Planck Constraints on General Primordial Isocurvature Perturbations and Curvaton Model

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on behalf of the Planck collaboration
Planck 2013 results. XXII. Constraints on inflation


(Affiliations can be found after the references)

Preprint online version: March 22, 2013
Motivation: Why to study isocurvature in the Planck Inflation paper?

1. An important test of inflationary models.
   - Single field inflation (with one degree of freedom) can produce only the primordial curvature perturbation, i.e., adiabatic primordial perturbations, since exciting isocurvature perturbations requires additional degrees of freedom.
   - Therefore a detection of primordial isocurvature perturbations would point to more complicated models of inflation, such as multi-field inflationary scenarios which can produce a (possibly correlated) mixture of curvature and isocurvature perturbations.
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3. The determination of the “standard” $\Lambda$CDM model parameters, $\Omega_b h^2$, $\Omega_c h^2$, $\tau$, $\theta$, $A_s$, $n_s$, $(H_0, \Omega_\Lambda)$ could be significantly affected by an undetected isocurvature contribution.
   - Need to check how allowing for general initial conditions for perturbations affects the basic results.
General phenomenological models studied

Flat $\Lambda$CDM model with power law primordial spectra for the adiabatic mode, for **one** isocurvature mode at a time, and for their correlation,

$$\mathcal{P}(k) = \begin{pmatrix} \mathcal{P}_{RR}(k) & \mathcal{P}_{RI}(k) \\ \mathcal{P}_{IR}(k) & \mathcal{P}_{II}(k) \end{pmatrix},$$

where $I$ can be any of the non-singular, i.e., non-decaying isocurvature modes:

- **CDI** (cold dark mater density isocurvature mode).
- **NDI** (neutrino density isocurvature mode).
- **NVI** (neutrino velocity isocurvature mode).
- **BDI** (There can be also baryon density isocurvature mode, which is indistinguishable from CDI by the CMB observations. Above, the CDI mode can be regarded to include also baryons as: $I_{\text{effective}} = I_{\text{CDI}} + (\Omega_b/\Omega_c)I_{\text{BDI}}$).
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In the general models we have:

- 4 background params (as in the adiabatic $\Lambda$CDM model): $\Omega_b h^2, \Omega_c h^2, \tau, \theta$.
- 2 adiabatic perturbation parameters describing the power law spectrum $P_{RR}(k)$. 
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- In addition, 4 extra perturbation parameters: 2 describing the isocurvature power law spectrum $P_{\mathcal{I}\mathcal{I}}(k)$, and 2 for the correlation spectrum $P_{\mathcal{R}\mathcal{I}}(k) = P_{\mathcal{I}\mathcal{R}}(k)$. 

Planck constraints on isocurvature and curvaton
General phenomenological models studied

Flat ΛCDM model with power law primordial spectra for the adiabatic mode, for one isocurvature mode at a time, and for their correlation,

\[ P(k) = \begin{pmatrix} P_{RR}(k) & P_{RI}(k) \\ P_{IR}(k) & P_{II}(k) \end{pmatrix}, \]

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- 4 background params (as in the adiabatic ΛCDM model): \( \Omega_b h^2, \Omega_c h^2, \tau, \theta \).
- 2 adiabatic perturbation parameters describing the power law spectrum \( P_{RR}(k) \).
- In addition, 4 extra perturbation parameters: 2 describing the isocurvature power law spectrum \( P_{II}(k) \), and 2 for the correlation spectrum \( P_{RI}(k) = P_{IR}(k) \).

Theoretically motivated special cases with only 1 extra param

Curvaton, axion.
How to parametrize power law primordial spectra?

For each spectrum specify 2 parameters:
- amplitude at the pivot scale $k_0$, $P(k_0)$
- spectral index $n = \frac{\ln P}{\ln k} + 1$

ALARM! Cannot be used in MCMC, if $n_{\text{iso}}$ or $n_{\text{cor}}$ (or $n_t$) are free.
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For each spectrum specify 2 parameters:
- amplitude at scale \( k_1, P(k_1) \)
- amplitude at scale \( k_2, P(k_2) \)

From these the spectral index \( n \) and amplitude \( P(k_0) \) can be calculated as **derived** parameters.

Idea presented in 2004 in H. Kurki-Suonio, V. Muhonen, and J. Valiviita, Phys. Rev. D 71, 063005, and used ever since in most of isocurvature studies.
68% & 95% C.L. constraints on primordial powers at two scales

For perturbations, our primary MCMC parameters with uniform priors are the powers at $k_1 = 0.002 \text{ Mpc}^{-1}$, i.e., $P_{RR}^{(1)}, P_{II}^{(1)}, P_{RI}^{(1)}$ and at $k_2 = 0.100 \text{ Mpc}^{-1}$, i.e., $P_{RR}^{(2)}, P_{II}^{(2)}, |P_{RI}^{(2)}|$. 

19.8 20.0 20.2 20.4 20.6 20.8 21.0 21.2 21.4
10^9 P^{(1)}_{RR}
10^9 P^{(2)}_{RR}
10^9 P^{(1)}_{II}
10^9 P^{(2)}_{II}
10^9 P^{(1)}_{RI}
10^9 P^{(2)}_{RI}

19.0 19.5 20.0 20.5 21.0 21.5 22.0 22.5 23.0
10^{-1} |P^{(2)}_{RI}|

The Planck collaboration (Jussi Väliiviita, Uni. Helsinki) presents Planck constraints on isocurvature and curvaton.

ESTEC 4 April 2013 6 / 20
\( C_\ell \) from scale-invariant spectra with equal primordial amplitude

Note the \((k/k_{eq})^{-2}\), i.e., \(\ell^{-2}\) damping of CDI compared to the other modes, in particular compared to the adiabatic mode.

⇒ With CMB \(C^{TT}_\ell\), the CDI spectral index can never be constrained to much less than \(n_{iso} \approx n_{ad} + 2 \approx 3\), if the data are almost “adiabatic”.

Planck collab. (Jussi Väliviita, Uni. Helsinki)
Primordial isocurvature fraction and spectral index

\[ \beta_{iso}(k) = \frac{P_{II}(k)}{[P_{RR}(k) + P_{II}(k)]} \]

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$$\beta_{\text{iso}}(k) = \frac{P_{\text{II}}(k)}{[P_{\text{RR}}(k) + P_{\text{II}}(k)]}$$


$$n_{\text{iso}} \equiv n_{\text{II}} = \frac{d(\ln P_{\text{II}})}{d(\ln k)} + 1$$
Primordial correlation fraction

\[ \cos \Delta (k) = \frac{P_{RI}(k)}{\sqrt{P_{RR}(k)P_{II}(k)}} \]

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Planck constraints on isocurvature and curvaton

ESTEC 4 April 2013
Non-adiabaticity of the CMB temperature fluctuations

The non-adiabaticity fraction in today’s CMB \((\delta T)^2\)

\[
\alpha_{\text{non–ad}} = \frac{\langle (\delta T_{\text{non–ad}})^2 \rangle}{\langle (\delta T_{\text{total}})^2 \rangle} = \frac{\sum_{\ell=2}^{2500} (2\ell + 1)(C^{TT}_{II,\ell} + C^{TT}_{RI,\ell})}{\sum_{\ell=2}^{2500} (2\ell + 1)C^{TT}_{\text{tot,}\ell}}
\]
95% C.L. constraints and $\Delta \chi^2_{\text{eff}} = \chi^2_{\text{eff,ISO,}\text{best}} - \chi^2_{\text{eff,adiabatic,}\text{best}}$

<table>
<thead>
<tr>
<th>Primordial isoc. fraction</th>
<th>Isoc. frac. in CMB</th>
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</tr>
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<tbody>
<tr>
<td>(k=0.002) (\beta_{\text{iso}}) (0.050) (0.100)</td>
<td>(\alpha_{\text{non-ad}})</td>
<td>(-7% \ldots +2%)</td>
<td>(-4.6)</td>
<td>(-4.7) (-0.2)</td>
</tr>
<tr>
<td>CDI</td>
<td>&lt;0.08 &lt;0.39 &lt;0.60</td>
<td>-7% ... +2%</td>
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<td>-4.7</td>
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<tr>
<td>NDI</td>
<td>&lt;0.27 &lt;0.27 &lt;0.32</td>
<td>-9% ... +1%</td>
<td>-4.2</td>
<td>-3.8</td>
</tr>
<tr>
<td>NVI</td>
<td>&lt;0.18 &lt;0.14 &lt;0.17</td>
<td>-5% ... +4%</td>
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These models have 4 extra parameters compared to the pure adiabatic model.

*) Low-\(\ell\) TT = Planck TT likelihood at \(\ell = 2 \rightarrow 49\) (\texttt{commander.v4.1_lm49.clik})

**) High-\(\ell\) TT = Planck TT likelihood at \(\ell = 50 \rightarrow 2500\) (\texttt{CAMspec.v6.2TN.2013_02_26.clik})

***) The rest (NOT IN THE TABLE) = WMAP-9 TE and EE (\texttt{lowlike.v222.clik})
Temperature $C_\ell$ spectra of the best-fit models

**Planck constraints on isocurvature and curvaton**

Planck collab. (Jussi Väliiviita, Uni. Helsinki)
Effect on the estimation of background parameters

In March 2013 with Planck+WP data

Planck constraints on isocurvature and curvaton

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Larger values of $H_0$ and $\Omega_\Lambda$ are acceptable than in the pure adiabatic model.
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$$n_{ad} \equiv n_{RR} = \frac{d\ln P_{RR}}{d\ln k} + 1$$
A theoretically motivated special case: \textbf{curvaton}

The Planck collaboration studied the curvaton scenario with assumptions:

1. The average curvaton field value, $\chi^*$, is sufficiently below the Planck mass at the time when cosmologically interesting scales exit the horizon during inflation.

2. The curvature perturbation from inflaton is negligible compared to the curvaton perturbation at horizon exit during inflation.

3. The same is true for any inflaton decay products after reheating. This means that after reheating the Universe is homogeneous, except for the spatially varying entropy (i.e., isocurvature perturbation) due to the curvaton field perturbations.

4. Later, CDM is created from the curvaton decay.

This curvaton scenario leads to fully correlated primordial curvature and isocurvature perturbations with the same shape of their power spectra:

$$P_{RI} = +\sqrt{P_{RR}P_{II}}, \quad n_{iso} = n_{ad}.$$ 

Hence only one extra parameter compared to the adiabatic model.
The isocurvature fraction in this curvaton model will be

$$\beta_{iso} = \frac{9(1 - r_D)^2}{r_D^2 + 9(1 - r_D)^2}, \quad \text{where} \quad r_D = \frac{3\rho_{\text{curvaton}}}{3\rho_{\text{curvaton}} + 4\rho_{\text{radiation}}},$$

and the non-linearity parameter (assuming sudden decay of curvaton and quadratic potential) will be

$$f_{NL}^{\text{local}} = \frac{5}{4r_D} - \frac{5}{3} - \frac{5r_D}{6}.$$ 

If the curvaton totally dominates the energy density at curvaton’s decay time, then $r_D = 1$, $f_{NL}^{\text{local}} = -5/4$, and the curvaton perturbations are converted to pure adiabatic ones with no residual isocurvature.
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With **Planck**+WP data we find \( \beta_{\text{iso}} < 0.0025 \) at 95\% C.L.

5 times tighter than the WMAP9 constraint, 3 times tighter than WMAP9+ACT+SPT!

- This corresponds to \( 0.98 < r_D < 1 \) and \( -1.25 < f_{\text{NL}}^{\text{local}} < -1.21 \).
- Fits within \( 1\sigma \) **Planck** constraint \( f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8 \).
Summary on the general & special isoc. cases

95% C.L. constraints and $\Delta \chi^2_{\text{eff}} = \chi^2_{\text{eff,ISO,\text{best}}} - \chi^2_{\text{eff,adiabatic,best}}$

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Special CDI cases

| “axion” | $<0.0360$ | $<0.0390$ | $<0.0400$ | $<2\%$ | $0$ |
| “curvaton” | $<0.0025$ | $<0.0025$ | $<0.0025$ | $0\% \ldots +3\%$ | $0$ |
| anti-corr. | $<0.0087$ | $<0.0087$ | $<0.0087$ | $-6\% \ldots 0\%$ | $-1.3$ |

The general cases IN COLOR have 4 extra parameters.
The special cases have 1 **extra parameter** compared to the adiabatic model.

“axion” = no correlation ($P_{RI} = 0$), $n_{\text{iso}} = 1$.
“curvaton” = +100% correlation ($P_{RI} = +\sqrt{P_{RR}P_{II}}$), $n_{\text{iso}} = n_{\text{ad}}$.
anti-corr. = -100% correlation ($P_{RI} = -\sqrt{P_{RR}P_{II}}$), $n_{\text{iso}} = n_{\text{ad}}$. 
The Planck collaboration studied in a general phenomenological set-up all non-singular primordial isocurvature modes, CDI (and BDI), NDI, NVI, one at a time, possibly correlated with adiabatic perturbations:

- Acoustic peak structure in the Planck data is “adiabatic” to high precision.
- The power of Planck low-\( \ell \) spectrum is about 10% below the prediction of the best-fit adiabatic \( \Lambda \)CDM model.
  - A negatively correlated isocurvature component improves the fit to low-\( \ell \) data (\( \Delta \chi^2_{\text{eff}} \approx -4.5 \)), without affecting too much the high-\( \ell \) spectrum.
  - This \( \ell \sim 10...30 \) “anomaly” leads to relatively large isocurvature fractions (-5% — -9%) to be allowed in the CMB temperature variance.
- Determination of standard cosmological parameters is only mildly affected by allowing for an isocurvature contribution! A great improvement over WMAP.
Conclusions

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In addition, the Planck collaboration studied theoretically motivated models:

- Both curvaton and axion models worsen the fit to the Planck data, since they increase the power at low-\(\ell\).
  - This leads to the stringent constraints on these models:
    - primord. isoc. frac. \(\beta_{\text{iso}} < 0.25\%\) (curvaton) and \(\beta_{\text{iso}} < 3.9\%\) (axion).
  - A model similar to curvaton, but with 100% anticorrelation improves the fit moderately (\(\Delta \chi^2_{\text{eff}} \approx -1.3\)), and leads to \(\beta_{\text{iso}} < 8.7\%\).
The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada.