Global, fully multi-fluid simulations of radially stratified protoplanetary disks

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The People

- Donna Lee (DIAS/TCD) (grad student)
- Dr Wayne O'Keeffe (former grad student)

The Organisations

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- PRACE
- Irish Centre for High End Computing

Accretion Disks are Awkward

Credit: J. Bally (University of Colorado) and H. Throop (SWRI)

- Accretion disks:
  - “Small”
  - Weakly ionized
  - Possibly turbulent

Weakly ionized systems

Initial considerations:

1. Want to use the continuum approximation
2. Take account of differing motions between neutrals and charged species
3. Don't want to solve the Poisson equation

Approximations:

1. The velocity of the fluid as a whole is the velocity of the neutrals
2. For charged species, collisions with neutrals dominate
3. Inertia of the charged species is negligible

Generalised Ohm's Law

- Generalised Ohm's law (in principle) removes requirement for Poisson equation
- Derive this from the N momentum equations for charged species

$$\alpha_i \rho_i (\mathbf{E} + \frac{1}{c}\mathbf{v}_i \times \mathbf{B}) + \mathbf{f}_{i1} = 0, \tag{1}$$

$$f_{ij} = \rho_i \rho_j K_{ij} (\mathbf{v}_j - \mathbf{v}_i). \tag{2}$$

The multifluid MHD equations

The equations for our (isothermal) weakly ionized system are then

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = 0, \quad (3)$$

$$\frac{\partial \rho_1 \mathbf{v}_1}{\partial t} + \nabla \cdot \left(\rho_1 \mathbf{v}_1 \mathbf{v}_1 + a^2 \rho_1 \mathbf{I}\right) = \mathbf{J} \times \mathbf{B}, \quad (4)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{v}_1 \mathbf{B} - \mathbf{B} \mathbf{v}_1) = \nabla \times \left(r_0 \frac{(\mathbf{J} \cdot \mathbf{B})\mathbf{B}}{B^2} - r_1 \frac{\mathbf{J} \times \mathbf{B}}{B} \quad (5)\right.$$

$$\left. + r_2 \frac{\mathbf{B} \times (\mathbf{J} \times \mathbf{B})}{B^2} \right), \quad (6)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (7)$$

$$\nabla \times \mathbf{B} = \mathbf{J}. \quad (8)$$

Numerical set-up

Aim

- O'Keeffe & Downes (2014) published first fully multifluid sims
- Extend this study to investigate impact of radially varying parameters
- Investigate appropriate canonical time-scales of the problem
  - Is the orbital time, or growth time of the MRI, always appropriate?
  - What are turbulent time-scales?

Focus here on time-scales

Numerical set-up

- (Quasi-)Global simulations
- Cartesian grid
- Weakly ionised multifluid approximation (3 fluids)
- Wavekilling boundaries
- Radially stratified ionisation and density

Density distribution

- Neutral Density:

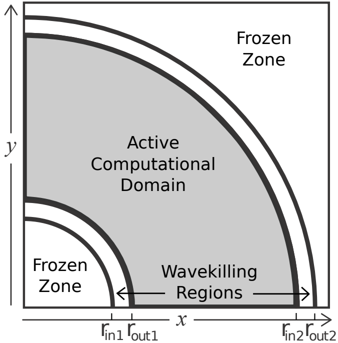
$$\rho_{\rm n}(r) = \begin{cases} \frac{\rho_0}{(0.8r_{\rm in})^3} & \text{if } r \leq 0.8r_{\rm in}, \\ \frac{\rho_0}{r^3} & \text{otherwise} \end{cases}$$

  where $\rho_0 = 2.33 \times 10^{-10}\,\mathrm{g\,cm^{-3}}$.
- Ionisation fraction quadratic in r, fitted to match Salmeron & Wardle (2003)

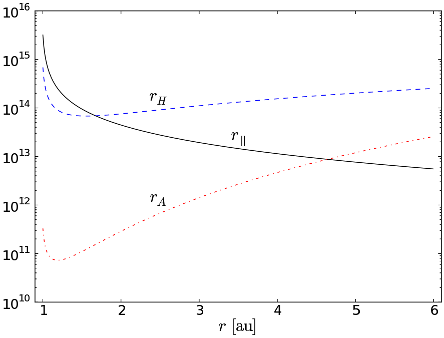
The other stuff

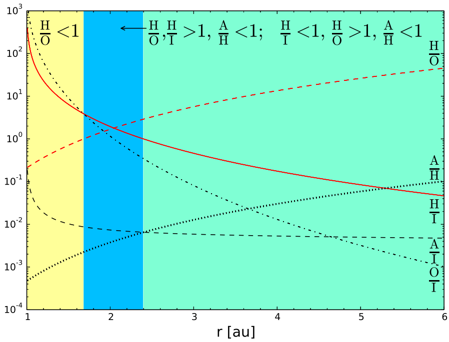
- Initial temperature 280 K
- Net initial magnetic flux of 100 mG
- Radial range: 1 AU - 6 AU
- Resolution $512 \times 512 \times 64$
- Well resolved for MRI (Hawley 2013).

Schematic of set-up

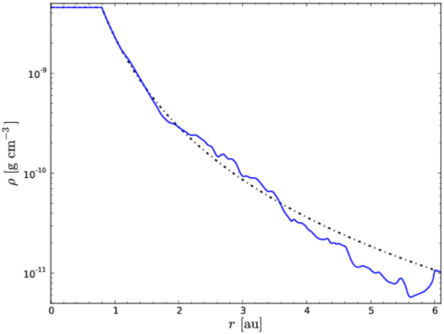


Multifluid effects



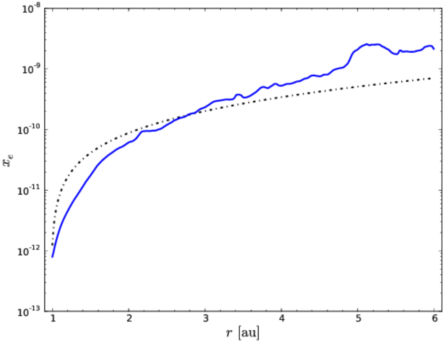


Neutral Density evolution



Initial and final neutral densities

Ionisation fraction evolution



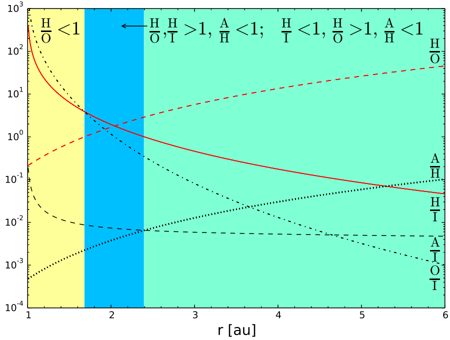
Relevant Time-scales

- Typically take the orbital period as representative time-scale
- Often combine with Alfvén speed to get length-scale
- Ignores turbulent cascade, Whistler waves

Two other possibilities

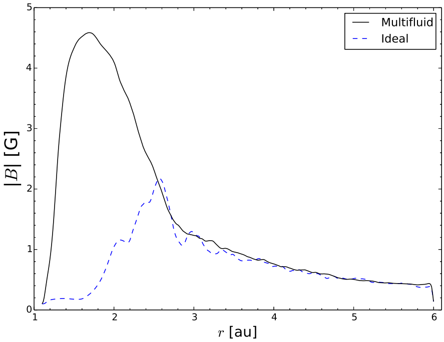
- Eddy turn-over time
  - $t_l \sim \dfrac{l}{v(l)} \sim l^{(3-\alpha)/2}$
- Mean/median turbulent speed with ionisation profile length-scale
  - Typical MRI velocity $\sim 10^4\,\mathrm{cm\,s^{-1}}$
  - At 2 AU equilibrium ionisation length-scale $\leq$ 1 AU (Salmeron & Wardle 2003)
  - Time-scale: $t_R \sim 10^9$ s, $t_z \ll t_R$ (e.g. Lesur et al 2014)
  - If ionisation varies rapidly then time-scale can be less than orbital period

Resistivities



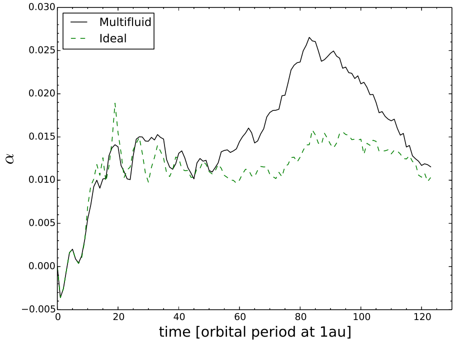
Evidence of strongly varying resistivities

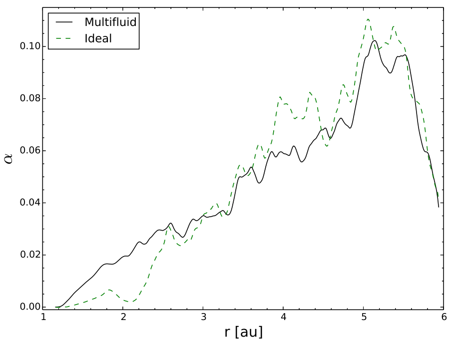
Magnetic field evolution



Multifluid effects particularly significant at low r

Anomalous viscosity





Conclusions

- Resistivities can be strongly varying
- Hall dominated region can be large (no need to dominate induction term to have impact)
- Magnetic field in ideal MHD does not approximate multifluid structure well
- Correct time-scale for modelling may not be the orbital one
  - Radial time-scale may be short
  - Vertical time-scale may be much shorter

Conclusions

Turbulence is hard.

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