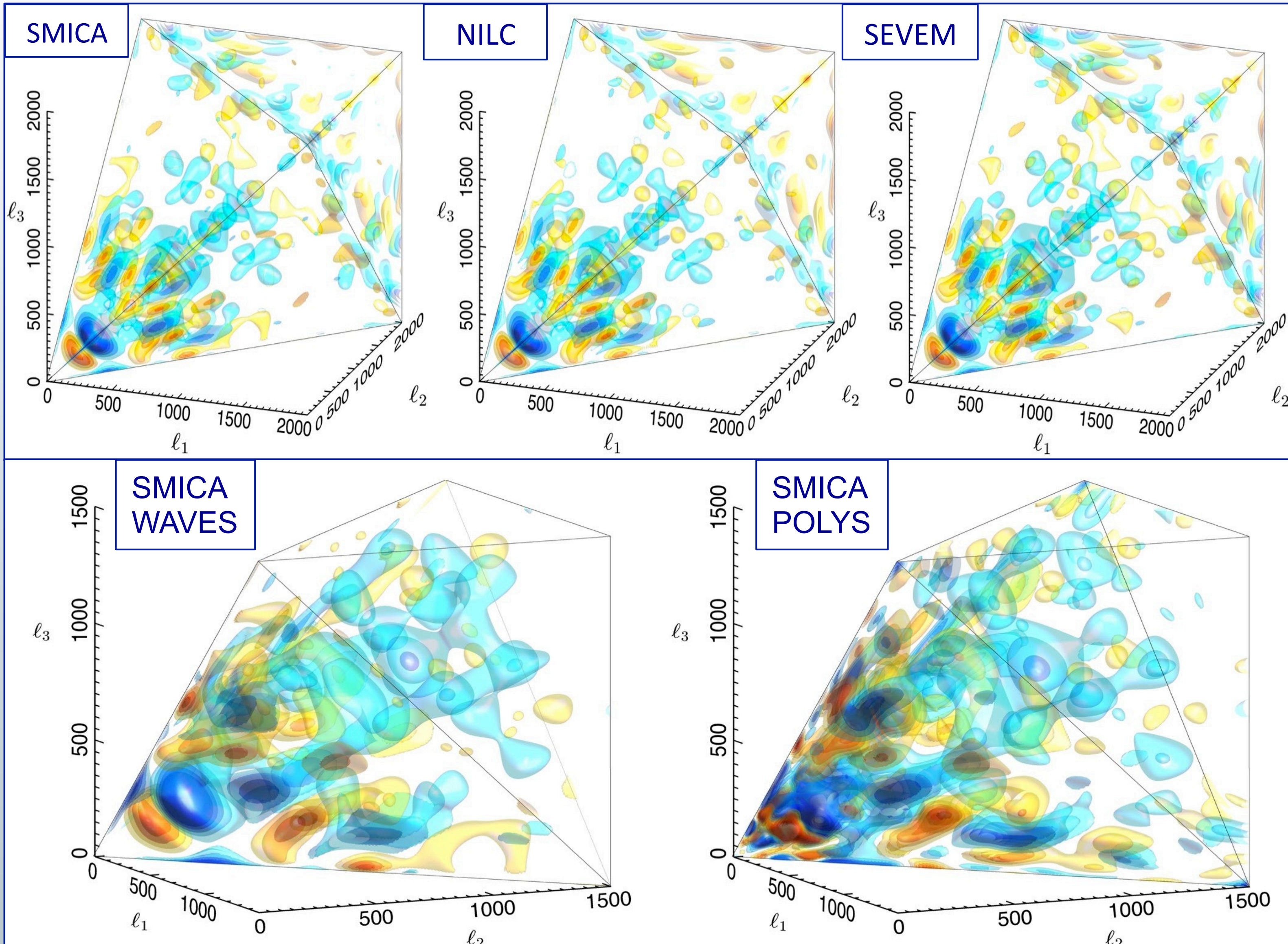


$$\left(\beta_0, \beta_1, \ldots, \beta_n\right)$$

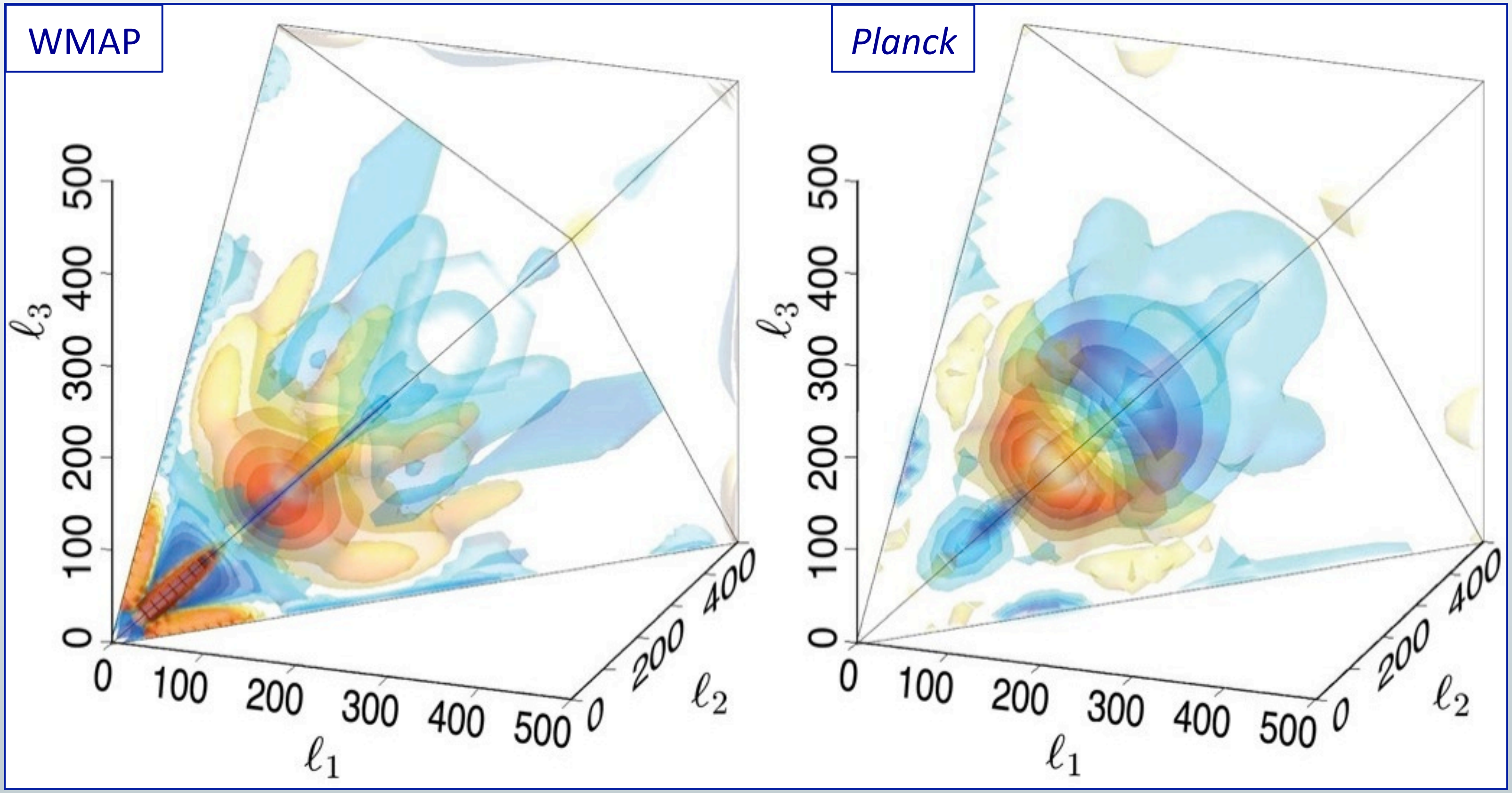
f_{NL} from *Planck* data



The *Planck* bispectrum



WMAP vs. Planck



Consistency checks

We perform several additional tests of robustness of the results:

- ✓ SMICA, dependence on l_{max}
- ✓ SMICA, dependence on sky coverage (*)
- ✓ f_{NL} from raw single-frequency channels (70,100,143,217 Ghz) (*)
- ✓ Jackknife analysis (*)
- ✓ Study of FFP6 lensed simulations including foreground residuals from SMICA and NILC (*)

(*) = modal pipeline was used for this test

See B. Van Tent's talk

“Non-standard” shapes

- *All we need from the data are the mode amplitudes β. We can then estimate f_{NL} for all the shapes that we can accurately expand in our basis.*

- We compute α and f_{NL} for a large number of models, including

  - ✓ Equilateral family (DBI, EFT, ghost)
  - ✓ Flattened shapes (non-Bunch Davies)
  - ✓ Feature models (oscillatory bispectra, scale-dependent)
  - ✓ Vector models
  - ✓ Quasi-single-field

- Results are validated (besides previous tests in the local, equil, ortho direction) using waves and polynomial expansions, changing lmax and mask, and using SMICA, NILC and SEVEM foreground cleaned data.

- No evidence for NG found, constraints on parameters from the models above

See P. Shellard’s talk

Summary

- Modal method: expand the bispectrum in a complete orthonormal set of separable basis templates ("modes"). It allows: f_{NL} estimation in full generality and model independent (smoothed) bispectrum resconstruction
- The modal estimator was one of three optimal bispectrum pipelines used to measure f_{NL} in Planck data (the other two being the "KSW" and binned method).
- The main steps of the analysis were summarized:
  - ✓ Cross validation of the pipelines on simulations
  - ✓ Measurement of the mode spectrum
  - ✓ fNL estimation: loc., equil., ortho., isw-lens, using (KSW,binned,modes) × (SMICA,NILC,SEVEM,C-R)
  - ✓ Bispectrum reconstruction
  - ✓ Consistency checks on data
- The modal pipeline provided also constraints on a large number of primordial shapes, beyond the standard local, equilateral, orthogonal analysis.

The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.