CMB optimal constraints on Non-Gaussianity in isocurvature perturbations

Toyokazu Sekiguchi (Nagoya Univ.)

References:

- C. Hikage, K. Kawasaki, TS & T. Takahashi [arXiv: 1211.1095, 1212.6001]
- TS & N. Sugiyama [arXiv: 1303.4626]

“Universe as seen by Planck”@ESTEC
April 04, 2013

No non-Gaussianity from Planck

Type	$f_{\rm NL}(1\sigma)$
Local	2.7 ± 5.8
Equilateral	-42 ± 75
Orthogonal	-25 ± 39
DBI	11 ± 69
EFT1	8 ± 73
EFT2	19 ± 57
Ghost	-23 ± 88
WarmS	4 ± 33

Is this the end of the story...?

Beyond f_{NL}

Planck analysis mainly focuses on the purely adiabatic perturbations at bispectrum level (*except for* τ_{NL})

$$\langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\rangle \propto f_{\rm NL}$$

However, there may be other types of non-Gaussianity

- *Non-Gaussianity in isocurvature perturbations*
- g_{NL} *(trispectrum)*
- *...*

→ Tightest constraints have been derived from WMAP

WMAP 9yr constraints on g_{NL}

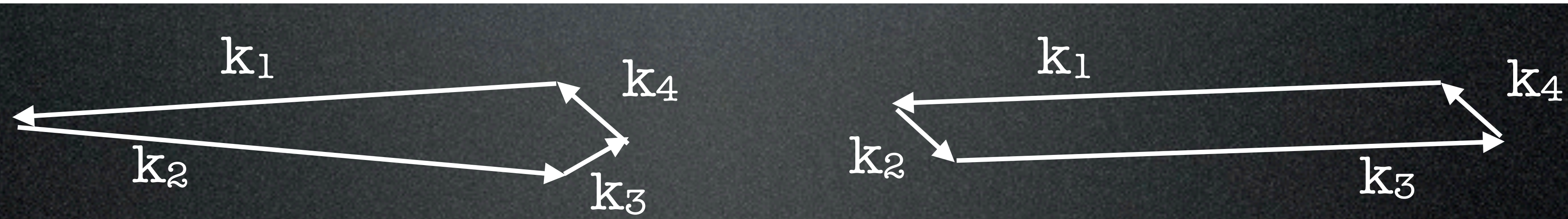
TS & Sugiyama arXiv:1303.4626

Local-type non-Gaussian perturbations (higher-order)

$$\Phi(\vec{x}) = \Phi_{\rm G}(\vec{x}) + f_{\rm NL}\left[\Phi_{\rm G}(\vec{x})^2 - \langle\Phi_{\rm G}(\vec{x})^2\rangle\right] + g_{\rm NL}\Phi_{\rm G}(\vec{x})^3$$

Primordial trispectrum

$$\langle\Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\Phi(\vec{k}_4)\rangle_{\rm conn} = 6g_{\rm NL}\left[P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3) + (3\ \text{perms})\right](2\pi)^3\delta^{(3)}(\vec{k}_{1234})$$



Optimal constraints from WMAP 9yr (temperature V+W)

$$g_{\rm NL} = (-3.3 \pm 2.2) \times 10^5$$

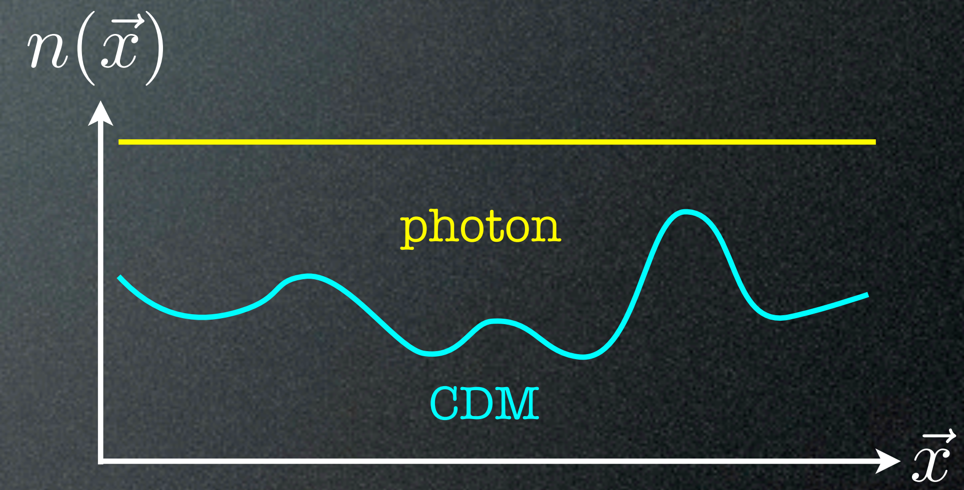
cf. Planck forecast (Fisher matrix): $\Delta g_{NL} = 6.7\times10^4$

Isocurvature perturbations

Relative entropy perturbations btw. photon and CDM/neutrinos

$$S_{\mathrm{CDM/b/}\nu}(\vec{x}) = \delta \ln \frac{n_{\mathrm{CDM/b/}\nu}(\vec{x})}{n_\gamma(\vec{x})}$$

vanishes for single-source model



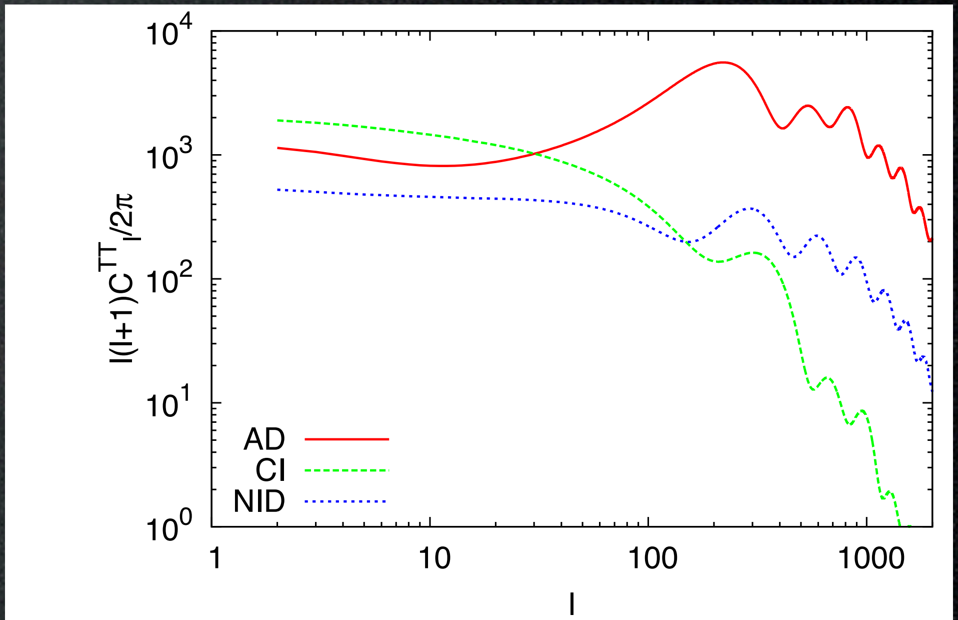
Planck constraints on power spectrum

(CDM isocurvature)

$\alpha < 0.04$ (uncorrelated γ=0)

$\alpha < 0.0025$ (anti-correlated γ=-1)

$$\alpha \approx \frac{P_S}{P_\Phi} \qquad \gamma = \frac{P_{S\Phi}}{\sqrt{P_S P_\Phi}}$$

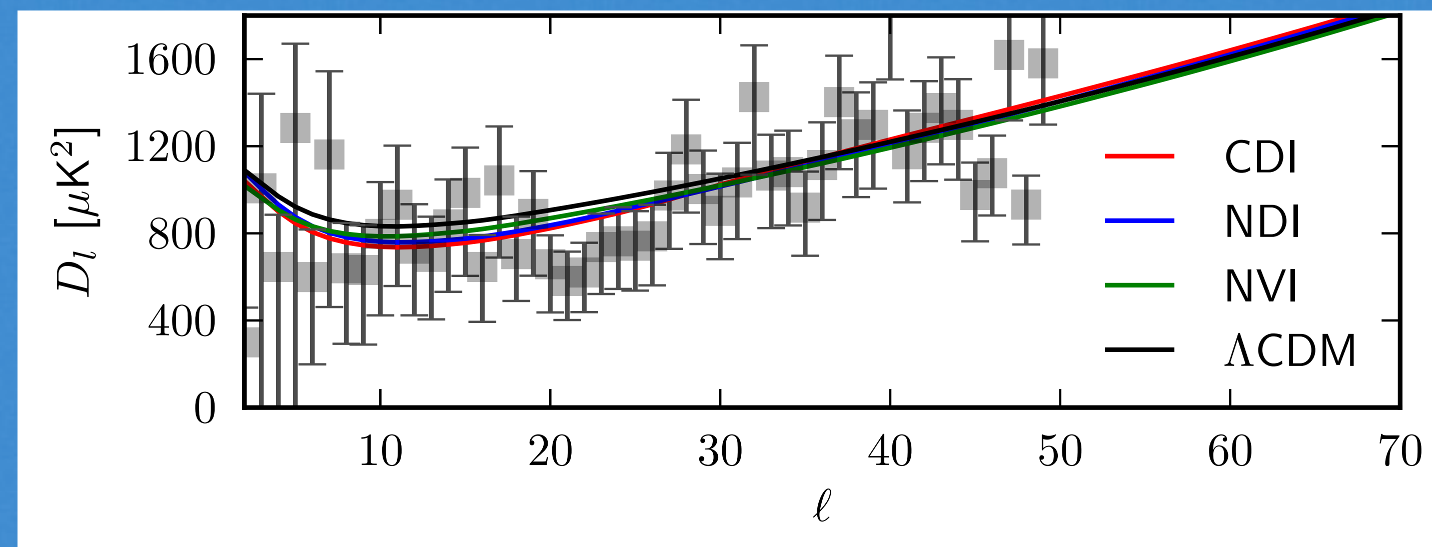


Isocurvature perturbations

Relative entropy perturb

$$S_{\mathrm{CDM/b/\nu}}(\vec{x}) = \delta \ln \frac{n_{\mathrm{CI}}}{}$$

vanishes for single-sour



Hints from low-ell power suppression?

Planck constraints on power spectrum

(CDM isocurvature)

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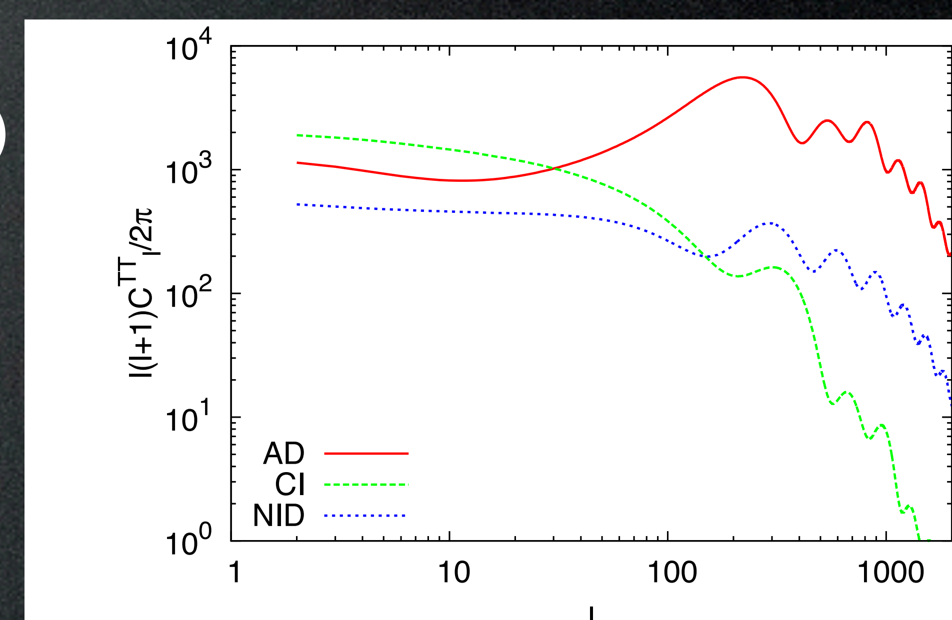


Non-Gaussianity in isocurvature perturbations

Extension of local-type NG to non-adiabatic perturbations

$$\Phi(\vec{x}) = \Phi_G(\vec{x}) + f_{\rm NL}(\Phi_G^2(\vec{x}) - \langle\Phi_G^2\rangle)$$
$$S(\vec{x}) = S_G(\vec{x}) + f_{\rm NL}^{\rm (ISO)}(S_G^2(\vec{x})^2 - \langle S_G^2\rangle)$$

e.g., Linde & Mukhanov 1997

Primordial bispectrum

Axion type (uncorrelated with Φ)

Kawasaki, TS+ 2008; Hikage+ 2009

$$\begin{aligned}\langle S(\vec{k}_1)S(\vec{k}_2)S(\vec{k}_3)\rangle &\sim 2f_{\rm NL}^{\rm (ISO)}[P_S(k_1)P_S(k_2) + (2 \text{ perms.})] \\ &\sim \alpha^2 f_{\rm NL}^{\rm (ISO)}[P_\Phi(k_1)P_\Phi(k_2) + (2 \text{ perms.})]\end{aligned}$$

Curvaton type (totally correlated)

Langlois, Vernizzi & Wands 2008;
Kawasaki TS+ 2009

$$\langle S(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\rangle \propto \alpha f_{\rm NL}^{\rm (ISO)}$$

cf. general case: six distinct bispectra (Langlois & Tent 2011)

Studies on isocurvature NG

Theoretical models

curvaton scenario: Linde & Mukhanov 1996; Boubekeur & Lyth 2005; Langlois, Vernizzi & Wands 2008; Kawasaki+ 2009; Moroi & Takahashi 2009, Kobayashi Mukohyama 2009; ...

axion model: Kawasaki+ 2008; Hikage+ 2009; ...

Affleck-Dine mechanism: Kawasaki+ 2009

multi-field inflation: Langlois+ 2008,...

modulated reheating: Boubekeur & Creminelli 2006; Takahashi, Yamaguchi, Yokoyama 2009

neutrino isocurvature: Kawasaki 2012; Kawakami+ 2012

....

Observational constraints

Fisher matrix forecast: Hikage+ 2009; Langlois & Tent 2011, 2012; Kawakami+ 2012

Minkowski functionals: Hikage+ 2009

Optimal bispectrum estimator: NONE!

Data & Analysis

Optimal estimator Komatsu, Spergel, Wandelt 2005; Creminelli+ 2006; Yadav+ 2007, 2008

$$\hat{f}_{\rm NL}^{(X)} = \sum_Y \mathcal{N}^{-1}_{(XY)} \sum_{\{l,m\}} B^{(Y)m_1m_2m_3}_{l_1l_2l_3} [\tilde{a}_{l_1m_1}\tilde{a}_{l_2m_2}\tilde{a}_{l_3m_3} - 3\langle \tilde{a}_{l_1m_1}\tilde{a}_{l_2m_2}\rangle \tilde{a}_{l_3m_3}]$$

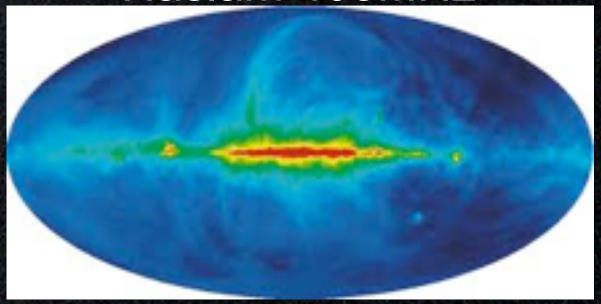
$$\tilde{a}_{lm} = (C^{-1}a)_{lm} \qquad X, Y = \Phi, S$$

- full inverse-covariance filtering of maps Smith+ 2007
- normalization determined by exact NG CMB simulation for local-type Elsner & Wandelt 2009

Data: WMAP 7yr Jarosik+ 2011; Gold+ 2011

- Temperature maps at V+W bands
- KQ75y7 conservative sky cut (f_{sky}=72%)
- Template marginalization of Galactic foregrounds (synch, free-free, thermal dust)

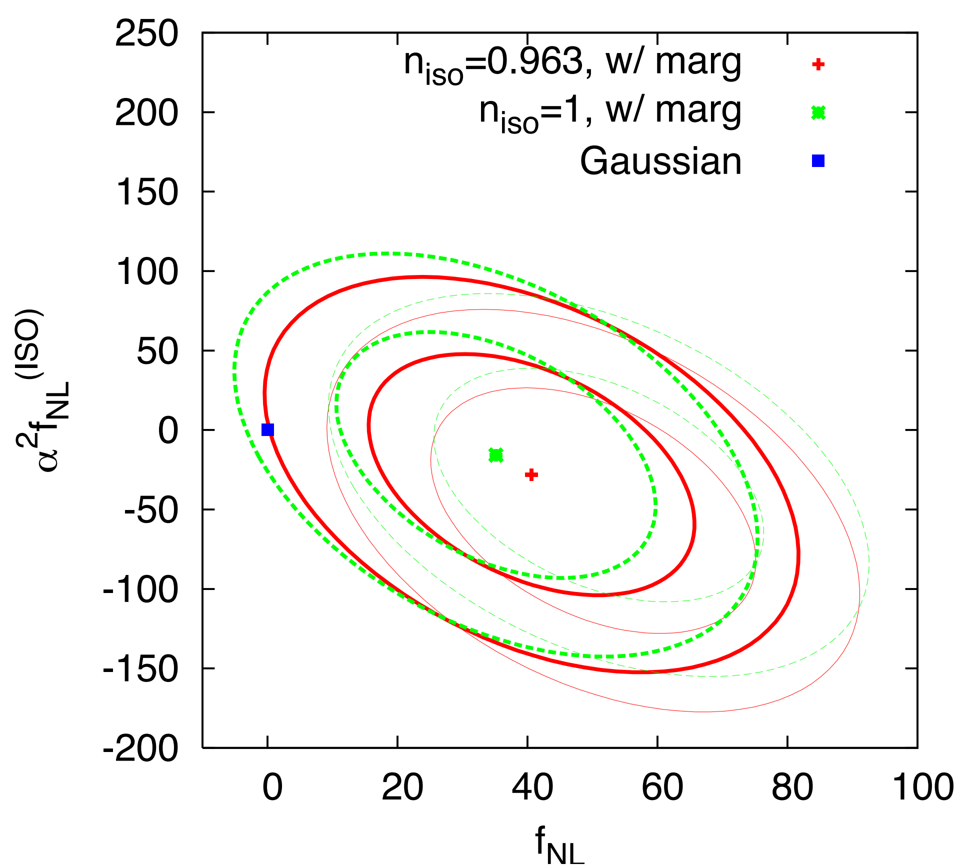
Haslam 408MHz



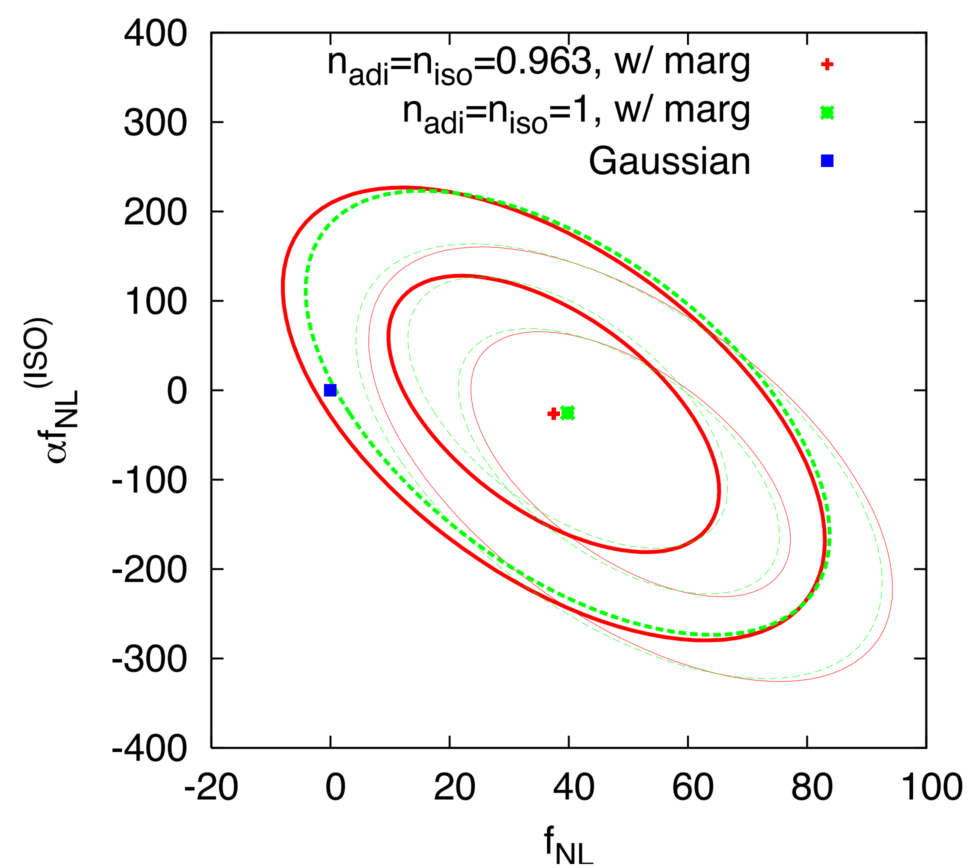
Result: CDM isocurvature

C. Hikage, M. Kawasaki, TS, T.Takahashi, arXiv:1211.1095, 1202.6001

Uncorrelated case (axion type)



Correlated case (curvaton type)



$$f_{\rm NL} = 36 \pm 23$$
$$\alpha^2 f_{\rm NL}^{\rm (ISO)} = -39 \pm 69$$
(1 sigma)

(for $n_{\rm iso} = n_{\rm adi} = 0.963$)

$$f_{\rm NL} = 37 \pm 25$$
$$\alpha f_{\rm NL}^{\rm (ISO)} = -26 \pm 144$$
(1 sigma)

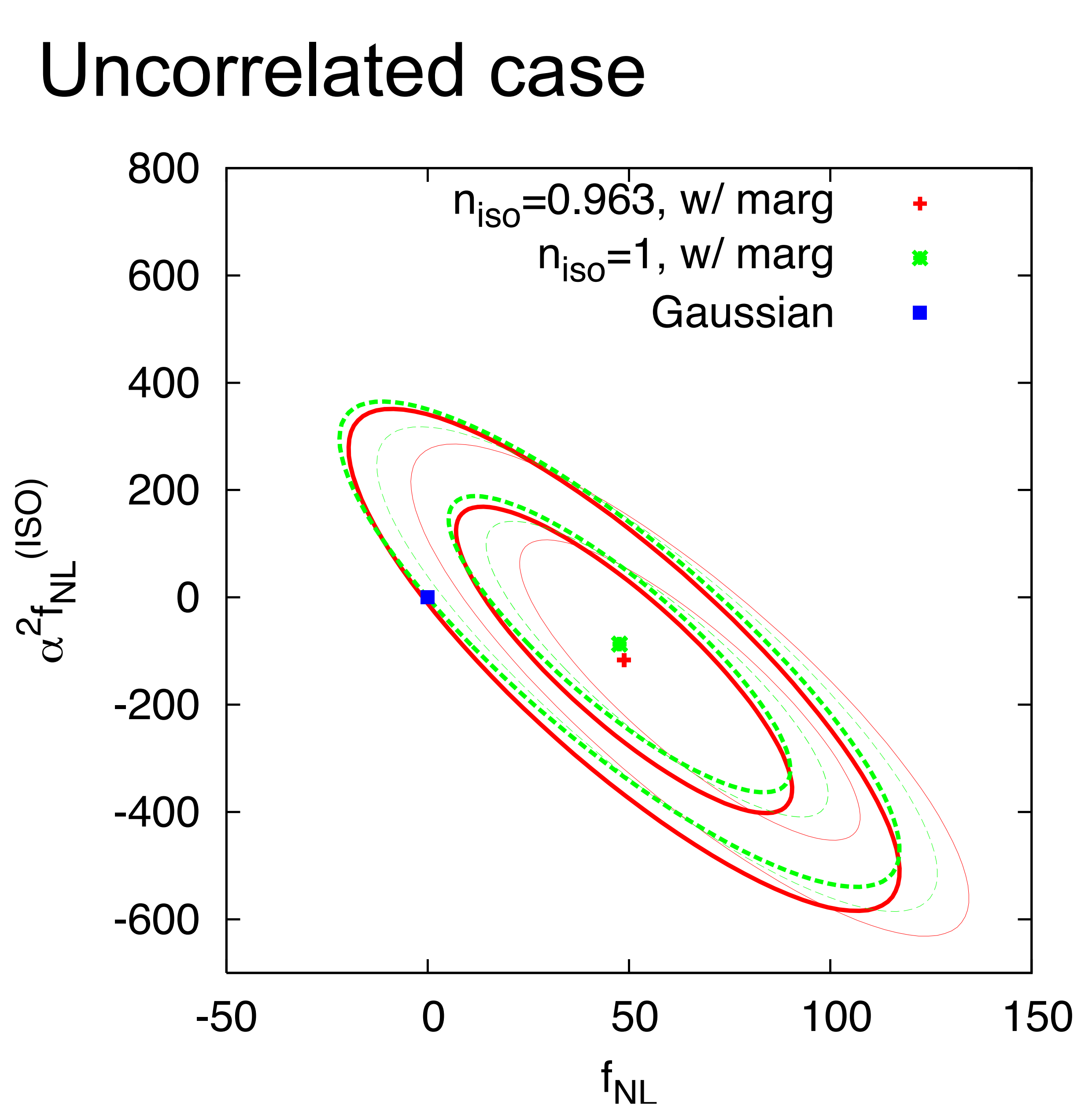
(for $n_{\rm iso} = n_{\rm adi} = 0.963$)

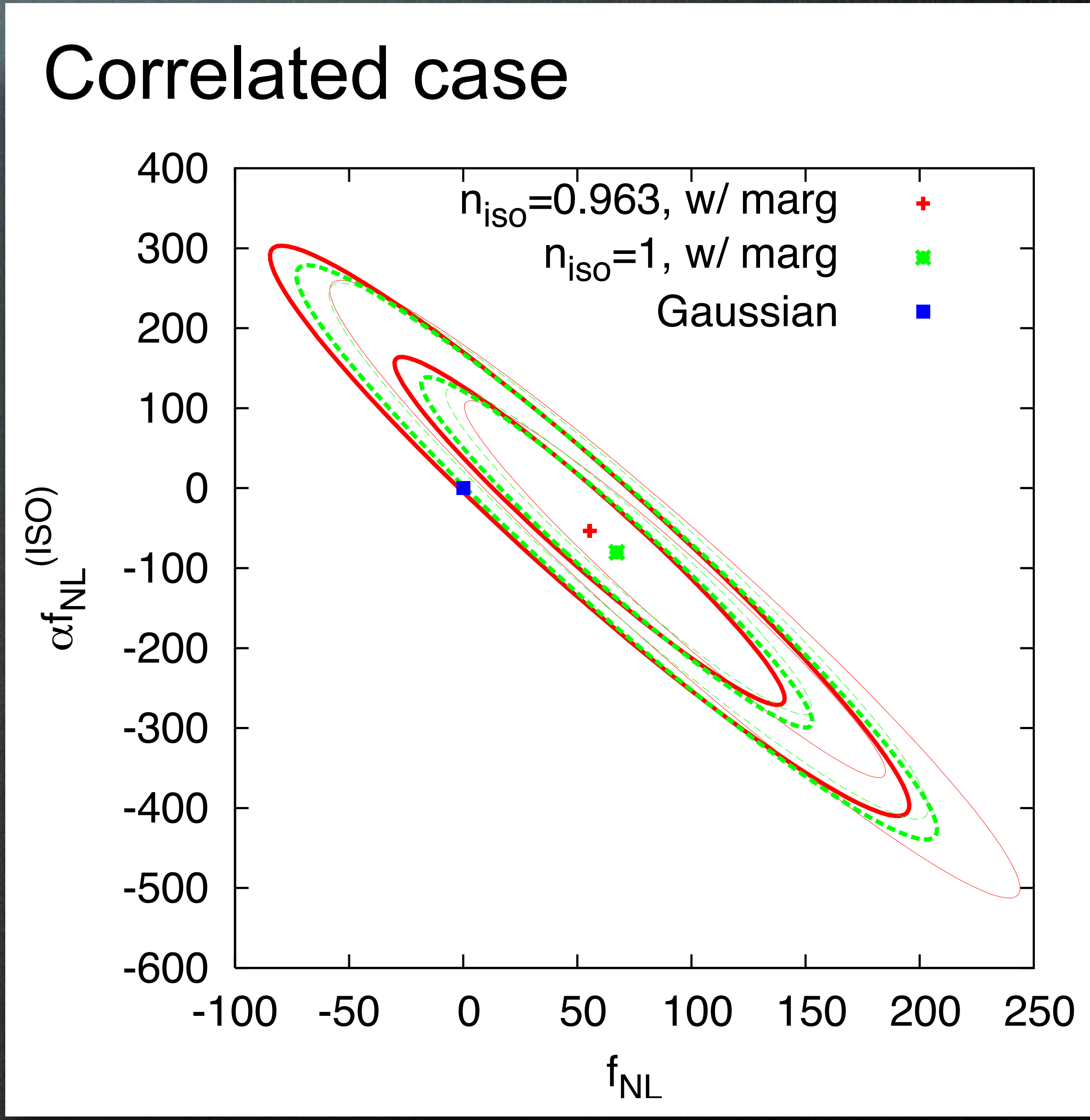
cf. Fisher matrix forecast Hikage+ 2010

$$\Delta(\alpha^2 f_{\rm NL}^{\rm (ISO)}) = 60$$

Result: neutrino density isocurvature

C. Hikage, M. Kawasaki, TS, T.Takahashi, arXiv:1202.6001





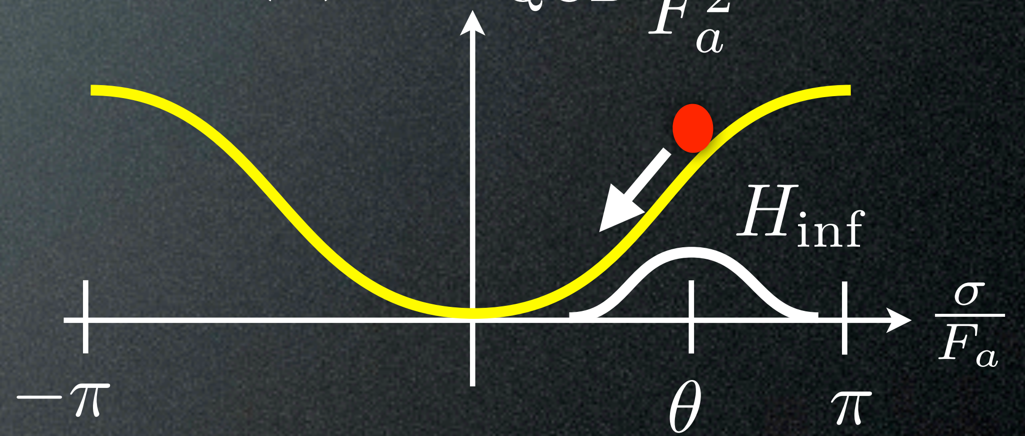
Consistent with Gaussianity

Application: axion model

M. Kawasaki, K. Nakayama, TS, T. Suyama, F. Takahashi 2008

- Axion field σ has a nearly-quadratic potential

$$V(\sigma) \approx \Lambda_{\rm QCD}^4 \frac{\sigma^2}{F_a^2}$$



F_a: axion decay constant

θ: initial misalignment angle

$H_{\rm inf}$: Hubble rate at inflation

- The amplitude of coherent oscillation is determined on the total uniform-density at $m_{\rm axion} = H(t)$.

- Energy density on the uniform density slice

$$\rho_{\rm axion}(\vec{x}) \propto \left[\sigma_i + \delta\sigma(\vec{x})\right]^2$$

with $\sigma_i = F_a\theta, \quad \sqrt{\langle\delta\sigma^2\rangle} \simeq H_{\rm inf}/2\pi$

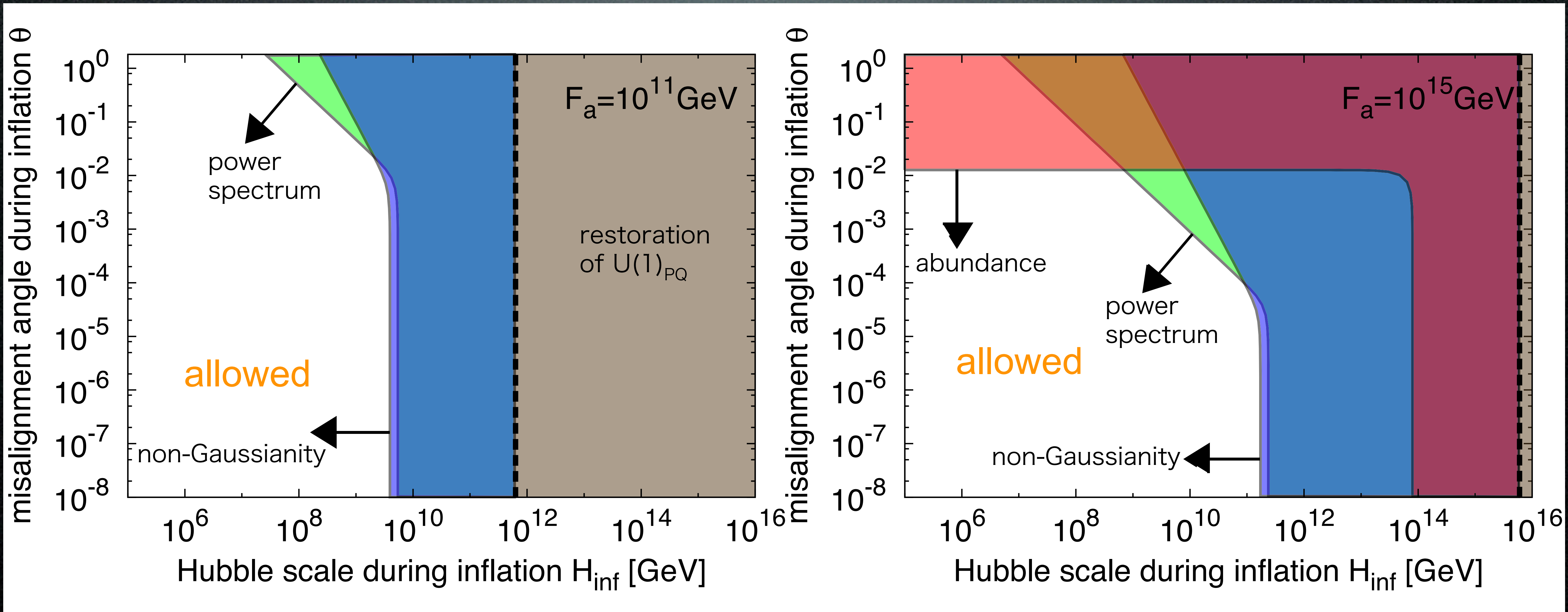
- **Uncorrelated** isocurvature perturbations

$$S_{\rm CDM}(\vec{x}) \propto S_\sigma(\vec{x}) \propto 2\sigma_i\delta\sigma(\vec{x}) + \delta\sigma(\vec{x})^2$$

→ NG is local-type

$$\langle S_{\rm CDM}(\vec{x})\Phi(\vec{x})\rangle = 0$$

Application: axion model (cont'd)

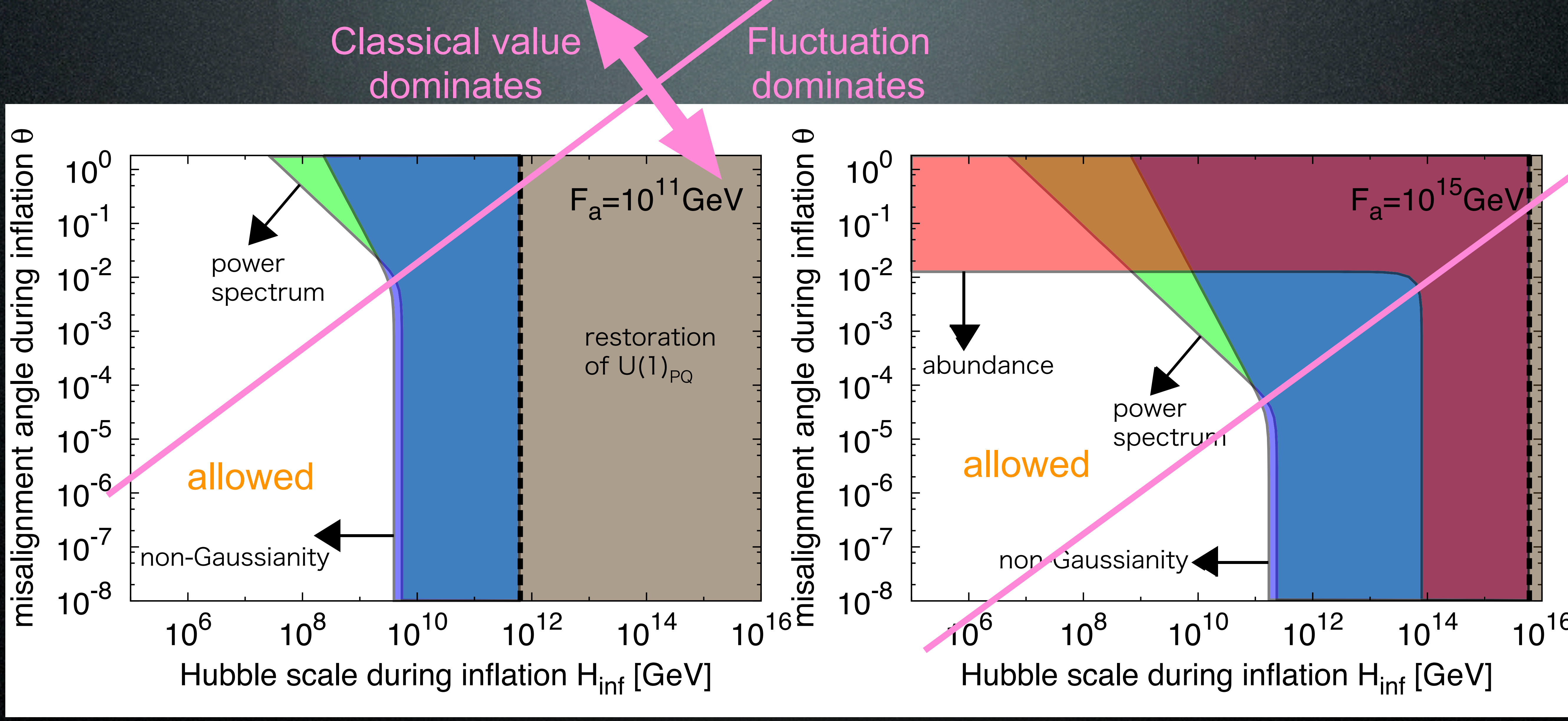


- NG in isocurvature perturbation marginally improves the constraint on H_{inf} when the misalignment angle θ is small.

- Parameter dependences differ by whether fluctuation or the classical field value dominates.

$$\langle \rho_{\rm axion} \rangle \propto a_i^2 + \langle \delta a^2 \rangle = (F_a \theta)^2 + (H_{\rm inf}/2\pi)^2$$

Application: axion model (cont'd)



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Conclusion

- CMB constraints on two extensions of the local-type non-Gaussianity are studied.
  - Isocurvature perturbations (CDM/neutrino, correlation w/ Φ)
  - g_{NL}
- WMAP data is consistent with Gaussian primordial perturbations even these extensions are allowed. (Some of) the constraints will be upgraded by Planck data.
- Constraints give implications to particle physics models e.g. axion.

Thank you for your attention!

How to constrain f_{NL} optimally

- NG is manifested in the CMB bispectrum.

$$\langle a^{(\mathrm{th})}_{l_1 m_1} a^{(\mathrm{th})}_{l_2 m_2} a^{(\mathrm{th})}_{l_3 m_3} \rangle \equiv B^{m_1 m_2 m_3}_{l_1 l_2 l_3} \propto f_{\mathrm{NL}}$$

- Estimator of f_{NL} can be constructed from cubic product of CMB anisotropy with suitable weight ("matched filtering") [Komatsu, Spergel, Wandelt (05), Yadav+ (07, 08)].

$$\hat{f}_{\mathrm{NL}} = \frac{1}{\mathcal{N}} \sum_{\{l,m\}} B^{m_1 m_2 m_3}_{l_1 l_2 l_3} (C^{-1} a^{(\mathrm{obs})})_{l_1 m_1} (C^{-1} a^{(\mathrm{obs})})_{l_2 m_2} (C^{-1} a^{(\mathrm{obs})})_{l_3 m_3}$$

$C_{lm,l'm'} = C^S_{lm,l'm'} + C^N_{lm,l'm'}$: total (signal+noise) covariance

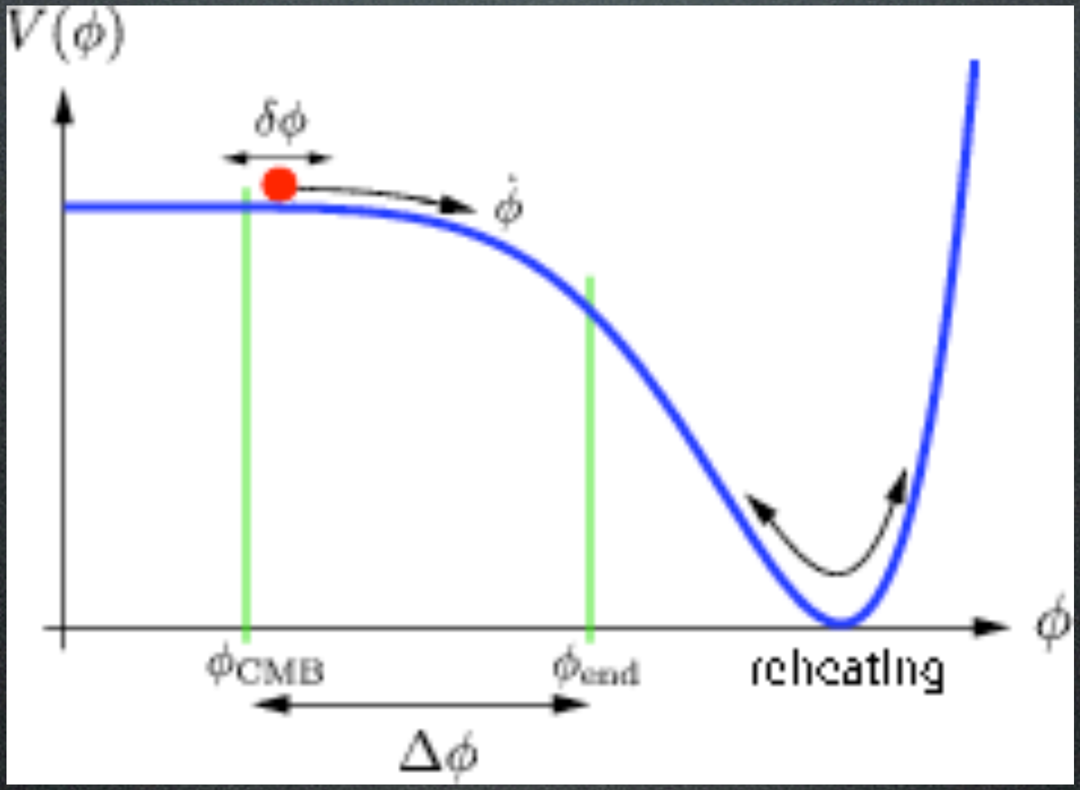
← off-diagonal due to
inhomogeneous noise, sky cuts

- Normalization can be determined from simulations.

$$\mathcal{N} = \sum_{\{l,m\}} B^{m_1 m_2 m_3}_{l_1 l_2 l_3} \langle (C^{-1} a^{(\mathrm{sim})})_{l_1 m_1} (C^{-1} a^{(\mathrm{sim})})_{l_2 m_2} (C^{-1} a^{(\mathrm{sim})})_{l_3 m_3} \rangle_{f_{\mathrm{NL}}=1}$$

Single-field slow-roll inflation model

- Standard class of inflation models
- Potential energy of a scalar field (inflaton) drives the accelerated expansion.
- Slow-roll: inflaton rolls down a flat potential during inflation.

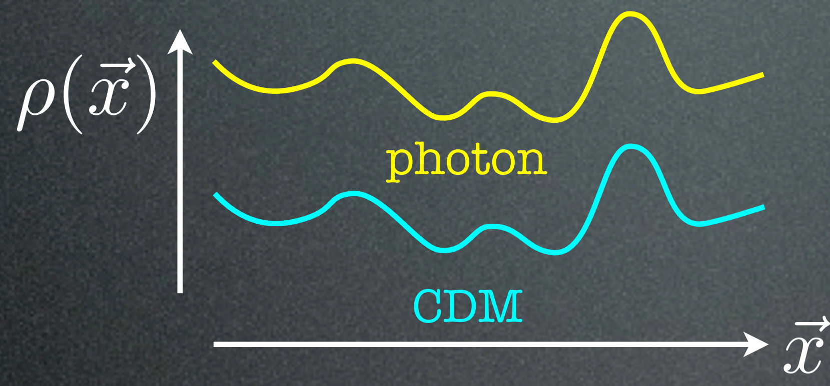


- Initial perturbations are generated only from the fluctuations of inflaton field.

Prediction of single-field slow-roll inflation

Initial perturbations should be ...

- Adiabatic



curvature perturbations

$$\zeta(\vec{x}) \sim \frac{\delta\rho_\gamma(\vec{x})}{\bar{\rho}_\gamma}$$

- Gaussian

$$\zeta(\vec{x}) = N(\vec{x}) - \bar{N}$$
$$= \frac{dN}{d\phi}\delta\phi(\vec{x}) + \frac{1}{2}\frac{d^2N}{d\phi^2}\delta\phi(\vec{x})^2 + \cdots$$

N=ln(a): e-folding number

- Nearly scale-invariant in amplitude

$$\zeta(\vec{k}) \propto \delta\phi(\vec{k}) \simeq \frac{H}{2\pi}$$

$\rightarrow$ match with current observations

Implications of deviation

- If non-Gaussianity is detected,
  - Single-field slow-roll inflation model is ruled out.
  - Multiple degrees of freedom during inflation?
  - Other mechanisms for perturbation generation than inflation?

$\rightarrow$ Probe for not only beginning of our Universe, but also physics at very high energy scales

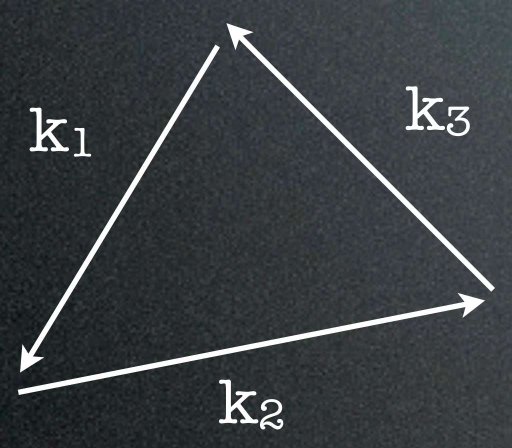
- Non-adiabatic (isocurvature) perturbation is another probe.

Signals of non-Gaussianity

- Non-zero n-point correlation functions (n>3)

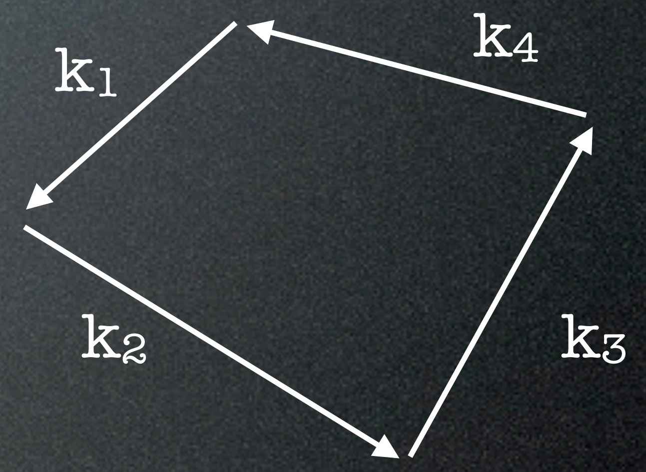
bispectrum

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle$$

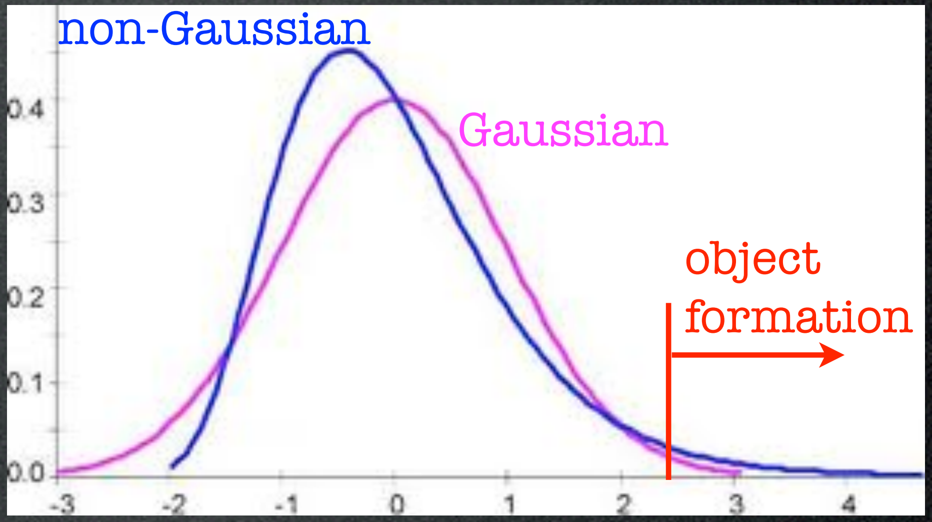


trispectrum

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\zeta(\vec{k}_4)\rangle_{\text{connected}}$$



- Enhancement in formation of rare objects



Signals of non-Gaussianity

- Non-zero n-point correlation functions (n>3)

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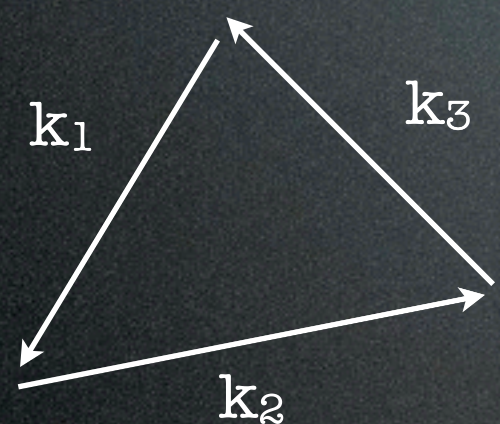
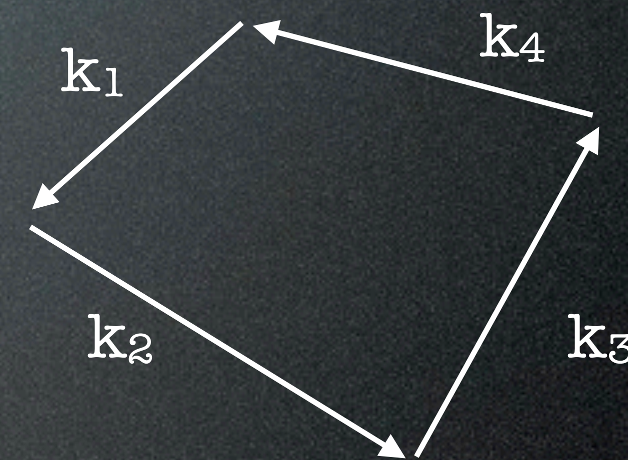


trispectrum

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\zeta(\vec{k}_4)\rangle_{\text{connected}}$$



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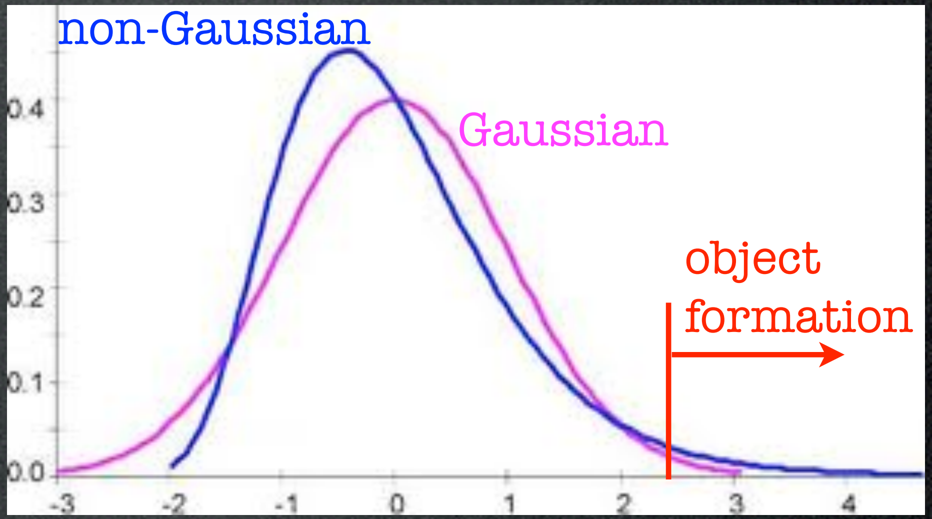


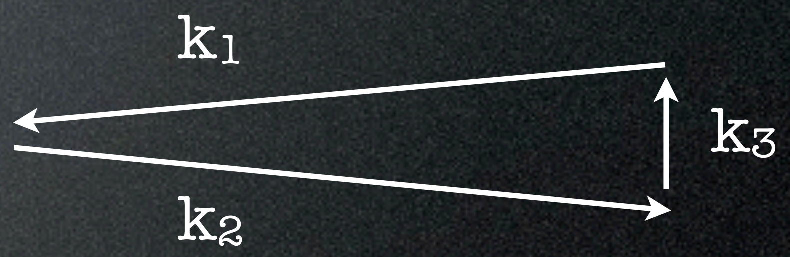
Local-type non-Gaussianity

- A specific type of non-Gaussianity

$$\zeta(\vec{x}) = \zeta_G(\vec{x}) + f_{\rm NL}\zeta_G(\vec{x})^2$$

→ coupling btw. modes at very large & very short scales

→ large signal at squeezed configuration



- Single-field inflation models predict small undetectable non-Gaussianities.

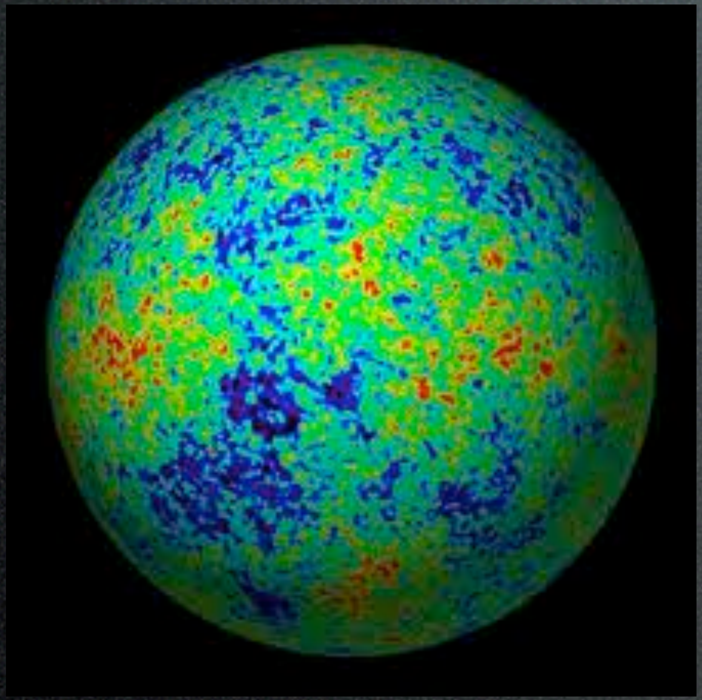
$$f_{\rm NL} \simeq (1 - n_s) = \mathcal{O}(0.01)$$

- Large $f_{\rm NL}$ is predicted by many theoretical models

  curvaton scenarios[Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (01)], modulated reheating[Dvali, Gruzinov, Zaldarriaga; Kofman (03)], ...

Cosmic Microwave Background (CMB)

- Photons scattered when the Universe becomes neutral.
- Anisotropy in CMB carries an imprint of initial perturbations.



$$T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

$$a_{lm} \sim \sum_{\vec{k}} g_l(k) \zeta(\vec{k}) Y^*_{lm}(\hat{k})$$

transfer function

- Linear perturbation theory, well-understood physics!

  $\longrightarrow$ Easy to extract information of initial perturbations

CMB signatures of non-Gaussianity

- CMB bispectrum: (indirect) measure of primordial bispectrum.

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = b_{l_1 l_2 l_3} G^{l_1 l_2 l_3}_{m_1 m_2 m_3}$$

↖ coupling of angular momenta

$$G^{l_1 l_2 l_3}_{m_1 m_2 m_3} = \int d\hat{n}\ Y_{l_1 m_1}(\hat{n}) Y_{l_2 m_2}(\hat{n}) Y_{l_3 m_3}(\hat{n})$$

- reduced bispectrum

$$b_{l_1 l_2 l_3} \sim \sum_{\vec{k}_1 \vec{k}_2 \vec{k}_3} g_{l_1}(k_1) g_{l_2}(k_2) g_{l_3}(k_3) \langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \rangle$$

$$= f_{\rm NL} \hat{b}_{l_1 l_2 l_3}$$

$\rightarrow$ We can make template bispectrum for $f_{\rm NL}$.

- From data, $f_{\rm NL}$ can be optimally estimated from data by matched filtering.

Implications of isocurvature perturbations

- In inflationary universe
  - Initial perturbations for structure formation are generated from vacuum fluctuations of light (scalar) fields.
  - If a single field sources the perturbations, no isocurvature perturbations can be generated at super-horizon scales.

- Detection of nonzero isocurvature perturbations
  - Single-field model is ruled out.
  - Multiple degrees of freedom exist during inflation.

- Non-Gaussianity?

$$S(\vec{x}) = S_{\rm G}(\vec{x}) + f_{\rm NL}^{\rm (ISO)} S_{\rm G}^2(\vec{x}) \quad \longleftrightarrow \quad \Phi(\vec{x}) = \Phi(\vec{x}) + f_{\rm NL}\Phi(\vec{x})^2$$

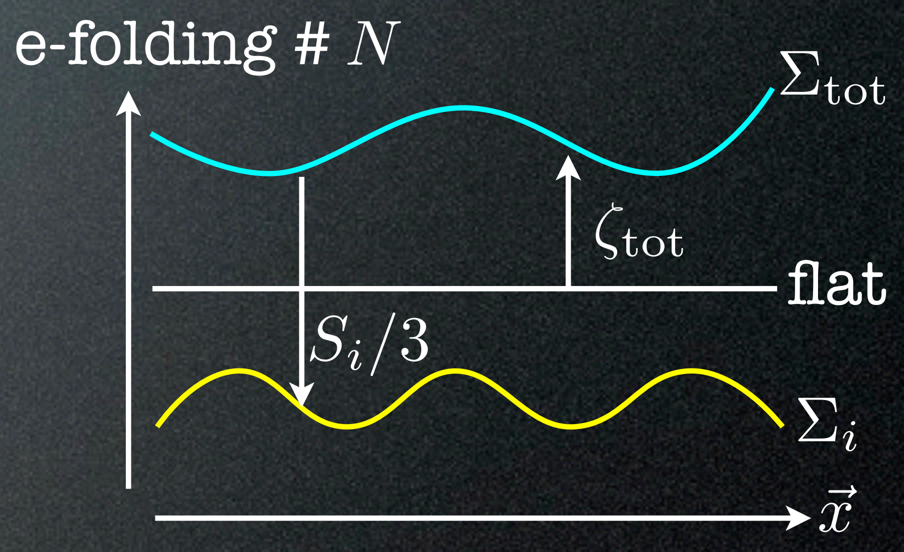
Additional information beyond power spectrum.

Delta-N formalism

Starobinsky (85), Salopek & Bond (90), Sasaki & Stewart (96)

- Delta-N formalism
  - For each fluid i, we can define its uniform-density hyper-surface Σ_i .
  - curvature perturbation on Σ_i :
    Difference in e-folding numbers btw. the initially flat hyper-surface and Σ_i

$$\zeta_i(\vec{x}) = N_{\Sigma_i}(\vec{x}) - N_{\Sigma_{\rm flat}}(\vec{x})$$



  - energy density in nonlinear formalism

$$\rho_i(\vec{x}) = \bar{\rho}_i e^{3(1+w_i)[\zeta_i(\vec{x})-\delta N(\vec{x})]}$$

- curvature and isocurvature perturbations

$$\zeta = \zeta_{\rm tot}$$

$$S_i = 3(\zeta_i - \zeta_{\rm tot})$$

This definition is fully nonlinear.
At linear order,

$$S_i = \left(\frac{1}{(1+w_i)} \frac{\delta\rho_i}{\bar{\rho}_i} - \frac{4}{3} \frac{\delta\rho_\gamma}{\bar{\rho}_\gamma} \right) .$$

Example(1): curvaton model

Linde & Mukhanov (96), Boubekeur & Lyth (05),
Langlois, Vernizzi & Wands (08), Kawasaki+ (09),
Moroi & Takahashi (09),..

- A spectator field during inflation (curvaton) decays into radiation (and matter) after inflation and contributes to primordial perturbations.

Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi (01)

- Setup:

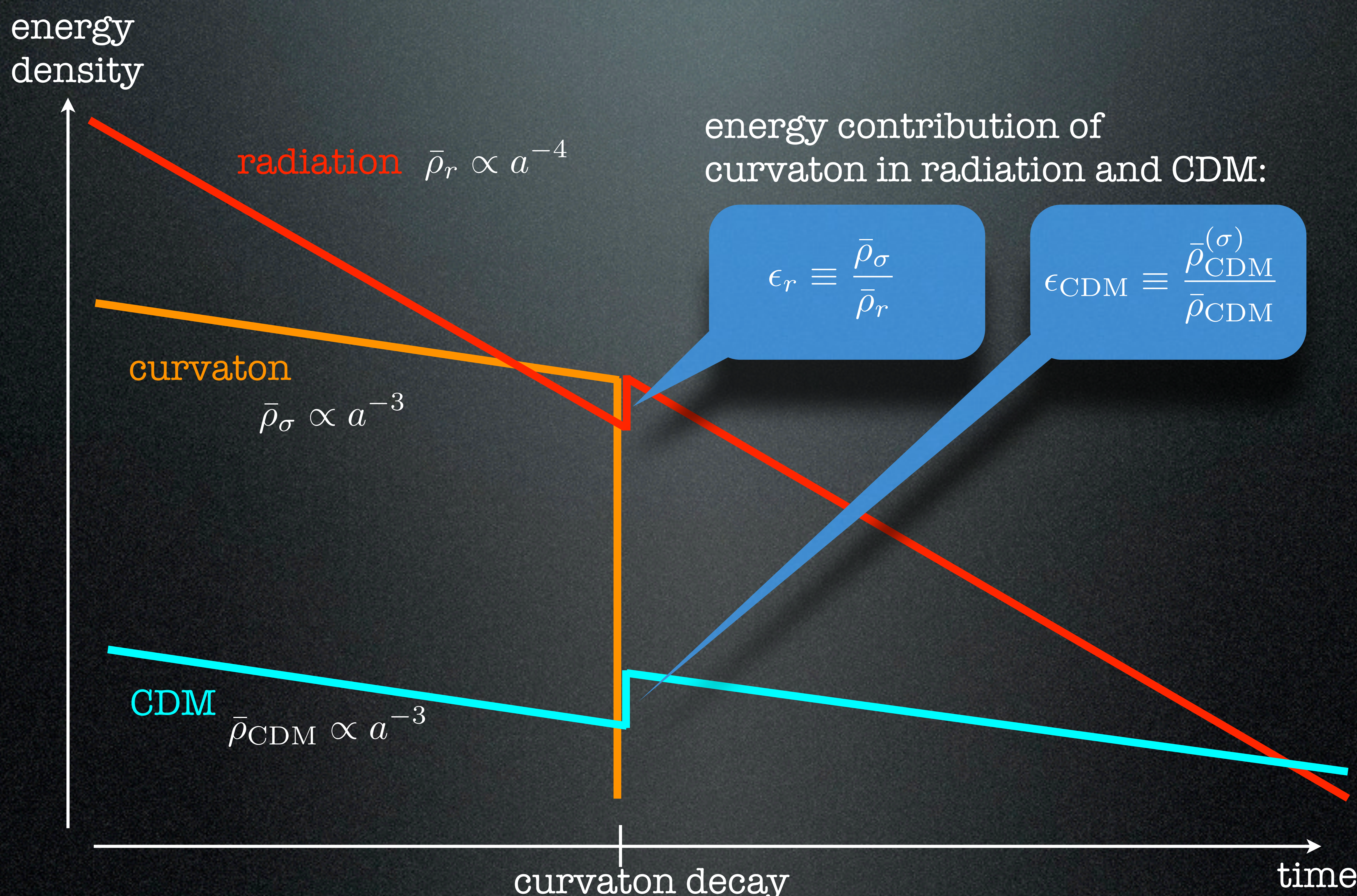
Kawasaki, Nakayama, TS, Suyama, Takahashi [arXiv:0905.2237]

— During decay, the Universe is dominated by radiation and curvaton.
(CDM energy density is negligible)

— Curvaton mostly decays into radiation.
However, curvaton also decays into CDM with nonzero branching ratio.

— Some fraction of CDM is generated when curvaton is subdominant.
The rest of CDM is generated directly from the curvaton decay.

Schematic picture



curvaton model (cont'd)

- At H=Γ, decay occurs synchronously on the uniform density hyper-surface of total matter.
  - — energy conservation (sudden decay approx.):

$$\text{radiation:} \quad 1 = (1-\epsilon_r)e^{4(\zeta_\phi-\zeta_r)} + \epsilon_r e^{3(\zeta_\sigma-\zeta_r)}$$

$$\text{CDM:} \quad e^{3(\zeta_{\rm CDM}-\zeta_r)} = (1-\epsilon_{\rm CDM})e^{3(\zeta_\phi-\zeta_r)} + \epsilon_{\rm CDM}e^{3(\zeta_\sigma-\zeta_r)}$$

from inflaton   from curvaton

- **correlated** curvature and isocurvature perturbations

(2nd order)

$$\zeta \approx \zeta_\phi + \frac{rS_\sigma}{3} + \frac{3}{2r}\left(\frac{rS_\sigma}{3}\right)^2$$

$$S_{\rm CDM} \approx (\epsilon_{\rm CDM}-r)S_\sigma + \frac{1}{\epsilon_{\rm CDM}-r}\{(\epsilon_{\rm CDM}-r)S_\sigma\}^2$$

$r(\simeq \frac{3}{4}\epsilon_r),\ \epsilon_{\rm CDM} \ll 1$

induced NG

- Even if fluctuations generated during inflation ($\zeta_\phi,\ S_\sigma$) are Gaussian, NG is induced from S_σ . Induced NG is local-type.

Application(2): curvaton model

- Correlated isocurvature perturbations are generated.

$$\zeta \approx \zeta_\phi + \frac{rS_\sigma}{3} + \frac{3}{2r}\left(\frac{rS_\sigma}{3}\right)^2$$

$$S_{\rm CDM} \approx (\epsilon_{\rm CDM} - r)S_\sigma + \frac{1}{\epsilon_{\rm CDM} - r}\{(\epsilon_{\rm CDM} - r)S_\sigma\}^2$$

- amplitude of isocurvature power spectrum

$$\alpha = \frac{9A}{r^2}\left[\epsilon_{\rm CDM} - r\right]^2$$

- adiabatic non-Gaussianity

$$f_{\rm NL} = \frac{5A^2}{2r}$$

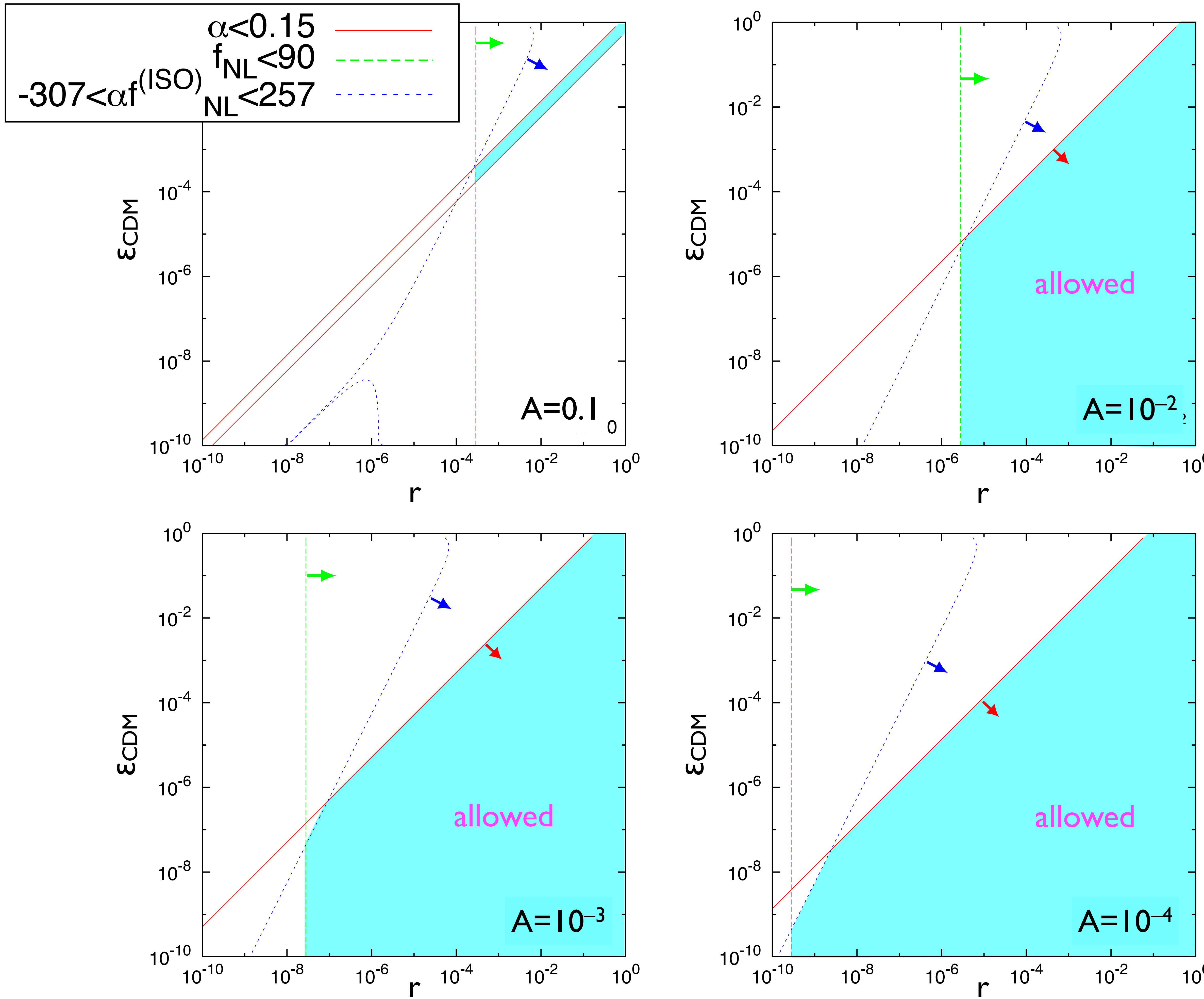
- isocurvature non-Gaussianity

$$\alpha f_{\rm NL}^{\rm (ISO)} = \frac{9A^2}{2r^2}\left[\epsilon_{\rm CDM} - r\right]$$

Parameters:

$$r \simeq \frac{3}{4}\frac{\bar\rho_\sigma}{\bar\rho_r}, \quad \epsilon_{\rm CDM} \simeq \frac{\bar\rho^{(\sigma)}_{\rm CDM}}{\bar\rho_{\rm CDM}}$$

$$A = \frac{\langle (rS_\sigma/3)^2\rangle}{\langle \zeta^2\rangle}$$



α<0.15
f_{NL}<90
-307<α$f^{(ISO)}_{NL}$<257
A=0.1
A=10^{-2}
A=10^{-3}
A=10^{-4}
allowed
allowed
allowed
ε_{CDM}
r

Extra radiation?

Kawasaki, Miyamoto, Nakayama, TS [arXiv: 1107.4962]
Kawakami, Kawasaki, Miyamoto, Nakayama, TS [arXiv:1202.4890]

- Neutrino energy density $\rho_\nu = N_{\rm eff} \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \rho_\gamma$
  - In standard cosmology, $N_{\rm eff} \simeq 3$.
- Observational constraints
  - abundance of light elements (2 sigma)
    $N_{\rm eff} = 3.68^{+0.80}_{-0.70}$ [Izotov & Thuan (10)]
  - CMB power spectrum (1 sigma)
    $N_{\rm eff} = 4.56 \pm 0.75$ WMAP+ACT [Dunkley+ (2010)]
    $N_{\rm eff} = 3.86 \pm 0.42$ WMAP+SPT [Keisler+ (2011)]

- Isocurvature perturbation in "dark radiation (active neutrinos+extra rad.)"
  - Very weak interaction of extra rad. with SM particles
    - Different origin & initial fluctuation? Never be in thermal equilibrium?
  - Isocurvature perturbation in active neutrinos can be generated from the Affleck-Dine mechanism with large lepton asymmetry.

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➡Can be tested by Planck

$\Delta N_{\rm eff} = 0.1$

[Ichikawa, TS, Takahashi (08)]

- Isocurvature perturbation in "dark radiation (active neutrinos+extra rad.)"
  - Very weak interaction of extra rad. with SM particles
    - Different origin & initial fluctuation? Never be in thermal equilibrium?
  - Isocurvature perturbation in active neutrinos can be generated from the Affleck-Dine mechanism with large lepton asymmetry.

Simulation: method

- NG CMB simulation (local type) [Liguori+(03), Elsner & Wandelt (09)]

$$a_{lm} = \int d\hat{r}\, Y^*_{lm}(\hat{r}) \int_{l.o.s} dr\, r^2\, \underline{\alpha_l(r)}\,\underline{X(\vec{r})}$$

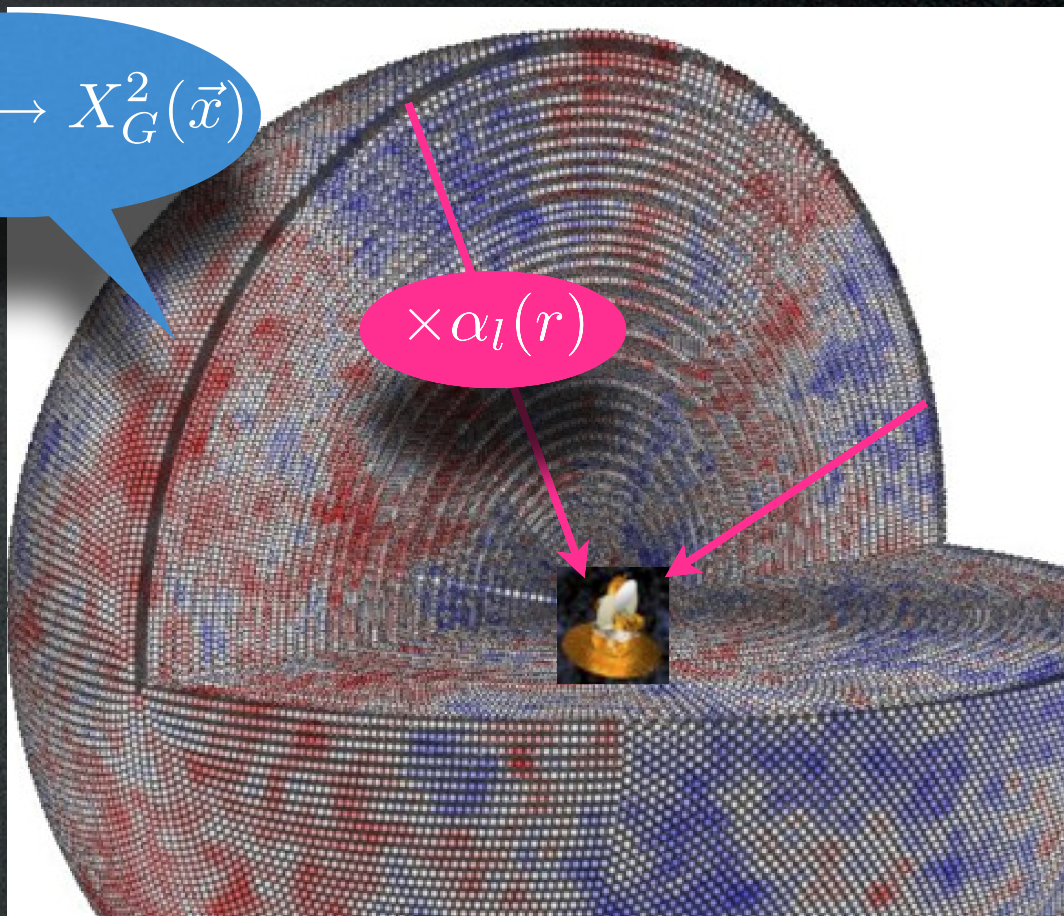
transfer function in real space

initial perturbation

$$X(\vec{r}) = X_G(\vec{r}) + f_{\rm NL} X_G(\vec{r})^2$$

Simulation procedure:

- Set concentric spherical shells covering the observable Universe.
- Randomly realize $X_G(\vec{r})$ on the shells and square it to get $X_G(\vec{r})^2$.
- Integrate along the line of sight with transfer function $\alpha_l(r)$.



(c) Elsner & Wandelt (09)

Non-Gaussian CMB simulation

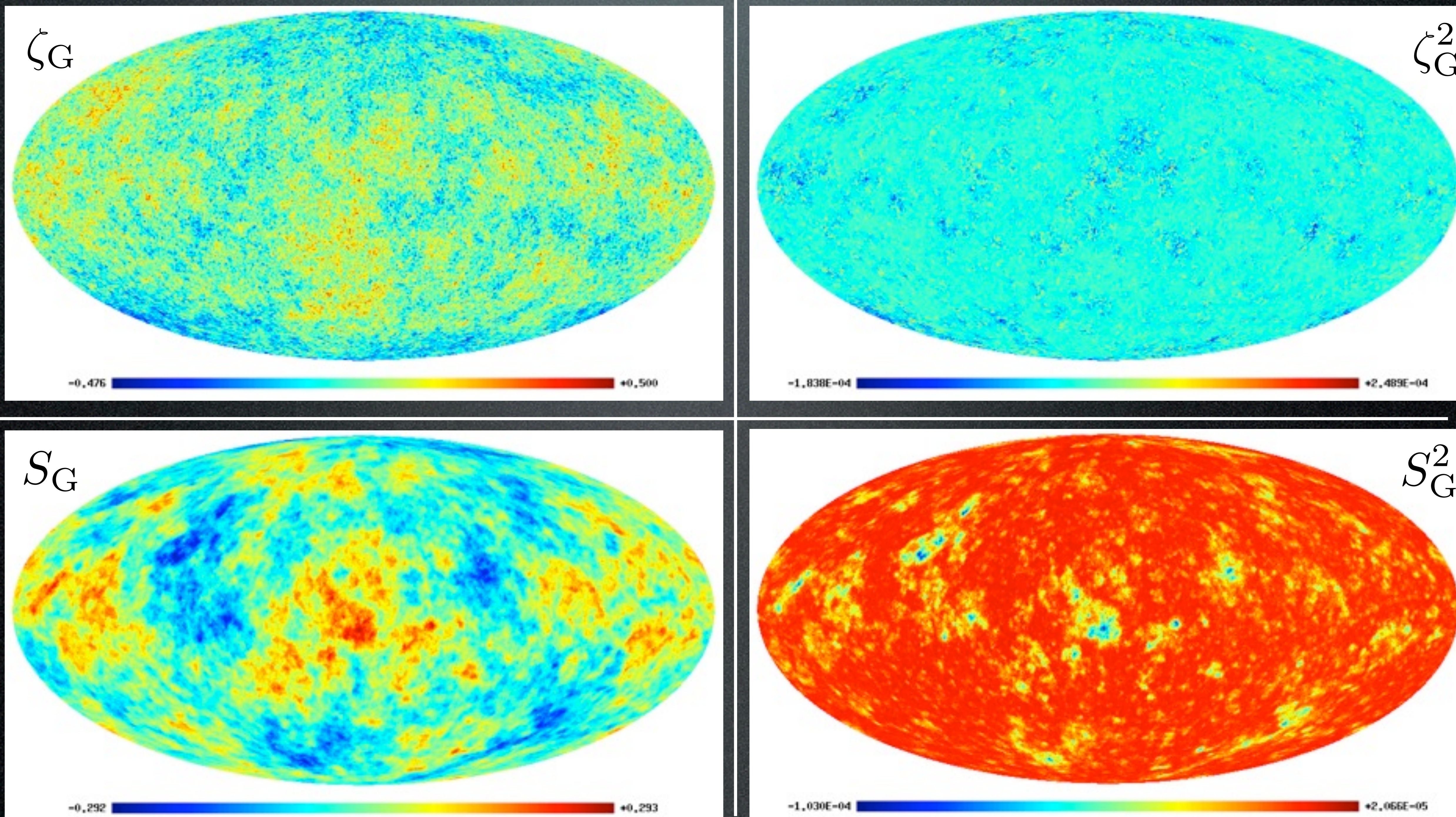
$$X = \{\zeta,\ S\}$$

$$X = X_{\rm G} + f_{\rm NL}^{(\rm X)} X_{\rm G}^2$$

Gaussian | non-Gaussian

AD

ISO



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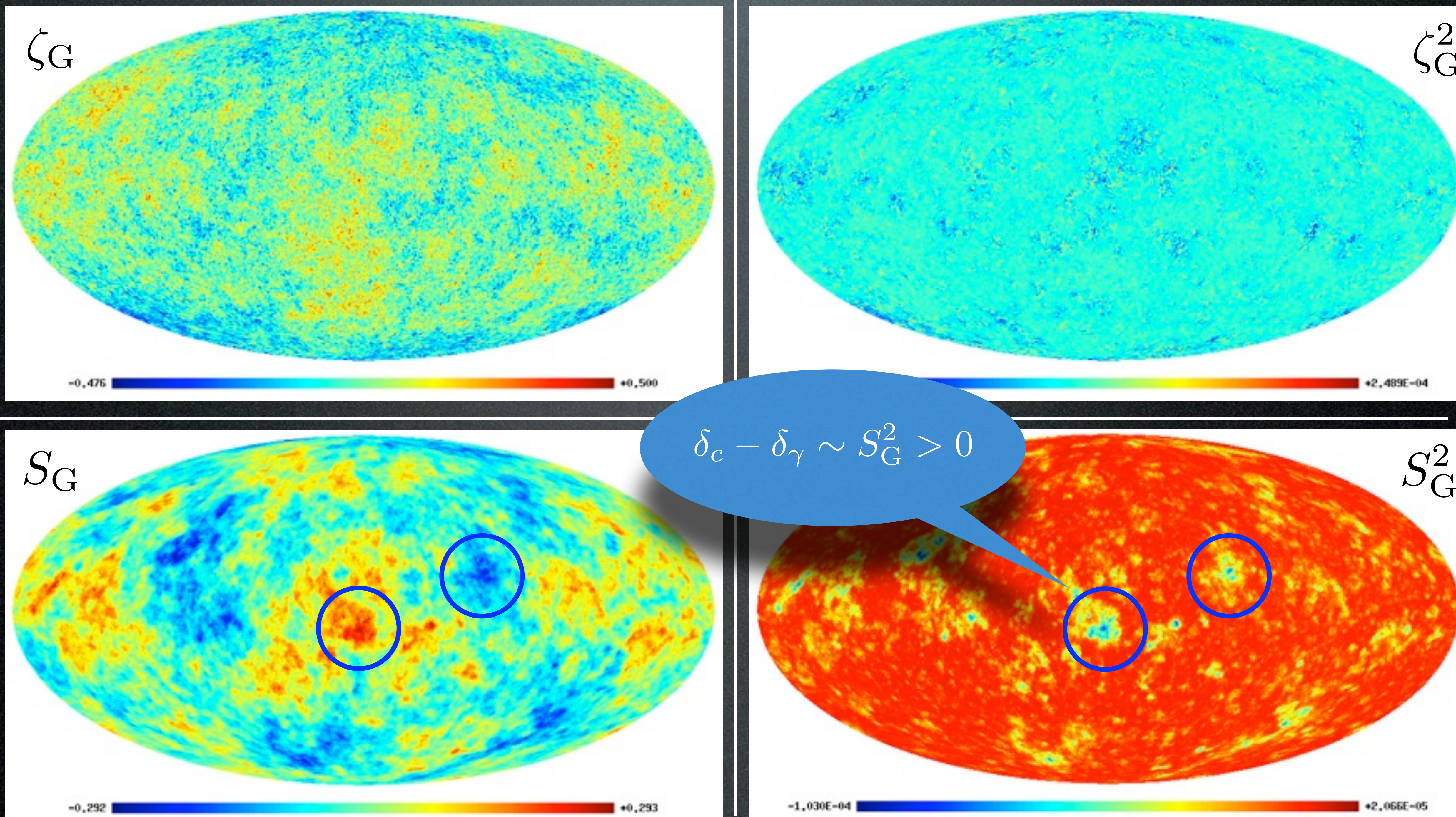
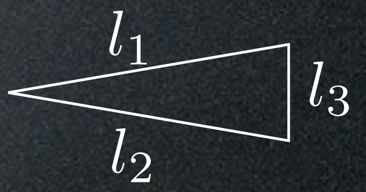
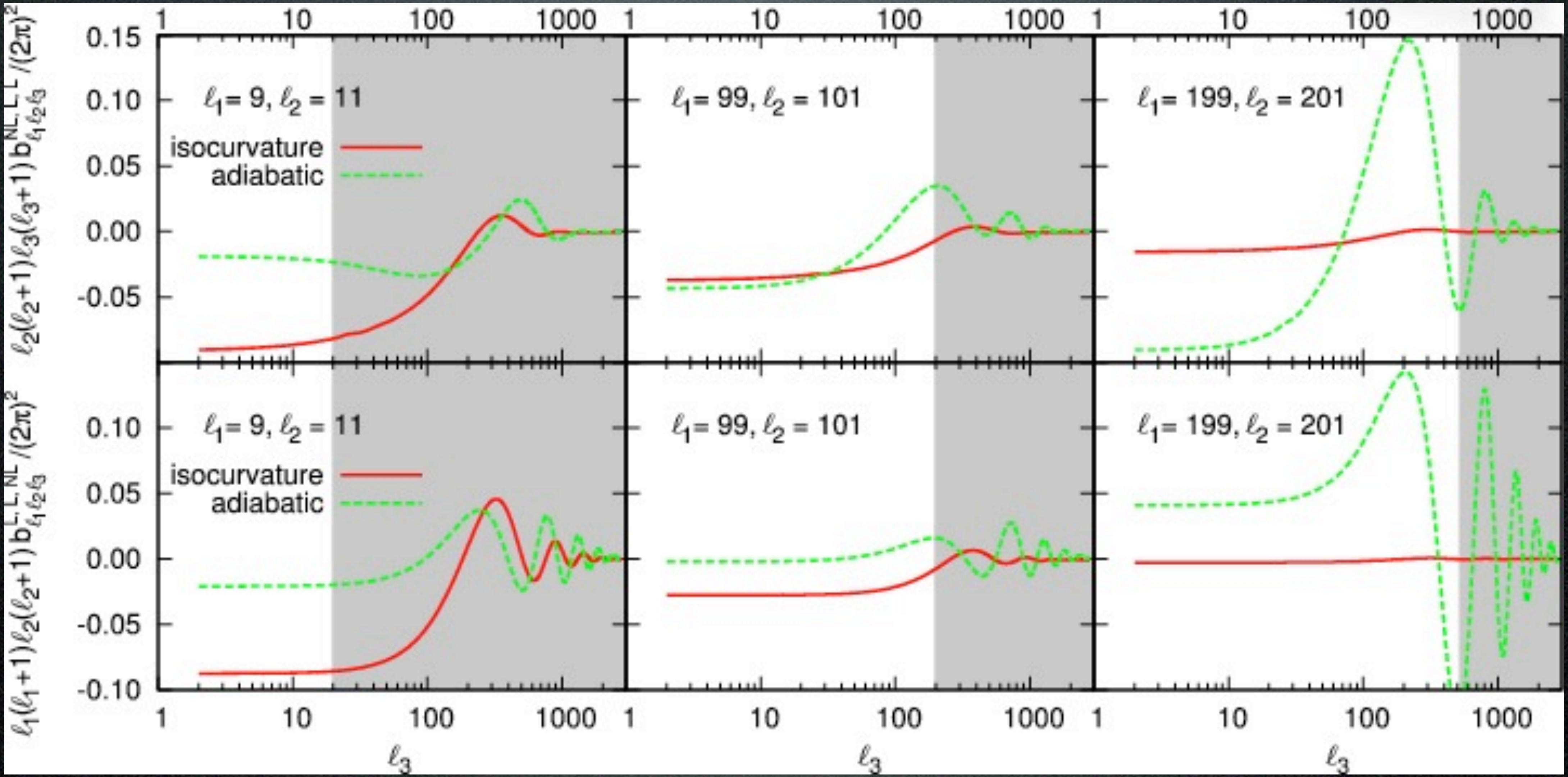


AD vs ISO: CMB bispectrum

M. Kawasaki, K. Nakayama, TS, T. Suyama, F. Takahashi (08)

- model: uncorrelated CDM isocurvature
- bispectrum in isosceles triangular configuration ($l_1 \simeq l_2$)





➡ Distinct in spectral shape from adiabatic bispectrum.

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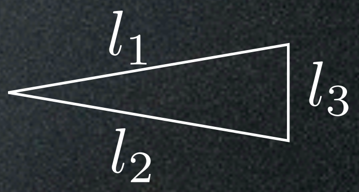

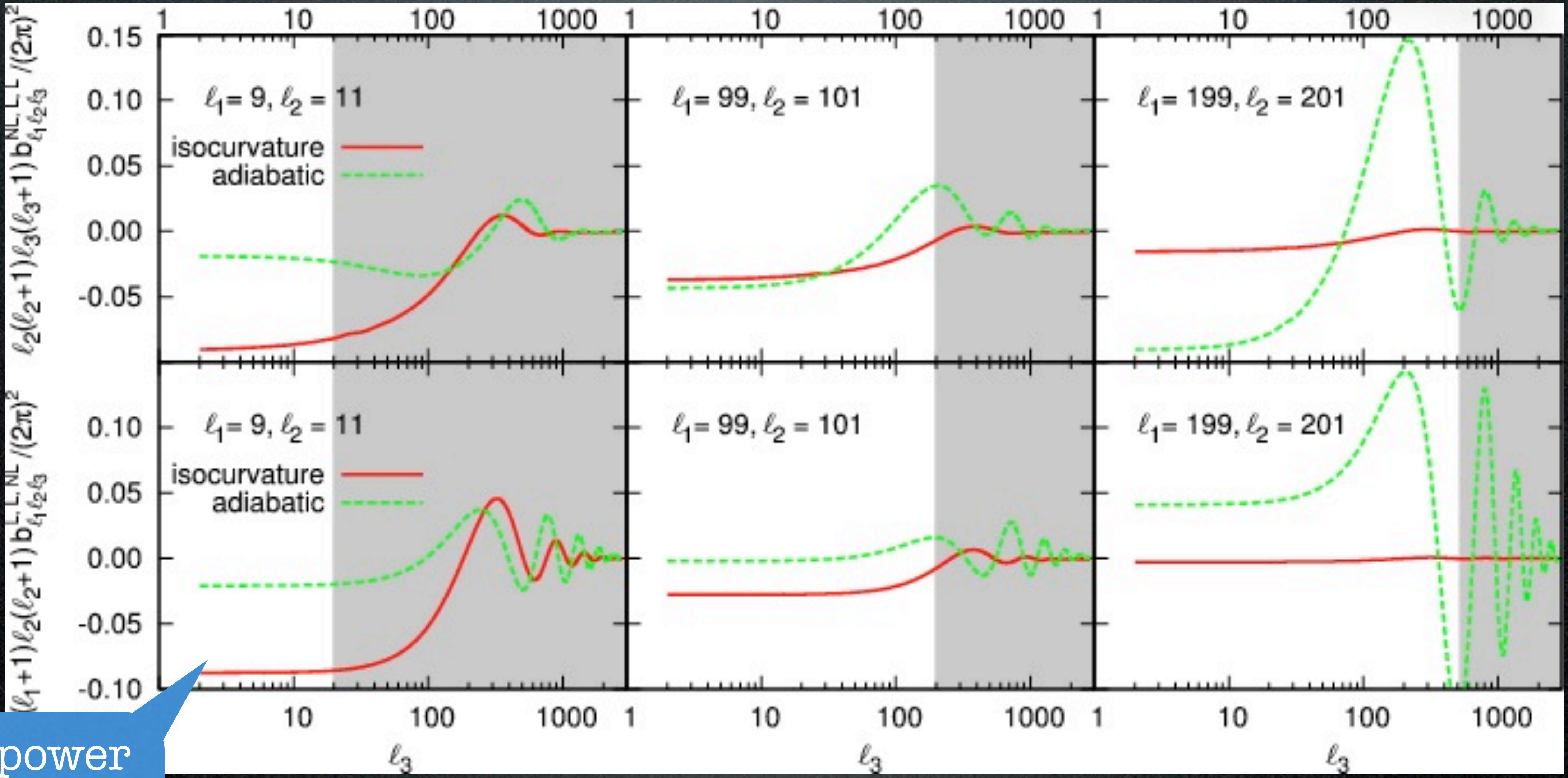

large power
at low ell

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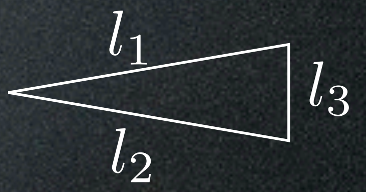

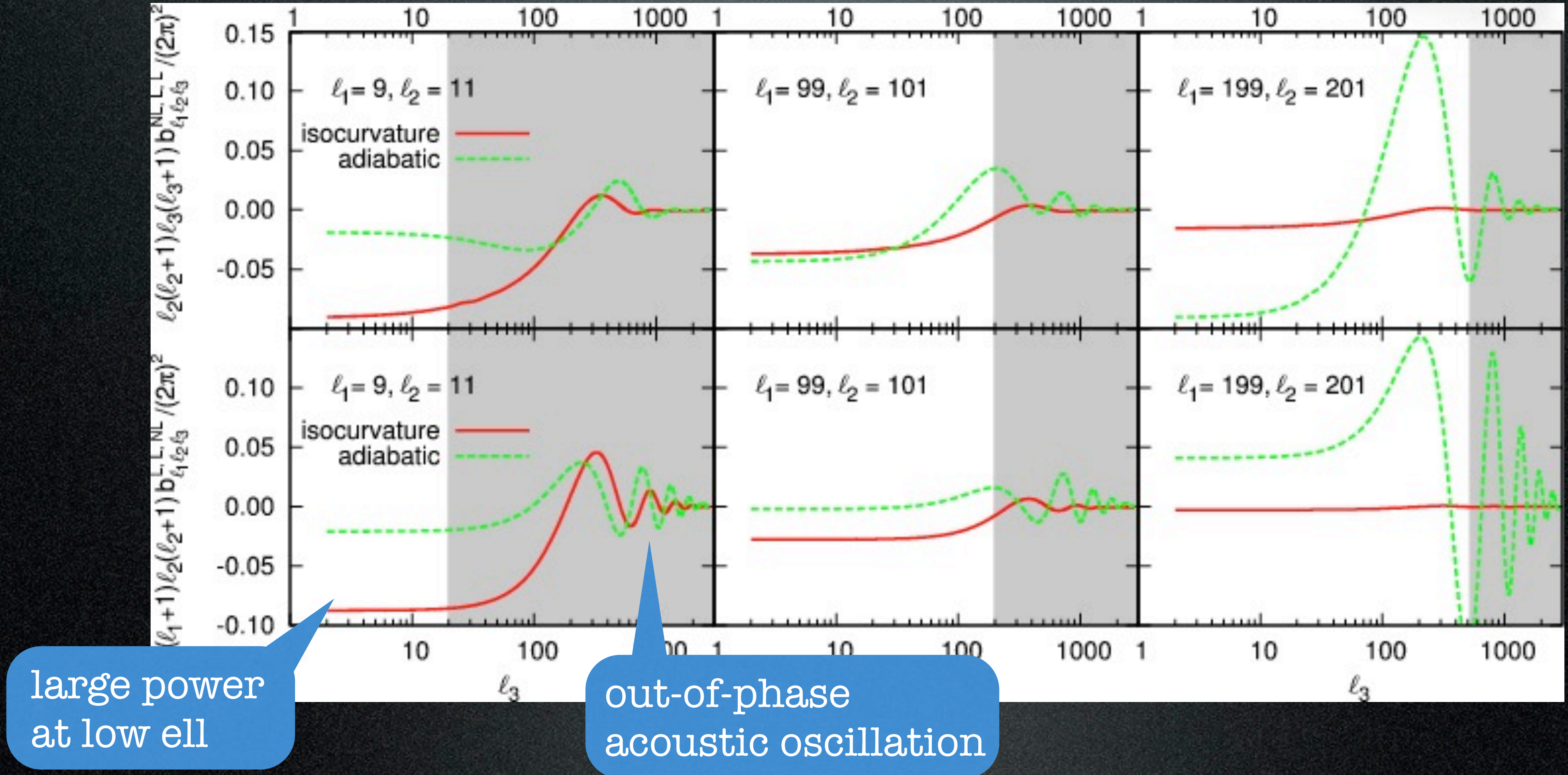

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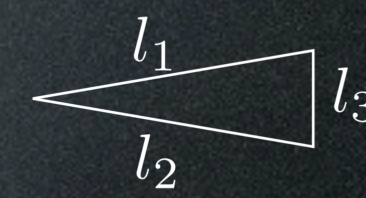
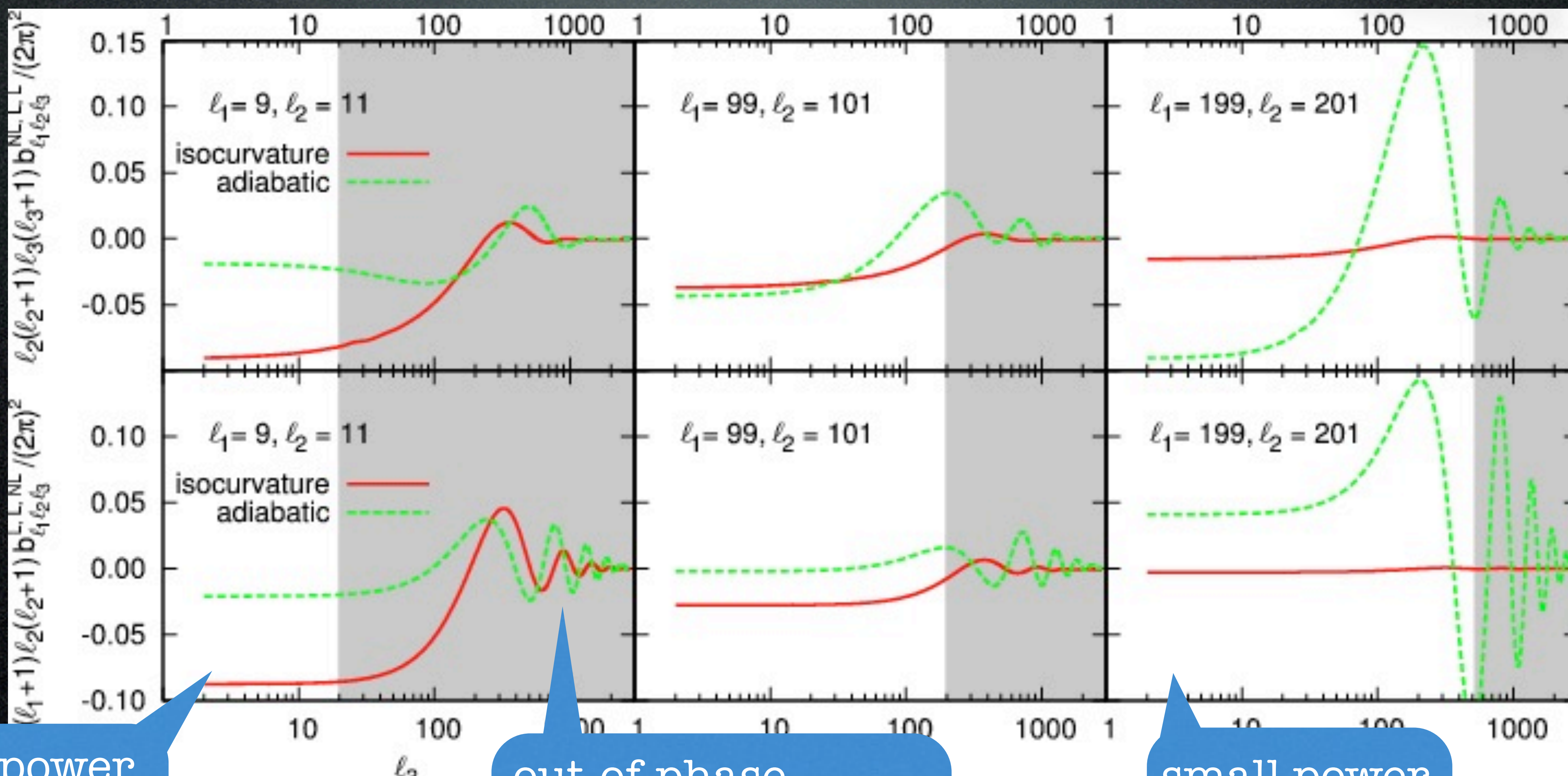




large power at low ell

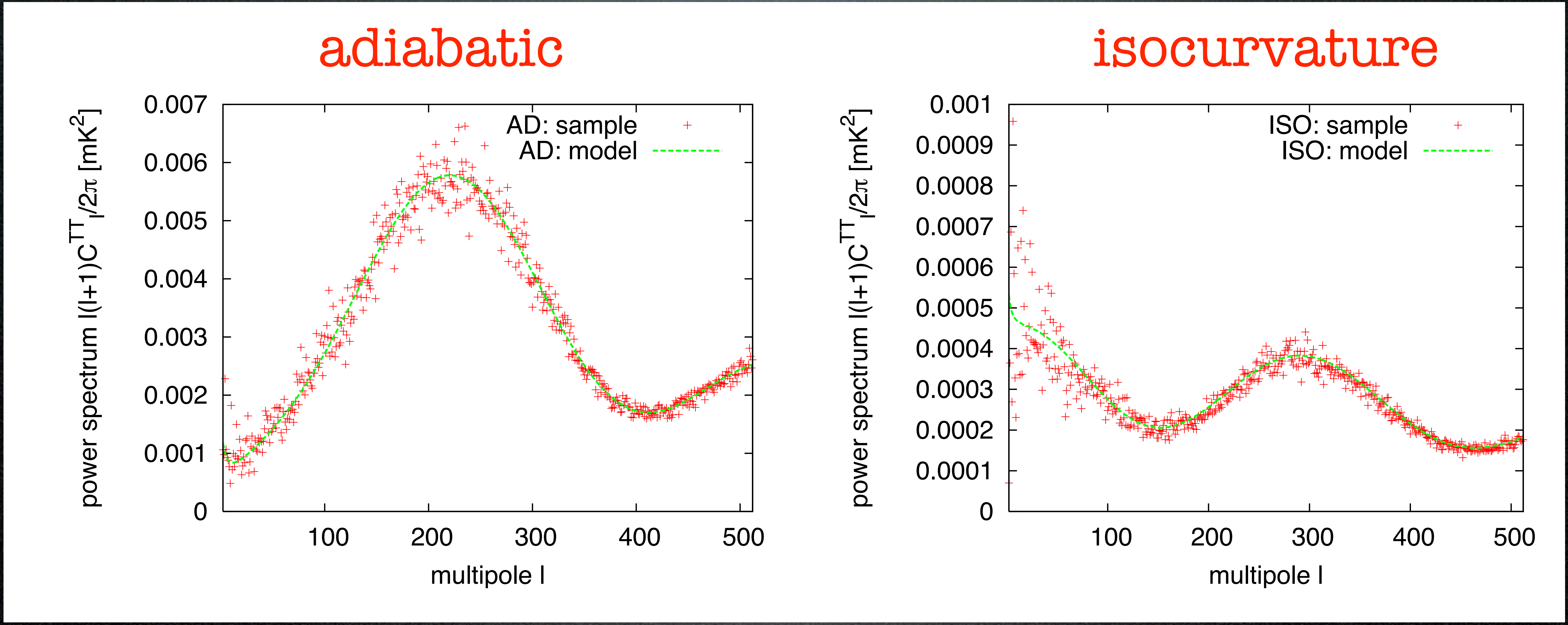
out-of-phase acoustic oscillation

small power at large ell

➡ Distinct in spectral shape from adiabatic bispectrum.

Simulation: check(1)

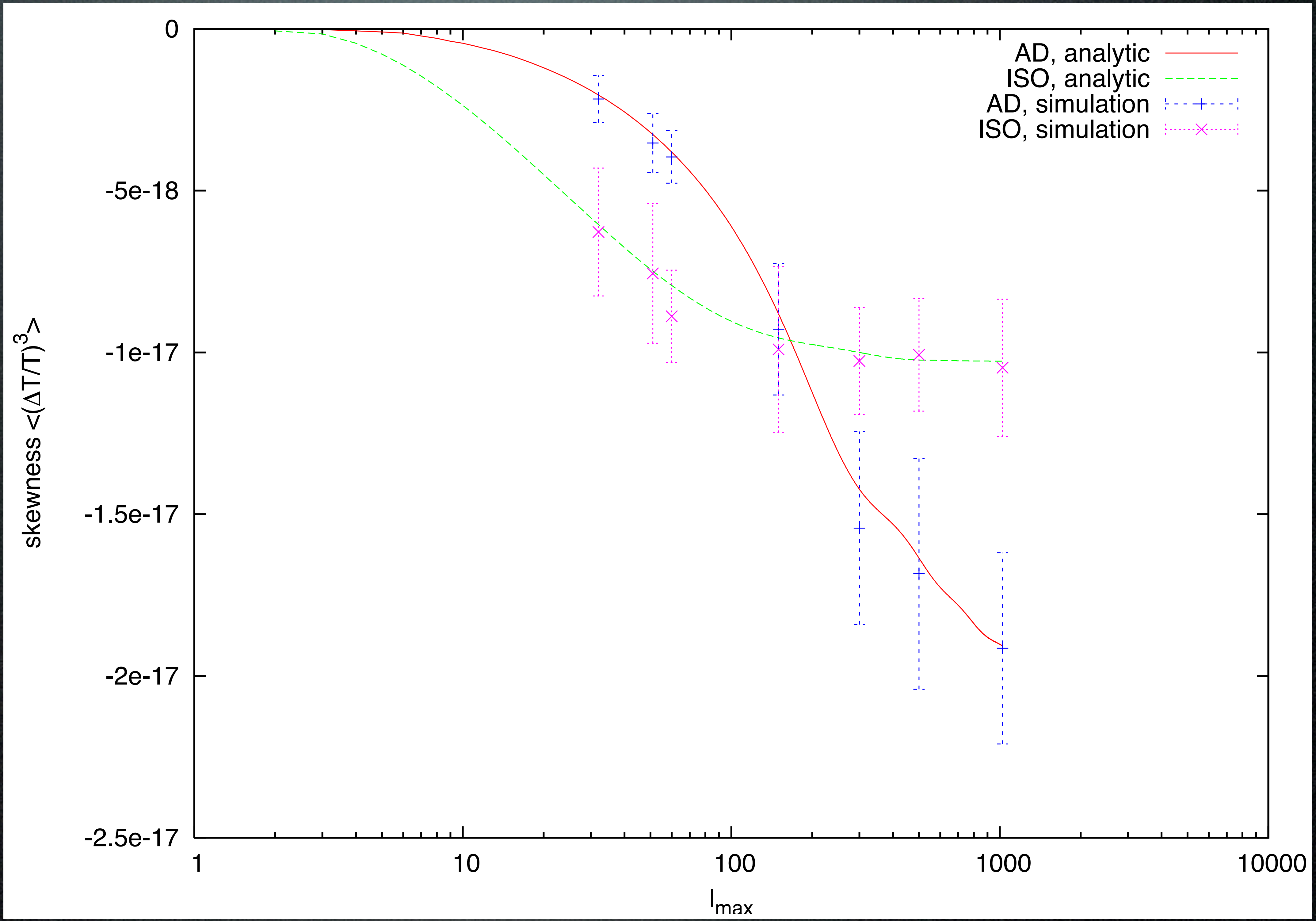
- Variance of simulated a_lm: $\frac{1}{2l+1}\sum_{m=-l}^{l}|a_{lm}|^2$



→Simulation is OK

Simulation: check(2)

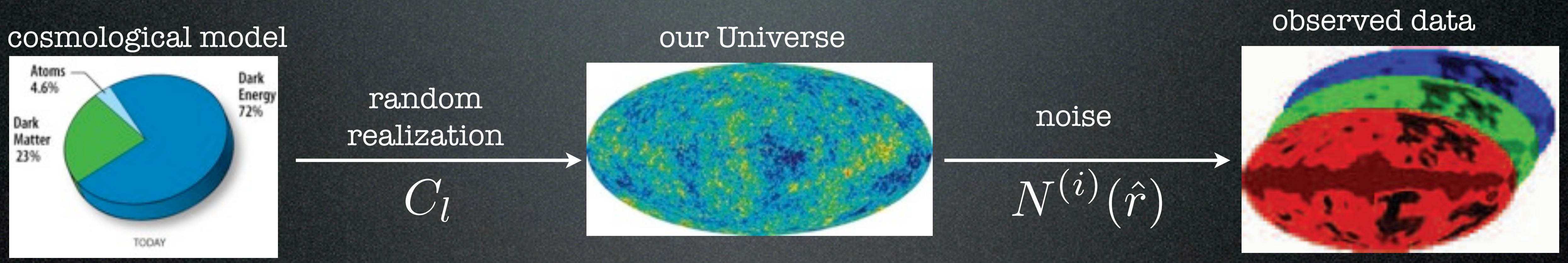
- Skewness:



→Simulation is OK.

Inverse-variance weighting(1)

- Optimally weighted map: $\tilde{a} = [C+N]^{-1} d$
  - Our universe: random realization
  - Large variance means less reliability.
- Why $(C+N)^{-1}$ weighting? Why not N^{-1}?



  - Both variance should be taken into account
  - Universally required in optimal estimation

- Direct inversion is practically impossible in realistic time-scales

  Need O(N_pix^3) arithmetics(!)

Inverse variance weighting(2)

- Conjugate gradient (CG) method [Oh, Spergel, Hinshaw(99)]

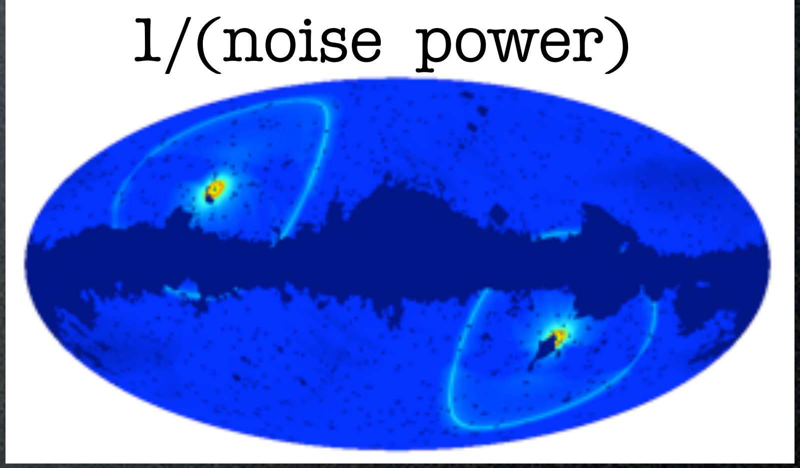
  Solve a linear equation $(C^{-1}+N^{-1})\tilde{a} = C^{-1}N^{-1}d$

- Simple CG converges very slowly

  (C+N) is correlated at large angular scales (small l's)

  ← inhomogeneous noise + sky cuts

  Good pre-conditioner close to $(C+N)^{-1}$ is required.



- Multi-grid preconditioning [Smith+(07)]

  Use $(C+N)^{-1}$ coarsified to $N_{side}/2$ as pre-conditioner at N_{side}.

  → O(10) speedup

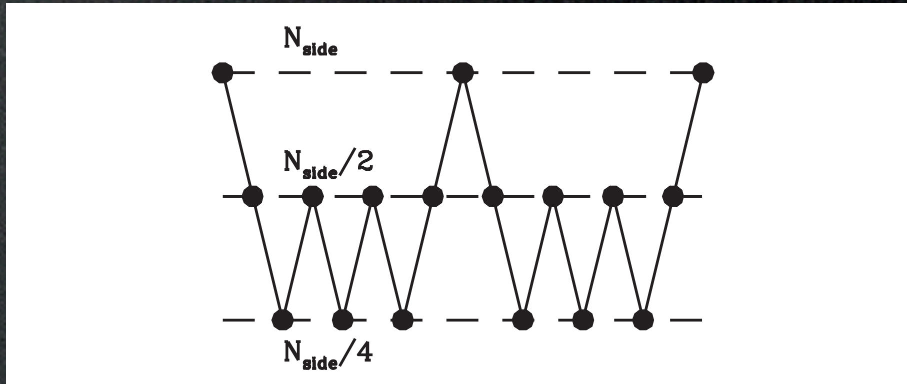
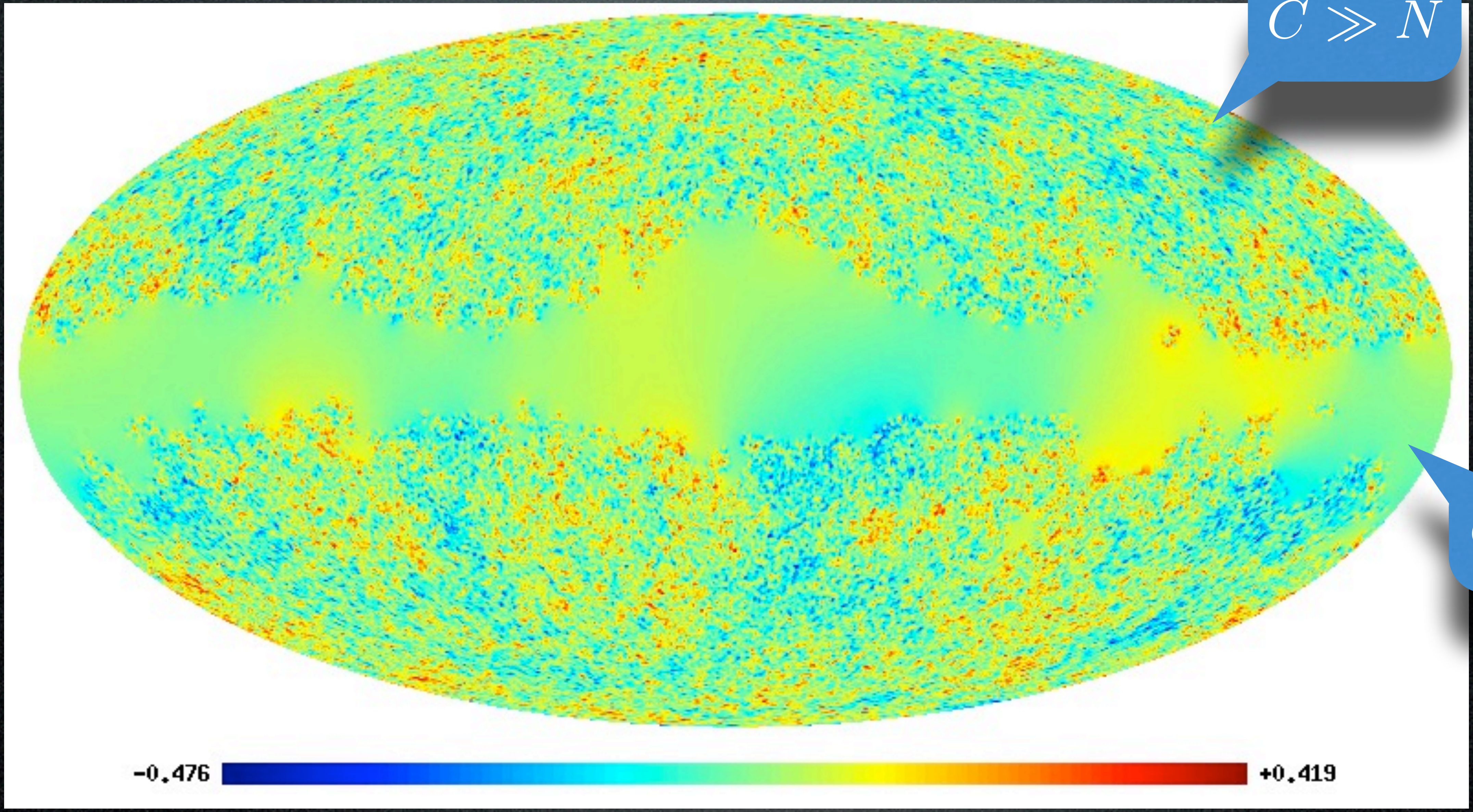


FIG. 21. Sequence of coarsifying and decoarsifying operations

Filtered map

- Wiener filtered map from WMAP V+W band

$$a = C[C+N]^{-1}d = C\tilde{a}$$



$C \gg N$

$C \ll N$

Table of constraints: uncorrelated case

	setups	$f_{\rm NL}$	$\alpha^2 f_{\rm NL}^{(\rm ISO)}$
CI, $n_{\rm iso} = 0.963$	w/o template marginalization	43 ± 21	13 ± 66
		(50 ± 23)	(-51 ± 72)
	w/ template marginalization	37 ± 21	22 ± 64
		(41 ± 23)	(-28 ± 71)
CI, $n_{\rm iso} = 1$	w/o template marginalization	46 ± 21	26 ± 63
		(51 ± 23)	(-34 ± 69)
	w/ template marginalization	33 ± 21	30 ± 66
		(35 ± 23)	(-15 ± 72)
NID, $n_{\rm iso} = 0.963$	w/o template marginalization	43 ± 21	191 ± 140
		(65 ± 39)	(-173 ± 261)
	w/ template marginalization	34 ± 21	164 ± 143
		(48 ± 39)	(-116 ± 266)
NID, $n_{\rm iso} = 1$	w/o template marginalization	40 ± 21	178 ± 137
		(57 ± 40)	(-133 ± 257)
	w/ template marginalization	36 ± 21	175 ± 137
		(48 ± 40)	(-87 ± 257)

Table 4: Constraints on $f_{\rm NL}$ and $\alpha^2 f_{\rm NL}^{(\rm ISO)}$ at 1σ level for the cases of uncorrelated isocurvature perturbations. A value with (without) parenthesis is a constraint on a nonlinearity parameter without (with) marginalization of the other one.

Table of constraints: correlated case

setups		$f_{\rm NL}$	$\alpha f_{\rm NL}^{\rm (ISO)}$
CI, $n_{\rm iso} = n_{\rm adi} = 0.963$	w/o template marginalization	41 ± 21	76 ± 114
		(50 ± 25)	(-82 ± 138)
	w/ template marginalization	34 ± 21	90 ± 120
		(37 ± 25)	(-26 ± 144)
CI, $n_{\rm iso} = n_{\rm adi} = 1$	w/o template marginalization	40 ± 21	70 ± 114
		(48 ± 25)	(-79 ± 138)
	w/ template marginalization	37 ± 21	99 ± 117
		(40 ± 25)	(-25 ± 141)
NID, $n_{\rm iso} = n_{\rm adi} = 0.963$	w/o template marginalization	45 ± 21	103 ± 55
		(93 ± 86)	(-126 ± 220)
	w/ template marginalization	35 ± 21	82 ± 54
		(55 ± 80)	(-53 ± 203)
NID, $n_{\rm iso} = n_{\rm adi} = 1$	w/o template marginalization	42 ± 21	99 ± 53
		(72 ± 75)	(-78 ± 191)
	w/ template marginalization	36 ± 21	86 ± 53
		(67 ± 80)	(-80 ± 204)

Table 5: Constraints on $f_{\rm NL}$ and $\alpha f_{\rm NL}^{\rm (ISO)}$ for the cases of correlated isocurvature perturbations.

Previous observational constraint

- Minkowski functional method [Hikage, Komatsu, Matsubara (06), Hikage+ (08)]
  - Topology of excursion set depends on skewness

    area fraction,

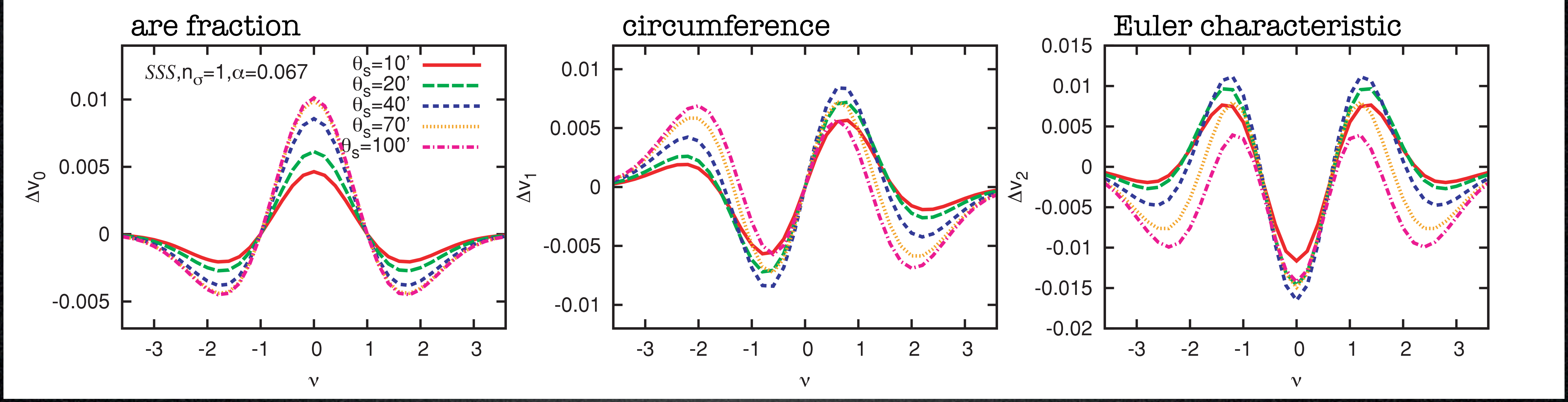
    circumference,...

$$S \sim \sum_{l_1 l_2 l_3} b_{l_1 l_2 l_3} W_{l_1}(\theta) W_{l_1}(\theta) W_{l_1}(\theta)$$

- WMAP5 constraint (uncorrelated isocurvature model)

[Hikage, Koyama, Matsubara & Takahashi (09)]

$$\alpha^2 f_{\rm NL}^{\rm (ISO)} = -15 \pm 60 \text{ (1 sigma)} \longleftarrow \quad \begin{array}{l} \alpha \sim P_S / P_\zeta \\ b^{\rm iso}_{l_1 l_2 l_3} \propto f_{\rm NL}^{\rm (ISO)} \alpha^2 \end{array}$$



[Hikage+(09)]

What's new in our analysis?

- Optimal constraints based on bispectrum
- Joint constraint on f_{NL} and $f_{NL}^{(ISO)}$
- Other types of isocurvature models than uncorrelated CDM one
  — correlated isocurvature models
  — neutrino density isocurvature

Analysis and validity check

- Analysis
  - Data: WMAP 7-year V+W temperature maps.
  - Conservative KQ75y7 mask (f_{sky}=72%)
  - Fiducial cosmological parameters: WMAP 7-year mean
  - Template marginalization of Galactic foregrounds

- validity check: purely adiabatic case ($f_{NL}^{(ISO)}$=0):

$$f_{\rm NL} = 31 \pm 21 \ \ (1 \text{ sigma})$$

➡ Consistent with the WMAP group.

cf. WMAP result [Komatsu+(11)]

Band	Foreground[b]	f_{NL}^{local}
V + W	Raw	59 ± 21
V + W	Clean	42 ± 21
V + W	Marg.[c]	32 ± 21
V	Marg.	43 ± 24
W	Marg.	39 ± 24

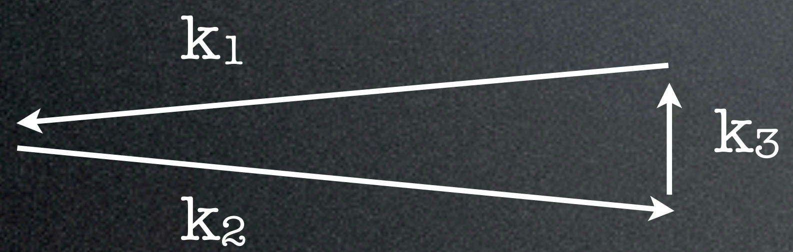
Local-type non-Gaussianity

- Local in real space

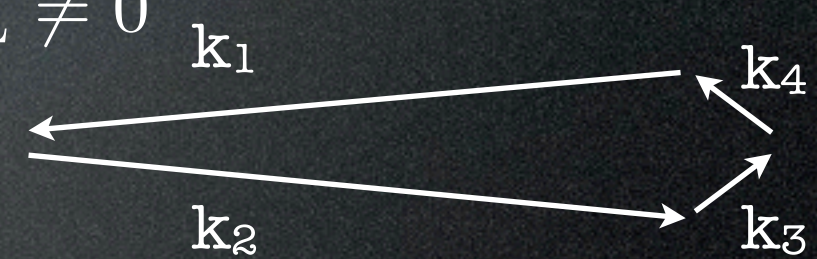
$$\zeta(\vec{x}) = \zeta_G(\vec{x}) + f_{\rm NL}\zeta_G(\vec{x})^2 + g_{\rm NL}\zeta_G(\vec{x})^3 + \cdots$$

- Signals are largest at squeezed configurations

$f_{\rm NL} \neq 0$



$g_{\rm NL} \neq 0$



- Single-field inflation models predict small undetectable non-Gaussianities.

$$f_{\rm NL} \simeq (1 - n_s) = \mathcal{O}(0.01), \qquad g_{\rm NL} = \mathcal{O}(10^{-4})$$

CMB Constraints on g_{NL}

WMAP constraints

- N-point pdf (Vielva & Sanz 2010): $g_{NL}/10^5=0.4\pm3.0$
- Kurtosis (Smidt+ 2010): $g_{NL}/10^5=0.5\pm3.9$
- Trispectrum (Fergusson+ 2010): $g_{NL}/10^5=1.6\pm7.0$
- Minkowski functionals (Hikage & Matsubara 2012): $g_{NL}/10^5=-1.9\pm6.4$
- Trispectrum+exact filtering (TS & Sugiyama 2013): $g_{NL}/10^5=-3.3\pm2.2$

Estimator of g_{NL}

Optimal estimator of g_{NL} Regan+ 2010; Fergusson+ 2010

$$\hat{g}_{\rm NL} = \frac{1}{N}\sum_{\{l,m\}} T^{m_1m_2m_3m_4}_{l_1l_2l_3l_4}\left[\tilde{a}_{l_1m_1}\tilde{a}_{l_2m_2}\tilde{a}_{l_3m_3}\tilde{a}_{l_4m_4}\right.$$
$$\left. -6\langle\tilde{a}_{l_1m_1}\tilde{a}_{l_2m_2}\rangle_{\rm G}\tilde{a}_{l_3m_3}\tilde{a}_{l_4m_4} + 3\langle\tilde{a}_{l_1m_1}\tilde{a}_{l_2m_2}\rangle\langle_{\rm G}\tilde{a}_{l_3m_3}\tilde{a}_{l_4m_4}\rangle_{\rm G}\right]$$

$$\tilde{a}_{lm} = (C^{-1}a)_{lm}$$