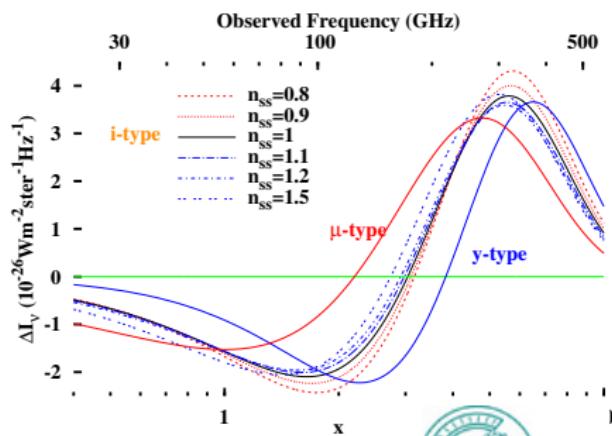
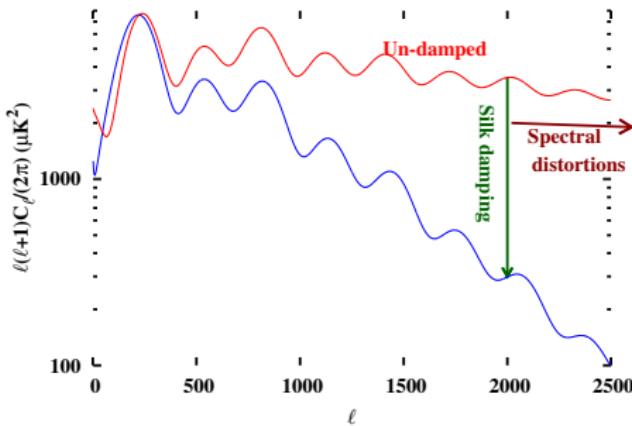


After Planck: The Road to Observing 17 e-Folds of Inflation

Rishi Khatri



with

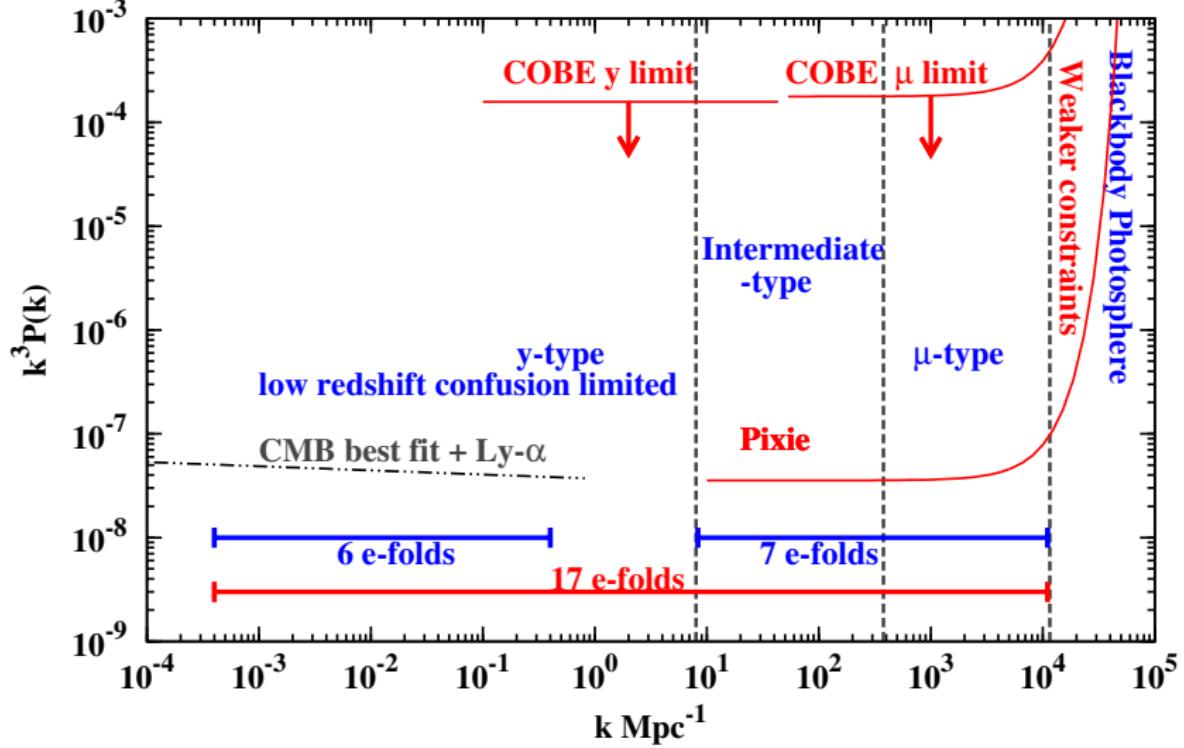
Jens Chluba (JHU)

Rashid Sunyaev(MPA)

Max-Planck-Institut
für Astrophysik



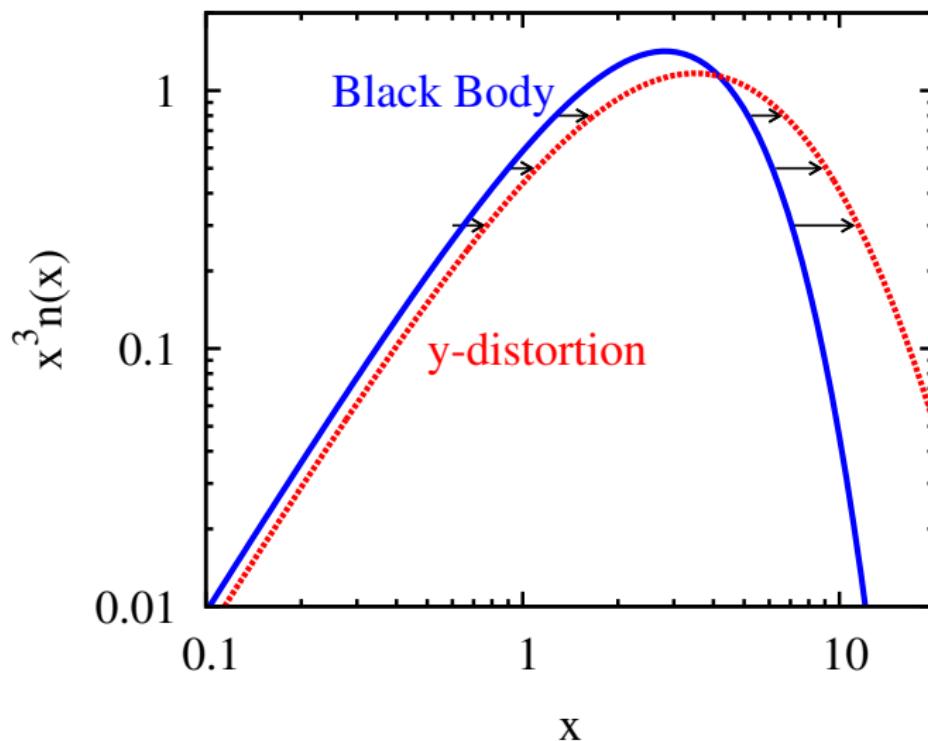
Going from 6-folds at present to 17 e-folds - almost 1/3 of inflation ?



y -distortion (Sunyaev-Zeldovich effect)

(Zeldovich and Sunyaev 1969)

COBE-FIRAS limit (95%): $y \lesssim 1.5 \times 10^{-5}$ (Fixsen et al. 1996)



Bose-Einstein spectrum

$$n(x) = \frac{1}{e^{x+\mu} - 1}$$

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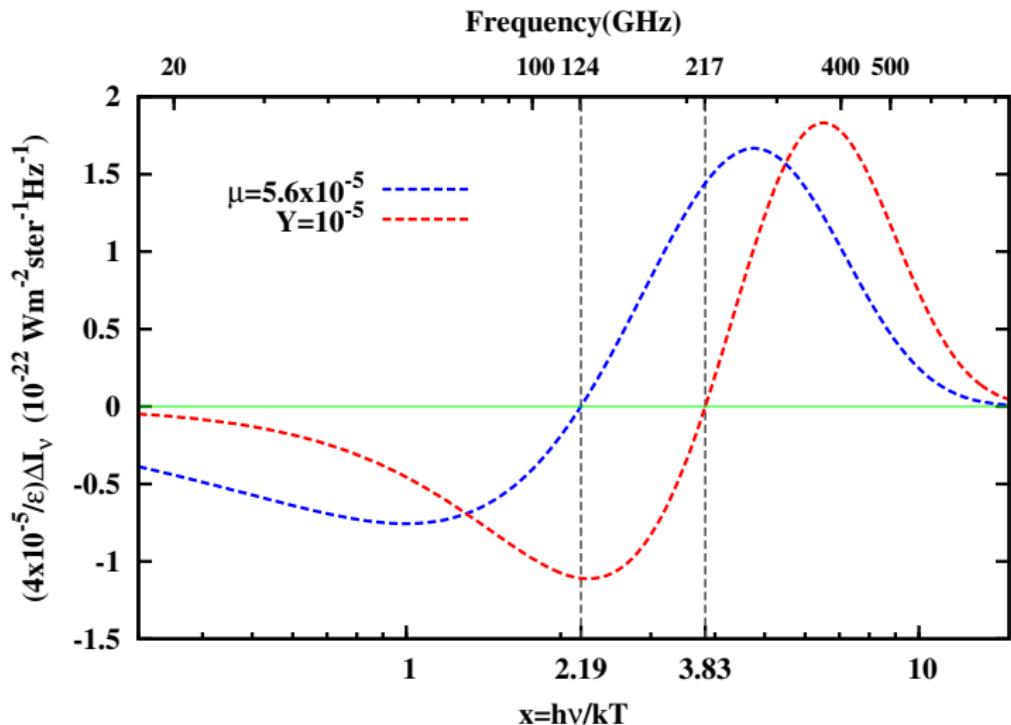
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Given two constraints, energy density (E) and number density (N) of photons, T, μ uniquely determined.

To get analytic solution, just need to determine rate of production of photons (when energy production rate is given)

μ -distortion: Bose-Einstein spectrum

COBE-FIRAS limit (95%): $\mu \lesssim 9 \times 10^{-5}$ (Fixsen et al. 1996)



y parameters

Sunyaev-Zeldovich effect:

$$y = \int dt \frac{k_B \sigma_T n_e}{m_e c} (T_e - T_\gamma)$$

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Recoil:

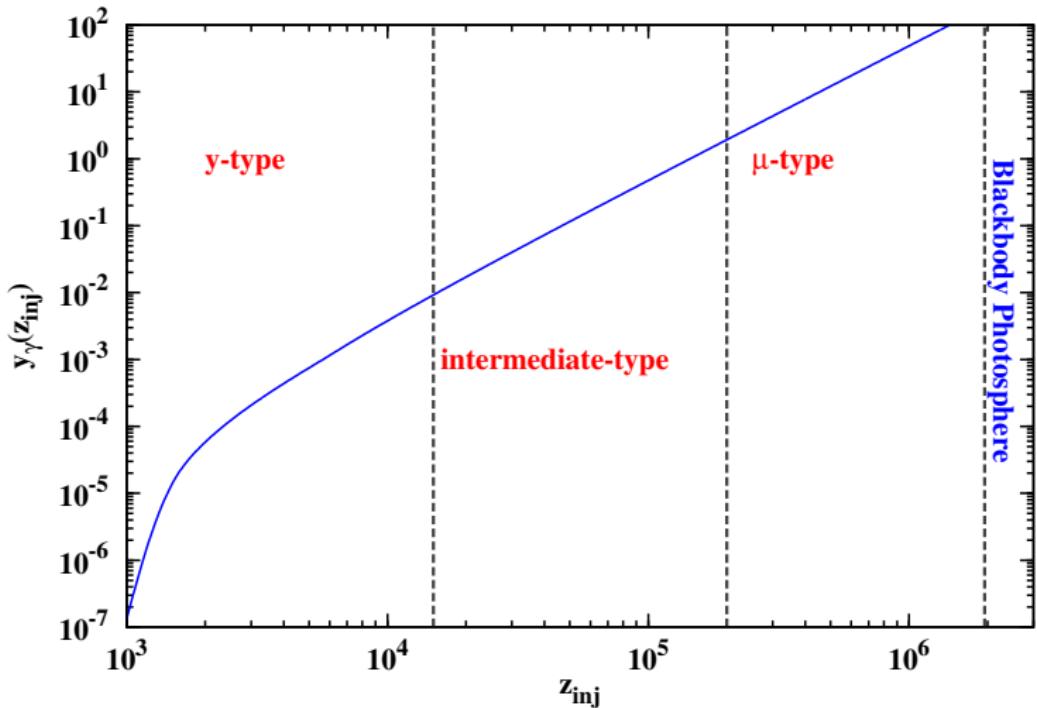
$$y_\gamma = \int dt \frac{k_B \sigma_T n_e}{m_e c} T_\gamma, \quad T_\gamma = 2.725(1+z)$$

Doppler effect:

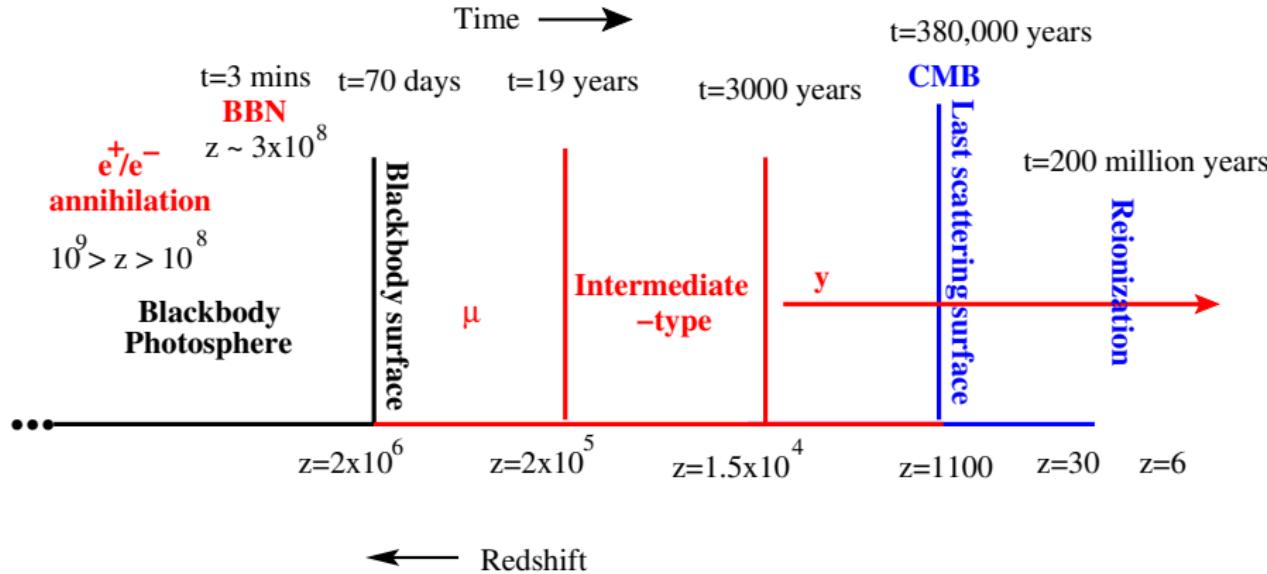
$$y_e = \int dt \frac{k_B \sigma_T n_e}{m_e c} T_e$$

In early Universe $y_\gamma \approx y_e$

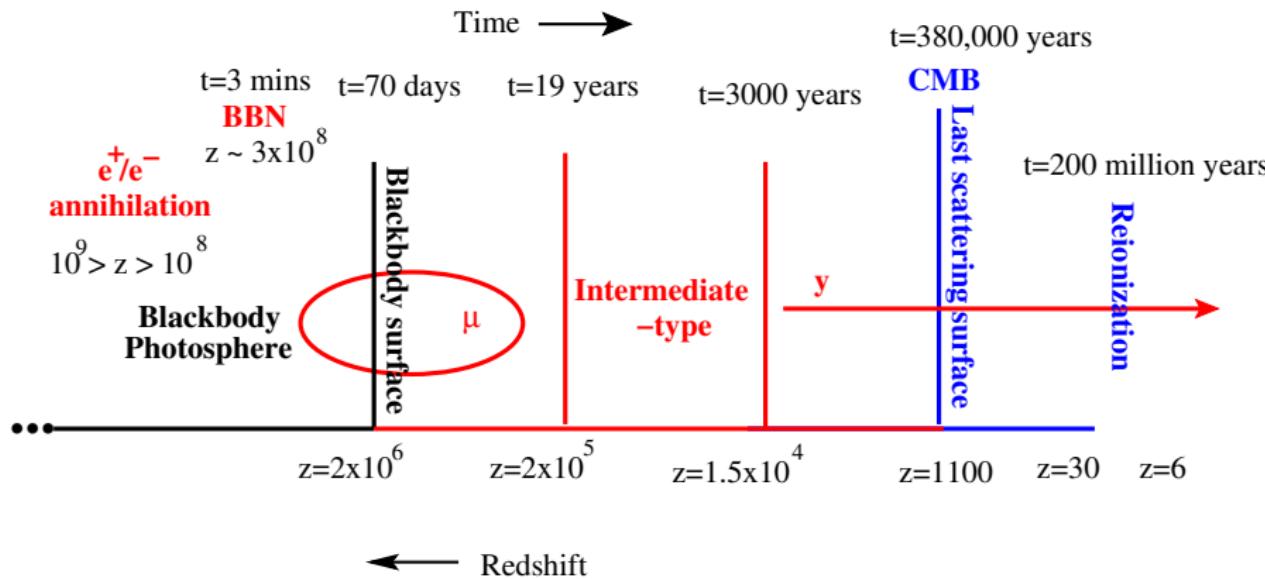
$$y\gamma = \int_{z_{\text{inj}}}^0 dt \frac{k_B \sigma_T n_e}{m_e c} T_\gamma$$



Cosmic Photosphere



μ -type distortions



Compton + double Compton + bremsstrahlung
Analytic solution: $\mu = 1.4 \int \frac{dQ}{dz} e^{-\mathcal{T}(z)} dz$
(Sunyaev and Zeldovich 1970)

Solutions for $\mathcal{T}(Z)$

Old solutions

(Sunyaev and Zeldovich 1970, Danese and de Zotti 1982)

Extension of old solutions to include both double Compton and bremsstrahlung

$$\mathcal{T}(z) \approx \left[\left(\frac{1+z}{1+z_{dC}} \right)^5 + \left(\frac{1+z}{1+z_{br}} \right)^{5/2} \right]^{1/2} + \varepsilon \ln \left[\left(\frac{1+z}{1+z_{\varepsilon}} \right)^{5/4} + \sqrt{1 + \left(\frac{1+z}{1+z_{\varepsilon}} \right)^{5/2}} \right]$$

This solution has accuracy of $\sim 10\%$, $z_{dC} \approx 1.96 \times 10^6$

Numerical studies: Illarionov and Sunyaev 1975, Burigana, Danese, de Zotti 1991, Hu and Silk 1993, Chluba and Sunyaev 2012

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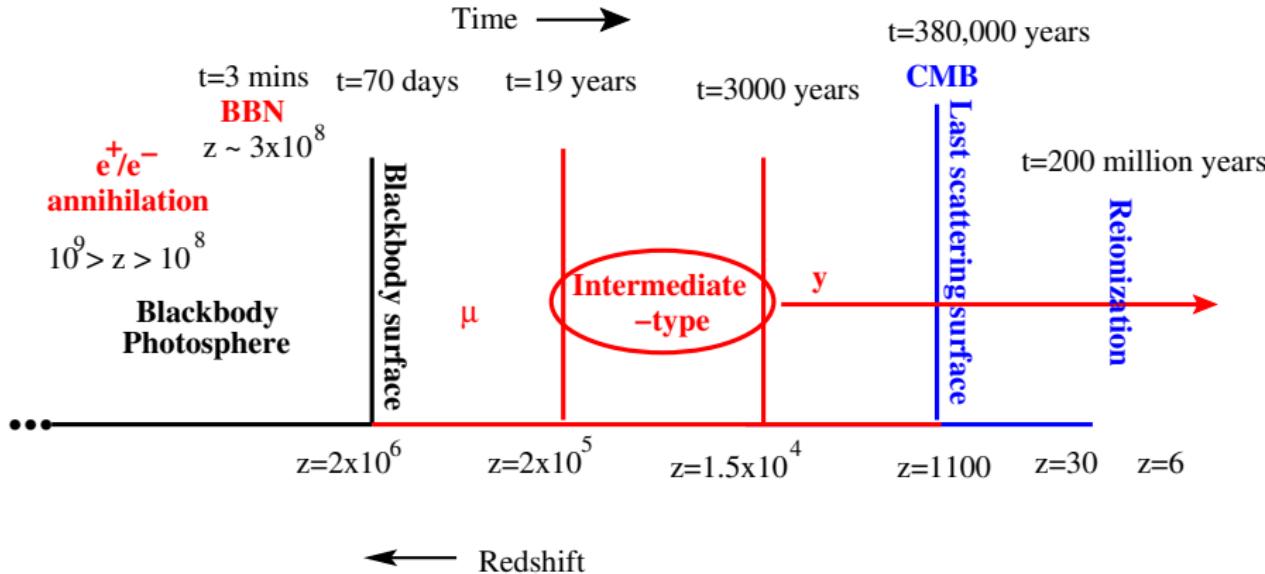
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New solution, accuracy $\sim 1\%$

(Khatri and Sunyaev 2012a)

$$\begin{aligned} \mathcal{T}(z) \approx & 1.007 \left[\left(\frac{1+z}{1+z_{dC}} \right)^5 + \left(\frac{1+z}{1+z_{br}} \right)^{5/2} \right]^{1/2} + 1.007 \varepsilon \ln \left[\left(\frac{1+z}{1+z_\varepsilon} \right)^{5/4} + \sqrt{1 + \left(\frac{1+z}{1+z_\varepsilon} \right)^{5/2}} \right] \\ & + \left[\left(\frac{1+z}{1+z_{dC'}} \right)^3 + \left(\frac{1+z}{1+z_{br'}} \right)^{1/2} \right], \end{aligned}$$

Intermediate-type distortions



intermediate-type distortions: Numerically solve Kompaneets equation

Intermediate-type distortions (*Khatri and Sunayev 2012b*)

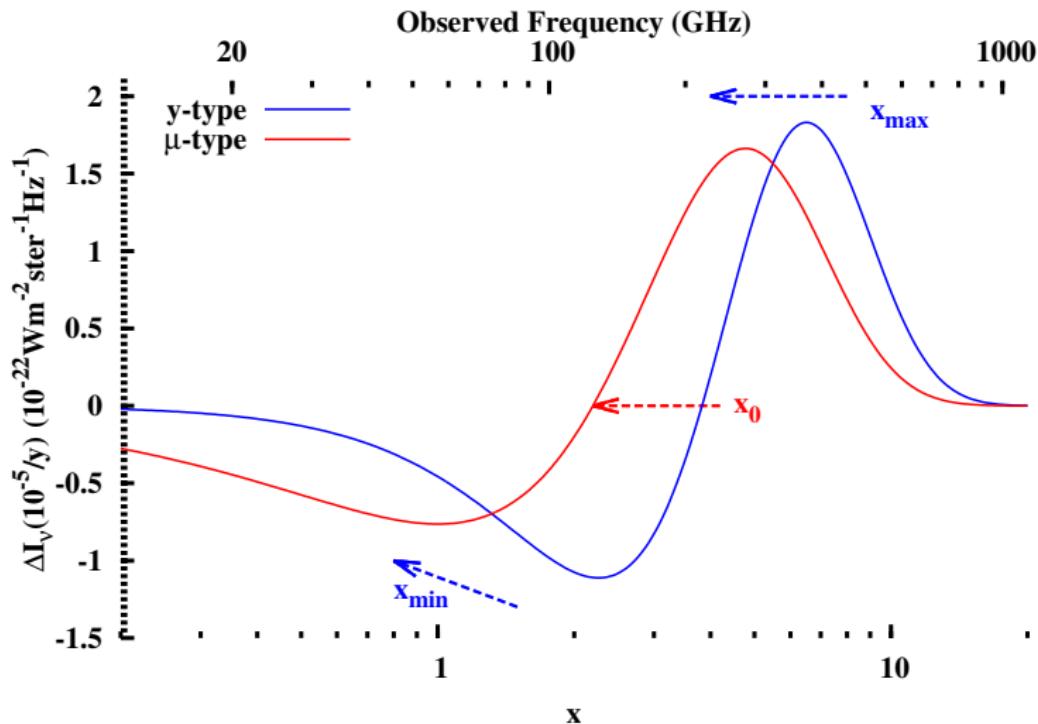
Solve Kompaneets equation with initial condition of y-type solution.

$$\frac{\partial n}{\partial y_\gamma} = \frac{1}{x^2} \frac{\partial}{\partial x} x^4 \left(n + n^2 + \frac{T_e}{T} \frac{\partial n}{\partial x} \right), \quad \frac{T_e}{T} = \frac{\int (n + n^2) x^4 dx}{4 \int n x^3 dx}$$

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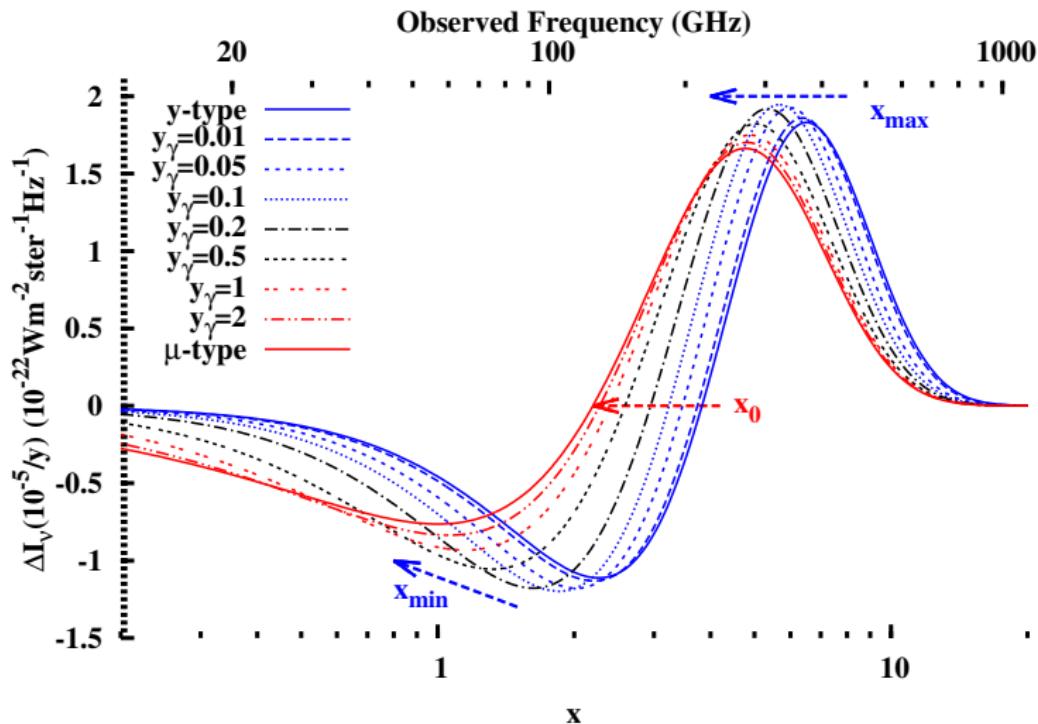
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Algorithm for fast solution, $\sim 1\%$ level accuracy

(Khatri and Sunyaev 2012b, arXiv:1207.6654)

- ▶ Calculate μ type distortion using the analytic solution, integrating upto the redshift when $y_\gamma = 2$.

$$n_{\mu-type} = 1.4 n_\mu \int_{\infty}^{z(y_\gamma=2)} \frac{dQ}{dz} e^{-\mathcal{T}} \quad (1)$$

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$$n_{i-type} = \frac{1}{Q_{num}} \sum_i \frac{dQ}{dy_\gamma}(y_\gamma^i) \delta y_\gamma^i n(y_\gamma^i) \quad (2)$$

<http://www.mpa-garching.mpg.de/khatri/idistort.html>

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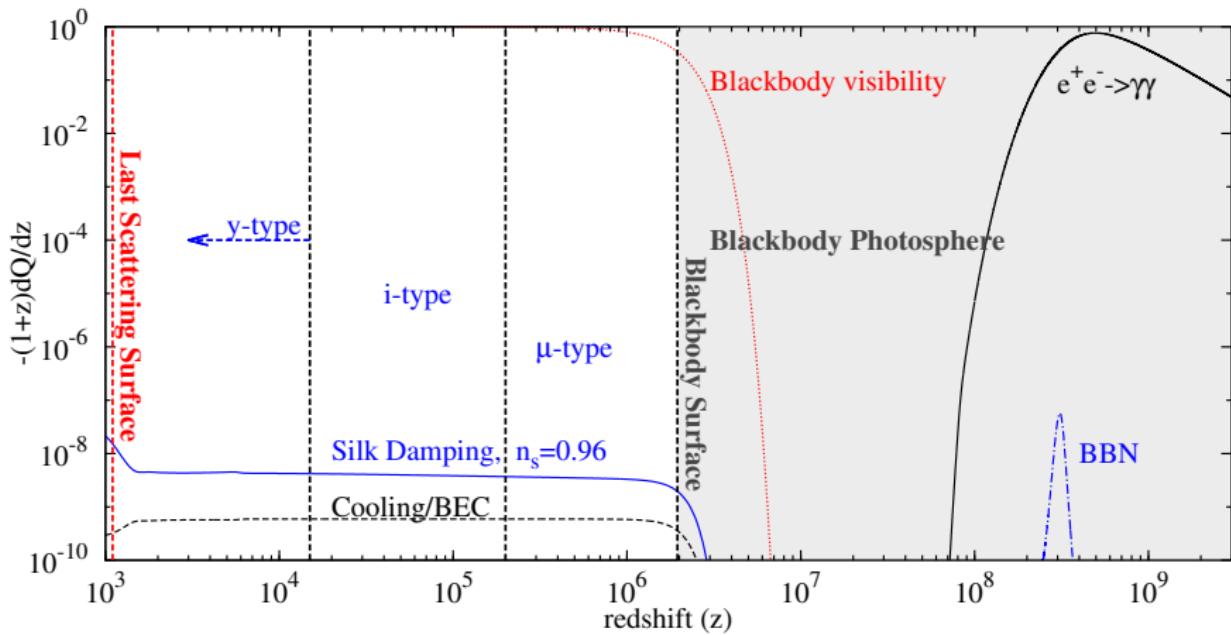
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- ▶ Add rest of the energy to y -type distortions.

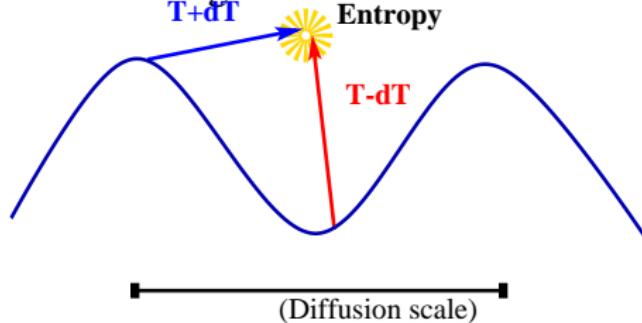
$$n_{y-type} = 0.25 n_y \int_{z(y_\gamma=0.01)}^{z=0} \frac{dQ}{dz} \quad (3)$$

The general picture



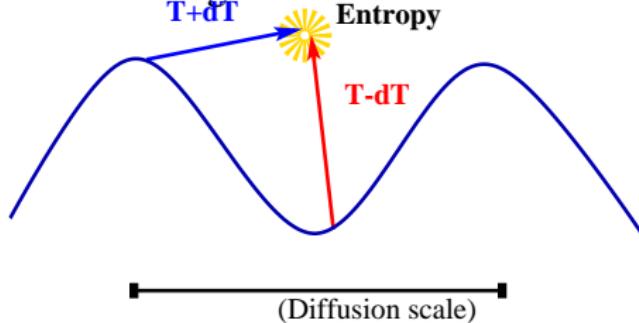
Silk damping

Photon diffusion \longrightarrow mixing of blackbodies



Silk damping

Photon diffusion \rightarrow mixing of blackbodies

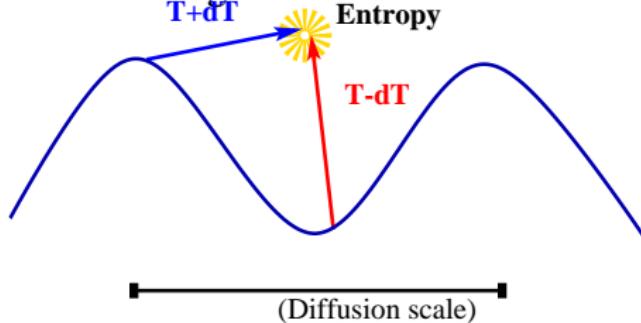


Mixing of blackbodies gives γ -type distortion

Zeldovich, Illarionov & Sunyaev 1972, Chluba & Sunyaev 2004

Silk damping

Photon diffusion → mixing of blackbodies



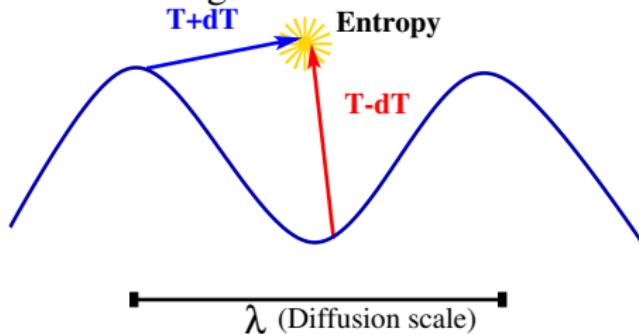
Mixing of blackbodies gives y-type distortion

Zeldovich, Illarionov & Sunyaev 1972, Chluba & Sunyaev 2004

$$\begin{aligned}
< n_{\text{Planck}} > &= \frac{1}{e^{\frac{h\nu}{kT}} - 1} + \left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle T \frac{\partial n_{\text{Pl}}}{\partial T} + \frac{1}{2} \left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle T^4 \frac{\partial}{\partial T} \frac{1}{T^2} \frac{\partial n_{\text{Pl}}}{\partial T} \\
&= n_{\text{Planck}} \left(T + \left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle \right) + \frac{1}{2} \left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle Y(\text{SZ})
\end{aligned}$$

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Photon diffusion → mixing of blackbodies



Apply mixing of blackbodies result to CMB

Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012

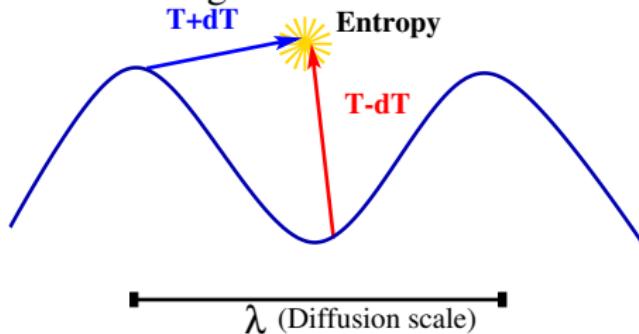
$$\frac{d}{dt} \left. \frac{\Delta E}{E_\gamma} \right|_{\text{distortion}} \approx - \frac{d}{dt} 2 \int \frac{k^2 dk}{2\pi^2} P_i(k) [\Theta_0^2 + 3\Theta_1^2 + (\ell > 1 \text{ terms})]$$

$$\frac{\Delta T}{T} = \sum_{\ell} (-i)^{\ell} (2\ell + 1) P_{\ell} \Theta_{\ell}$$

Tight coupling: density $\Theta_0 \propto \sin(kr_s)$, velocity $\Theta_1 \propto \cos(kr_s)$

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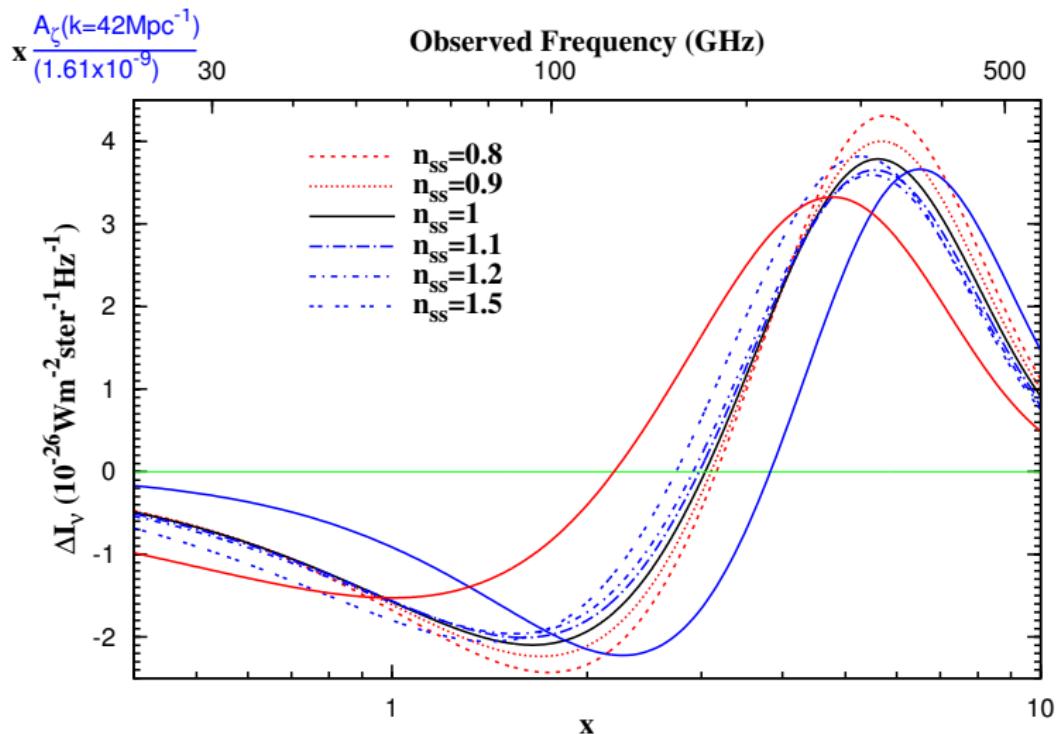
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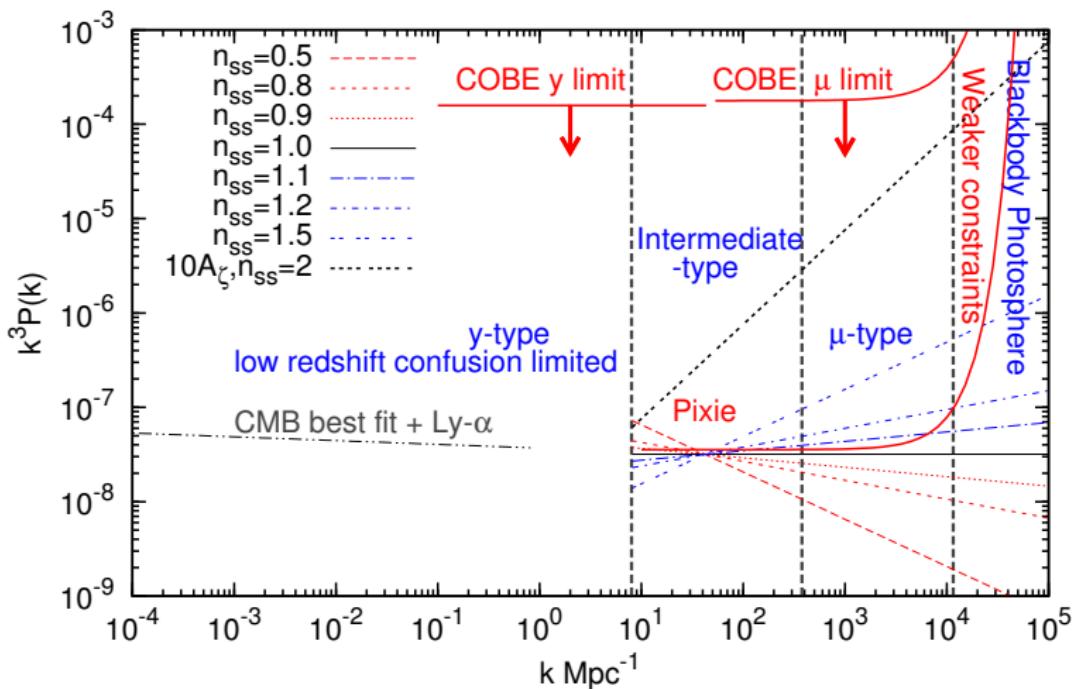
Total energy in the standing wave is independent of time

Silk damping (*Khatri and Sunayev 2012b*)



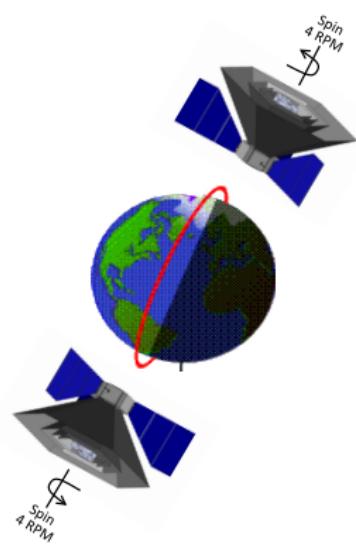
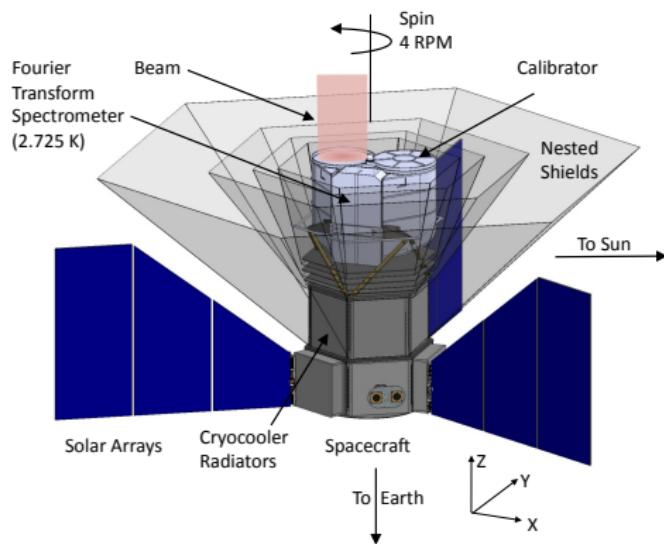
Pivot point $k_0 = 42 \text{ Mpc}^{-1}$

$$P_\zeta = (A_\zeta 2\pi^2/k^3)(k/k_0)^{n_s - 1 + \frac{1}{2}dn_s/d\ln k(\ln k/k_0)}$$



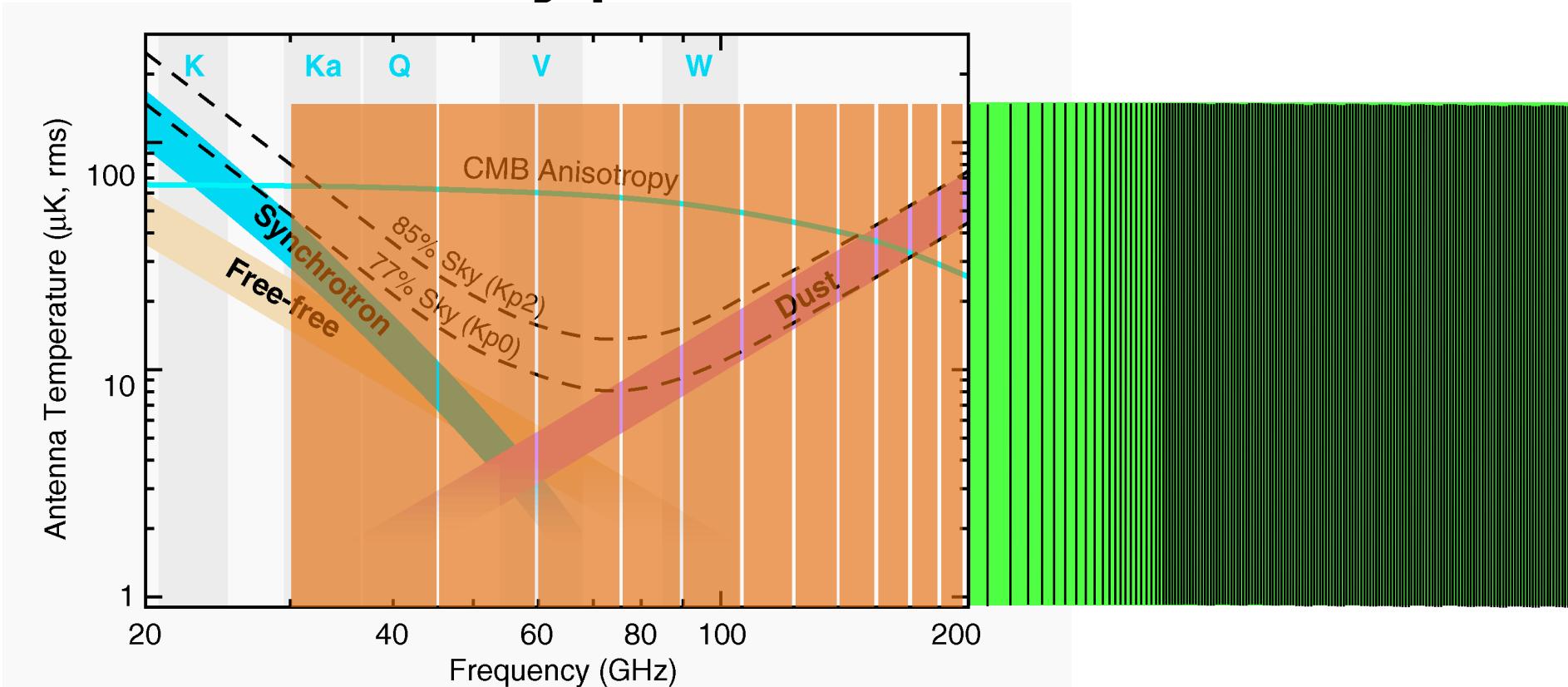
Spectrum: Pixie will improve over the COBE precision by at least 3 orders of magnitude

Kogut et al. 2011



PIXIE vs Typ Pol

Courtesy: Dale Fixsen



10 bands 30 GHz through 200 GHz ...

PLUS 390 more bands to 6 THz

FTS gets extra bands for free: why not use them?

Fisher matrix forecasts

Model:

$$\Delta I_V = t I_V^t + y I_V^y + I_V^{\text{damping}}(n_s, A_\zeta, dn_s/d\ln k).$$

Marginalise over temperature (t) and SZ effect (y)

I_V^{damping} contains i -type and μ -type distortions

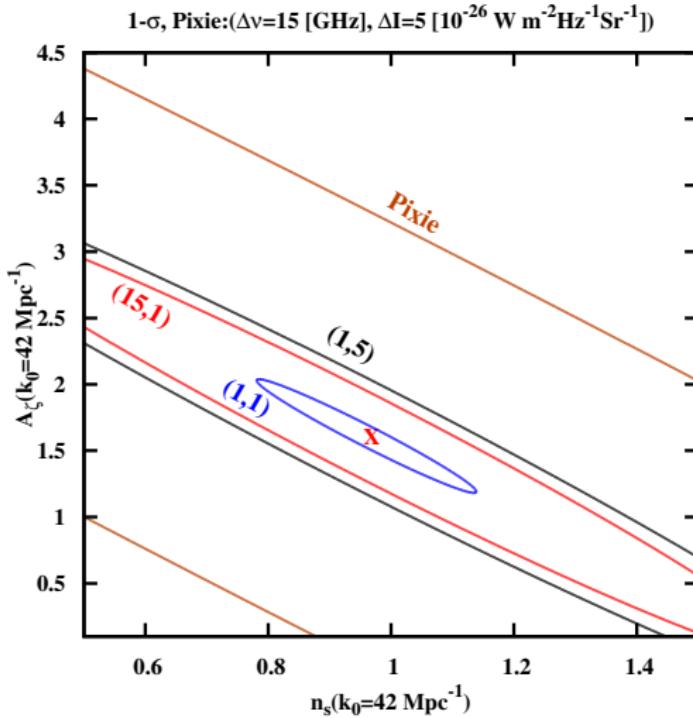
Fisher matrix forecasts

(Khatri and Sunyaev 2013)

Pixie-like experiments:

$(x,y) \equiv (\text{Resolution GHz}, \delta I(v) = 10^{-26} \text{W m}^{-2} \text{Sr}^{-1} \text{Hz}^{-1})$

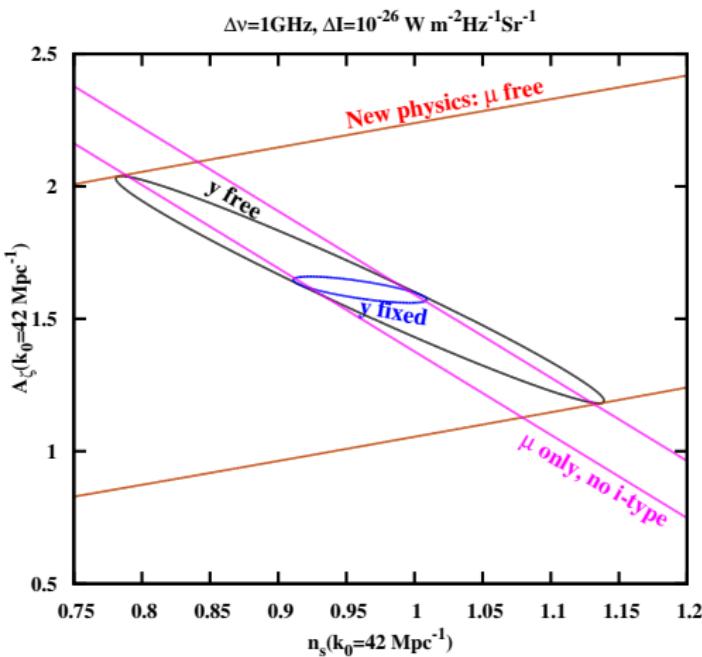
Pixie=(15,5)



Importance of *i*-type distortions,degenracies

(Khatri and Sunyaev 2013)

Information in the shape of *i*-type distortions breaks the $A_\zeta - n_s$ degeneracy



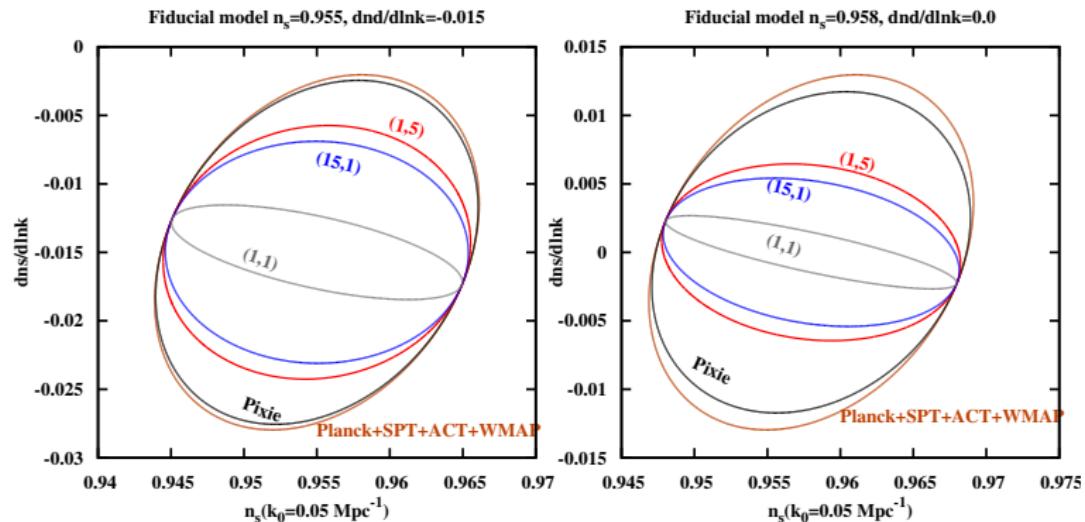
Fisher matrix forecasts with Planck+SPT+ACT+WMAP-pol

(Khatri and Sunyaev 2013)

Planck parameters, running spectrum, Pivot point $k_0 = 0.05$

$(x,y) \equiv (\text{Resolution GHz}, \delta I(v) = 10^{-26} \text{Wm}^{-2} \text{Sr}^{-1} \text{Hz}^{-1})$

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- ▶ i -type distortions are quite powerful in removing degeneracies between power spectrum parameters. The extra information comes from the shape of the i -type distortion

The future is bright

CMB spectrum is very rich in information about the early Universe,
late time Universe and fundamental physics

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This information is accesible and within reach of experiments in near
future like Pixie

Public code/pre-calculated numerical solutions

Example mathematica code + high precision pre-calculated numerical solutions for i-type distortions available at
<http://www.mpa-garching.mpg.de/~khatri/idistort.html>
Fortran version soon.

Numerical Kompaneets+double Compton+bremsstrahlung solver:
CosmoTherm code by Jens Chluba
www.chluba.de/CosmoTherm

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Tashiro and Sugiyama 2008, Carr et al. 2010
- ▶ Quantum wave function collapse: $\frac{dQ}{dz} \propto (1+z)^{-4}$
Lochan, Das and Bassi 2012

Summary continued

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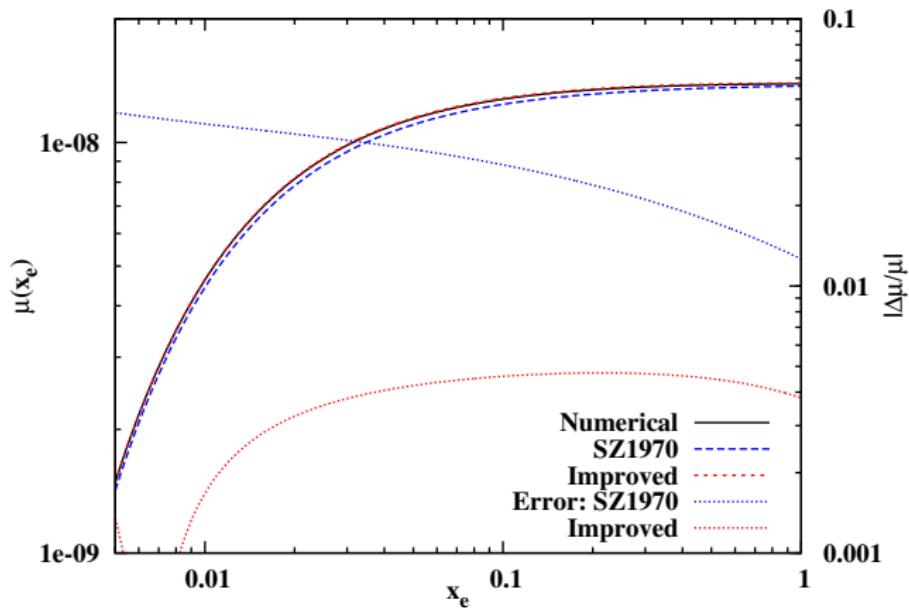
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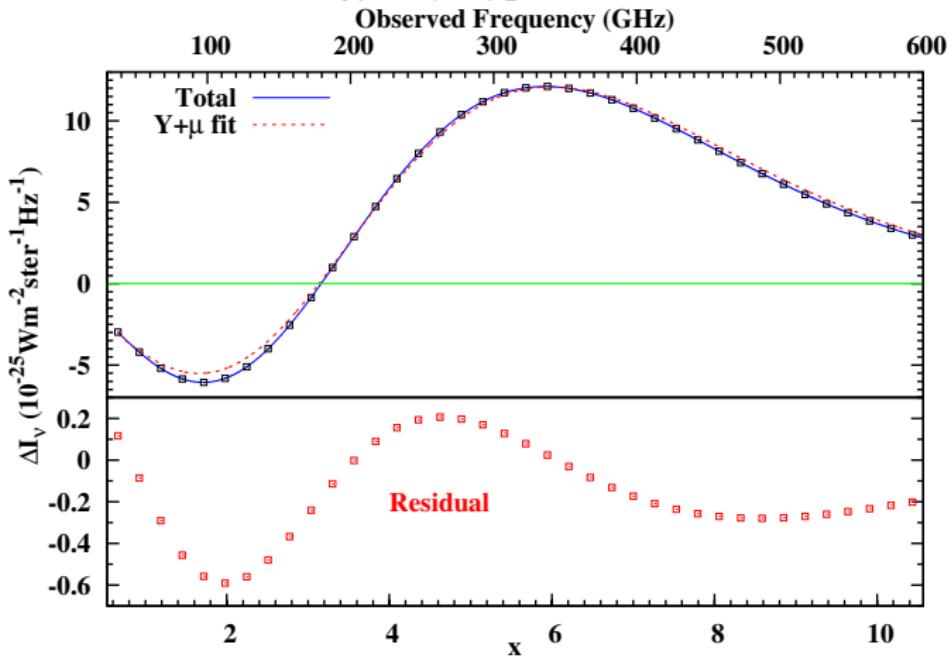
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- ▶ Primordial non-gaussianity on extremely small scales
Pajer and Zaldarriaga 2012, Ganc and Komatsu 2012

Accuracy of new solutions is better than 1%



$y+\mu$ cannot fully mimic i -type distortion

μ type and intermediate-type distortions are not independent. For Silk damping, intermediate-type distortions must contain about the same amount of energy as μ -type distortions.



Blackbody photosphere

