Athena Mirror
Version 3.1
Defocused
Point Spread Function

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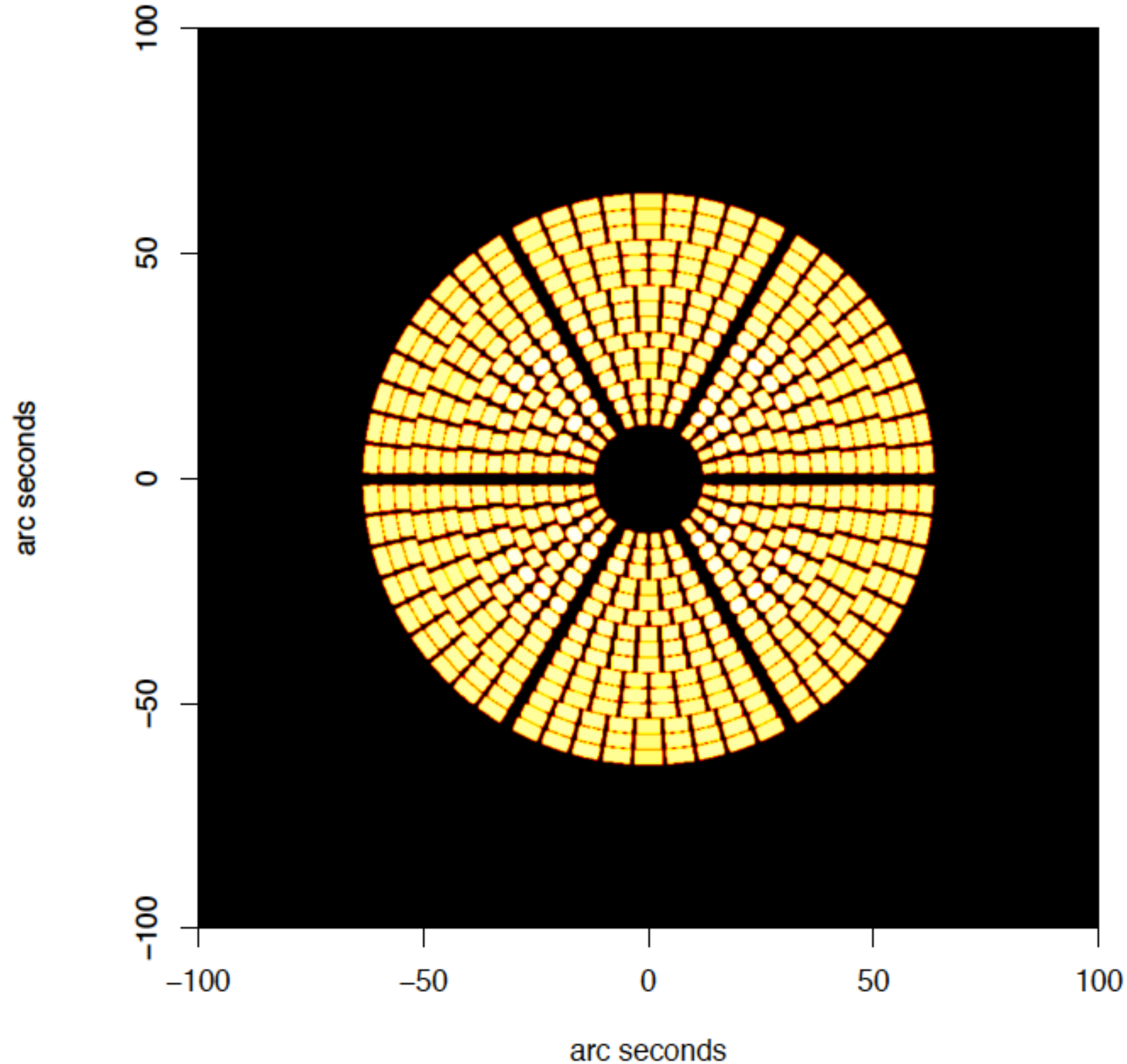
Defocused PSF

- If the detector is moved forward (towards the mirror, intra-focal) or backwards (away from the mirror, extra-focal) the PSF produced is a projection of the module aperture distribution in the mirror aperture.
- The scaling factor depends on the axial displacement, Delz, the radius of the module in the aperture, R, and the focal length, F:
  - Scaling factor = Delz/sqrt(F^2-R^2)
- In the absence of figure errors, module alignment errors and scattering from surface roughness the PSF is a faithful rendering of the module aperture distribution and if Delz=0 the PSF collapses to a point.
- When figure errors are included the defocused image of each module aperture is spread by the module PSF.
- The brightness of each module aperture image in the defocused PSF is modulated by the effective area of the module at the appropriate X-ray energy.
- The defocused PSF can be calculated analytically using the module aperture layout, the PSF parameters of each row as a function of energy and the module effective areas as a function of energy.
- The full defocused PSF including module alignment errors and scattering from surface roughness can be rendered by ray tracing – accuracy/acuity limited by the Monte Carlo errors arising from the finite number of rays traced.

Perfect Analytical Defocused PSF

- Delz = 35 mm forward
- Zero module alignment errors and scattering from surface
- Figure errors very small to produce HEW of ~0.25 arc seconds
- Pixel size 0.25 arc seconds
- X-ray energy zero – reflectivity 1.0 – area geometric area of module



Module PSF

- The in-plane and out-of-plane PSF profiles of a single XOU have been fitted using a pseudo-Voigt distribution – Cosine document CR-SPOEQU-ME60, G. Vacanti, 2020-07-10
- In order to produce a simulated single module PSF to match the measured profiles, the figure gradient errors in the ray tracing simulation are now generated using a pseudo-Voigt distribution

$$F(t) = \frac{(1-\alpha)}{\sigma_g\sqrt{2\pi}} \exp\left(\frac{-t^2}{2\sigma_g^2}\right) + \frac{\alpha}{\sigma_c \pi}\left(1+\frac{t^2}{\sigma_c^2}\right)^{-1}$$

$$\sigma_g = \frac{FWHM}{2\sqrt{2\ln(2)}},\ \sigma_c = \frac{FWHM}{2}$$

$$\int_{-\infty}^{+\infty} F(t)dt = 1$$

- Cosine fitted parameters for XOU row 8: in-plane: α=0.7978, σ_c=17.3216, out-of-plane α=0.2312, σ_c=5.2800
- The figure gradient distributions required to match the Cosine profile fit and produce a full aperture (all modules) HEW of 5 arc seconds: in-plane σ_c=0.62 arc seconds, α=0.7, out-of-plane σ_c=12.5 arc seconds, α=0.14
- Generate PSF predictions for XOUs in other rows using the same figure error parameters σ_c and α
- Scale σ_c values to generate different HEW and FWHM values for the full aperture (using alignment etc. on next slide) – table to right

Full Aperture PSF

sigma_c	HEW	FWHM
0.61	4.95	2.87
0.62	5.01	2.90
0.64	5.14	2.92
0.77	5.97	3.15
0.84	6.42	3.32
0.85	6.49	3.35
0.86	6.55	3.36

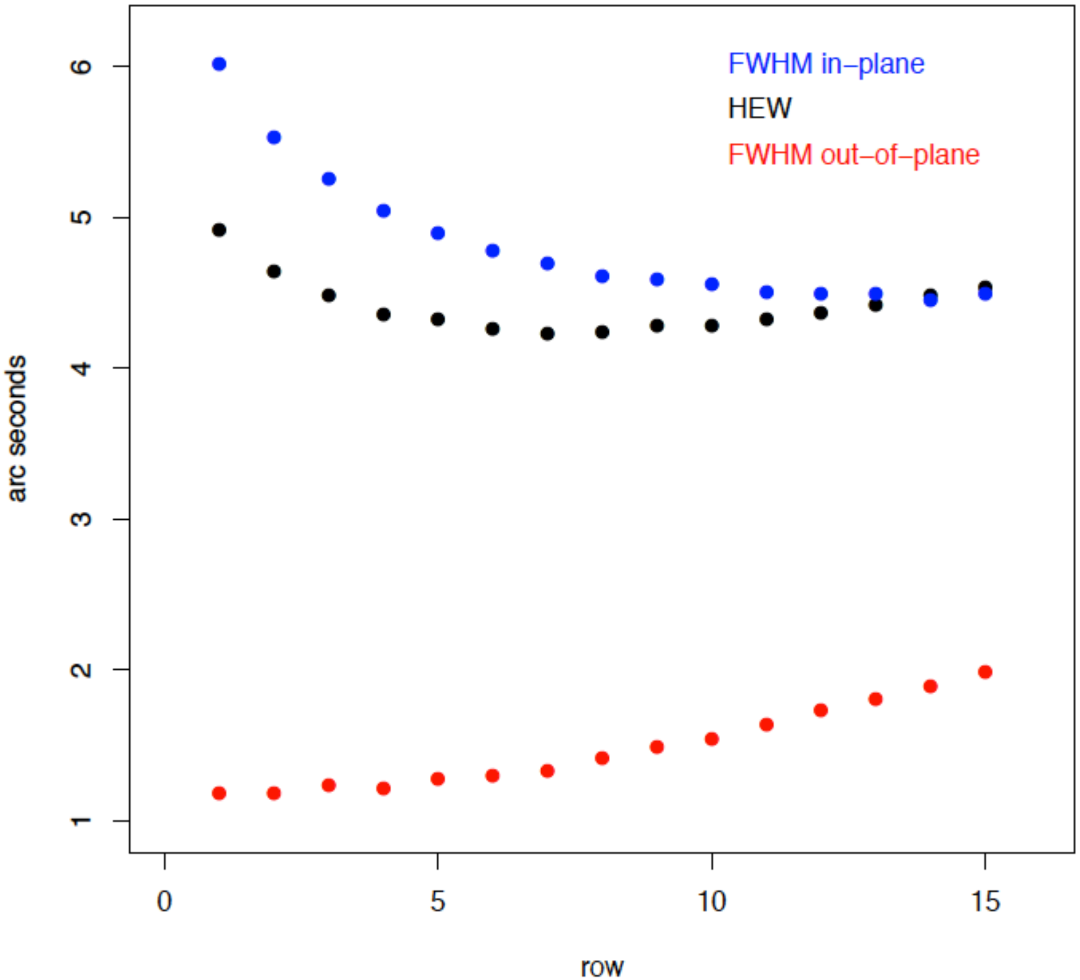
Predicted module PSF for each row

- Row 8 figure error parameters: σ_c=0.62 arc seconds, α=0.7, out-of-plane σ_c=12.5 arc seconds, α=0.14
- Generate predictions for XOUs in other rows using the same figure error parameters σ_c and α
- Fit the in-plane and out--of-plane module PSF with a pseudo-Voigt profile
- For inner row 1 the in-plane curvature errors dominate – ratio inFWHM/outFWHM=5.1
- For outer row 15 the out-of-plane curvature/roundness errors dominate because the angular displacement in the focal plane caused by out-of-plane errors scales with grazing angle - ratio inFWHM/outFWHM=2.3

row	in FWHM	in alpha	out FWHM	out alpha	HEW	Area 1 keV
1	6.01	0.65	1.18	0.84	4.92	12.99
2	5.54	0.68	1.18	0.75	4.64	18.64
3	5.26	0.71	1.23	0.64	4.48	23.93
4	5.04	0.72	1.22	0.51	4.36	22.19
5	4.90	0.74	1.28	0.46	4.32	26.02
6	4.78	0.75	1.29	0.38	4.27	23.81
7	4.69	0.76	1.33	0.32	4.23	22.11
8	4.61	0.77	1.41	0.29	4.24	24.39
9	4.59	0.78	1.49	0.29	4.28	32.15
10	4.56	0.77	1.55	0.26	4.28	29.90
11	4.51	0.79	1.64	0.26	4.32	32.08
12	4.50	0.80	1.73	0.25	4.37	34.10
13	4.50	0.79	1.80	0.24	4.42	31.47
14	4.45	0.81	1.89	0.24	4.48	32.91
15	4.49	0.79	1.99	0.23	4.53	34.18

Module PSF FWHM and HEW 1 keV

- The in-plane FWHM decreases from row 1 to row 15
- The out-of-plane FWHM increases from row 1 to row 15
- The HEW is minimum for rows 7-8
- The full aperture including all alignment errors - HEW=5 arc seconds
- Using the Cosine fit module PSF the out-of-plane width has a significant effect on the HEW for outer rows



Module PSF 2-D Model

- The in-plane and out-of-plane 1-D profiles of the module PSF are modelled using the pseudo-Voigt distribution

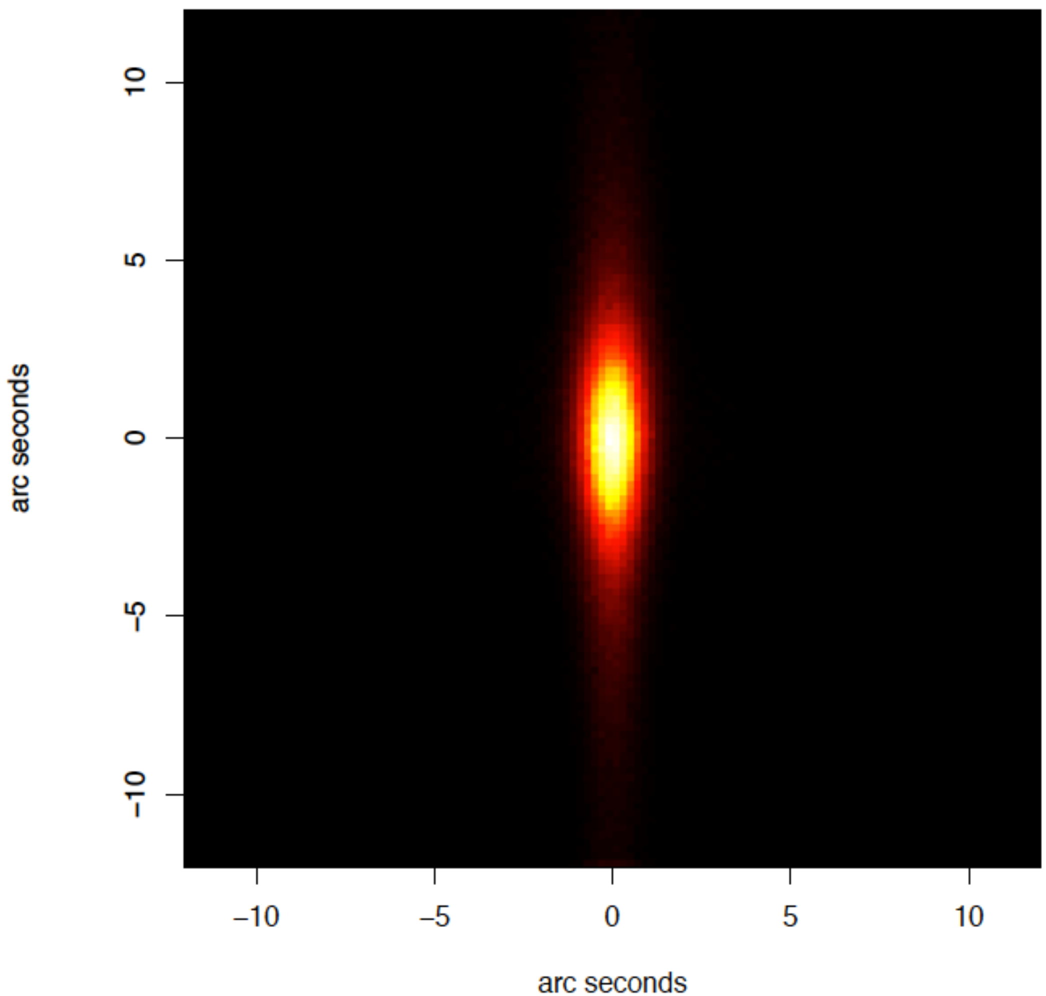
$$F(t) = \frac{(1-\alpha)}{\sigma_g\sqrt{2\pi}} \exp\left(\frac{-t^2}{2\sigma_g^2}\right) + \frac{\alpha}{\sigma_c \pi}\left(1 + \frac{t^2}{\sigma_c^2}\right)^{-1}$$

$$\sigma_g = \frac{FWHM}{2\sqrt{2\ln(2)}},\ \sigma_c = \frac{FWHM}{2}$$

$$\int_{-\infty}^{+\infty} F(t)dt = 1$$

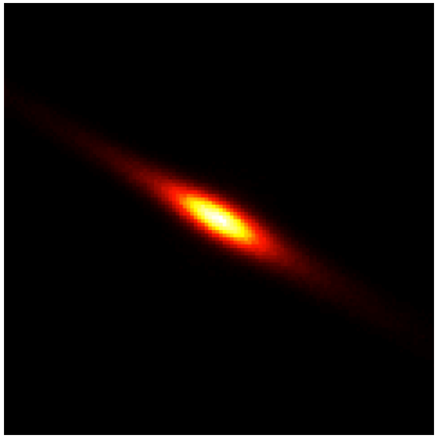
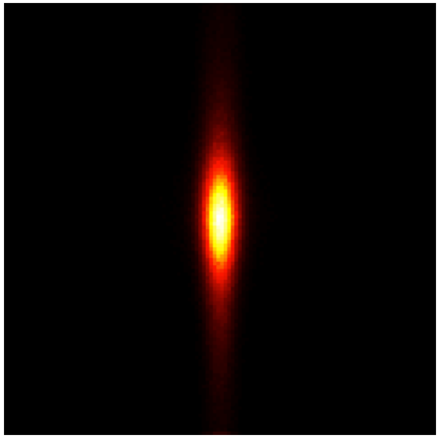
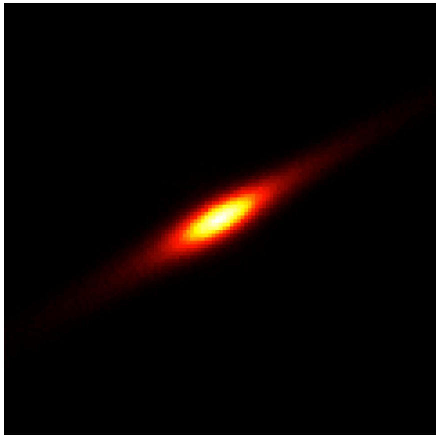
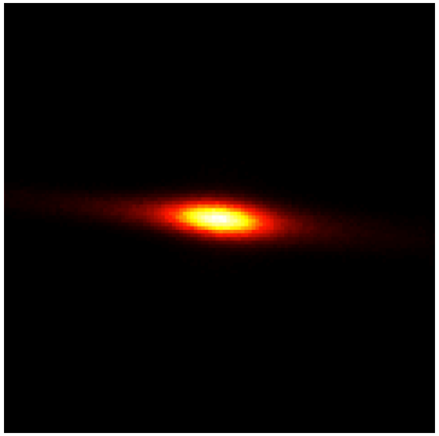
- The 2-D PSF is given by the product P(x,y) = F(x)F(y) - Cosine document CR-SPOEQU-ME60, G. Vacanti, 2020-07-10
- The parameters fitted to the ray traced module PSF, in FWHM, in alpha, out FWHM and out alpha are given as a function of row in the table above.

Single Module PSF



- In-plane width (y axis on plot) is larger than out-of-plane width (x axis on plot) because the reflections are at grazing incidence
- In-plane spread caused by axial figure gradient errors
- Out-of-plane spread caused by azimuthal figure gradient (circularity) errors
- Both in-plane and out-of-plane figure gradient errors have been modelled using a pseudo-Voigt distribution to match the module PSF fit produced by Cosine
- The PSF plotted is for the module in row 8 at position angle 90 degrees

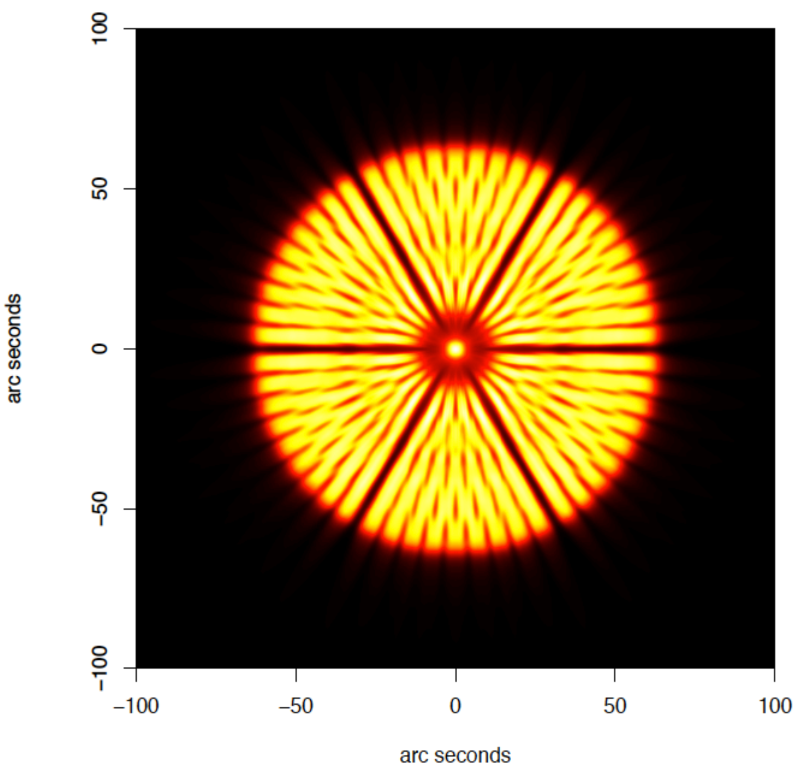
Module PSF Distributions Around Row

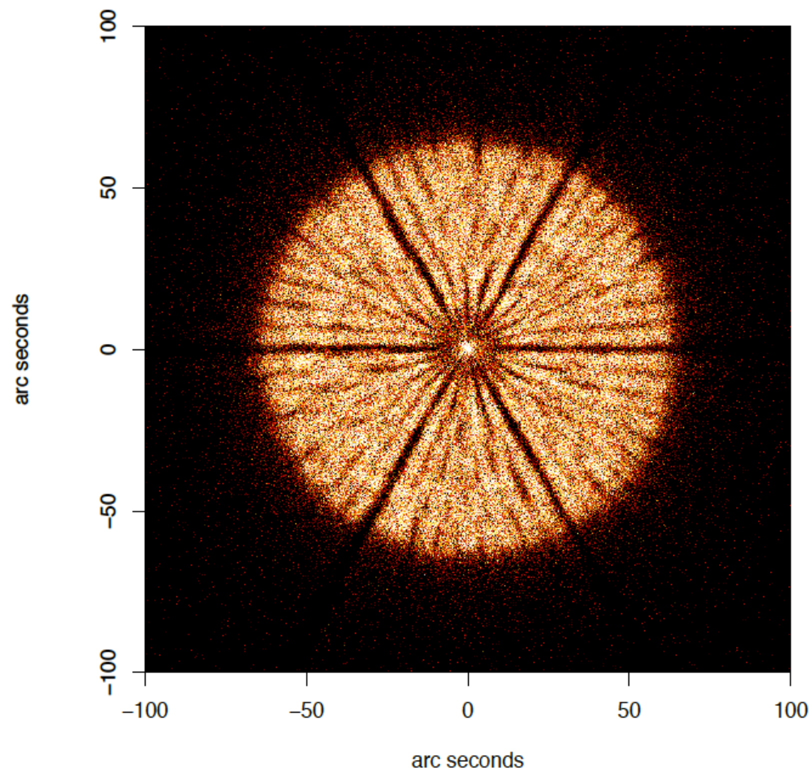





- The in-plane and out-of-plane gradient errors are modelled using a pseudo-Voigt distribution
- Every module has different PSF width and index parameters
- The position angle of the module PSF distribution depends on the azimuthal position of the module in the full mirror aperture

Defocused PSF 1 keV

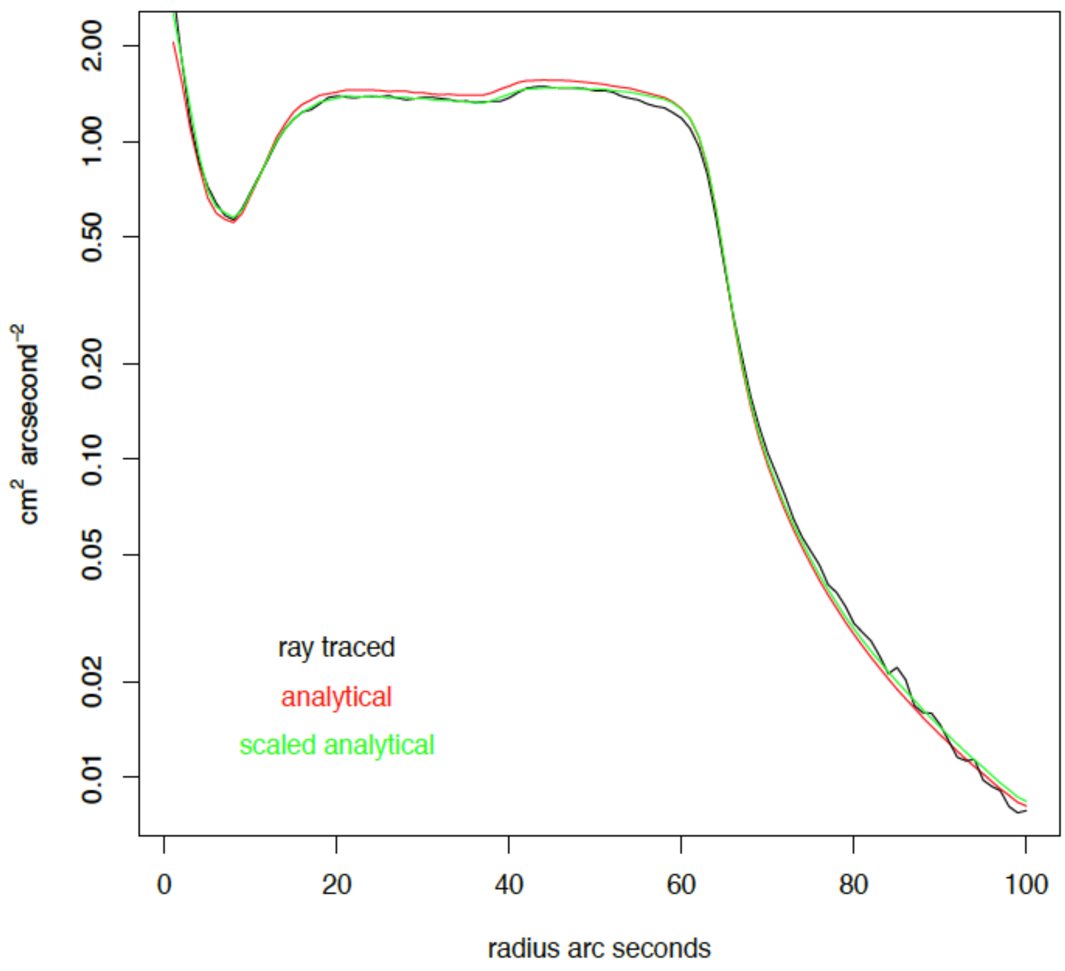
- Detector 35 mm forward of focal plane – pixel size 0.25 arc seconds
- The bright central spot is produced by the overlapping scattering wings from the individual module PSFs
- All detail in analytical PSF (left) is real
- The Monte Carlo noise in the ray tracing rendering (right) masks the detail





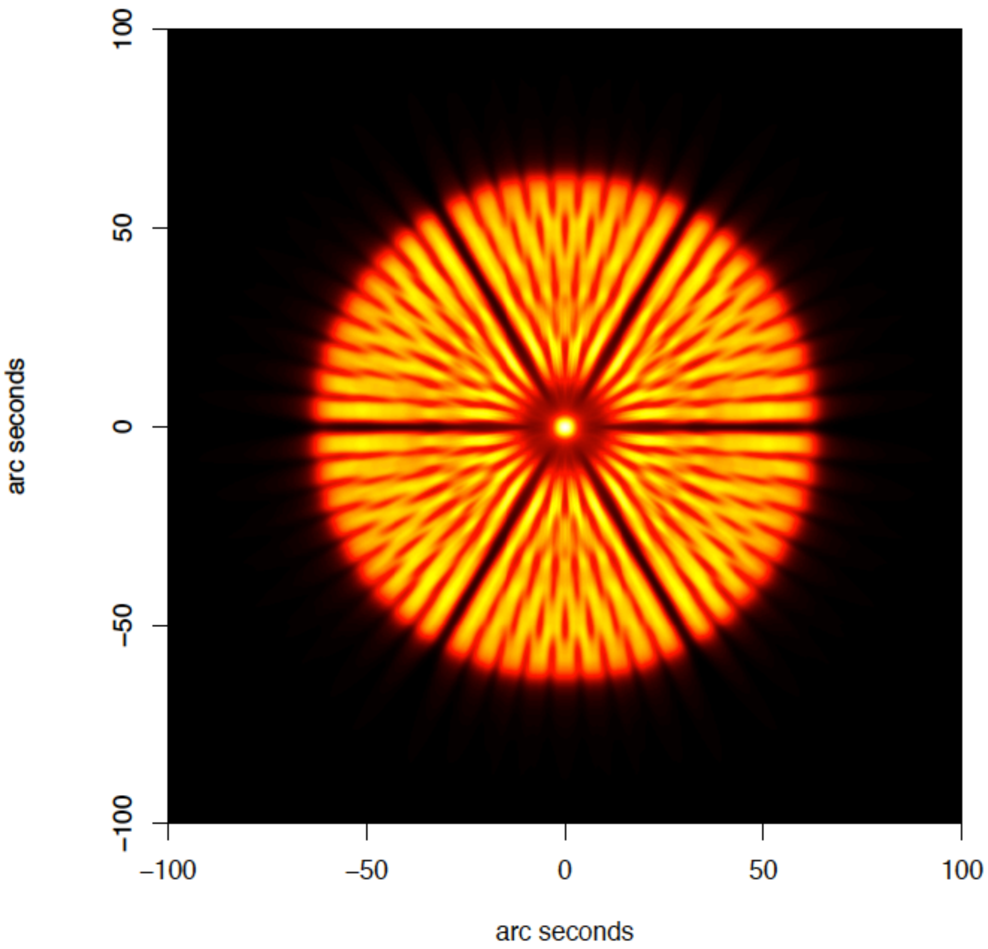
Radial Distribution 1 keV

- Compare the radial surface brightness of the analytical (red) and ray traced (black)
- At 1 keV the analytical and ray traced radial distributions are very close because the X-ray scattering fraction is small (a few percent)
- The bright central spot and the scattering wings at large radii are slightly higher in the ray traced PSF
- Scale the analytical surface brightness as a function of radius to produce the scaled version (green)



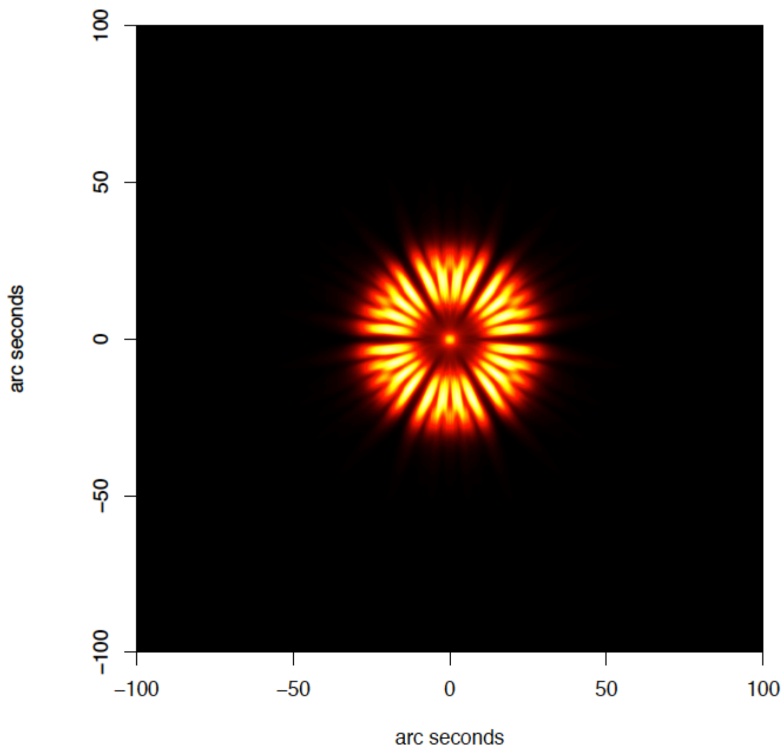
Scaled Analytical PSF 1 keV

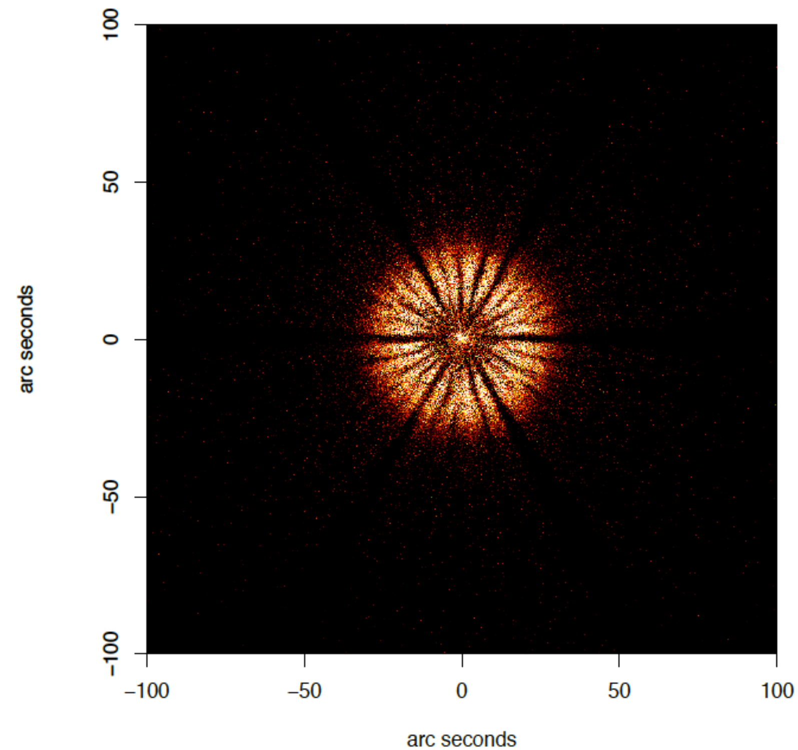
- The analytical PSF scaled to match the radial surface brightness of the ray traced rendering
- All the real detail of the analytical PSF is retained
- The brightness of the central spot and scattering wings at large radii now include the influence of X-ray scattering and the module alignment errors that are included in the ray tracing



Defocused PSF 7 keV

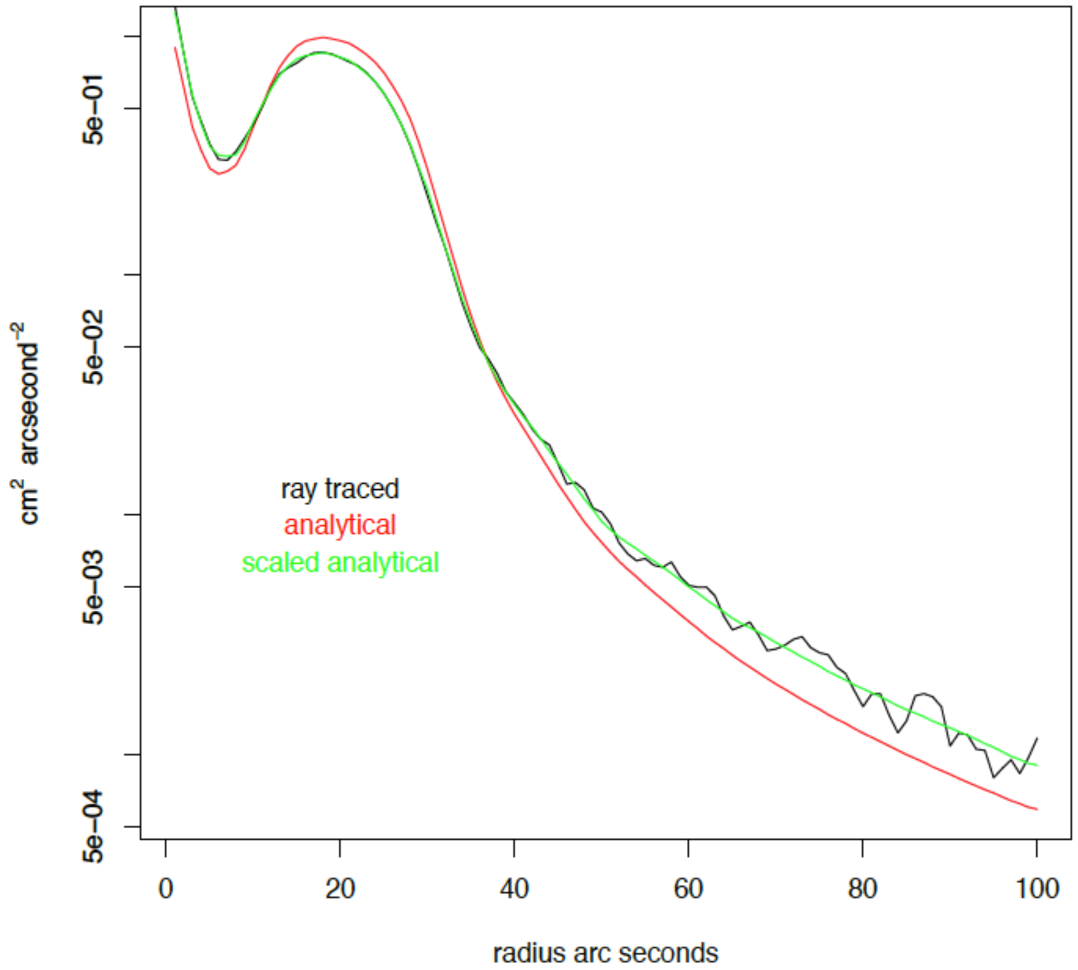
- Detector 35 mm forward of focal plane – pixel size 0.25 arc seconds
- The bright central spot is produced by the overlapping scattering wings from the individual module PSFs
- All detail in analytical PSF (left) is real
- The Monte Carlo noise in the ray tracing rendering (right) masks the detail





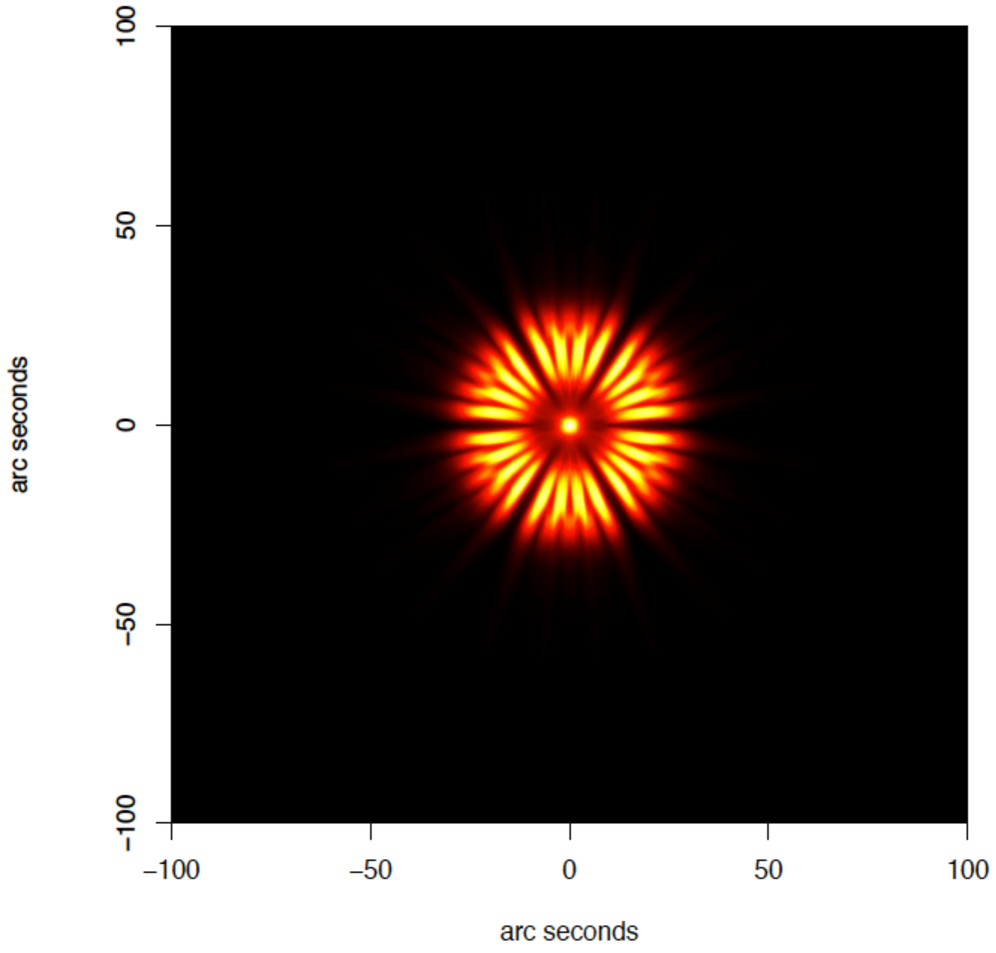
Radial Distribution 7 keV

- Compare the radial surface brightness of the analytical (red) and ray traced (black)
- At 7 keV the analytical and ray traced radial distributions are not so close because the X-ray scattering fraction is larger
- The bright central spot and the scattering wings at large radii are higher in the ray traced PSF
- Scale the analytical surface brightness as a function of radius to produce the scaled version (green)

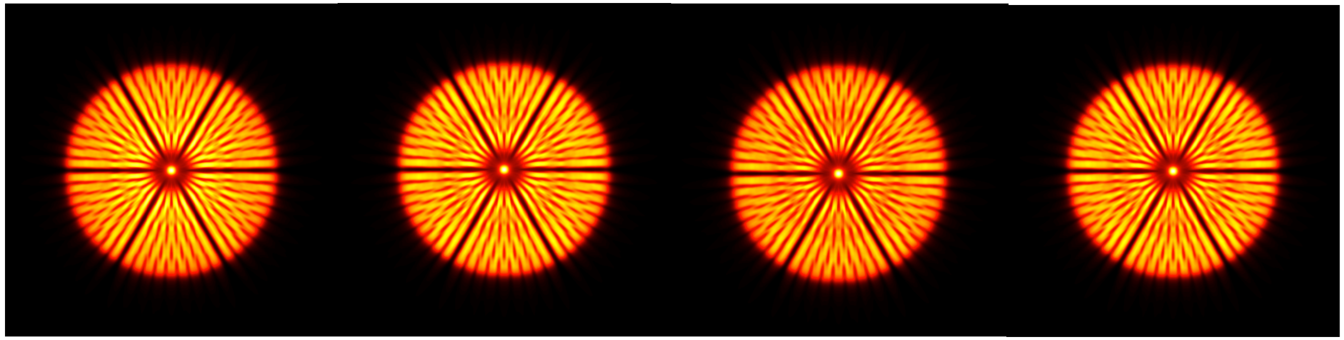


Scaled Analytical PSF 7 keV

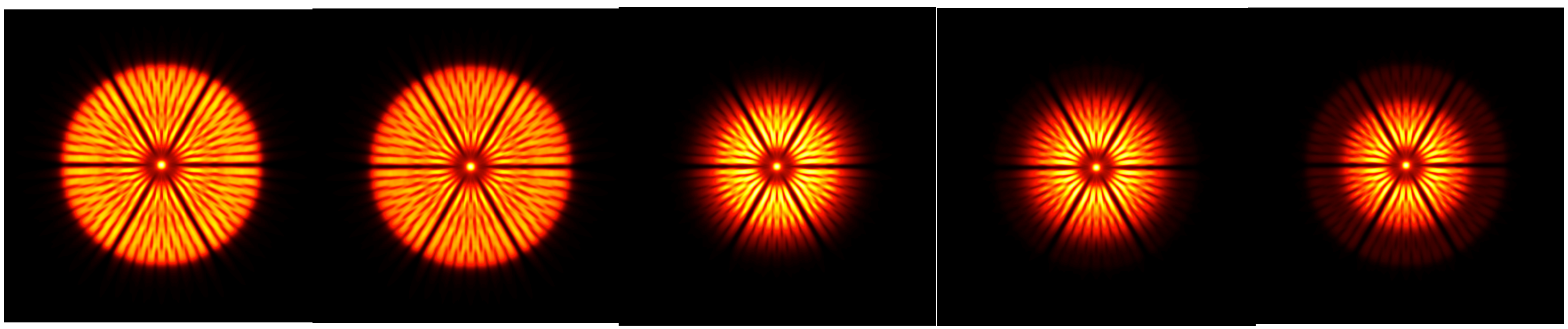
- The analytical PSF scaled to match the radial surface brightness of the ray traced rendering
- All the real detail of the analytical PSF is retained
- The brightness of the central spot and scattering wings at large radii now include the influence of X-ray scattering and the module alignment errors that are included in the ray tracing



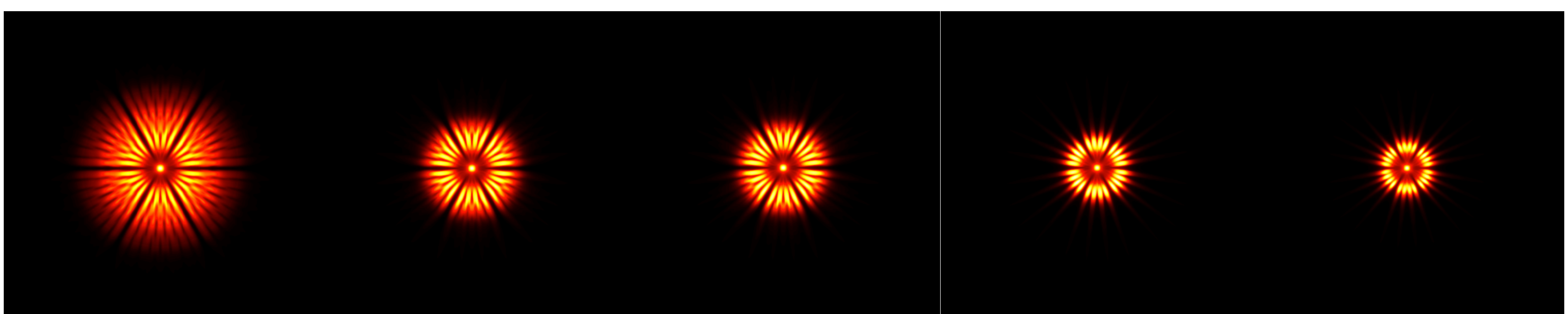
Scaled Analytical vs. Energy



0.2 keV 0.35 keV 0.75 keV 1 keV



1.25 keV 1.5 keV 2 keV 2.5 keV 3 keV



4 keV 6.5 keV 7 keV 10 keV 12.5 keV