The Hipparcos and Tycho Catalogues

The Hipparcos and Tycho Catalogues

Astrometric and Photometric Star Catalogues

derived from the

ESA Hipparcos Space Astrometry Mission

A Collaboration Between the European Space Agency

and

the FAST, NDAC, TDAC and INCA Consortia

and the Hipparcos Industrial Consortium led by

Matra Marconi Space and Alenia Spazio

> European Space Agency Agence spatiale européenne

Cover illustration: an impression of selected stars in their true positions around the Sun, as determined by Hipparcos, and viewed from a distant vantage point. Inset: sky map of the differences in parallax between the final NDAC and FAST sphere solutions, in equatorial coordinates.

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Volume 3

Construction of the Hipparcos Catalogue

Compiled by:

F. van Leeuwen, L. Lindegren & F. Mignard

with the support of

members of the NDAC and FAST Data Reduction Consortia

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Foreword

In March 1980, the Hipparcos mission was accepted within the scientific programme of the European Space Agency. It was approved on the basis of performance analyses predicting a standard error in parallax, at visual magnitude 9, of about 2.0 milliarcsec (mas), assuming an observing programme of 100 000 stars. The standard errors actually achieved are about 40 per cent smaller than these predictions, and the programme includes nearly 20 per cent more stars, most of them selected on astrophysical grounds; moreover, a wealth of accurate photometric data, variability and multiplicity information has been extracted, which was not anticipated in the original project. The Tycho experiment, resulting in a separate astrometric and photometric catalogue of over one million stars, was also totally unforeseen in 1980. Thus, it is no exaggeration to claim that the Hipparcos mission has achieved its original goals, and much more.

The planning and execution of the data reductions for Hipparcos required an immense and concerted effort from the astronomical community, bringing together expertise not only in many areas of astronomy but also in mechanics, numerical methods, geodesy, and related fields. The reduction task was unusual among astronomical satellite projects in the sense that it was an entity that could not be subdivided: no small subset of stars could be reduced separately. It was therefore necessary to regard the data reductions as an integral part of the project, which thus logically ends with the present publication of the Hipparcos and Tycho Catalogues.

Even before acceptance of the mission in 1980, there had been two independent groups of scientists planning to reduce all the observations of the satellite and produce an astrometric catalogue. When, in 1981, ESA issued an Announcement of Opportunity to participate in the processing of the scientific data, the two groups consolidated into the present data reduction consortia-Fundamental Astronomy by Space Techniques, under the leadership of J. Kovalevsky, and Northern Data Analysis Consortium, originally led by E. Høg and, from 1990, by L. Lindegren. These groups were subsequently entrusted with the task of producing a single output catalogue under the supervision of the Hipparcos Science Team. Between 1981 and 1989 the consortia developed independent software for the comprehensive simulation and reductions of the satellite data. Numerous comparisons were made between the partial reductions of simulated data, from which errors in the mathematics and algorithms were identified and corrected. Such comparisons continued during the mission, now using the real observations. Finally, two catalogues were produced that differed only slightly, and a rigorous method was developed to combine them into a single, agreed Hipparcos Catalogue-the only one that is published.

The point of making two independent reductions was not obvious to everybody certainly judging by the many times we were asked what we would do if the two catalogues turned out to be different! Our point was simply that any significant difference in the results must be due to some error in the method or software, and that such errors should then be found and corrected. In retrospect, it was an extremely good idea to duplicate the main reductions. Not only did this eliminate many errors that might otherwise have gone unnoticed, but it was also found that the combined catalogue was superior to either of the consortia catalogues in terms of accuracy and reliability, for reasons which could be understood (and which are explained in this volume). he full-scale scientific exi

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The full-scale scientific exploitation of the Hipparcos mission can now begin. Some users will perhaps at first be confused by the wealth of information, the rich diversity of results, the intricate relationships between the different parts of the catalogue, and the sometimes very detailed descriptions of what the data represent. Indeed, the Hipparcos Catalogue is vastly more complex than any previous astrometric catalogue. Apart from 'traditional' astrometric data—positions, proper motions, and parallaxes—the catalogue provides accurate photometric results, light curves and variability analysis, detailed information on resolved double and multiple stars, astrometric binaries, minor planets, etc. The complexity of the catalogue reflects the non-trivial nature of celestial objects revealed by an instrument of pioneering excellence.

The full complexity of the data analysis, in particular the multiple inter-relationships of the various results, was not fully appreciated before launch. It demanded great flexibility and ingenuity within the data analysis teams to cope successfully with this complexity, with the additional complications brought by the unforeseen satellite orbit, and to converge towards a single goal in a very short space of time. The linking of the combined catalogue to the extragalactic reference frame, making the Hipparcos proper motions inertial and enabling the positions to be compared immediately with radio catalogues, was another example of an immensely successful collaboration involving many more institutes throughout the world and the completion of a very difficult task according to a tight schedule. Finally, the preparation of the results, their verification and presentation on various media—printed volumes, machine-readable files, *Celestia 2000*—was itself a formidable task.

A huge effort was thus invested in making these results accessible to the scientific community, and to do so in a form preserving as much as practicable of the scientifically useful information hidden in the raw data stream. However, the extraction of this information was necessarily based on certain assumptions, e.g. that stars generally move in straight lines through space. When these assumptions were clearly contradicted by the data, alternative assumptions (models) had to be used, thus, for example, uniformly accelerated motion or orbital motion. The division between different models is always a matter of compromise between random and systematic errors, and thus to some extent arbitrary. Similar considerations applied to all aspects of the processing, for instance regarding the choice between constant and variable models in the photometric reductions. The additional information in the Epoch Photometry Annex, the Intermediate Astrometric Data, and the Transit Data, is provided partly with a view to allow these considerations to be re-assessed by the users of the catalogue.

The published Hipparcos Catalogue represents the reduction consortia's interpretation of the satellite data in terms of a certain range of models and criteria for selecting between them. For most astrophysical applications it can be taken for granted that the interpretation is reasonable and adequate. In other cases the user may wish to understand precisely what has been done, why it was done in that way, and how these choices are reflected in the final data. The present volume is intended to provide an account of the reductions which is by no means complete, but sufficient to permit a detailed understanding of the properties of the catalogue.

- J. Kovalevsky, FAST Consortium Leader
- L. Lindegren, NDAC Consortium Leader
- E. Høg, former NDAC Consortium Leader

1. INTRODUCTION

The Hipparcos data reductions were the responsibility of two scientific consortia, FAST and NDAC, supervised by the ESA-appointed Hipparcos Science Team. Two other consortia, TDAC and INCA, were responsible for the Tycho data reductions and the preparation of the Hipparcos Input Catalogue respectively. In this chapter the motivation for presenting descriptions of the various processes employed in the data reduction by FAST and NDAC is outlined. A general overview of the data reductions is provided, with references to specific chapters where more details can be found, and a summary of various other aspects is presented, such as the preparation of the Hipparcos Input Catalogue and the role of comparison activities.

1.1. The Purpose of this Volume

In addition to the tasks of production and description of the final Hipparcos and Tycho Catalogues, the Hipparcos Science Team placed considerable importance on the full documentation of the Hipparcos satellite operations (Volume 2), and in a detailed description of the procedures used to reduce, calibrate and verify the data contained in the final Hipparcos and Tycho Catalogues.

Several reasons motivated the preparation of this documentation. In the first instance, the scientific method demands a careful and thorough explanation of the steps involved in any scientific experiment, and in this respect the Hipparcos mission is no exception. For many catalogue users, the precise methods adopted for the data analysis will be of little interest, but for certain applications, a careful understanding of the data reduction steps, the instrument calibration, the reduction algorithms, and the associated assumptions and numerical constants, will be relevant in assessing the limitations of the astrometric, photometric, and associated data presented in the catalogues. Similarly, the steps that have been undertaken to place the resulting catalogues on an extragalactic reference system, and to verify the quality of the resulting data, is important information that must be preserved for future catalogue users.

Second, the compilation of the adopted methods, assumptions, and complications of the data analysis was considered as an important contribution to a future astrometric space mission, where target accuracies of microarcseconds have already been proposed. This volume should provide many pointers to the difficulties, and possible solutions, to be faced by such a mission in the future.

At the same time, the documentation should serve to illustrate the intricate complexities of achieving milliarcsec astrometry, and may therefore more easily illustrate the profound and dedicated commitment and considerable scientific involvement which has been invested in the Hipparcos project, and the challenges faced in bringing the largest data analysis problem ever undertaken in astronomy to a rapid and successful conclusion.

1.2. Pre-Launch Preparations

As described in the Prologue to Volume 2, the first ideas for carrying out astrometric measurements from space were presented in 1966. Lengthy and careful studies resulted in an ESA Phase A study report, on which selection of the Hipparcos mission as a programme within ESA's scientific programme was finally based. Following this selection in 1980, organisation of the scientific aspects was discussed in detail, and in 1981 ESA, in consultation with the interested scientific community, issued two 'Announcements of Opportunity': the first for a scientific consortium willing to undertake the compilation of the Hipparcos Input Catalogue; the second, for one or more groups willing to undertake the main mission data analysis, leading to the construction of the final Hipparcos Catalogue. As reflected in the interest shown by the scientific community during the Phase A studies, one consortium (subsequently called the INCA Consortium, and led by Dr Catherine Turon of the Observatoire de Paris, Meudon) duly responded to the first announcement; two teams (the NDAC Consortium and the FAST Consortium) responded to the second. The FAST Consortium was led by Professor Jean Kovalevsky of the Observatoire de la Côte d'Azur, CERGA, Grasse, France. The NDAC Consortium was initially led by Professor Erik Høg of the Copenhagen University Observatory, Denmark. It was subsequently led by Dr Lennart Lindegren of the Lund Observatory, Sweden, following the inclusion of the Tycho experiment within the Hipparcos programme, and the consequent formation of the Tycho Data Analysis Consortium, led by Professor Høg.

From 1981 ESA organised a Hipparcos Science Team, under the chairmanship of the ESA Hipparcos Project Scientist, Dr Michael Perryman. The four scientific consortia (INCA, NDAC, FAST, and TDAC) thereafter worked autonomously under their respective consortia leaders, with the coordination of all of the scientific tasks being undertaken by the Hipparcos Science Team. The Science Team included representatives from each of the Consortia (including the leaders); its terms of reference were to supervise and take responsibility for all of the scientific aspects of the mission, including the definition of the entire satellite observing programme, monitoring and approval of the satellite's scientific performance, the preparation and testing of the data analysis software, the eventual creation of the final mission products including the production of a single agreed-upon final catalogue, and the overall policy for preliminary and final data distribution.

Preparations leading up to the satellite launch were described in the three volume ESA SP-1111 (1989) 'The Hipparcos Mission': Volume 1 dealing with the Hipparcos satellite, Volume 2 dealing with the preparation of the Hipparcos Input Catalogue, and Volume 3 dealing with the preparations for the data analysis.

Following termination of the satellite observations in August 1993, after the satellite had been in orbit for just four years, and with the completion of the final Hipparcos and Tycho Catalogues announced by the Hipparcos Science Team on 8 August 1996, all of

the original scientific goals of the Hipparcos mission had been met, and indeed in all cases, significantly exceeded. More target stars, a higher astrometric accuracy, and a substantial photometric data base have been realised. The original cost envelope for the mission was exceeded by less than 15 per cent, a cost over-run largely attributable to the one-year launch delay imposed by the Ariane launcher programme. The complex data analysis system—the global treatment of 1000 Gigabits of data was considered as the largest single data reduction problem undertaken in astronomy to date—was completed according to the originally foreseen time schedule announced before launch for the main Hipparcos Catalogue, and one year in advance of the pre-launch expectations for the Tycho Catalogue.

These achievements may be attributed to a variety of factors and important organisational aspects:

(a) a clear set of scientific goals was established by the scientific community, and endorsed by the ESA advisory bodies at the time of the project's selection by ESA in 1980. These were considered as inflexible by the ESA Project Team and, in turn, by industry. Specifications were established at the highest level—thus, a mean sky accuracy in the five astrometric parameters at 9 mag of 2 mas was demanded—as well as at all intermediate levels. With the scientific importance of the mission critically dependent on the final accuracy, the spirit prevalent within the entire project was that 2 mas was the requirement, anything worse was unacceptable;

(b) many of the intermediate specifications were formulated based upon extensive simulations and studies already carried out during the Phase A study of the mission, many of them relying critically on the studies carried out by the scientists who would eventually take responsibility for the satellite data analysis;

(c) responsibility for all of the scientific aspects was taken by a single committee, the Hipparcos Science Team, a non-political group committed to the mission goals and hence its scientific success. All other bodies involved in the scientific aspects—the scientific proposal selection committee, the four scientific consortia, and a variety of working groups, all reported directly to this Science Team. This organisation is shown schematically in Figure 1.1. The Hipparcos Science Team was in turn, responsible for all scientific decisions during the satellite development phase, for overseeing the timely preparation of the observing programme catalogue and the data analysis software, and for controlling all other interfaces with ESA and ESOC having a potential impact on the scientific conduct. The majority of the members of the Science Team were involved with the Hipparcos project as their primary research effort during a period of about 16 years since formal approval of the project by ESA;

(d) members of the Hipparcos Science Team were closely involved in project decisions which affected any aspect of the scientific performances, in formal project reviews, and also as direct consultants to industry during the satellite's detailed definition phase, assisting the prime contractor in its interpretation and implementation of the ESA project specifications;

(e) all of the scientific aspects of the Hipparcos mission, apart from the overall scientific coordination of the project led by the ESA Project Scientist, were entrusted to the scientific community, under their responsibility and financial authority, although with the Hipparcos Science Team coordinating their activities and schedule at the highest level;



Figure 1.1. The organisation of the scientific aspects of the Hipparcos mission. The Hipparcos Science Team was responsible for the scientific aspects of the ESA/ESOC and industrial development efforts (left-hand boxes), the overall coordination and synchronisation of the scientific consortia activities (top boxes), the comparison activities between the parallel reduction of the main mission data (right-hand boxes), and the coordination of the working group activities devoted to the unification of specific results generated as part of the data analysis (bottom boxes).

(f) in turn, ESA took financial responsibility for the entire satellite (spacecraft and payload), and entrusted its development, manufacture, testing and calibration (on-ground and in-orbit) to the industrial prime contractor. The overall system approach to the satellite as a single entity, adopted by ESA and the prime contractor (Matra, France, subsequently Matra Marconi Space)—including error analysis and allocation, and procurement, integration, verification and calibration of the payload—was a substantial and crucial factor contributing to the eventual success of the mission;

(g) the parallel development of the satellite, the observing programme, and the software and management system for the on-ground data analysis, was crucial. Thus, the deadline for observing proposals terminated in 1982 (at a time when launch was scheduled for 1988) despite various suggestions to keep it undefined for longer. Careful optimisation of the observing programme, and its optimisation with respect to the satellite's operational and observational capabilities, occupied a team of 30 or so people—some working full time, and some part time—for 6 years. But as a result, the Hipparcos Input Catalogue and the associated observations of nearly 120 000 programme stars worked smoothly and flawlessly. In retrospect, the early deadline imposed on the observing proposals, allowing extensive and meticulous preparations of the Input Catalogue, was without doubt a correct decision;

(h) similarly, development of the software for the data analysis tasks started in the two main data reduction teams (the FAST and NDAC Consortia) in 1981, in parallel with the development of data simulation software. Consequently, not only was the software finalised and tested pre-launch, but very significant guidance was provided by both consortia, to ESA and to industry directly, in the area of satellite design and operation.

The efficiency of the consortia's preparations were evident from their results: even in spite of the post-launch problems, the first great-circle reductions were completed within a month or so after the start of the routine acquisition of data, and the first 'sphere solution' was reported just one year later;

(i) the data distribution system established by ESOC was prepared in parallel with the data reception software being developed by the consortia. This ensured that, when data first started flowing from the satellite—at 24 kbit/s—it could be received and treated almost immediately by the consortia;

(j) an 'Agreement', or Memorandum of Understanding, was drawn up at an early stage between ESA and the four scientific consortia involved in the project, setting out deliverables and schedules for all groups, and their respective 'rights' in terms of pre-release data. This included the agreement not to circulate, release, or publish preliminary data, or scientific results based on such preliminary data; this had the very beneficial effects of not propagating incorrect or misleading data into the literature, and not distracting the work of the catalogue finalisation by motivations to publish investigations into such preliminary data.

The accuracy analysis and error allocation budget for Hipparcos during the development phase was a highly complex activity, comprising diverse but inter-related aspects such as spacecraft attitude control and jitter, optical performance and stability, detector characteristics, spacecraft and payload thermal control, data rates, spacecraft and payload shielding (electromagnetic and particle/Cerenkov), straylight, satellite spin rate, scanning law, mission duration, and so on. Global missions like Hipparcos demand that target accuracies are met and, in turn, that a minimum operational lifetime is also achieved. Hipparcos was unusual amongst ESA missions in that the development of the spacecraft and payload was entrusted to a single prime contractor (rather than separate Principal Investigator groups providing the payload instruments).

All of this can be summarised by stating that a systems approach was adopted for Hipparcos, with all of the many complex tasks encountered in a satellite project viewed as part of the same system. A unique goal—the final catalogue, of the highest possible astrometric accuracy, precision and rigour—was also established early on as the final mission product; this ensured that the ultimate objectives of the mission were apparent to all, both inside and outside the project. The simple advisory and decision-making structure was efficient and successful, with a clear identification of responsibilities.

1.3. Preparation of the Observing Programme

A very challenging problem for Hipparcos was to identify the desired subset of programme stars (about 120000 could be accommodated) from amongst all those potentially observable (a few million down to about 12 mag). This required (a) an announcement of opportunity for observing proposals (500000 objects were eventually proposed for study); (b) scientific assessment and priority allocation by an *ad hoc* (independent) selection committee; (c) extensive mission simulations covering scientific and operational considerations; (d) a careful compromise between scientific desires and aspirations and technical capabilities (e.g. general requirements on the uniformity of the overall sky distribution of programme stars, and the inability to observe many faint stars in a small region of the sky); (e) an extensive, laborious, and complex programme for the compilation of the requisite *a priori* astrometric and photometric data.

The details of the preparation of the Hipparcos Input Catalogue, published as ESA SP-1136 in March 1992 (and subsequently on CD-ROM) have been described in ESA SP-1111 Volume 2, and some key aspects of the observing programme are summarised in Chapter 3.

1.4. Methodology and Organisation of the Data Analysis

The data analysis problem for Hipparcos was both global and complex, and is the subject of the remainder of this volume. Both of these aspects have influenced how the data analysis was undertaken, and how the final mission products have been made available. It was not considered possible, or appropriate, for example, to circulate widely preliminary astrometric data, for which the errors, both internal, external, or systematic had been neither confirmed nor qualified. Neither was it possible to circulate subsets of the raw data to Principal Investigators: the complex inter-relationship between the data acquired by the satellite throughout the mission was itself the key to the eventual determination of the astrometric parameters. The scientific community, many members of which had been eagerly anticipating the mission results for many years, had to wait patiently, and allow the data analysis teams to complete their work.

In practice, the Hipparcos reduction problem was broken down into a series of three 'steps': (1) solving for one-dimensional positions on a 'reference great circle'; (2) reconstructing the origins of these reference great circles; and (3) back-substitution of the one-dimensional coordinates within the reference great circle system in order to estimate the astrometric parameters. The overall flow of data through the data analysis chain is illustrated in Figure 1.2, and the details of this analysis are the subject of the remainder of this volume. A more detailed synopsis of the entire data reduction procedure is given in Chapter 4, which itself gives reference to details covered in subsequent chapters.

It may be noted that the sequential approach to the data analysis problem introduces approximations in the projections onto the reference great circles, and to an extent decouples the solution of the astrometric parameters from the problem of the satellite attitude determination. Truly global reduction algorithms for the Hipparcos data were studied; they could possibly lead to small improvements in the overall astrometric accuracies and the suppression of certain potential systematic errors, but were not adopted due to time, schedule and computer resource constraints. On the other hand the sequential approach also had one major advantage: that a comparison between the two parallel reduction schemes could be undertaken at numerous well-defined points, permitting errors to be identified and rectified before subjecting the results of that step of the processing to the next.

The treatment of error sources such as chromatic terms, timing errors, relativistic (metric) effects, orbit corrections and Earth ephemeris, secular acceleration, effects of double and multiple stars (including astrometric binaries), computational rounding errors, and so on, resulted in a complex analysis problem which required careful evaluation, and iteration, before the results could be considered final and free from systematic errors at the level of a few tenths of milliarcsec.



Figure 1.2. The organisation of the data processing. The main mission data (left-hand part) were analysed in parallel by the FAST and NDAC Consortia, with comparison activities leading to the establishment of a single agreed-upon Hipparcos Catalogue. The main mission data processing is the subject of this volume. The Tycho data processing (right-hand part) are the subject of Volume 4.

This considerable complexity in the data analysis motivated the original selection of two data analysis groups who would undertake the entire analysis tasks in parallel. This was a highly unusual feature of the mission. However, in brief, this approach proved to be a remarkably powerful solution to the problem of cross-verification, identification of software coding errors or incorrect comprehension of interface specifications, etc., as well as providing important information on the final data quality, and the possible contribution of modelling terms to the final accuracy estimations. Many errors or imperfections were rapidly identified in this way. It is difficult to overemphasize how important and successful this has been for Hipparcos. The power of this approach has been repeatedly stressed by the Hipparcos scientific consortia, and at the time of writing a similar approach has been proposed in the scientific management plan of the COBRAS/SAMBA (microwave background) mission.

The necessity of the two independent reductions may be qualified further. Aside from the fact that complex problems generally benefit from an independent approach to their solution, the nature of the Hipparcos data means that any future re-analysis of the raw satellite data seems highly unlikely. Confidence by the scientific community in the results of the processing is very important. Unlike many other types of astronomical observations, astrometric data have a crucial historical relevance: a new experiment with a more modern instrument cannot simply be expected to reproduce or confirm measurements that were made previously. One specific example may suffice: as of the time of writing, FK5 and Hipparcos proper motions have not been fully reconciled: one very likely explanation seems to be that the existence of (astrometric) binaries and the corresponding photocentric motion due to orbital effects means that proper motions measured at one epoch will not necessarily agree with, or will not necessarily be superficially consistent with, proper motions at another. All efforts to eliminate artificially induced random or systematic errors within the Hipparcos data have been made, and independent reductions of the satellite data, along with appropriate crossverifications, offered powerful possibilities of controlling such errors.

As evident from the introduction to Volume 1, the complexities of the data analysis demanded the formation of data analysis consortia comprising members and institutes throughout Europe contributing a range of knowledge, interest, and expertise. The geographical distribution of participating scientists involved its own complexities of management and coordination. Regular meetings took place between the members of each consortium, of the Hipparcos Science Team, and latterly between working groups involved in the preparation of the final mission products (see Figure 1.1). In the early 1980's, additional communication between participating scientists took place by normal mail or, in urgent cases, by telex. From the mid-1980's the widespread availability and efficiency of electronic mail revolutionised daily working practices. It is not possible to imagine the final mission products having been finalised so efficiently in the absence of electronic mail communications.

Nevertheless, the geographical distribution of participants posed problems for information flow. A centralised data processing institute would certainly have overcome some of these problems, advantages including (i) centralisation of expertise and improved possibilities for the exchange of ideas; (ii) ease of communications (even in the age of fast computer networks meetings are necessary, and the problem of defining and controlling interfaces of different tasks is complicated by geographical separation); (iii) centralisation of documentation and the consequent improvement in the exchange of information (the problem of keeping large numbers of individuals in many different institutes up to date with a large, rapidly moving project was a formidable one, and was absolutely crucial at all stages of the project); (iv) exchange of data (in the multi-step, sequential processing of the Hipparcos and Tycho data, large quantities of data had to be moved from institute to institute). In this approach the need for two independent reduction groups might have been relaxed, with critical steps perhaps being undertaken by two or more separate individuals or groups within the central institute.

On the other hand the disadvantages of a centralised institute would have been numerous, including: (i) the difficulties of attracting and retaining the necessary individuals to work away from their home institutes for prolonged periods of time; (ii) making this approach attractive to participating countries or institutes, both financially and intellectually. Although the European-wide distribution of the Hipparcos data analysis effort had its complications, the advantages of the large-scale collaboration of individuals and institutes with various competences at various stages of the project was indispensable.

1.5. Comparisons

As mentioned above, the sequential approach for the data reductions allowed the identification of a number of key steps at which rigorous comparisons of the intermediate data could be undertaken. These comparison activities had not been carefully specified in advance of launch, but grew up naturally as the processing advanced, with the comparison activities being undertaken by the individual(s) or institute(s) possessing the capabilities or resources necessary to carry out the work. A simplified diagram illustrating the main aspects of the comparison activities is shown in Figure 1.3.

All intermediate data were not compared. Rather, various data subsets, including 'difficult' great circles, were identified, and evaluation and analysis of these cases pursued until all features had been explained. It should also be noted that the eventual outcome of each comparison task was never complete agreement on the numerical values derived at each step: the independence of the parallel reduction groups was paramount, and so many different assumptions, numerical algorithms, approximations, divisions of data sets, etc., occurred such that this was not a realistic (or desirable) product of the comparison exercises. The main objective was to ensure that the outcomes of each step were consistent with their predicted errors, and with the models adopted for the data analysis.

The entire comparison exercise identified numerous errors, shortcomings, imperfections, and incorrect assumptions. Like the parallel data reductions themselves, it is difficult to overemphasise the importance of these tasks in achieving the final, agreedupon catalogue.

1.6. The Final Results Data Base and the Final Mission Products

Although the data analysis activities were undertaken within each consortium, the final mission results are a combination of data derived at numerous separate institutes. Thus the astrometric data, independently generated within the FAST and NDAC Consortia (at CNES/CERGA and Lund respectively) were combined into a single final catalogue



Figure 1.3. The organisation of the comparisons. Only the principle features of the main mission comparisons are indicated. The left and right columns indicate schematically the flow of data through the NDAC and FAST Consortia respectively, and the institute at which the corresponding software was developed (in the FAST Consortium, the main chain of the data analysis was entirely implemented at CNES, Toulouse). The central column indicates the location at which the corresponding comparison activities were carried out (ARI = Astronomisches Rechen-Institut, Heidelberg; Bologna = Università di Bologna; CERGA = Observatoire de la Côte d'Azur, CERGA; Copenhagen = Copenhagen University Observatory; Delft = Delft Geodetic Institute; Geneva = Observatoire de Genève; Lund = Lund Observatory; RGO = Royal Greenwich Observatory, Cambridge; Torino = Centro di Studi Sui Sistemi.)

at the Observatoire de Paris-Meudon. Photometric data were unified into a single photometric catalogue at the RGO, while corresponding light curves were produced at the Observatoire de Genève using periods determined there and at the Royal Greenwich Observatory. Double and multiple star parameters were derived within institutes in Italy, at CERGA, and at ARI (Heidelberg) for the FAST Consortium, and at Lund Observatory for the NDAC Consortium. The final Hipparcos Catalogue includes Tycho photometry generated at Tübingen and Strasbourg, and transformed photometric colour indices produced at the Observatoire de Genève. Each catalogue iteration produced intermediate astrometric catalogues which evolved in parameters and precision. The final astrometric data resulted from a rotation of the Hipparcos internal reference system to the ICRS, using a final prescription based on a substantial coordinated effort within the Hipparcos 'Reference Frame Working Group'. Details of the final stages of the Hipparcos Catalogue production are given in Chapters 16–18.

To keep track of these large data sets, and their updates, a central 'Hipparcos Results Data Base' was set up, during the mission operations, at SRON Utrecht, under the responsibility of Dr Hans Schrijver. Using the SYBASE data base system, intermediate and final astrometric and photometric data were compiled into this centralised data base, and critically examined for quality and consistency with all other available astrometric and photometric data, including ground-based results. The comprehensive centralised system maintained an account of the various updates, drawing together the various elements into a final single data base system.

Generation of the final mission products was based on this final results data base. Definition of the form, content, format, and inter-relationship of the final mission products was a task handled by the Documentation Working Group. Starting with the early results of the first iterations of the catalogues, the concept for these final mission products drew together the parallel evolution of the Hipparcos and Tycho results, resulting in a comprehensive series of mission products which aims to be fully interconsistent and properly documented. Converging to this series of final mission products, in an agreed format, was a substantial effort which occupied the members of the Documentation Working Group for several years.

In parallel with the final catalogue production, considerable effort was devoted to the task of catalogue and data verification based, for example, on comparisons with the best-available catalogues of ground-based positions, proper motions, and parallaxes. The results of these verification activities are presented in Chapters 19–22.

1.7. Astrophysical Exploitation

With the Hipparcos programme of 120 000 stars, many of the target objects were known, in advance, as objects of astrophysical or astrometric 'interest'. In many cases their spectral types and/or multi-colour photometry, and details of their multiplicity or (coarse) photometric variability, were known. Metallicities, luminosity types, and many radial velocities were known or are in the process of being acquired as part of dedicated support programmes. Nevertheless, it must be pointed out that much of this 'auxiliary' material is of very inhomogeneous quality: when the final Hipparcos Catalogue is published, two-dimensional MK spectral types will be available for some 60 000 of the 120 000 programme stars; while radial velocities will only be available for some 20 000 of the programme stars (although many others have meanwhile been acquired by associated principal investigators).

The absence of radial velocities for the majority of the Hipparcos objects (let alone for the one million Tycho objects) is considered unfortunate—radial velocities provide the third space velocity component of the star, and high velocity accuracy can be achieved. The radial velocity is a very important supplementary piece of information for any kinematical or dynamical interpretation of the proper motion data. At the same time, repeated radial velocity measurements provide a powerful method of inferring and characterising double or multiple systems (and consequently, for mass determinations). And finally, radial velocities will be of significance in the assessment of secular (perspective) acceleration, the contribution to the apparent photocentric motion due to the (apparent) time-dependent proper motion, an effect which will attain increasing significance with improved astrometric measurements in the future.

Efforts were made by the Hipparcos Science Team to coordinate the acquisition and inclusion of the radial velocities within the final Hipparcos Catalogue. Unfortunately, this (ground-based) aspect was never incorporated within ESA's scientific mandate

for the mission. Formal and less formal attempts to acquire, compile, or support independent national efforts to acquire these data were also largely unsuccessful; it proved difficult for the Hipparcos project to present a convincing case to relevant funding authorities. Nevertheless, it must be concluded that for any future astrometric mission, a parallel effort directed at the acquisition of complementary photometric, spectroscopic, or radial velocity data should be considered very carefully, in order to provide the homogeneous observational data necessary for a complete astrophysical exploitation of the resulting astrometric data.

1.8. Data 'Rights' and Related Issues

The question of data rights, publication policies, and early release of data, are complex issues which face the conduct of any space mission and, of course, all scientific experiments conducted as large collaborations. Much energy is devoted to these issues, for which there is rarely a clear-cut right or wrong answer.

The Hipparcos Science Team debated this question at an early stage. The earliest thoughts were directed at the release of preliminary astrometric data two or three years into the mission. As the complexities of the real data analysis became apparent, and the huge effort that had to be devoted to the preparation and documentation of the results became evident, the Hipparcos Science Team realised the dangers of this approach. Preparing the data for release, even in preliminary form, would have taken critical effort away from the principal task at hand—that of completing the final catalogue as carefully and rapidly as possible. More importantly, it was considered that it would undoubtedly have led to great confusion (and criticism) of the results from users unfamiliar with the details of the Hipparcos project. Before the final iterations the errors were poorly categorised, and the coupling of errors between parallaxes, proper motions, and double/multiple stars would have created many problems at the level of the scientific interpretation; furthermore the positions and proper motions would not have been on any well-defined reference system. For an experiment aiming at high-precision astrometry, these shortcomings would have been unacceptable. The Hipparcos Science Team considered that the benefits of releasing only final results convincingly outweighed the prospects of distributing preliminary data. It is to be hoped that any such perception of 'delays' will be considered appropriately in an historical context.

M.A.C. Perryman

2. MISSION OPERATIONS TIME-LINE

This chapter provides a summary of activities and conditions that influenced the quantity and quality of the data return, and shows them in the form of a mission operations time-line. Due to its orbital problems, the mission operated under far from ideal conditions, and was subject to excessive radiation which ultimately destroyed vital parts of the electronic hardware and brought the mission to an end. In order to overcome the worst of these conditions and to recover the mission in the best possible way, there was intense collaboration and exchange of calibration and other results between the data reduction consortia and the European Space Operations Centre in Darmstadt, Germany, coordinated by the European Space Research and Technology Centre in Noordwijk, The Netherlands. Most topics described in this chapter are dealt with more extensively in Volume 2 and various chapters of the current volume, most notably Chapter 8.

2.1. Introduction

The data quality and data return of the Hipparcos mission were affected by many different factors. Some led to improvements in the quantity and the quality of the data, such as the inclusion of additional ground stations and improved instrument calibrations; some were routine, such as refocusing, and gyro de-storage; while others were unwelcome side effects of the orbit Hipparcos was forced to use, such as large background variations, gyro failures, and interruptions of the real-time attitude determination near perigee. This chapter provides a summary of those events and their place on the time-line of the mission, as a general reference point for the many calibration results presented in this volume. In order to facilitate such comparisons, all figures in the current volume showing calibrated quantities over the length of the mission, are shown on the same horizontal scale as the summary figures in this chapter, such as the overall summary presented in Figure 2.1. The origin of the time scale is 1989 Jan 0.0 = JD 2 447 526.5.

2.2. Activities Leading to Improvements of the Data Quality

Over the entire mission length there was intensive collaboration between the reduction consortia, the input catalogue consortium, and the operations team at ESOC, aiming at improving the quality and quantity of the data return.



Figure 2.1. Summary of various events that affected the Hipparcos mission. Table 2.1 lists chronologically events that had direct bearing on the data reductions, and explains some of the symbols in the figure. The ground stations used during the Hipparcos mission were: O: Odenwald; P1, P2: Perth (two receiver dishes); K1, K2: Kourou (two receiver dishes); G: Goldstone.

Additional Ground Stations

The elliptical orbit Hipparcos occupied meant that contact with the satellite from the Odenwald ground station was only possible for limited amounts of time, leading to severe degradation of the mission. Additional ground stations were brought into operation at very short notice: first Perth, then Kourou, and later Goldstone. Of these, Odenwald and Perth were fully dedicated, while Kourou was also used during Ariane rocket launches and was no longer used when the Goldstone station became reliably operational. Goldstone also had other obligations, mainly towards the end of the mission, when the Kourou station was again used. Only the Kourou station could sometimes keep contact with the satellite during the perigee passage, but for almost all of the time there was no contact between satellite and ground station for 1 to 2 hours around perigee, and no observations were possible around that time. Two different receiver dishes were used in Perth and in Kourou (indicated in Figure 2.2 by P1, P2 and K1, K2 respectively). Figure 2.1 shows the use of the ground stations throughout the mission.

The final overall coverage is shown as part of Figure 2.4. Further details of the ground station commissioning can be found in Volume 2, Chapter 4. Chapter 8 in this volume presents the verification and calibration of the ground station delay times.

Start	End	Code	Event
328		I1	Correction of coil current calibration matrix
382		I2	Analogue mode anomaly detected
389		I3	Grid rotation calibration implemented
612		T1	Payload thermal control anomaly
632		T2	Change from mechanism drive electronics unit 1 to 2
592	596	а	Attitude lost, faulty attitude initialization
755		T3	Change from thermal control electronics unit 1 to 2
819		V	Spurious undervoltage
900		S	Gas tanks swapped
940		Н	Gyro 3 heater failure
1001	1004	b	Uplink command error
1013		I4	Anomalous image dissector tube voltages
1161	1163	с	Data lost due to tape fault
1285		T4	Thermal control electronics 2 anomaly
1285	1288	d	Non-z-gyro patch introduced
1315	1396	е	Suspension of data acquisitions
1453	1466	f	Gyro 2 anomaly, sun-pointing for recovery

Table 2.1. Summary of the main events.	. Start and end times are given in days from 1989 Jan 0.
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Instrument Calibration Upgrades

Various instrument calibrations were performed by ESOC and/or the data reduction consortia, some on a regular basis (the coil current calibration matrix, the main grid modulation coefficients), while others were produced as one-off inputs (grid rotation, basic angle, star mapper single-slit response functions). The latter type of calibrations all took place during the first few months of the mission, when the grid rotation and the proper pointing of the image dissector tube were established. By the end of January 1990 the instrument was known sufficiently well and data accumulation was no longer adversely affected by inadequate instrument descriptions. These calibrations are described in Chapter 5 of Volume 2.

Catalogue Updates

A major influence on the mission performance were the updates of the input catalogue. All updates were supplied by the input catalogue consortium, which acted on information supplied by the data reduction consortia as well as by ESOC. The first positional and photometric updates were obtained from the star mapper processing, later updates used preliminary sphere solution results. Large updates were confirmed by plate examinations before being implemented. These catalogue updates led to improved attitude convergence and pointing accuracy of the image dissector tube. Further details can be found in Volume 2, Chapter 8.

Hardware Calibrations

The calibration of the gyro orientations as supplied to ESOC by the NDAC consortium, provided a more accurate separation of gyro drift and therefore an improvement in the

implementation of the gyro data in the real-time attitude determination. This implementation took place in June 1991 and can be recognized in Figure 13.4 in Volume 2 from the discontinuity in the drift values. Further details on the gyros can be found in Chapter 8, and in Volume 2, Chapters 13 to 15.

The calibration of the thruster firing performance as described in Chapter 8 was supplied to ESOC, but required no adjustment of operational parameters.

2.3. Routine Operational Phase

Refocusing

A wide range of data checks were carried out by ESOC on a routine basis and are described in Volume 2, Chapter 10. These data checks provided information on the performance of the main detector in terms of modulation coefficients and total signal intensity. From this information was derived the requirement to refocus the instrument, which happened at the start of the mission about every four weeks, later in the mission at longer intervals. Refocusing affected all data reductions relying on the amplitude or the phase of the modulated signal: great circle reductions (Chapter 9), ac-photometry (Chapter 14), double star processing (Chapter 13) and optical transfer function calibration (Chapter 5). For this reason, most (but not all) refocusing took place before or after the collection of useful data, i.e. close to perigee. The refocusing times are indicated in Figure 2.1.

Occultations

The lengths of Earth occultations near perigee was much longer than was foreseen for the nominal mission and led at some times to a temporary loss of attitude. In order to limit the impact of the occultations, ESOC experimented with decreasing the time-window during which the shutters were closed. As a result, there was less attitude loss associated with occultations, but the price was an exponential increase in the background for the star mapper and the image dissector tube detectors shortly before closing the shutters. This was difficult to accommodate in the routine data reductions, and was dealt with afterwards for the main mission photometric data (see Chapter 14).

Background Levels

The unforeseen orbit of Hipparcos, and the coincidence of the mission with a period of high solar activity, led to large variations in the background signal of primarily the star mapper detectors, as shown in Figure 2.2. The main contributor to the variations were the high energy electrons in the van Allen belts, which were encountered at least 4 times a day. These background levels influenced badly the performance of the real-time attitude determination, as it made the detection of a large number of fainter reference star transits in the star mappers impossible, thus reducing the number of available reference points. Figure 2.2 shows the background as the equivalent of stellar magnitude, indicating that at times near perigee even most of the brightest stars were undetectable.



Figure 2.2. Star mapper background levels in the V_T channel at different orbital phases over the mission. The orbital phases were measured from apogee. The top graph also shows events of high solar activity (just below the level of 5 mag).

Thruster Firings

The cold-gas for the thruster firings was supplied from one of two gas-tanks. The pressure in those tanks was monitored, and in mid-June 1992 (day 900) the first tank was almost empty and swapped for the second tank (see Section 8.4 and Figure 8.9).

The thruster firing strategy was changed twice during the mission, using the experience obtained during the early parts of the mission. The first change involved a decrease in the minimum firing length from 4/75 s to 2/75 s. This took place around day 570. The second change affected only the firings around the *z*-axis: these firings were limited to those cases where the requirement for a firing translated into a firing length of at least 8/75 s. This took place around day 760. The main effect of these changes was a decrease in the amount of attitude disturbance, due to a decrease in the number and length of the thruster firings.

Ground Station Coverage Patterns

The orbital period of the satellite was intentionally close to 4/9 days, which led to repetitions in ground station coverage. This was most clearly so by the end of 1992 (around day 1050), when small changes in the orbital period meant almost exact repeats of ground station coverage patterns over a period of several weeks (see also Section 8.1 and Figure 8.2). This repeating pattern led to the use of the 4-day period in the examination of the data return statistics, as shown in Figure 2.4. Neighbouring periods of 4 days were little affected by the variations in coverage patterns that existed from one orbit to the next.

Gyro De-storage

Hipparcos was equipped with 5 gyros, of which three were needed for its nominal operations, the remaining two being redundant. In order to ensure that the redundant gyros were still in good working order they were subjected to a de-storage procedure once every 4 weeks. This procedure consisted of a 1-minute long spin-up to nominal spin frequency, followed by a period of approximately 2 hours of normal operations, followed by a 1-minute spin-down back to its storage configuration. As a result of those actions, the satellite was subjected to additional torques, most noticeably during the spin-up and spin-down phases. Such (short) stretches of data were lost (see also Chapter 7 and Chapter 8).

De-storage for redundant gyro 3 was stopped when it was found out that its heater had broken down.

Micrometeorites

External hits of the satellite, possibly by micrometeoroids, were recognized in the gyro readout records as discontinuities not associated with thruster firings. Two fairly substantial hits and 10 to 15 smaller ones were recorded. The larger hits were roughly equivalent to thruster firings lasting 0.2 s, causing a change in rotation rate of the order of a few arcsec s^{-1} , the smaller ones were mostly about ten times smaller.



Figure 2.3. Sky coverage of the nominal scanning law for times affected by data loss or sun-pointing recovery. See Table 2.1 for the meaning of labels a, b, c, d.

2.4. Events and Failures Leading to Loss or Degradation of Data

The main events that led to some degradation of the Hipparcos data were generally associated with failures: from the orbital problems due to the apogee boost motor failure, through various thermal control electronics anomalies to the final failure of the gyro electronics and the on-board computer. Table 2.1 lists chronologically the events that were most noticeable in the data reduction and calibration results, summarizing also some of the events mentioned in the previous two sections. Figure 2.1 shows these events along the mission time-line. Below follows a brief description of the consequence of some types of failures and anomalies.

Data Gaps and Sun-pointing

Gaps in the scanning of the sky were caused by uplink command errors and by hardware failures. The main effect of such data gaps was in the final results: as the satellite described a pre-defined scanning law, every data gap would cause a gap in the sky coverage, meaning that some stars along a 'strip' of sky, had missed their chance of being observed. Such events can sometimes be recognized from sky-maps showing the total number of observations per object. The astrometric and photometric data of the stars involved suffered an inevitable deterioration, and this was most serious for objects near to the ecliptic plane, for which the density of observations was already lower than in other parts of the sky, and for objects affected by two or more of these events. The distribution of sky coverage affected by data-gaps before the interruption of observations on day 1315 is shown in Figure 2.2. The effect of data gaps can be seen in Figure 14.16 for the photometric data.

Sun-pointing had three effects. It caused a disruption of the scanning law, with the same effects as for data gaps; a narrow strip in a different part of the sky was very densely scanned instead; the changed exposure of the satellite to sunlight caused changes in temperature of the spacecraft, leading to problems with calibrations. Thus, especially in photometry, sun-pointing data were often unreliable, while consisting on the other hand

of often longer stretches of observations. Data obtained immediately following a sunpointing mode period was subject to rapid changes in spacecraft temperature. From the astrometric point of view, the sun-pointing data only contributed to the determination of position and proper motion in ecliptic latitude, the displacements due to parallax and the longitude components being perpendicular to the scan direction and thus not measurable on the main grid.

Thermal Anomalies

Thermal anomalies were associated with heater failures, and caused a drift in the temperature of the payload on a few occasions. Such thermal anomalies had therefore the same effect as refocusing, most noticeably the thermal control electronics failure indicated by T3 in Table 2.1 and Figure 2.1. This failure caused a run-away effect for the focus, and the recovery, through employing a redundant heater, brought the focus abruptly back in line (see Figure 14.3, the discontinuity at day 755 associated with event T3). This was accommodated in the reductions by associating the time of recovery with a pseudo-refocusing event, so that calibrations relying on running solutions would implement an appropriate break at that position in time.

Gyro Failures

Gyro failures were first indicated by noise bursts, i.e. sudden increases in the noise on the gyro readings. This affected the attitude control possibilities, in particular around perigee when no ground-station contact and star mapper transits were available as an additional control. The actual failure was accompanied by spin-downs and haphazard torques working on the spacecraft, resulting in loss of control and recovery to sunpointing. The failure of both *z*-axis gyros led to the need for operating the spacecraft on two gyros only, one of which (gyro 3) had a failed heater. The consequence of this was increased sensitivity in gyro drift to temperature fluctuations in the spacecraft, and the presence of low amplitude modulated noise on the readings (see Chapter 8 and Volume 2, Chapters 13 to 15).

2.5. Data Return

Many of the points mentioned in the previous sections contributed in a positive or negative way to the overall data return. Primarily, however, this was determined by ground station coverage, occultations, and the success rate of the real-time attitude determination convergence. The overall data return over the mission (in units of 4 days) is shown in Figure 2.4; it averaged just over 60 per cent.

Real-Time Attitude Determination Convergence

Successful data collection could only start after the real-time attitude determination loop had converged: when predicting oncoming transits of star mapper stars, it would find them at the expected time and place. After a perigee passage the attitude control would often be lost, and in need of ground-based assistance to converge again (see Volume 2). There were no clear patterns recognized in the time and effort it took to re-converge the



Figure 2.4. The data return summary over intervals of 4 days (close to 9 orbits). The top graph shows the fraction of time lost due to 'no attitude convergence'. The bottom graph shows between the upper boundary and curve 'a' the fraction of time lost due to occultations during data coverage. Curve 'b' (dotted) shows the maximum possible data return (ground-station coverage minus occultations). Curve 'c' shows the actual data return. The difference between 'b' and 'c' is the same as the curve in the upper graph. Curve 'd' shows the fraction of data contained in data sets of less than 1200 frames, usually too short to be included in the final results. The difference between curves 'a' and 'b' shows the fraction of time lost due to 'no ground station coverage'.

attitude (in other words, with how far the satellite had drifted away from its assumed attitude), no relations with perigee height were detected (low perigee caused drag and could have influenced the satellite pointing). There was, however, almost certainly a relation with high background level in the star mapper detectors near perigee, effectively extending the period without attitude control. Other influences were occultations very shortly before perigee, effectively increasing the time span without observations.

F. van Leeuwen, M.A.C. Perryman
3. OBSERVING PROGRAMME

The Hipparcos observing programme was defined, during successive steps, over the period 1982–1991, on the basis of scientific proposals submitted to ESA, while taking into account the operational requirements and the observing possibilities of the satellite. Considerable attention was paid to the selection of stars in order to enhance, as far as possible, the scientific return expected from the mission. In parallel, extensive ground-based programmes were organised to obtain, before launch, good positional and photometric data about the programme stars. This information was used to optimise the observations by the satellite, through proper positioning of the instantaneous field of view, and careful determination of the observing time to be devoted to each star, which were observed only one-by-one. The Hipparcos Input Catalogue, published in 1992, contained the most up-to-date, comprehensive and homogeneous information on the 118 209 stars selected for observation with Hipparcos at the time of the satellite launch.

3.1. Introduction

The Hipparcos mission was primarily designed to provide a uniform whole-sky catalogue of stellar positions, proper motions and parallaxes. However, from the very beginning, it was recognised that a major enhancement of the scientific return might result from also selecting stars on the basis of their relevance to major astrophysical questions. The resulting catalogue has enormous value for a wide variety of detailed astrometric and astrophysical studies. Compared with existing stellar catalogues, the Hipparcos Catalogue offers a significant improvement on the errors of these quantities, absolute rather than relative parallaxes and proper motions, a relatively dense reference network, and homogeneous sky coverage. Some of the most spectacular advances expected from the mission were always expected to arise from the significant increase in the precision of measurements of trigonometric parallaxes compared with typical Earthbased observations, and from the very much larger number—and the very much wider variety—of stars which were measurable.

The construction of the Hipparcos Input Catalogue, which included, with relevant data, all stars retained for the Hipparcos Observing programme, is decribed in detail in Perryman & Turon (1989). It was published in printed form (Turon *et al.* 1992a), a tape version (Turon *et al.* 1992b, 1993) is deposited at the Centre de Données astronomiques de Strasbourg, and it can be interrogated through their WWW pages (VizieR and

SIMBAD). A CD-ROM version with extensive interrogation, sampling, and mapping facilities was also released (Turon *et al.* 1994).

3.2. The Stellar Inputs

Scientific Proposals

In answer to an Invitation for Proposals issued by ESA in 1982 to the scientific community, 214 proposals were submitted, comprising suggestions for the observation of both stars and minor planets. Amongst the scientific proposals submitted, programmes to determine distances, motions, luminosities, masses, radii, and ages of a wide range of stellar types including white dwarfs, normal dwarfs, giants, radio and X-ray stars, variables and binary stars were well represented. Studies of star cluster dynamics and distances, stellar physics (including studies of atmospheric convection and mass-transfer phenomena) and studies of the interstellar medium were proposed. Determination of the optical reference frame and its relationship to the radio and infrared reference frames was proposed, and major collaborative projects between the Hubble Space Telescope, VLBI and other important ground-based astrometric and astrophysical programmes were initiated. Proposals also covered studies of solar system dynamics, including the dynamics, structures and masses of asteroids, the major planets and certain planetary satellites; Earth rotation, polar motion and continental drift; and lunar occultation phenomena. Galactic dynamics and evolution, dynamics of the Magellanic Clouds, determination of the extragalactic distance scale from Cepheids, and investigations of the validity of general relativity are other examples of the broad scientific interest generated by the Hipparcos mission.

Altogether, the proposals submitted amounted to about 500 000 stars. It was eventually recognised by the INCA Consortium, through an extensive automated use of the SIMBAD database (Egret *et al.* 1991) and manual cross-identifications (Turon, Gómez & Crifo 1989), that there were many redundancies in the stars proposed: finally, about 210 000 individual objects were contained in the 214 proposals submitted.

In addition to stars, 48 minor planets and three satellites of major planets (Europa, Titan and Iapetus) were observed by Hipparcos, mainly for improving the definition of the dynamical reference system and for linking it to the Hipparcos reference system. Preparatory work undertaken within the INCA Consortium to optimise their observability with the satellite is described by Bec-Borsenberger (1992).

The INCA 'Survey'

The 'Survey' is a basic list of bright stars, largely complete to a given magnitude limit, resulting from a compromise between various, possibly conflicting, requirements: (i) the satellite operations and the data reductions required a list of about $50\ 000 - 60\ 000$ stars with $V \leq 9$ mag and with good positions (to better than about 1 arcsec), uniformly distributed over the celestial sphere; (ii) from a purely scientific point of view, it was considered highly desirable to define a sample over the whole celestial sphere, complete to the faintest possible magnitude limit, in order to enhance future statistical uses of the whole catalogue.

In practice, stars were selected automatically from the SIMBAD Data Base of the Centre de Données astronomiques de Strasbourg (CDS), considered to be essentially complete down to about V = 9.0 mag (Egret *et al.* 1991), using the following criteria:

 $V \le 7.9 + 1.1 \sin |b|$ for spectral types earlier or equal to G5 $V \le 7.3 + 1.1 \sin |b|$ for spectral types later than G5

When no spectral type was available, the break was taken at B - V = 0.8 mag. Special attention was subsequently given to variable stars, for which the SIMBAD magnitude is usually the one at maximum brightness.

The choice of the above limits was made after the study of the statistical properties of various possible selections obtained from SIMBAD (Crifo *et al.* 1985, Crifo 1988, Turon *et al.* 1989a). In order to reduce the very high contribution of red giant stars (43 per cent, mostly situated between 300 and 500 pc) in favour of A, F, and early G-type stars, statistically closer to the Sun, and for which the ages may be better predicted, a brighter limiting magnitude was chosen for late-type stars than for early-type stars. The magnitude difference was a constant, adjusted in order to have the bulk of giant stars within 200 pc in the galactic plane, thereby avoiding the most disturbing interstellar clouds.

As a result, about 55 000 objects were selected. This sample of stars was then processed during the numerical simulations of the mission just like any other proposal. However, special care was taken to maintain its statistical properties as much as possible.

Due to uncertainties in the knowledge of magnitudes and spectral types, inevitably some stars were erroneously included or rejected from the selected sample. The effect of these errors has been estimated to be about 1000 missed stars, with some 2500 incorrectly included. The sample finally retained contains 52 000 stars, 95 per cent of them being closer than 500 pc. Less than 6 per cent of the complete sample failed to be retained after the selection process, due to operational constraints on the satellite (Gómez *et al.* 1989).

Additional INCA Proposals

In addition to the 214 proposals submitted to ESA by the worldwide astronomical community, five additional proposals were defined during the course of the work of the INCA Consortium. In particular, it was necessary to make a global and dedicated study of all proposals submitted on certain specific topics in order to optimise their observation by Hipparcos. This was implemented for programmes dealing with stars in the Magellanic Clouds (Prévot 1989) and in galactic open clusters (Mermilliod & Turon 1989), for stars used for the geometrical calibration for the Hubble Space Telescope (within the cluster NGC 188), and for programmes for linking the Hipparcos system to an extragalactic reference system, i.e. radio stars and stars around compact extragalactic radio sources (Argue *et al.* 1984, Jahreiß *et al.* 1992).

These proposals were made in close cooperation with members of the data analysis consortia, and after detailed studies on proximity effects, and on the requirements of the link to an extragalactic reference system (Turon *et al.* 1989b).

3.3. From Scientific Proposals to a Tentative Input Catalogue

The steps taken to arrive at the composition of the final Hipparcos Input Catalogue were not at all obvious at the outset of the project, and the final inclusion or rejection of some objects was rather arbitrary. The main steps taken were as follows:

(a) a Scientific Selection Committee, appointed by ESA, ranked the proposal, or subsets of the proposals, in five priority classes, from objects with a high scientific interest which were to be included in the Hipparcos Input Catalogue if at all possible (priority 1), through to objects which were not to be retained in the Hipparcos Input Catalogue selection process unless there were no other competing stars in the relevant area of sky (priority 5). Different priorities were often awarded for a given proposal for different magnitude ranges, since it was known that the observation of fainter objects would be expensive in terms of observing time, and that only a decreasing percentage of all stars in the sky at fainter magnitudes could be included;

(b) based on these recommendations, the INCA Consortium constructed distributions of the proposed objects as a function of priority, magnitude and position on the sky. After the first round of priority allocations, it was immediately obvious that a large amount of work was needed to achieve a sky and magnitude distribution better suited to the satellite's capabilities;

(c) methods were developed within the INCA Consortium to simulate numerically the observation with Hipparcos, and to control the observing time allocated to each star throughout the mission. This allowed the Consortium to establish the feasibility of observations of any given star, according to its magnitude and the detailed properties of its surroundings, as well as the expected precision of the astrometric parameters. Different algorithms prescribing the allocation of observing time as a function of magnitude were studied at the start of the work, allowing a decision to be made on the total number of stars to be retained in the Input Catalogue as a function of magnitude, based on the final expected accuracies implied by these distributions. Nine successive selections were submitted to a chain of numerical simulations (Crézé 1985, Crézé & Chareton 1988, Crézé *et al.* 1989), allowing the statistical representation of the various proposed programmes and the expected precision on the astrometric parameters to be assessed;

(d) the proposers were given the opportunity to express their comments, first on the priorities allocated to their proposed programmes, and later, once a close-to-final star selection was obtained, on the individual stars retained from their proposal. This dialogue, albeit unusual, was felt desirable for two important reasons: (1) the first round of recommendations from the ESA Selection Committee and the corresponding treatment of the data by the INCA Consortium was necessarily somewhat statistical in nature. It was realised that such a coarse treatment might be satisfactory for many proposals, but quite unsuitable for others, and (2) since the observing programme of Hipparcos remained fixed throughout the satellite lifetime (it was not possible to add new objects to the observing list throughout the mission, nor to undertake new rounds of proposals) it was important to satisfy the scientific requirements of each proposal from the very outset and to check, further on, that the final star selection would not exclude one specifically important object in the opinion of the proposer;

3.4. Resulting Catalogue Content

Global Content

The sky distributions of all candidate stars and of stars selected in the final catalogue are shown in Figures 3.1 and 3.2 respectively. Though much smoother than the distribution of proposed stars, the distribution of observed stars still shows a concentration along the galactic plane. This was allowed by the fact that, in these regions, the only stars which were observable are relatively bright, and their individual target observing time relatively small. The global distribution of selected stars versus *Hp* magnitude (i.e. the magnitude in the Hipparcos band) is given in Table 3.1, along with the percentage of success obtained for priority 1 stars and for all survey stars (the *Hp* band has an effective wavelength close to that of the V band of the Johnson system, but much wider, as shown in Figure 14.1, and in Table 1.3.1 in Volume 1). The bulk of stars observed by Hipparcos are brighter than Hp = 10 mag, and few of them are fainter than 12 mag. The effect of the weight placed on high priority stars is also clear from the comparison of columns 2 and 5.

Astrometric Programmes

The general 'success' of the astrometric programmes was very high, since they contained mainly bright stars spread all over the sky or over large areas. The number of stars proposed for each main programme and the percentage of observed stars in each case, are given in Table 3.2. Particular attention was paid to the inclusion of fundamental stars (FK5, FK5 extension, IRS), and to guarantee that radio and 'link' stars (stars in the close neighbourhood of quasars compact in the optical and radio wavelengths) would be observed by Hipparcos in an optimum way, in order to prepare the link of the Hipparcos reference frame to an extragalactic reference system, via VLBI and Hubble Space Telescope observations.

Astrophysical Programmes

As described in Section 3.2, a very large variety of astrophysical programmes was proposed for observation on Hipparcos. Table 3.3 shows the mean rate of inclusion for the main categories of programmes. The Hipparcos Input Catalogue contains field stars of almost all spectral types and luminosity classes belonging to various stellar populations, most types of binary and variable stars, very specific objects such as white dwarfs, central stars of planetary nebulae, and Wolf-Rayet stars, stars in about 280 open clusters, and stars in the Magellanic Clouds (Gómez 1988; Gómez & Crifo 1988). In most cases, the closest stars of each category were retained. The result is that more than 85 per



Figure 3.1. Sky distribution of candidate stars shown as a function of galactic coordinates. The most prominent feature is the concentration of candidate stars along the galactic plane. Stellar densities refer to the number of stars in an area of $6^{\circ}.4 \times 6^{\circ}.4$.



Figure 3.2. Sky distribution, in galactic coordinates, of all selected stars. Stellar densities refer to the number of stars in an area of $6^{\circ}.4 \times 6^{\circ}.4$.

Magnitude (<i>Hp</i>)	Entries in INCA Data Base	Entries in Input Catalogue	Global success (per cent)	Success of priority 1 stars (per cent)	Success of survey stars (per cent)
< 6	4 200	4 200	100	100	100
6 - 7	8 5 4 0	8 5 1 0	99	99	99
7 - 8	24 160	22250	92	98	93
8 - 9	55290	41 100	74	96	93
9 - 10	70970	29410	41	91	91
10 - 11	36270	9 3 3 0	26	83	-
11 - 12	10190	2 930	29	86	-
≥ 12	5140	650	12	44	-
Total	214 760	118 380	55	94	94
Retained				72 500	52 800

Table 3.1. Final selection of stars in the Hipparcos Input Catalogue, and the global percentage of success for priority 1 and survey stars, as a function of the Hipparcos magnitude, *Hp*.

Table 3.2. Success of the main astrometric proposals in the Input Catalogue.

Catalogue or Proposal	Number of proposed stars	Success (per cent)
FK5	1 535	100
FK5 extension	2 013	99.8
NPZT	1 718	99.4
AGK3R	21 499	98.2
SRS	20 495	96.8
IRS Supplement	1 900	95
GC	33 100	90
Selected radio stars	189	98
Selected link stars	175	95
Photographic link stars	1 000	42
Lunar occultations	15 300	50
Jupiter occultations	4 900	42
Uranus and Neptune occultations	23	39
Pluto occultations	290	41
Parallax standard stars	64	95

Type of Proposal	Success (per cent)
Luminosity calibration	>70
Stellar masses	>95
Stellar atmospheres	>90
Stellar structure	>90
Galactic structure ⁽¹⁾	>50
Galactic structure ⁽²⁾	>80
Magellanic Clouds	>50

Table 3.3. Success rates of various categories of astrophysical programmes.

 1 if the number of proposed stars was $\geq 10\;000$

 2 if the number of proposed stars was < 10 000

Spectral Type Distance (pc) Total <100 100-500 500-1000 > 1000 O-B 170 6050 2530 1980 10730 A0-A9 1260 15910 1330 270 18770 F0-F9 12 400 13150 110 250 25910 G0-K1.5 14200 23 560 2150 530 40 4 40 3 3 8 0 10470 5 5 0 0 900 20250 K2-M8 Total 31410 69140 11620 3930 116 100

Table 3.4. Distribution of the selected stars as a function of spectral types and distance estimates (the total number is not 118 000, as it was impossible to make even a rough estimate of the distance of some stars).

cent of the selected stars are closer than 500 pc. The distribution of the selected stars by spectral types, with a rough estimate of the distance, is given in Table 3.4.

3.5. Tests of the Hipparcos Input Catalogue by Satellite Observations

Tests of the Hipparcos Input Catalogue Data

The Hipparcos Input Catalogue was probably the first catalogue ever tested before its publication, and with an instrument allowing a much higher accuracy on positions, proper motions, magnitudes and colours to be achieved. The specifications of ESA were standard errors of 1.5 arcsec on the positions at epoch 1990 (with a somewhat better accuracy on positions for a subset of stars used for real-time satellite attitude determination), and 0.5 mag on the *B* or *V* magnitudes for all programme stars. From comparisons made using the preliminary sphere solution obtained from 30 months of Hipparcos data (see Chapter 16), the accuracies achieved in the Hipparcos Input Catalogue were largely within these specifications: 0.3 arcsec on the positions, and 0.25 mag on *Hp* magnitudes, with accuracies of 0.02 mag or better for more than one third of the catalogue (Turon *et al.* 1995).

In the final Hipparcos Catalogue, 263 entries have no astrometric solution, 14 of these also have no photometric solution. The main reasons for these problems were that the position and/or proper motion and/or magnitude was wrong in the Hipparcos Input Catalogue and that, as a result, either the sky background was measured, or the star was too faint at the epochs of observation for an astrometric solution to be obtained. The stars entering these categories are mainly high proper motion stars and large amplitude variable stars. A few of them are components of double or multiple systems. Notes in the Hipparcos Catalogue indicate these problem stars.

In addition to these entries where no star was observed by the satellite, a small number of identification errors were detected, where the satellite observed a star which is not the star quoted in the Hipparcos Input Catalogue (at least with respect to the crossidentifications tabulated for such a star). Again, these entries are mainly high proper motion stars and large amplitude variable stars. Notes in the Hipparcos Catalogue also indicate these problems.

C. Turon

4. OVERVIEW OF THE DATA ANALYSIS

This chapter provides a general overview of the data analysis leading from the raw satellite data to the Hipparcos Catalogue and associated annexes—the details of the various processes are presented in the following chapters. The basic concept for the data analysis had already been established at an early stage in the mission planning, and the reductions were subsequently carried out in a series of sequential processes, each substantially decreasing the corresponding data volume. Certain aspects of the analysis were in practice dictated by the relatively limited computer resources at the time of development and earliest implementations. Much effort was devoted during the preparatory phases to creating reduction software that was both fast and highly reliable. The practical organisation of the treatment of the data in both consortia, FAST and NDAC, is also summarised.

4.1. Main Stages of the Data Reduction

Input Data Stream

The input data stream consisted of the scientific data from the satellite, auxiliary data (principally the satellite orbital parameters) and various satellite 'housekeeping' data. These data streams were prepared by ESOC from the original telemetry data, and put on 9-track, 6250 bpi tapes (see Volume 2, Chapter 9) at a rate of just under 1 tape per day. Most of the data was provided in a highly compressed form, with some 1350 tapes in total delivered by ESOC to each consortium. The total amount of compressed science data relevant for the construction of the Hipparcos Catalogue was around 70 Gigabytes.

Reduction Processes

A 'three-step' reduction scheme was first proposed by L. Lindegren, of Lund Observatory (Sweden), during the feasibility study of the space astrometry mission. It consisted of three major steps: the great-circle reduction, the sphere solution and the determination of the astrometric parameters. Although a direct, global solution of the mission data would have been possible in principal, it was totally impossible in practice. The threestep method allowed the overall analysis problem to be decomposed into three sequential steps, allowing the processing of the satellite data to be considered as feasible. It led to a marginal degradation of accuracy compared with a theoretically optimum reduction system, and the same general approach was adopted by both consortia. However, steps 2 and 3 were combined to one step by NDAC in the final implementation, which also comprised the processes for star mapper data reduction, attitude reconstruction and image dissector tube data reduction, plus a wide range of off-line processes (see below).

In practice, the processing did not follow an exclusively sequential structure—thus, for example, results from the sphere solution were used to iteratively improve the attitude reconstruction and great-circle reduction, by feeding back improved stellar positions into the reduction process, while results from the great-circle reduction were used as a quality control of the attitude reconstruction. Several other processes ran largely in parallel with the main data processing, but with a sometimes complex interaction with other processes:

- the double star processing (Chapter 13) interacted at many points with main reduction processes: results from the star mapper processing provided precise starting coordinates for double stars with separations larger than 1.5 arcsec, image dissector tube data processing provided the input data for the double star processing, the great-circle reduction provided relative reference positions for the measurements, and the sphere solution results allowed these to be transformed into absolute positions;
- the photometric reductions (Chapter 14) proceeded rather independently of the astrometric tasks, although the photometric results were used in all three main processes, in particular providing colour determinations obtained with the star mapper data;
- comparisons between various calibration parameters obtained at various stages of the data processing led to a better understanding of the instrument and thus of the reliability of the reduced data. These aspects are presented in Chapter 8 and 10;
- special treatment was required for the processing of minor planets and planetary satellites (Chapter 15), from the first processing to the presentation of the final results;
- at all stages of the reductions, a catalogue of stellar parameters (positional and photometric) was used, and was regularly updated using intermediate mission results.

The rest of this section provides some more details about the reduction processes and references to the chapters in the current volume where full descriptions can be found. The processes are here divided into the main reduction chain, the parallel processes, and other aspects.

Main Reduction Chain

Part A. Processing of photon counts (Chapters 5 to 7): The aim of the first reduction processes was to derive from the image dissector tube counts, in combination with the star mapper data (and to some extent gyro data), the phases of the modulated signals at a given reference time during the star transits, and with respect to a chosen reference line on the grid. In addition, the satellite attitude angles at the chosen reference time were needed for further processing, as well as for the first processing of the image dissector tube data. The processing proceeded as follows:

- preliminary investigation of the satellite attitude to provide input data required for the star mapper processing, processing of the star mapper data to transit times (Chapter 6);
- (2) using catalogue positions and star mapper transit times, the attitude for each observational frame with image dissector tube data was reconstructed with an accuracy of better than 0.1 arcsec (Chapter 7);
- (3) using the reconstructed attitude results, the image dissector tube data were reduced to provide phase and amplitude information for the astrometry and photometry respectively (Chapter 5).

The output of the latter two processes formed the input for Part B, the great-circle reductions (Chapter 9). Parallel processes in Part A were the star mapper and image dissector tube photometric reductions, catalogue improvements, and a range of calibrations. Data were also prepared for the double star processing. The total amount of data at this stage was, on average for each observation frame of 2.13 s, 4.5 transit times, their accuracies and amplitudes, the three satellite attitude angles with accuracies, as well as full timing information. The reduction in data volume from the original photon counts was about a factor of 100. On average, about 50 000 transit times were obtained per orbit (these typically constituting one 'reference great circle').

Part B. The great-circle reductions (Chapter 9): The task of the great-circle reductions was to combine the 50 000 transit times obtained over one orbital period into abscissae along a reference great circle, for the 2 000 different stars observed during that period. This was obtained through a further refinement of the along-scan satellite attitude, through projection onto a reference great circle using the results of process (2) described under Part A, and through calibration of the large-scale distortions of the projections on the main detector. Also calibrated was the basic angle (Chapter 10) between the two fields of view, which enabled the linking of different parts of the sky at the milliarcsec level. The monitoring of the various calibrations was used as part of the overall quality control.

Double star coordinates were generally not carried along in the great-circle reduction: the reference phase of the modulation for a double star was a combination of two signals, the result of which depended on the orientation of the modulating grid with respect to the orientation of the double system. They had to be treated independently, using, however, the scan-phase information obtained in the great-circle reduction.

The output of each reference great circle reduction was a reference pole, a preliminary zero-phase (relative to some celestial reference frame), and abscissae, and their accuracies, for some 2000 stars. This represented the data collected for single stars over an interval of between 6–9 hours.

Part C. Sphere solution and astrometric parameters (Chapter 11): The task of the sphere solution was to establish a consistent system of zero-phases for all reference great circles, and subsequently derive from the combined abscissae, corrected for the zero-points, the five astrometric parameters: the two components of position, the two components of proper motion and the parallax. In the process, effects that could not be detected at earlier levels in the data reductions were calibrated: certain harmonics in the great-circle solutions could enter due to occultation-gaps, and chromaticity corrections could only be obtained at this level.

The final astrometric parameter determination was carried out after the merging of the data from the two consortia (Chapter 17). At this stage non-linear proper motion cases were also detected and solved for.

Parallel Processes

A series of parallel processes used data from the main processing chain, and sometimes provided information to it. The two principal parallel processes were the double star processing and the photometric calibrations.

Double star processing (Chapter 13): The character of the modulated signal obtained with the main detector was such that all information on double stars had to be processed as a separate, dedicated task. Input data came from the image dissector tube data processing, the image dissector tube photometric calibrations, the great-circle calibration results and the reference great circle zero-point calibrations obtained as part of the sphere solution. Further information for the detection of double stars was provided through accumulated statistics from the image dissector tube data processing. Data from the optical transfer function (Section 5.9), describing the characteristics of the modulated signal as a function of position in the field of view and as a function of star colour, was incorporated in the reduced image dissector tube data results. These data were not only supplied for double stars known *a priori*, but for every star observed, since many unknown double stars were present in the observing programme.

The double star processing consisted of two stages: (1) recognition of stars as double (or multiple); (2) solving the double star parameters (separation, orientation, magnitude difference). By using data from the sphere solution and the great-circle reductions, the double star parameters could be entered into the final sphere solution. Checks on the reliability of this fitting were made through treating some single stars with the same processing, and comparing the results with those obtained in the normal processing.

Photometric calibrations (Chapter 14): Two types of photometric calibrations were carried out: for the star mapper data, and for the image dissector tube data. The first was of moderate accuracy, and intended to provide colour information for stars without reliable colour information in the Hipparcos Input Catalogue. This could only be done for stars brighter than approximately 10 mag. The star mapper photometry also provided a background signal relative to which the background signal in the main detector could be determined.

The image dissector tube photometry provided accurate photometric data in a broad band, eminently suitable for use as 'epoch photometry' for the detection and investigation of variable stars. It was also used as calibration information in the double star processing.

Catalogue updates (Chapters 6 and 16): The quality of the data reductions was considerably improved through the work done on updating the catalogue information: improved positions provided a more accurate attitude reconstruction, allowed a better distinction of the pointing of the instantaneous field of view, and a more reliable great-circle reduction (through removal of grid-step ambiguities). Catalogue updates were initially provided through accumulated star mapper reduction results: the attitude reconstruction was more accurate than that represented by the positions of stars in the Hipparcos Input Catalogue, and the residual transit times left after attitude reconstruction were used to correct those positions. The star mapper data also provided improved

colour information as described in the previous section. Finally, the star mapper data provided improved parameters for some 1200 double stars. At later stages in the processing, improvements came from the first preliminary sphere solutions, at which point positional accuracies were reached that no longer had any negative influence on their use in the reduction processes.

Catalogue updates were also provided to the satellite operations team at ESOC for improved satellite performance.

Other Aspects

Calibrations (Chapters 8 and 10): Calibrations were carried out at almost every stage of the data reductions, and comparisons between various calibration results led to a much better understanding of the instrument and the external influences on it. In many cases calibrations were performed to a level well beyond the strict requirements of the data processing, providing information that can be of use in future space missions.

Interfaces with the Tycho Catalogue reductions: The Tycho data reductions required the attitude reconstruction from the main mission. As the Tycho reductions used the star mapper data, their reduction processes and results were incorporated in comparison exercises. The Tycho photometry was essential in establishing the reference colours for the final main mission reductions.

Comparisons: Careful and extensive comparisons were carried out throughout the data reductions, an important process which began before the start of the mission using simulated data, and which was to some extent facilitated by the common adoption of the same sequence of reduction processes by both consortia. The main comparison tasks were:

- (1) star mapper reductions: detections, transit times, intensities, and error estimates;
- (2) attitude reconstruction: coverage, general performance, outliers;
- (3) image dissector tube data reductions: phase and amplitude estimates, results from 'partially observed stars', accuracies;
- (4) great-circle reduction: abscissae, accuracies, slit ambiguities;
- (5) sphere reconstruction: accuracies, rotations;
- (6) double stars reductions: detections, parameter comparisons;
- (7) photometric reductions: biases, error estimates, data rejection and flagging, barycentric corrections;
- (8) variability investigations: detections, periods, zero-phases;
- (9) satellite orbital parameters, and determination of celestial directions.

All these comparisons had important consequences for the mission results: they revealed numerous errors or possibilities for improvement in the reductions, stimulating the data reduction groups to produce improved and optimum results—this element of (good hearted) competition was an important factor in obtaining the final high quality of the Hipparcos Catalogue results.



Figure 4.1. General scheme of the interactions between tasks in the FAST Hipparcos data reduction organisation. DMCS: Data Management and Command System.

4.2. Organisation of the Data Reductions in FAST

The data analysis by the FAST Consortium followed the principles described above, being essentially an iterative process during which intermediate quantities such as attitude and instrumental calibrations status were improved together with the five astrometric parameters (see Figure 4.1). The process was iterated about ten times, either when new observational data was included or simply when a better working star catalogue was available from earlier reductions. On several occasions, software was improved between two successive iterations, partly as a result of the comparison exercises, also thereby contributing to the general improvement of the results.

The various tasks were prepared in different scientific institutes before the launch of the satellite. They were verified using simulated data prepared in CERGA (Observatoire de la Côte d'Azur), and put in a format that was pre-defined in an architecture document and an interface document where all the interactions and data exchanges between software were foreseen. The actual exploitation of the software was carried out systematically on the CNES CDC computer, so that the software had to be transferable, irrespective of the computer used in the contributing institutes. Thus each piece of software had to be verified first in the responsible institute then, once transferred, in CNES. An ensemble of acceptance tests was set up for each software module, and acceptance was declared after the tests had been fully satisfied. An inter-institute committee, referred to as the Software Advisory Group (SWAG), which included representatives of CNES (J.L. Pieplu and C. Huc), CERGA (J. Kovalevsky and J.L. Falin) and other participating institutes (E. Canuto, CSS; R. Hering, ARI; H. Kok, Delft; and F.P. Murgolo, CSATA) played a major role in building the interfaces, in enforcing inter-institute cooperation, and in transforming independent software into a coherent data processing system.

The role of the Software Advisory Group continued after satellite launch. The software accepted on the basis of simulated data, had to be proved adequate to treat the real data. All the software modules, separately and in combination, had to be confronted with unforeseen problems or perturbations, and with the consequences of the highly eccentric orbit such as the absence of data during perigee passages, the increase of photon noise in the van Allen belts, long eclipses, and the large variability of torques with time.

It took more than one year to rewrite some of the software and to re-perform all the acceptance tests on the CNES computer using some data sets chosen in such a way that all different peculiarities of the orbit and of the data recovery were fully covered. Thus the actual systematic data treatment started only in May 1991, 1.5 years after the actual start of the mission. The automatic mass treatment organised in CNES, and controlled by the 'Data Management and Command System' was very efficient, and the lag rapidly diminished. However additional improvements, in particular in attitude determination and great-circle reduction, continued to be made for at least another year and a half.

The FAST reduction structure can be described by grouping the various tasks and related software into five categories:

- (1) data reception: this consisted of transforming ground-based data (the Hipparcos Input Catalogue, and planetary ephemerides) and the satellite data transmitted by ESOC into a form directly accessible by the various software modules;
- (2) 'first-look' and calibrations: every week, the first-look task in SRON, Utrecht treated one data set (data acquired during one satellite orbit) to validate the data sent by ESOC almost in real time, and to provide the first calibrations to the FAST data reductions. Later, these calibrations were improved using the results of the main data reductions;
- (3) mass treatment: this consisted of transforming the raw data into star abscissae on a reference great circle, carried out data set by data set. Two different chains were used: one for the first treatment of the data, the second for the iterations;
- (4) synthesis: all the available results on data sets, whether obtained from the first or iterated treatments, were combined to determine a consistent mesh of reference great circles with their origins, followed by the astrometric parameters of each star;
- (5) off-line tasks: these included photometry, double and multiple star treatment, minor planet reductions, and the improvement of calibrations.

Each of these tasks, and their interactions, is described hereafter.

Data Management and Command System

The decision to perform all the data treatment on a single large centralised computer, and to do it as automatically as possible, required the setting up of a software system which would be able, using simple commands, to process any subsystem for any data set, and to access all intermediate or final files at any time.

Processing all the data, over the whole mission, demanded the correct and coherent execution of more than 30 000 jobs, accessing 80 classes of files, comprising a total of more than 120 000 files. In order to minimise the consequences of mistakes, to control the progress of the processing, and to optimise the mass production, the scientific software and the subsystems were embedded in a 'Data Management and Command System'. The main functions of this system were as follows:

- (1) to activate the different parts of the reduction software according to a pre-set operation scheme. The organisation of commands allowed the activation of the subsystems sequentially or individually, on one or several data sets. Special care was taken to verify the feasibility of the commands, in accordance with the operation scheme and the available computer resources;
- (2) to manage the scientific and operational data within a data bank on disks and magnetic tapes. The Data Management and Command System identified and extracted from the data bank the data necessary to execute a task and stored the results. The storage resources were divided into different areas in order to avoid interferences between parts of the main reduction. A copy of the files was created and managed in a separate archival area;
- (3) to allocate dynamically and control the computer resources needed by the subsystems during operation;
- (4) to update the consultation area with new results and to manage the access requests. This consultation facility of intermediate and final results through the Data Management and Command System was intensively used by the CERGA team in Grasse, throughout the reductions, for the scientific control of the processing. It was through this facility that all off-line tasks received the intermediate data that was needed;
- (5) to allow the debugging, correction of errors, implementation and tests of modifications or software improvements in a separate resource environment.

The size of the Data Management and Command System was about 150000 lines of code, including comments. It represented roughly half of the complete reduction system, the other half covering the preparation tasks and the scientific software. It managed 4000 high-density magnetic tapes and used 1.5 Gigabytes of disk space. The reduction system worked on the multi-user CDC 992 of the CNES computing centre at Toulouse. The complete reduction of the Hipparcos data at CNES used approximately 1500 hours of CPU time of this 24 Mips machine.

Data Reception and Preparation

The data received from ESOC was first checked for consistency, then divided into different files for subsequent use. From ESOC quality flags the limits of the data sets

were automatically determined. After a few months, the test parameters were made less stringent because it appeared that the flagging of possibly bad data was sometimes unnecessarily severe. After this, the quantity of data retained by FAST and NDAC was quite comparable. The preparation of data included the following items:

- (1) a full description of the data set was made containing the observation times, the stars observed, the satellite gas jet actuation times and durations of telemetry gaps, eclipse times, the relation between on-board and on-ground times in the form of a third-order polynomial and some statistics. This 'Mission Control File' was used by all FAST software modules throughout the reductions;
- (2) several files included all information concerning the stars observed in the data set: in particular *a priori* geometric and apparent positions of stars at their times of observation in the great circle reference frame, various partial derivatives with respect to astrometric and calibration parameters, reference astrometric parameters to be improved, grid coordinates derived from coil-currents, etc. These files were updated at each iteration;
- (3) photon counts from the main grid, with details of the observing strategy, and the star mapper photon counts with time indicators. These files were used only in the first treatment;
- (4) orbit and on-ground attitude files, and minor planet ephemerides.

First Look

In the FAST Consortium's first-look facility at SRON, Utrecht, data from one orbit per week were received from ESOC within a few days after data acquisition. Using a special version of the FAST data reduction software, these data were analysed within 24 hours of reception; relevant results were distributed to FAST, ESOC and INCA. This special processing served a number of purposes:

- (1) a check on the integrity of the data produced by the instrument and processed by ESOC. More particularly, it was verified that the data could be processed without problems by the FAST main reduction software;
- (2) a check on the correctness of the reduction software. Especially in the early phases of the data reduction, numerous corrections were proposed. In parallel, results were used in the various comparisons between the consortia processing results;
- (3) as a result of the great-circle reductions, geometric calibrations of the main field of view, including the basic angle, were obtained. This provided an excellent method of verifying the stability of the instrument. These results were sent to FAST and ESOC; the latter used the calibration, in particular the grid rotation, to improve the real-time pointing of the instrument;
- (4) a number of other quick calibrations were performed: geometry of the star mapper field of view, photometry of the main field, photometry of the star mapper, singleslit response of the star mapper. Results of all calibrations were sent to the FAST main reduction to serve as a first approximation to be improved after the availability of the complete set of measurements. Results of some calibrations were also sent regularly to ESOC; for example, the calibrated modulation factors were sent weekly in order to allow ESOC to monitor the focusing of the instrument;
- (5) the level of all star signals were compared with their expected values. In particular, those cases where no signal was measured (which could have been due to an

incorrect position in the Hipparcos Input Catalogue) were signalled to the Input Catalogue Consortium.

Calibrations

The various parameters describing the instrument and the instrumental effects on the data had to be calibrated since the quality of the results of the reduction depended critically on the quality of calibration. So, in FAST, great care and much work was devoted to improve the calibrations so as to use the latest and optimised values at each iteration. Conversely, better intermediate solutions were used to re-run the calibration software in order to further improve the values of the parameters. The main quantities which were calibrated fell into one of the following categories:

- (1) image dissector tube data: modulation coefficients M_1, M_2 , the phases g_1 and g_2 for single stars, and a first photometric calibration as a function of position on the grid and star colour (see Chapter 5). The calibrations were essentially used in the determination the grid coordinates, in the double and multiple system processing, and in the photometric analysis;
- (2) star mapper: the shapes and the position of the star mapper grids with respect to the main grid were calibrated with an accuracy significantly better than the precision of the star mapper observations (see Chapter 6);
- (3) optical parameters of the instrument: these included the basic angle and the gridto-field transformation over the main grid. They were performed as a part of the great-circle reduction (see Chapter 9) although the synthesis and analysis of these results as a function of time were an important part of the calibration task.

Most of these parameters were not stable with time (see Chapter 10) so that the incorporation of the calibrations had to be time dependent. In FAST, the following structure was adopted. The full mission was divided into 33 periods whose duration varied from 6 to 40 days depending on the speed of variation of parameters (mainly the basic angle and the field rotation) also taking into account the refocusing times. Two of these periods concerned sun-pointing situations. For each of these periods, the calibrated quantities were presented, whenever suitable, as analytical functions (polynomials) of position on the grid, colour and magnitude of the star, and time. For the star mapper calibrations, needing lower precision, only 9 periods ranging between 6 and 332 days were adopted. All were periodically updated when better data became available.

Mass Treatment

Within the Data Management and Command System, every data set was treated continuously in a single run from the preparation of the data to the great-circle reduction inclusively, proceeding first through the star mapper data processing, and the determination of the attitude, then the image dissector tube data processing and grid coordinate determination and, finally, the computation of grid coordinates. A number of intermediate results used in other tasks were stored in files: for example the results of the image dissector tube data processing were provided to all the off-line tasks, the attitude to the double star processing and minor planet task, data for various calibrations, etc. Results of the image dissector tube and star mapper photon count treatment and the attitude file were re-used in the iteration mass treatment. The latter therefore included, in addition to a simplified preparation task, a re-evaluation of the attitude and of the grid coordinates and a new reduction on great circles.

Synthesis

Whenever a sufficient number of new or iterated great circle results were obtained, they were sorted star-by-star. The data of some 42 000 stars, selected for their brightness, absence of duplicity, and quality and number of observations, and referred to as 'primary stars' in FAST, were used as an input to the sphere solution software (see Chapter 11). Then, the abscissae of the stars were corrected using the re-determined origins of the great circles and used to compute the astrometric parameters of stars. The latter work was shared by CERGA on the CNES computer, and Astronomisches Rechen-Institut (ARI) in Heidelberg. At this level, the results of the double and multiple star processing were introduced in the equations to determine their astrometric parameters.

The ensemble of the mass production and the corresponding synthesis was called a 'run'. Eight such runs were made during the four years of processing. Four iterations were made during the first 18-month data treatment and three involved more data during the global course of the data reductions. In addition, during the final year, several other syntheses were performed without re-doing the totality of mass processing, but introducing various individual improvements after re-processing some data sets by the mass treatment, or in modifying the status of some stars (primary/secondary or double/single).

Off-Line Tasks

In principle, off-line tasks were performed outside the Data Management and Command System, but the input data were prepared by it. The organisation was as follows:

Minor Planets: The data included the abscissae on great circles as determined by the great-circle reduction task, five parameter modulation coefficients, various data describing the reference great circles, and the orbit of the satellite (position and velocity). The files were prepared in CNES and sent to the Bureau des Longitudes where the corresponding analysis was carried out.

Photometry: This task was essentially based on treated photon counts and modulation coefficients prepared by the Data Management and Command System. The photometry task itself was executed on the CNES computer by the CERGA team. Very good communications existed with the Royal Greenwich Observatory (RGO) team of NDAC, so that comparisons of results were made very frequently and allowed a quick convergence towards comparable results.

Double and Multiple Stars: The complex activities related to the double and multiple star reductions was divided into two separate phases:

(a) relative astrometry of stellar systems, in which the relative positions and intensities of components were determined. Again, the data was prepared within the Data Management and Command System, then sent to the relevant groups. In Italy, it was sent to CSATA (Bari) where further preparation of the data was performed, the results being sent to the Istituto Astrofisica Spaziale in Frascati and the Torino Observatory. The results were compared in Torino and formed the first set of results. A second set was obtained in CERGA on the CNES computer. The FAST ensemble of results was evaluated in CERGA and the best results were sent to Lund for the final merging of double and multiple star data. Multiple star reductions were made independently in Torino and CERGA, then checked, and the results which improved the goodness-of-fit in the computation of the astrometric parameters were retained and sent to Lund for merging;

(b) absolute astrometry of stellar systems, in which the astrometric parameters were determined. This was performed in ARI, Heidelberg. In addition, ARI analysed the residuals of the astrometric parameter reduction, and identified some astrometric double stars and 'variability induced movers'. It also contributed to what are called 'stochastic solutions' for unresolved, probably double or multiple, stars.

4.3. Organisation of the Data Reductions in NDAC

The three main groups of data reduction processes, as described in Section 4.1, corresponded in NDAC with three groups that were each responsible for the development, testing and implementation of their part of the reduction software. The connections between the groups were specified in an interface document, and all data transfer was made by 9-track tape (later also by DAT cartridges) in the agreed standard formats.

The responsibilities were divided as follows:

- (1) Royal Greenwich Observatory, UK (RGO): Part A (star mapper processing, attitude reconstruction, image dissector tube data processing), photometric reductions and photometric variability investigations;
- (2) Copenhagen University Observatory, Denmark (CUO): Part B (great-circle reductions) and experiments with global solution;
- (3) Lund Observatory, Sweden (LO): Part C (sphere solution and astrometric parameter determination) and double star processing.

In all cases the same people who had designed, developed and tested the software also implemented the software, which gave NDAC some advantages of flexibility. All software was implemented on small, semi-dedicated computers (MicroVAX, Sun, and HP/Apollo workstations). The number of people involved at any one time has always been small: at RGO it varied between 3 and 4, at CUO between 1 and 2, and at LO only two people were involved. This reduced greatly the need for meetings, as most problems could easily be solved over the phone or later through electronic mail between the two or three people concerned. A great deal of testing took place before the start of the mission through the use of simulated data produced at the RGO.

The biggest strain on the processing of the data was at the RGO, primarily due to the sheer volume of data arriving and its preparation for the data processing. Approximately 120 to 140 man-days were spent getting the data across from tape to disk in the required format. All data was transferred to optical disk for direct access. A total of 120 disks of 2 Gigabytes each were used to store the complete mission data. This process involved also a first visual inspection of the data quality through an interactive display of the gyro data. Display packages for other data streams were also available and used extensively in the beginning of the mission in order to understand some anomalies.

From the moment the data was on disk, the processing was semi-automatic: programmes were created to produce command files for data processing according to log files of the processing done so far. The command files could keep the processing of many data sets for time intervals of up to several days, the only requirement being the supervision of the occasional change of magnetic tape or disk. The data reductions produced small selections of control graphs, showing the quality of the input data and of the reduced data. A log was kept of every process, which was automatically updated at completion. The standard reductions at this stage were completed by writing the necessary files for the great-circle reductions to tape and sending the data to CUO. During the first 18 months of the mission, updated catalogue data was also sent to CUO.

At CUO the tapes with input for the great-circle reductions were read and the data reorganised on disk, allowing easy access for processing. The reduction of a greatcircle set was carried out by performing a chain of processes where the first processes read data from the input files, and the last process wrote the output files. During the processing of a set, several intermediate files were created and used, but these files were all deleted after the output results were accepted. The definition and initialisation of the great-circle set and its reduction were controlled by a number of global parameters which could be changed by editing an option file. Each of the programs used in the reduction of the great-circle sets could be run as stand-alone programs, but the large number of files and tapes involved made it difficult, and indeed undesirable, to manage and monitor the great-circle reduction processes manually. A data management system of programs and directories was therefore set up to control the use of the data files and tapes throughout the mission. Utility programs were also available for plotting results such as the attitude updates or the data in any column of the intermediate files versus record number. From CUO the satellite three-axis attitude for the entire mission was delivered to TDAC, combining results from RGO and CUO.

All the input data for the sphere solution and determination of astrometric parameters, performed at LO, could be contained on a few magnetic tapes and the complete set of files could be stored on disk for the processing. Thus no special data management system was required for this task. However, because there was considerable experimentation with the solution programs and the modelling of various effects, some care was needed to preserve the different program versions and the corresponding data sets. Each new run was given a sequential number (the final sphere solution was number 370), which was used to identify successive catalogue versions as well as the programs by which they were generated. At major milestones of the processing, the whole directory tree containing data and programs was saved on tape cartridges. The solution program also generated files of intermediate abscissa data, used as input to the catalogue merging.

The double star processing at LO used much larger data sets as input, including the signal parameters for the individual field of view crossings from RGO, the complete attitude files from CUO, and a number of photometric and geometric calibration files from RGO, CUO, and the sphere solution at LO. The generation of the 'case history files' from these data streams involved some rather complex juggling of data between disks and tapes (replaced by DAT cartridges in the latter part of the mission). This was however only done a few times throughout the mission, and the subsequent solution of double and multiple stars could then be made one object at a time, using dedicated software for the different object types.

5. IMAGE DISSECTOR TUBE DATA PROCESSING

The image dissector tube data formed the main data stream for the Hipparcos mission. This detector assembly was situated behind the modulating grid and was capable of observing the light transmitted through a very small area of this grid, the so-called instantaneous field of view. This chapter describes the characteristics of the image dissector tube data and the reduction steps that prepared these data for further processing in the great circle reduction (as described in Chapter 9), the double star treatment (as described in Chapter 13) and the photometric treatment (as described in Chapter 14). The description of the reductions will use some information described in later chapters, most notably the results and description of the attitude reconstruction (as described in Chapter 7).

5.1. Description of the Measurements and Other Input Data

The Grid

A spherical glass surface, matching the curvature of optimal focus, was located in the focal plane of the telescope. The modulation grid, built up from 168 by 46 elements referred to as scan-fields, was engraved on this surface. Each scan-field contained 16 transparent lines. The method of engraving these lines meant that when projected or-thogonally on a flat plane normal to the optical axis of the curved glass on which it was engraved, these grids are strictly rectangular, but while projected on the spherical surface of the focal plane, they are slightly distorted. These distortions could largely be compensated by expressing grid coordinates as direction cosines rather than angular displacements, as was done by NDAC, or as a two-dimensional fourth order model which includes simultaneously the actual grid distortion and the field-to-grid transformation, as done by FAST. Further (very small) corrections (the so-called medium-scale distortions) for the positions on the grid were determined and applied at the level of the phase determination during the great-circle reductions by NDAC (Section 10.3), and applied at each interlacing period according to the ground based measurements by FAST (Section 5.9).

The grid contained a small number of defects, identified during the pre-launch verification. Some of these defects were recognised in the data analysis in Utrecht ('First Look' analysis). The grid defects were accounted for in the FAST processing by rejecting data obtained close to a grid defect. No special measures were taken for this purpose by NDAC. The amount of data affected was very small.

The grid had a total of $168 \times 16 = 2688$ lines, with an average width of $3.13 \,\mu\text{m}$ and an average separation of $8.20 \,\mu\text{m}$. More detailed specifications can be found in Chapter 2 of Volume 2. When projected on the sky, this gave a grid period of 1.2074 arcsec.

It was established early in the mission that the grid was oriented at an angle of 5 arcmin from the normal to the scanning circle. This was taken into account for the observations from mid-January 1990 onwards, but caused a small amount of degradation of the data accumulated during the first six weeks of the mission before this date.

The Instantaneous Field of View and Pointing

The instantaneous field of view (IFOV) allowed for the observation of objects within a small (30 arcsec diameter) area on the grid. It was directed towards the predicted position of a star by the satellite's on-board computer, using ground-based apparentcoordinate predictions and the satellite based real-time attitude determination (see Chapter 7). Thus, at times when problems occurred with the real-time attitude determination, the pointing of the IFOV was directly affected. Also, when *a priori* coordinates of an object were wrong by more than a few arcsec, then this also influenced the effective pointing. For this reason, many *a priori* coordinates were improved during the mission, initially using mainly results from the star mapper data reductions (see Chapter 6 and Volume 2, Chapter 8), and later using results of early sphere solutions.

The pointing of the IFOV was controlled with so-called coils currents, calibrated onboard the satellite and verified at ESOC at regular intervals during the mission (see also Volume 2, Chapters 5 and 10). This provided the 11×11 element coils current calibration matrix that made it possible for the data reduction consortia to reconstruct the IFOV pointing for every object observed. In the NDAC reductions the coils current calibration matrix was represented by a third order two-dimensional polynomial. The remaining standard deviation of the calibration data with respect to the fitted surface was of the order of 0.4 arcsec. The coils current step-size corresponded to 1.16 arcsec, which was therefore the highest pointing accuracy of the IFOV. The reduction software always selected the calibration nearest in time to the observations. In NDAC, coils currents supplied with observations were translated into positions on the grid and compared with positions based on reconstructed attitude (Chapter 7) and improved stellar positions. This allowed for rejection of data affected by bad pointing. Two selection criteria were used: a 7 arcsec limit, which provided very good quality data but also a relatively large loss of data, and a 10 arcsec limit, which gave a slight deterioration of the data but only very small losses. The final data were all reduced with the 10 arcsec limit. Figure 5.1 shows the IFOV pointing performance over the mission. Time intervals with relatively bad pointing were often associated with gyro-related attitude determination problems.

In FAST, the 11×11 coils current calibration matrix was extended by third order Lagrange interpolation formula to a 41×41 mesh of coils current values, each giving a pair of *G* and *H* coordinate values. This mesh was then inverted into a 41×41 mesh of *G* and *H* values giving the corresponding coils currents. Such a table was constructed for every orbit and then used by linear interpolation formulae to obtain *G* and *H* from the coils current values provided by ESOC for each star observed at mid-time of each observation frame.



Figure 5.1. The observed pointing accuracy of the IFOV over the mission. The graphs show the fraction of observations for which the reconstructed IFOV pointing was within the indicated radius from the actual position of the stellar image. Only observations within 9.5 arcsec of the reconstructed IFOV pointing were used. Those outside were associated with failures of the real-time attitude determination convergence.

The sensitivity profile of the IFOV was calibrated by ESOC and showed that serious attenuation started at about 5 arcsec from its centre. The calibrated average IFOV is shown in Figure 5.2. There were variations with colour and with position in the field of view.

The sensitivity of the IFOV at a distance of 100 arcsec from the centre was still at a level of 0.04 per cent, allowing light from very bright stars to slightly disturb the measurements of fainter stars through the so-called 'veiling glare'. As virtually all bright stars were also programme stars, this effect could be predicted, and veiling-glare corrections were applied by FAST. No veiling-glare corrections were applied by NDAC, where instead all image dissector tube transits were checked *a posteriori* for possible coincidences with other stellar images.



Figure 5.2. The average instantaneous field of view profile.

Observing Strategy

The integration time for a single image dissector tube observation (the sampling period) was $T_1 = 1/1200$ s. The observing strategy, i.e. the allocation of image dissector tube samples to the programme stars and the controlling of the instantaneous field of view pointing, was built on this unit (see Table 8.2). The image dissector tube data were processed in blocks of samples collected over an interval of 32/15 s, referred to as an observational frame $(T_4 = 2560T_1)$. Within this time interval between one and ten stars could be observed quasi-simultaneously. The average number of programme stars in the 0.9×0.9 field was 4.8. The observational frame was split into 16 interlacing periods of 2/15 s ($T_3 = 160T_1$), during which every star would receive its designated fraction of observing time. The observing time was distributed in units of 8 sample periods (also referred to as 'slots', $T_2 = 8T_1$), mainly according to the brightness of the object to be observed and the competition for observing time from other objects. A 'slot' almost covered the passage across one grid period: at an average scan-velocity of 168.75 arcsec s^{-1} (equal to 360 arcsec per observational frame), the path-length of a stellar image over the grid during a 'slot' equalled 1.125 arcsec, just under the 1.2074 arcsec of the grid period.

When switching from one object to the next, a tiny but significant amount of time was lost from the first integration interval after repositioning. Every first sample in the first slot obtained immediately after a repositioning was therefore not used in the reductions. Special provisions were made for brighter stars entering or leaving the field, which were observed for only part of the observational frame in units of two 'slots' per interlacing period. These observations were referred to as 'partially observed stars', and have required special attention in the data reductions. Further details of the star observing strategy are given in Volume 2, Chapter 8.

Analogue Mode and Photon Counting Mode

Stars brighter than 1.5 mag were intended to be measured in analogue mode, fainter stars in photon-counting mode. It was discovered early in the mission that data obtained in the analogue mode was faulty due to a phase shift between the change-over



Figure 5.3. Verification of the intensity transfer function (ITF), using data for the brightest stars. The diagonal line is the expected relation (apart from an offset) for properly decompressed counts. Only counts for Sirius, at -1.5 mag, were still affected by uncorrected saturation.

from photon-counting to analogue mode (and back), and the switch of the instantaneous field of view between the stars concerned. Part of the data thus affected could still be reduced (this was only done by the NDAC consortium), but a proper calibration of the intensities obtained in analogue mode with respect to those obtained in photon-counting mode could not be obtained because of lack of data. The (very small) number of transits affected have not been included in the Hipparcos Epoch Photometry Annex, and concern only stars brighter than Hp = 1.5 with observations obtained before JD 2 447 925.0. It thus follows that all epoch photometry data presented in the Hipparcos Epoch Photometry Annex (HEPA, see Volume 1) were obtained in photon-counting mode, which means that some distortion of the signal could take place, primarily affecting the final photometric results of one or two of the brightest stars, due to saturation of the highest counts.

The Decompression of Photon Counts

The photon counts were compressed on-board the satellite using a semi-logarithmic scheme (see Volume 2, Chapters 3 and 9) to create a 1-byte integer in the range 0 to 255. A decompression law for the photon counts had been provided in the form of a table, relating the 255 possible compressed counts back to actual intensities. These relations were checked and partly re-calibrated, allowing for the effects of truncation and non-linearity. This was done early in the mission, using actual mission data. Figure 5.3 shows the relation for the decompressed counts and the stellar magnitudes for the brightest stars in the mission. Data from only a relatively short stretch of time could be used due to the changes in sensitivity of the image dissector tube tube detector (see Chapter 14). The selected counts were chosen very close to the maximum of the modulated signal, as based on the reconstruction of the phases. Before saturation set in, the noise introduced by the compression and decompression was well below the Poisson noise on the photon counts.

The Scanning Motion

The scanning motion of the satellite was never exactly around its z axis (see Chapter 7). Due to torques working on the satellite, accelerations existed around all three satellite axes, resulting in small amounts of rotation also around the x and y axes, as well as variations in the scan velocity around the z axis. The reduction process of the image dissector tube data used a reference position of the satellite at mid-time of the observational frame, and first and second derivatives with time of this position, to describe the position of the grid with respect to the position of a star at any time during the observational frame.

Quality Flag

A quality flag was added to the data by ESOC, based on the monitoring of the realtime attitude determination. However, during the first two years of the mission, this monitoring was unable to distinguish cases where only one field of view was properly converged in the attitude loop, and not the other. As a result, the use of this quality flag was limited, and did allow faulty data to enter early reduction stages. The quality flag was only used by FAST which, in addition, verified the relative positioning of the instantaneous field of view (using the coil currents) and of the stars, allowing rejection of most of the faulty observing frames in the photometric reduction. The effect on grid coordinates was negligible since this did not affect the modulation phases.

Information from the Catalogue

The star catalogue, initially a preliminary version of the Hipparcos Input Catalogue, provided initial positions, magnitudes and colours for the observed stars. The positions were important for the proper recognition of small scale distortions such as grid errors, and for a comparison between the pointing of the instantaneous field of view and the reconstructed position of the star on the grid. This catalogue was updated several times during the mission (see Volume 2, Section 8.3 and Table 8.3), first using additional ground-based data and later using star mapper data.

Magnitudes and colours were mainly important for the 'optical transfer function' calibration, described in Section 5.9, which allowed a distinction between single stars and double and multiple stars and were used in the photometric reductions and minor planet analysis.

5.2. The Signal Model

Five-Parameter Models

The image dissector tube photon counts N_k obtained for a star during an observational frame were described as a sequence of statistically independent and Poisson-distributed counts, having ideally a time-periodic expected value of $E(N_k) \equiv I_k$. This modulated

[5.4]	[5.1]	[5.3]
β_1	<i>b</i> ₁	$I_b + I_s$
β_2	$b_2^2 + b_3^2$	$I_s M_1$
β_3	$\arctan(-b_3/b_2)$ [+ π]	<i>g</i> ₁
β_4	$(b_2^2 - b_3^2)b_4 + 2b_2b_3b_5 / (b_2^2 + b_3^2)^{3/2}$	$(M_2/M_1)\cos 2(g_1 - g_2)$
β_5	$(b_2^2 - b_3^2)b_5 - 2b_2b_3b_4 / (b_2^2 + b_3^2)^{3/2}$	$(M_2/M_1)\sin 2(g_1-g_2)$

Table 5.1. Relations between the parameters in Equation 5.4, 5.1 and 5.3.

Table 5.2. Relations between the parameters in Equation 5.2, 5.1, 5.3 and 5.5.

[5.2]	[5.1]	[5.3]	[5.5]
	$\mathbf{a} = F_1(\mathbf{b})$		$\mathbf{a} = F_2(\mathbf{r})$
<i>a</i> ₁	<i>b</i> ₁	$I_b + I_s$	<i>r</i> ₁
a_2	$b_2^2 + b_3^2$	$I_s M_1$	r ₂
a ₃	$\arctan(-b_3/b_2)$ [+ π]	g_1	r ₃
a_4	$b_4^2 + b_5^2$	$I_s M_2$	μr_2
a_5	$\arctan(-b_5/b_4)$ [+ π]	$g_1 + g_2$	$r_3 + v$

signal was accurately represented by a first and second harmonic, five-parameter model (higher harmonics were not significant):

$$I_k(\mathbf{b}) = b_1 + b_2 \cos p_k + b_3 \sin p_k + b_4 \cos 2p_k + b_5 \sin 2p_k$$
[5.1]

An equivalent form, separating intensity and phase parameters, was used by FAST:

$$I_k(\mathbf{a}) = a_1 + a_2 \cos(p_k + a_3) + a_4 \cos 2(p_k + a_5)$$
[5.2]

which can also be given, expressed in parameters with direct physical interpretation, as:

$$I_k = I_b + I_s \ 1 + M_1 \cos(p_k + g_1) + M_2 \cos 2(p_k + g_1 + g_2)$$

$$[5.3]$$

In all cases the phases p_k were measured relative to the fiducial reference line of the main grid at mid-time of the observational frame. This reference line was positioned either halfway between slits 1344 and 1345 (NDAC) or in the middle of slit 1345 (FAST). In Equation 5.3, I_b represents the background signal (sky, radiation, dark current), I_s the signal intensity. g_1 is the actual phase of the first harmonic of the signal, relative to the fiducial reference line, at mid-frame time. M_1 and M_2 are the modulation coefficients for the first and second harmonics in the signal, while g_2 represents the phase difference between the first and second harmonic. The image dissector tube reductions aimed at the determination of $I_s + I_b$, g_1 , g_2 , M_1 and M_2 , using the photon counts obtained during an observational frame.

An expression slightly different from Equation 5.2 was used by NDAC, and was referred to as the β -parameter solution:

$$I_k(\beta) = \beta_1 + \beta_2 \cos(p_k + \beta_3) + \beta_4 \cos 2(p_k + \beta_3) + \beta_5 \sin 2(p_k + \beta_3)$$
 [5.4]

where β_4 and β_5 were pure instrument parameters, describing together the amplitude ratio and the phase difference between the first and second harmonics. The relations between the parameters in Equations 5.4, 5.1 and 5.3 are given in Table 5.1 (NDAC reductions). The relations between Equations 5.2, 5.1, 5.3 and 5.5 are given in Table 5.2 (FAST reductions).

Three-Parameter Models

For single stars the amplitude ratio $\mu = a_4/a_2$ and phase difference $v = a_5 - a_3$ were a function of the colour of the object and the position of the measurement on the grid. Their calibration allowed the following representation of the signal:

$$I_k(\mathbf{r}) = r_1 + r_2 \cos(p_k + r_3) + \mu r_2 \cos 2(p_k + r_3 + \nu)$$
[5.5]

as used in the FAST reductions. In the NDAC reductions the three-parameter model was contained in Equation 5.4 through the calibration of β_4 and β_5 . The calibration of these parameters is described in Section 5.9, dealing with the optical transfer function.

The Grid Phase of the Image Centre

The image centre was defined by the angular phase g_0 , and was referred to as the reference phase. In the NDAC processing $g_0 = \beta_3$ was chosen, since β_3 was less sensitive to slight focal variations (which could be due to temperature variations) than was β_5 , which was effectively the phase difference between the first and the second harmonic multiplied by the amplitude ratio of the second and first harmonics.

In FAST, g_0 was expressed as a linear combination of a_3 and a_5 (Equation 5.2):

$$g_0 = (1 - w)a_3 + wa_5$$

The choice of *w* which would minimise the variance of the combination would have been:

$$w = 4a_4^2 / (a_2^2 + 4a_4^2)$$

However, a_2 and a_4 are not known *a priori* and the actual observed values may differ from one observation frame to another. Some constant value had to be taken also in order to facilitate subsequent processing of grid coordinates and present a unique reference throughout the reduction for multiple star reduction. The choice was, using the notation of Equation 5.3:

$$w = 4M_{20}^2 / (M_{10}^2 + 4M_{20}^2)$$
[5.6]

where the zero subscript indicates that these values of the parameters M_1 and M_2 were taken as constant. They were chosen, after evaluating calibrations made by the 'First Look' task, for a mean colour index equal to 0.5. The values chosen were $M_{10} = 0.70$ and $M_{20} = 0.20$, leading to the rounded value w = 0.25.

This choice had at least three advantages:

- (1) to use the maximum amount of information present in a single star. Comparisons with other choices such as using only the first harmonic showed a significant improvement of the residuals in the great-circle reduction;
- (2) in the case of double stars, although Equation 5.6 is not optimum, it provided on average more information than any other value for *w*, especially when M_1 is close to zero as may happen in some cases;
- (3) tests made with 'First Look' results have shown that among all values of *w*, the one selected minimizes the rms of the great-circle reduction results.

5.3. Principles of the Image Dissector Tube Data Processing

The data processing in FAST and NDAC, although different in detail, proceeded along very similar lines. It aimed at estimating:

- the five photo-geometric parameters (**a** or β), together with their respective covariance matrix (**A** or **B**_{β}), to be used in subsequent double star and photometry processing;
- the three photo-geometric parameters of the single star model, to create a statistical test in charge of discriminating between single and multiple stars;
- the reference grid phase g_0 and its standard deviation, in NDAC referred to the fiducial reference line, in FAST referred to a reference slit number n_0 , to be updated in subsequent processing.

The cornerstone of all image dissector tube processing was the following assumption: if the second order Fourier expansion given by Equation 5.1 (or its derived forms) exactly models the expected value I_k of the image dissector tube count N_k of a star, and the relative phases p_k are exactly known for each sample k, then the maximum likelihood estimate $\hat{\mathbf{b}}$, together with its estimated Cramér-Rao bound covariance matrix **B**, tend asymptotically to be sufficient statistics of the image dissector tube samples of the observed star. The same assertion can be made for the alternative formulations leading to the parameter-covariance estimates ($\hat{\mathbf{a}}, \mathbf{A}$) and ($\hat{\boldsymbol{\beta}}, \mathbf{B}_{\beta}$). The five parameters, e.g. **b** in Equation 5.1, could be estimated by maximizing the logarithm of the likelihood function:

$$\ln L(\mathbf{b}) = [N_k \ln I_k(\mathbf{b}) - I_k(\mathbf{b}) - \ln(N_k!)]$$
[5.7]

while the covariance of $\hat{\mathbf{b}}$ could be estimated as the negative inverse of the Hessian matrix, $\mathbf{B} = -[\partial^2 \ln L/\partial \mathbf{b} \partial \mathbf{b}']^{-1}$.

The assumption of sufficiency means that the estimation residuals tend to be a zeromean white noise without any additional information on the modulated star signal. Consequently, any of the pairs ($\hat{\mathbf{b}}$, \mathbf{B}), ($\hat{\mathbf{a}}$, \mathbf{A}) or ($\hat{\boldsymbol{\beta}}$, \mathbf{B}_{β}), corresponding to 20 independent parameters, could be used for further processing, replacing the original photon counts and providing considerable data compression. Furthermore, the constrained estimates ($\hat{\mu}$, \hat{v}) or ($\hat{\beta}_4$, $\hat{\beta}_5$), corresponding to the three-parameter models, as well as the reference grid phase g_0 , could also be computed from the compressed data ($\hat{\mathbf{a}}$, \mathbf{A}) or ($\hat{\boldsymbol{\beta}}$, \mathbf{B}_{β}) without loss of precision.

Obviously, the modelling assumptions had to be carefully tested to ascertain that the estimated pair $(\hat{\mathbf{a}}, \mathbf{A})$ or $(\hat{\boldsymbol{\beta}}, \mathbf{B}_{\beta})$ was statistically sufficient; this was done from the estimation residuals, using a pair of suitable statistical indices, detailed in Section 5.7.

Image Dissector Tube Processing Steps

The image dissector tube processing was partitioned in four steps (here described as done by FAST, the NDAC processing proceeded along very similar lines):

• computation of the relative phase p_k for each photon count N_k ;



Figure 5.4 Flow chart of the FAST image dissector tube processing.

- maximum likelihood estimation of the five parameters **a** and model verification;
- Gauss-Markov estimation of the three parameters **r** and computation of the statistical index F_{35} , discriminating between single and multiple stars;
- veiling-glare correction of **a**, estimation of the reference grid phase g_0 and the slit number n_0 .

The flow chart of the FAST image dissector tube processing is shown in Figure 5.4.

5.4. Calculation of the Relative Phases

The first step in the data processing required assigning relative modulation phases to the individual samples. Relative phases p_k for each sample could be determined very accurately from the position of the star on the grid (accuracy better than 1 arcsec), the angular rates of the satellite axes and the resulting scan-velocity, and the grid geometry, with or without taking into account the medium-scale distortions. The two data consortia used different approaches to the calculation of p_k , but the aim was the same: providing reliable phases for solving Equation 5.1 or its equivalent.

The Relative Phases of Samples as Derived by NDAC

Let **f**, **w** and **z** be orthogonal unit vectors with **f** the relevant viewing direction at the centre of the field, **w** the nominal scanning direction and **z** the normal to the viewing plane. The 'proper direction' to a star (Chapter 12) can then be expressed relative to the instrument as:

$$\mathbf{u} = v\mathbf{f} + w\mathbf{w} + z\mathbf{z}$$
 [5.8]

where *w* and *z* are the field coordinates (direction cosines) along the scan and transverse to the scan, respectively, and $v^2 + w^2 + z^2 = 1$. (*w* in Equation 5.8 should not be confused with the weights *w* defined in Equation 5.6).

For the ideal grid, the modulation phase varies linearly with the field coordinate *w*. In practice, the reference phase may be calculated as:

$$p_k = -\frac{2\pi}{s}(w - w_0)$$
 [5.9]

where *w* is the field coordinate of the image at the time of the *k*th sample, w_0 is the coordinate at the frame mid-time, and *s* is the grid period. The sign in Equation 5.9 is negative because *w* decreases with time when the satellite spins in the nominal sense (positive about **z**), whereas p_k is by definition an increasing quantity.

Neither *w* nor w_0 were known with much precision at the time of the image dissector tube data processing, mainly due to uncertainties of the order of 1 arcsec in the star coordinates. The field coordinates were, however, sufficiently well known and the incremental field coordinate $w - w_0$ could be calculated to within a few milliarcsec for the duration of an observation frame from the scanning velocity. The variations of *w* and *z* with time were evaluated on the assumption that the angular velocity vector direction, but not necessarily the speed, remained constant throughout the frame.

Let **h** denote the unit vector along the instantaneous axis of rotation of the satellite and **u** the unit vector to a star at arbitrary time *t*, relative to axes fixed in the satellite. Then:

$$\mathbf{u} = (\mathbf{h}'\mathbf{u})\mathbf{h} + (\mathbf{h} \times \mathbf{u}) \times \mathbf{h}$$
 [5.10]

Now let \mathbf{u}_0 be the star direction at the mid-time of the frame. Assuming that \mathbf{h} remains fixed relative to the satellite, then $\mathbf{h}'\mathbf{u} = \mathbf{h}'\mathbf{u}_0$ and:

$$\mathbf{h} \times \mathbf{u} = \mathbf{h} \times \mathbf{u}_0 \cos D + \mathbf{h} \times (\mathbf{h} \times \mathbf{u}_0) \sin D$$
 [5.11]

where *D* is the angular phase of the spin at time *t* relative to mid-frame time. The direction of rotation about **h** is such that the relative spin phase decreases with time, i.e. $D = \psi_0 - \psi$, if ψ is the (increasing) attitude angle about the **z** axis. Combining Equations 5.10 and 5.11 gives:

$$\mathbf{u} = (\mathbf{h}'\mathbf{u}_0)\mathbf{h} + (\mathbf{h} \times \mathbf{u}_0) \times \mathbf{h} \cos D + \mathbf{h} \times \mathbf{u}_0 \sin D$$
 [5.12]

or:

$$\mathbf{u} = \mathbf{u}_0 + \mathbf{h} \times \mathbf{u}_0 \sin D + \mathbf{h} \times (\mathbf{h} \times \mathbf{u}_0) (1 - \cos D)$$
 [5.13]

The direction of the instantaneous axis of rotation can be expressed as:

$$\mathbf{h} = h_1 \mathbf{f} + h_2 \mathbf{w} + h_3 \mathbf{z}$$
 [5.14]

where h_1 and h_2 depend on the field of view, but h_3 does not. Scalar multiplication of Equation 5.13 by **w** gives, by means of Equations 5.10 and 5.14:

$$w = w_0 + (h_3 v_0 - h_1 z_0) \sin D + [h_2 h_1 v_0 - (1 - h_2^2) w_0 + h_2 h_3 z_0](1 - \cos D)$$
 [5.15]

where suffix zero denotes direction cosines at the mid-frame time. The coordinate z transverse to the scan is similarly obtained by scalar multiplication of Equation 5.13 by z:

$$z = z_0 + (h_1 w_0 - h_2 v_0) \sin D + [h_3 h_1 v_0 + h_3 h_2 w_0 - (1 - h_3^2) z_0](1 - \cos D)$$
 [5.16]

The attitude determination gives the three components of the total inertial angular velocity vector of the satellite at each frame mid-time, relative to axes fixed in the satellite. Let ω be the total angular velocity at mid-frame time and $\dot{\omega}_3$ the component about **h** of the total angular acceleration; the spin phase at time *t* is then given by $D = -\omega t - \frac{1}{2}\dot{\omega}_3 t^2$. Since v_0 , w_0 , z_0 , h_1 , h_2 , h_3 are assumed to be constant for the observation of a given object in a frame, the required expressions for the variations of *w* and *z* are given by Equations 5.15 and 5.16.

The first stage in the analysis of the 2560 individual image dissector tube sample counts for an observing frame was to calculate the relative spin phase D for the first sample in each slot. Since all the samples in one slot came from the same star, the phase increment was evaluated at the mid-time of each slot, and used throughout the slot to calculate the reference phases of individual sample counts.

Denote by $\tau_n = (8n - 1287.5) T_1$ the time at the centre of the first sample in slot *n*, relative to the mid-frame time, where *n* goes from 1 to 320 over the frame. The relative spin phase D_n of the first sample of slot *n* is calculated from:

$$D_n = -\omega\tau_n - \frac{1}{2}\dot{\omega}_3\tau_n^2 \tag{5.17}$$

Since $\tau_{n+1} = \tau_n + T_2$, where $T_2 = 8T_1$ is the duration of one slot, the relative spin phase at slot n + 1 can be written as:

$$D_{n+1} = D_n + \Delta D_n \tag{5.18}$$
where:

$$\Delta D_n = -(\omega + \dot{\omega}_3 \tau_n) T_2 - \frac{1}{2} \dot{\omega}_3 T_2^2$$
 [5.19]

Also from Equation 5.19:

$$\Delta D_{n+1} = \Delta D_n - \dot{\omega}_3 T_2^2 \tag{5.20}$$

The values of D_1 and ΔD_1 were evaluated at the start of the analysis of each frame, and the relative spin phases of the first samples of subsequent slots were obtained from the recurrence relations given by Equations 5.18, 5.19 and 5.20.

The calculation of reference phases was affected by inaccuracies in the field-to-grid transformation (i.e. in the assumed local value of the grid scale *s*), by variability inside the frame of the derivatives of the field coordinates, and by grid irregularities. The error contribution in the final estimation due to such inaccuracies was, however, negligible, as long as the specifications on angular velocity and field-to-grid transformation were met.

The methods described above were tested before launch using simulated data with and without photon noise. Noise levels introduced through the calculations of the relative phases were well below 0.1 mas, and therefore completely negligible.

Computation of the Relative Phases by FAST

Phase model: The estimate \hat{p}_k of the relative phase p_k for each sample *k* was performed so as to guarantee that the global effect of the estimated errors $\hat{p}_k - p_k$ on the reference phase g_0 would be well below the dispersion due to the average photon noise in an observational frame, which was $\simeq 10$ mas. The budget of the phase errors $\hat{p}_k - p_k$ was constrained to contribute no more than a few milliarcsec noise.

The estimation procedure was based on the following model:

$$p_k = \frac{2\pi}{s} \quad \int_{t_0}^{t_k} \omega(\tau; \mathbf{p}) d\tau + \delta \psi(t_k) + \delta p[G(t_k), H(t_k)] + \epsilon(t_k) \mod 2\pi$$
 [5.21]

where the different terms have the following meaning:

- the integral describes the image motion across the grid from mid-frame time t_0 to sample-time t_k , modelled by a given parametric function for the apparent scan velocity $\omega(t; \mathbf{p}) = dG(t)/dt$ depending on the attitude vector \mathbf{p} . In practice this took into account the combination of the low-frequency components of the attitude motion and the large-scale distortions of the instrument. The integral was computed by assuming during the whole frame a uniform motion with a velocity $\bar{\omega} = \omega(t_0; \hat{\mathbf{p}})$ estimated from the attitude reconstruction (see Chapter 7) performed just before the image dissector tube data processing, and from the large-scale distortion calibrations provided by the 'First Look' task months in advance of the mass processing (see Chapter 4);
- the component $\delta \psi(t_k)$, usually referred to as jitter, described those motion components that could not be modelled by the previous parametric model, such as vibrations induced by thruster firings. However, the satellite design had reduced jitter to a negligible level of a few milliarcsec, except immediately following thruster firings. For this reason, the observation frame following the start of such an actuation was omitted from the data reduction;

- the term δp described the medium-scale distortions of the grid as provided by the on-ground calibrations as one value per scan-field (see also Sections 5.1 and 10.3). The correction values were set once per interlacing period, during which time a star would cross just over one scan-field;
- the term $\epsilon(t_k)$, modelled as a zero-mean white noise term, collected all the high-frequency irregularities, e.g. those resulting from phase quantization (binning).

Since the jitter component $\delta \psi(t_k)$ could be assumed negligible, the above modelling assumptions could be simplified to the following estimate:

$$\hat{p}_k = \frac{2\pi}{s} \quad (k - 1280.5) T_1 \bar{\omega} + \delta p[G(\bar{t}_i), H(\bar{t}_i)] \mod 2\pi, \quad k = k_1(i) \dots k_2(i) \qquad [5.22]$$

where $k_1(i)$ and $k_2(i)$ are the first and last samples of the star within interlacing period *i* and \bar{t}_i is the mid-time of the interlacing period; $T_1 = 1/1200$ s is the IDT sampling period.

Computation of scan velocity: The computation of the apparent scan velocity $\bar{\omega} = dG(t)/dt$ at the mid-frame time t_0 was based on the estimated attitude angles and rates and on the calibrated transformation from the field angles (η, ζ) to grid coordinates (G, H). The field-to-grid transformation was written:

$$G = \sin \eta \cos \zeta + \Delta G(\eta, \zeta), \quad H = \sin \zeta + \Delta H(\eta, \zeta)$$
[5.23]

where the trigonometric terms account for the nominal transformation (Section 10.2) and the additional terms represent the large-scale distortions, in particular the rotation of the grid. Since the distortion terms had been calibrated, a pair of bi-cubic polynomials $g(\eta, \zeta; \mathbf{p}_1)$ and $h(\eta, \zeta; \mathbf{p}_2)$ could be used to compute *G* and *H* as:

$$G = \eta + g(\eta, \zeta; \mathbf{p}_1), \quad H = \zeta + h(\eta, \zeta; \mathbf{p}_2)$$

$$[5.24]$$

after calibration of the 10-component vectors \mathbf{p}_1 and \mathbf{p}_2 . Truncation to the third degree was justified since the fourth-degree terms were less than 1 mas at the field of view borders. Therefore, the apparent scan velocity was computed as:

$$\bar{\omega} = \dot{\eta} \quad 1 + \frac{\mathrm{d}g(\eta, \zeta; \mathbf{p}_1)}{\mathrm{d}\eta} + \dot{\zeta} \frac{\mathrm{d}g(\eta, \zeta; \mathbf{p}_1)}{\mathrm{d}\zeta}$$

$$[5.25]$$

evaluated for $t = t_0$. During the data preparation (Chapter 4) the apparent coordinates $v(t_0)$, $r(t_0)$ in the great-circle frame (see also Section 9.2) had been calculated for each star observed in a frame. The three attitude angles $\psi(t_0)$, $\phi(t_0)$ and $\theta(t_0)$ were obtained from the attitude reconstruction (Chapter 7). The field angles (η , ζ) at mid-frame time t_0 were computed from these angles according to the equations below, where the time argument has been omitted for simplicity:

$$\eta = \psi - v \pm \frac{1}{2}\gamma - \arcsin\left(\frac{\Delta}{\cos\zeta}\right)$$

$$\zeta = \arcsin\left\{\cos r\left[\sin\theta\cos(\psi - v) + \sin\phi\cos\theta\sin(\psi - v)\right] + \sin r\cos\phi\cos\theta\right\}$$
[5.26]

where the upper and lower sign refers to the preceding and following field of view. The basic angle, as calibrated beforehand in the great-circle reductions of the 'First Look' task, including a correction for chromaticity according to the colour index of the star, is given by:

$$\gamma = \gamma_0 + [(V - I) - 0.5]\gamma_1$$

and a second-order correction in the small angles ϕ , θ and *r* can be expressed as:

$$\Delta = \frac{1}{2}\cos r \quad \sin\phi\sin 2\theta\cos 2(\psi - v) + \frac{\sin^2\phi\cos^2\theta - \sin^2\theta}{1 + \cos\theta\cos\phi}\sin 2(\psi - v) + \sin r[\sin\phi\cos\theta\cos(\psi - v) - \sin\theta\sin(\psi - v)]$$

Unlike η , the transversal field angle ζ depends to first order on these angles.

Similar formulae give the angular field rates from the attitude rates $\dot{\phi}(t_0)$, $\dot{\theta}(t_0)$ and $\dot{\psi}(t_0)$ also provided by the attitude reconstruction (Chapter 7). They were simplified by assuming (without degradation of accuracy) that the rate $\dot{\psi}(t_0)$ of the FAST scan angle was equivalent with the inertial rate around the third axis:

$$\dot{\eta} = \cos\phi\cos\theta \quad \dot{\psi} + \frac{\theta\sin\phi + \phi\sin\theta}{1 + \cos\phi\cos\theta} \quad -\dot{\theta}\sin\phi \\ + \tan\zeta \sin(\psi - v) \quad \dot{\theta}\cos\phi - \dot{\psi}\sin\phi\cos\theta \quad -\cos(\psi - v) \quad \dot{\phi} - \dot{\psi}\sin\theta \\ \dot{\zeta} = \sin(\psi - v) \quad \dot{\phi} - \dot{\psi}\sin\theta \quad +\cos(\psi - v) \quad \dot{\theta}\cos\phi - \dot{\psi}\sin\phi\cos\theta$$
[5.27]

In their application, these formulae were computed through sufficiently accurate power expansions in the small angles ϕ , θ , and r.

5.5. Binning Techniques

At the time of the development and early implementation of the data reduction software, it was essential to look for ways of limiting the time spent on the processing of the 1200 image dissector tube samples received every second. Both data reduction consortia opted for a binning strategy, reducing the main processing tasks by a very considerable factor in both computing time and complexity.

The binning strategy assigns a reference phase p_i to all samples with phases p_k falling in a phase interval $p_i - dp$ to $p_i + dp$. With $p_k = p_i + \delta p_{k|i}$, and n_i samples falling in bin *i*, the expected count in bin *i* can be expressed as a modification of Equation 5.1:

$$\frac{1}{n_{i}} \qquad N_{k|i} = b_{1} + b_{2} \frac{1}{n_{i}} \quad \cos p_{i} \qquad \cos \delta p_{k|i} - \sin p_{i} \qquad \sin \delta p_{k|i} \\
+ b_{3} \frac{1}{n_{i}} \quad \sin p_{i} \qquad \cos \delta p_{k|i} + \cos p_{i} \qquad \sin \delta p_{k|i} \\
+ b_{4} \frac{1}{n_{i}} \quad \cos 2p_{i} \qquad \cos 2\delta p_{k|i} - \sin 2p_{i} \qquad \sin 2\delta p_{k|i} \\
+ b_{5} \frac{1}{n_{i}} \quad \sin 2p_{i} \qquad \cos 2\delta p_{k|i} + \cos 2p_{i} \qquad \sin 2\delta p_{k|i} \qquad [5.28]$$

The advantage of the binning arises from the δp_k being small, allowing approximations to be made for the sin $\delta p_{k|i}$ etc., and from the fact that the p_i values remain constant, and therefore need to be calculated only once for all transits.

Two different implementations were used: the FAST consortium applied a strategy using 64 bins, and neglected the effect of δp_k , in which case binning corresponded to a quantization of the relative phase. It was estimated that the error on the relative phase p_k resulting from the binning should be less than $\simeq 10$ mas, in which case the quantization

effects on the grid phase estimate would be negligible with respect to the photon noise. The quantization noise σ_q for *l* bins was given by:

$$\sigma_q = \frac{s}{2\sqrt{3}l} \tag{5.29}$$

where *s* is the grid-period of 1.2074 arcsec. For l > 35 it follows that $\sigma_q < 10$ mas. *l* was rounded up to the nearest power of 2, i.e. l = 64, giving $\sigma_q = 5.5$ mas and a noise contribution to the grid phase estimates less than 0.5 mas.

NDAC performed a substantial number of simulations and tests with various binning strategies, using both noise-free and Poisson-noise simulated data. The results of these tests showed that a substantial reduction in computing time without any significant loss of accuracy could be obtained using l = 12 bins and carrying along up to second order corrections for δp_k . The noise contribution to the phase estimates resulting from the binning this way, was less than 0.2 mas.

In the FAST approximation Equation 5.28 reduces to Equation 5.1, with p_k being replaced by the nearest p_i and $N_i = N_{k|i}$ being the total count per bin. In the NDAC approximation the equations obtained are:

$$\frac{1}{n_{i}} \qquad N_{k|i} = b_{1} + b_{2} \frac{1}{n_{i}} \quad \cos p_{i} \qquad (1 - \frac{1}{2} \delta p_{k|i}^{2}) - \sin p_{i} \qquad \delta p_{k|i} \\ + b_{3} \frac{1}{n_{i}} \quad \sin p_{i} \qquad (1 - \frac{1}{2} \delta p_{k|i}^{2}) + \cos p_{i} \qquad \delta p_{k|i} \\ + b_{4} \frac{1}{n_{i}} \quad \cos 2p_{i} \qquad (1 - 2\delta p_{k|i}^{2}) - \sin 2p_{i} \ 2 \qquad \delta p_{k|i} \\ + b_{5} \frac{1}{n_{i}} \quad \sin 2p_{i} \qquad (1 - 2\delta p_{k|i}^{2}) + \cos 2p_{i} \ 2 \qquad \delta p_{k|i} \qquad [5.30]$$

This no longer required repeated trigonometric calculations, as these calculations were the same for each transit.

Critical Binning Conditions

For certain values of the scan-velocity the relative grid phases p_k would assume only a limited number of different values. At nominal scan velocity (168.75 arcsec s⁻¹), the phase shift from one sample to the next was 41°.92. At a scanning speed of 170.456 arcsec s⁻¹, the phase shift to the next sample was 42°.35, which meant that the phase shifts of 17 samplings fitted in exactly 2 complete modulation cycles. Similar situations happened for scan velocities of 173.866 arcsec s⁻¹ (25 samplings in 3 cycles) and 167.178 arcsec s⁻¹ (26 samplings in 3 cycles).

In the case of 64 bins without phase correction, this left a large number of bins unoccupied, and could cause systematic differences between assumed bin-phases and actual mean bin-phases. The latter effect was, however, small with respect to other sources of noise, most notably the Poisson noise on the photon counts.

The scan velocity was changing continuously, and a resonance situation similar to that described above would normally not persist over more than a few frames. No measures were taken to remedy these phenomena by FAST, while in NDAC they were implicitly taken care of in the normal reductions. In FAST, the statistical index T (described later) was able to detect a non-negligible bias in the grid phase, for example due to resonances.

5.6. Solution of the Binned Equations

Five-Parameter Estimation by FAST

The parameter estimation aimed at performing the maximum likelihood estimation of the five-parameter model (Equation 5.2). The solution equation was obtained by looking for stationary points of the maximum likelihood functional (Equation 5.7):

$$\frac{\partial \ln L}{\partial \mathbf{a}} = \sum_{k} \frac{N_k - I_k(\mathbf{a})}{I_k(\mathbf{a})} \frac{\partial I_k(\mathbf{a})}{\partial \mathbf{a}} = 0$$
 [5.31]

Equation 5.31 is non-linear in **a**, but a unique asymptotic solution exists if the model were identifiable and error free. Under such assumptions, the above equations could be interpreted as the solution of a Gauss-Markov estimation of **a**, performed on the zero-mean uncorrelated residuals $e_k = N_k - I_k(\mathbf{a})$, having variance equal to $I_k(\mathbf{a})$. However, since the variance $I_k(\mathbf{a})$ depended on the unknown **a**, the maximum likelihood could only be solved by iterating the Gauss-Markov estimation and by disposing of a suitable starting estimate of $I_k(\mathbf{a})$. On the other hand, if I_k were linear in the parameters as it is in Equation 5.1, then the Gauss-Markov estimator should correspond to weighted least-squares.

These considerations led to the following procedure:

- (1) estimation of **b** by Fourier transforming the binned counts N_i , providing a first estimate $\tilde{\mathbf{b}}$;
- (2) weighted least-squares estimation of **b**, given the estimated variances $I_k(\hat{\mathbf{b}})$, hence providing the pair $(\hat{\mathbf{b}}, \mathbf{B})$ behaving like a sufficient statistic (even in the presence of veiling-glare);
- (3) non-linear transformation from the pair $(\hat{\mathbf{b}}, \mathbf{B})$ to the pair $(\hat{\mathbf{a}}, \mathbf{A})$; in the course of this transformation it would have been possible to eliminate the biasing effect of the veiling-glare, as will be shown later on. In fact, the veiling-glare correction was performed just at the end of the image dissector tube processing;
- (4) verification of the model hypotheses through statistical tests.

The Fourier transformation of the binned counts N_i used the equations:

$$\tilde{b}_{1} = \frac{1}{I} \int_{i=0}^{I-1} \mathcal{N}_{i}$$

$$\tilde{b}_{2} = \frac{2}{I} \int_{i=0}^{I-1} \mathcal{N}_{i} \cos(2\pi i/I), \quad \tilde{b}_{3} = \frac{2}{I} \int_{i=0}^{I-1} \mathcal{N}_{i} \sin(2\pi i/I) \quad [5.32]$$

$$\tilde{b}_{4} = \frac{2}{I} \int_{i=0}^{I-1} \mathcal{N}_{i} \cos(4\pi i/I), \quad \tilde{b}_{5} = \frac{2}{I} \int_{i=0}^{I-1} \mathcal{N}_{i} \sin(4\pi i/I)$$

The second estimation step also took advantage of the binning, by reducing the number of observation equations to a fixed number (l = 64) of the following form:

$$\mathbf{u}'_{i}\mathbf{b} + e_{i} = \mathcal{N}_{i}, \quad i = 0 \dots l - 1$$
 [5.33]

where $\mathbf{u}_i = [1 \cos(2\pi i/l) \sin(2\pi i/l) \cos(4\pi i/l) \sin(4\pi i/l)]'$ is the vector of coefficients for bin *i*. The variance of the residual e_i was assumed to be $\sigma_i^2 = (l/m) I_i(\tilde{\mathbf{b}})$, with *m* being the total number of samples used. By collecting the *l* rows in Equation 5.33 in matrix form, $\mathbf{U}\mathbf{b} + \mathbf{e} = \mathbf{N}$, the weighted least-squares estimate was obtained as:

$$\hat{\mathbf{b}} = (\mathbf{U}'\mathbf{S}^{-1}\mathbf{U})^{-1}\mathbf{U}'\mathbf{S}^{-1}\mathbf{N}$$
 [5.34]

where **S** is the diagonal covariance matrix of **e**.

The binning further simplified the computation of the (symmetric) Gram matrix $\mathbf{G} = \mathbf{U}'\mathbf{S}^{-1}\mathbf{U}$. 12 of the 15 different elements in \mathbf{G} could be expressed as linear combinations of the first three elements:

$$g_{11} = \frac{m}{l} \frac{l^{-1}}{I_i(\tilde{\mathbf{b}})}, \quad g_{12} = \frac{m}{l} \frac{l^{-1}}{I_i(\tilde{\mathbf{b}})}, \quad g_{13} = \frac{m}{l} \frac{l^{-1}}{I_i(\tilde{\mathbf{b}})} \frac{\sin(2\pi i/l)}{I_i(\tilde{\mathbf{b}})}$$
(5.35)

By collecting the combinations of these three independent terms into a 12-element vector γ and the twelve remaining elements of **G** into a vector **g**, a linear set of twelve equations results:

$$\boldsymbol{\gamma} = \mathbf{H}\mathbf{g}$$
 [5.36]

the non-zero elements of which are detailed in the following equation:

$m - b_1 g_{11} - b_2 g_{12} - b_3 g_{13}$	b_4	b_5											<i>g</i> 14	
$-b_3g_{12} + b_2g_{13}$	$2b_5$	$-2b_4$											<i>§</i> 15	
0		-1	2										<i>8</i> 23	
<i>B</i> 11				1	1								<i>8</i> 22	
0	-1			1	-1								<i>8</i> 33	
$-b_1g_{12}$	_		b_3	b_2		b_4	b_5						<i>8</i> 24	
<i>g</i> 13	-						1	-1					<i>8</i> 25	
$-b_1g_{13}$			b_2		b_3			b_4	b_5				<i>8</i> 34	
g_{12}						1			1				<i>8</i> 35	
0	<i>b</i> ₁				b_2		b_3			b_4	b_5		<i>g</i> 44	
<i>g</i> ₁₁										1		1	g_{45}	
0		<i>b</i> ₁				b_2		b_3			b_4	b_5	<i>8</i> 55	
													[5.3	37]

This system can be easily solved for the unknown $\mathbf{g} = \mathbf{H}^{-1}\boldsymbol{\gamma}$ by exploiting the quasitriangularity (Hessenberg form) of the matrix **H**. The six rows with numerical coefficients, corresponding to the entries g_{22} , g_{23} , g_{25} , g_{33} , g_{35} and g_{45} , arose from trigonometric equalities; the six rows having coefficients equal to the elements of **b** arose from the weighted averaging of trigonometric functions sampled at regular steps.

Five-Parameter Estimation by NDAC

Each bin contributed one observation equation for the solution of the parameters b_1 to b_5 . Each observation equation was given a weight according to the total count $N_{k|i}$ in the bin:

$$W_i = \frac{n_i}{1 + N_{k|i}}$$
[5.38]

The addition of one in the denominator prevented an infinite weight for $N_{k|i} = 0$. It can also be derived from an estimation of the most likely distribution to which an observed count belongs: when a total count of zero is observed, the most likely underlying distribution would have an expected value of one. Thus, in the NDAC

ParameterNDACFAST'dc' photometry β_1 $a_1 - I_b$ 'ac' photometry β_2 $(M_{10}a_2 + M_{20}a_4)/(M_{10}^2 + M_{20}^2)$ Reference phase β_3 $(1 - w)a_3 + wa_5$

Table 5.3. Definitions of astrometric and photometric parameters used in FAST and NDAC.

solution the only differences with treating every sampling on its own were the weight applied and the (very small) approximation of the reference phase. Because only twelve bins were used, the total number of samples per bin was usually not very low, and provided a reasonable estimate of the expected variance in a bin.

Transformation of the Solution

The sufficient statistics ($\hat{\mathbf{b}}$, \mathbf{B}) provided by the Gauss-Markov estimator were at the end transformed without degradation into the statistics ($\hat{\mathbf{a}}$, \mathbf{A}), defined by Equation 5.2 for FAST, or ($\hat{\boldsymbol{\beta}}$, \mathbf{B}_{β}), defined by Equation 5.4 for NDAC. The transformation of the parameter vectors was made according to Table 5.1 and 5.2. The transformation of \mathbf{B} to \mathbf{B}_{β} was made by means of the Jacobian $\mathbf{J} = \partial \boldsymbol{\beta} / \partial \mathbf{b}'$ according to:

$$\mathbf{B}_{\beta} = \mathbf{J}\mathbf{B}\mathbf{J}' \tag{5.39}$$

and a corresponding equation was used to calculate A.

The transformations from $\hat{\mathbf{b}}$ to $\hat{\boldsymbol{\beta}}$ (or $\hat{\mathbf{a}}$) led to biases in the amplitude estimates due to accumulation of squared errors. This was most noticeable for fainter stars, where the statistical correction led to an increase in the noise level on the transformed values. In particular, the correction to be subtracted from $\hat{\beta}_2$ was given by:

$$\Delta\beta_2 = (\sigma_2^2 + \sigma_3^2)/2\hat{\beta}_2$$
 [5.40]

where σ_2 and σ_3 are the estimated errors on \hat{b}_2 and \hat{b}_3 respectively. Typical corrections amounted to a few per cent for the faintest stars, and negligible corrections for brighter stars.

The Photometric Parameters

The final solution, expressed as either Equation 5.2 or Equation 5.4 provided the input for the further processing of the photometric data. The two consortia made slightly different choices here: NDAC opted for simple parameters directly obtained from the zero level and first harmonic Equation 5.4, while FAST used combined information from the first and second harmonics for both the phase estimate used in the greatcircle reductions and the 'ac' component used in the photometric reductions. The parameters, as used by the two groups, are summarised in Table 5.3, where M_{10} and M_{20} stand for the predicted modulation coefficients as derived from the optical transfer function calibration described in Section 5.9 and *w* was defined in Equation 5.6.

5.7. Statistical Tests of the Five-Parameter Solution

Two test parameters were derived at this stage: F and T. The first of these, F, tested the null hypothesis that no modulation was present in the signal, in other words, that the observed photon counts collected for a transit were a stationary Poisson white noise; hence F should appear significant when the photon-counts were sufficiently modulated. The second of these, T, tested the null hypothesis that the residuals left after application of the five-parameter model represented a zero-mean stationary white noise. Hence, T should appear significant when the residuals were modulated (failure of the 5-parameter model) or the original photon-counts were not Poisson distributed. The tests were carried out on the binned counts.

The Index T

In NDAC, a χ^2 value for the *T* statistic was derived for the binned mean counts, assuming them to be represented by $I_k(\hat{\mathbf{b}})$:

$$\chi^{2} = \prod_{i=1}^{l} \frac{(1/n_{i}) N_{k|i} - I_{k}(\hat{\mathbf{b}})^{2}}{(1/n_{i}) N_{k|i}}$$
[5.41]

where l = 12 is the number of bins and where the denominator represents the expected variance of the residuals. The χ^2 value was transformed into a pseudo-Gaussian variable, which under the null hypothesis should have a unit-normal distribution:

$$T = \frac{\overline{9d}}{2} - \frac{\chi^2}{d} - 1 + \frac{2}{9d}$$
 [5.42]

where *d* is the number of degrees of freedom, normally equal to l - 5 = 7 for the *T* statistic. Transits for which |T| > 4.0 were flagged in the Hipparcos Catalogue Epoch Photometry Annex as suspicious.

In FAST, a binned mean square error was computed as:

$$s^{2} = \int_{i=1}^{l} \frac{N_{i} - I_{i}(\hat{\mathbf{b}})^{2}}{(l/m) I_{i}(\tilde{\mathbf{b}})}$$
[5.43]

where l = 64 is the number of bins and *m* the total number of samples used. The statistic *T* was then computed as:

$$T = -\frac{\overline{l-5}}{2} - \frac{s^2}{l-5} - 1$$
 [5.44]

Under the null hypothesis, s^2 follows the chi-square distribution with l-5 = 59 degrees of freedom and T approaches asymptotically the unit-normal distribution. The null hypothesis was rejected if |T| > 3.5, which happened in less than 0.5 per cent of all transits.

The Index F

In NDAC, a χ^2 value for the *F* statistic was derived for the binned mean counts, assuming them to be stationary:

$$\chi^{2} = \frac{1}{\sum_{i=1}^{I} \frac{(1/n_{i}) N_{k|i} - \hat{b}_{1}^{2}}{(1/n_{i}) N_{k|i}}}$$
[5.45]

As for the *T* statistic, the χ^2 value was transformed into a pseudo-Gaussian variable, except that the number of degrees of freedom was *d* = 11.

In FAST the binned mean square of the estimated modulation components was computed as:

$$p^{2} = \prod_{i=1}^{l} \frac{I_{i}(\hat{\mathbf{b}}) - \hat{b}_{1}^{2}}{(l/m) I_{i}(\tilde{\mathbf{b}})}$$
[5.46]

having four degrees of freedom. The statistical index F was then defined as a Fisher ratio between the statistics in Equations 5.46 and 5.43:

$$F = \frac{p^2/4}{s^2/(l-5)}$$
 [5.47]

Since l - 5 was rather large, 4F almost followed a χ^2 distribution with four degrees of freedom. Accordingly, the null hypothesis was accepted (no modulation found) if F < 4.5. This happened in less than 1.5 per cent of all transits.

5.8. Veiling-Glare Correction by FAST

The purpose of the veiling-glare correction was to clean the five parameters of $\hat{\mathbf{a}}_j$ of programme star *j* from the possible perturbations of the intensity and phase estimates by brighter stars observed during the same frame. Such perturbations were due to the instantaneous field of view sensitivity all over the field of view.

The image dissector tube efficiency was taken from on-ground calibration and represented by 9 functions centred on 9 regularly distributed points of the grid. Each function was represented by 11 values given along radii every 30° . These values corresponded to distances to the centre expressed in arcsec such that their logarithm grows regularly from 1.5 to 4.0. The efficiency at a given point was computed by interpolating this three-parameter table using linear or quadratic interpolation formulae.

The most general way to implement the veiling-glare correction would have been to start from the observed data $(\hat{\mathbf{b}}_j, \mathbf{B}_j)$ for all transits in a frame, and directly estimate the corrected data $(\hat{\mathbf{a}}_i, \mathbf{A}_j)$. Under the following assumptions:

- veiling glare was only due to other programme stars observed in the same frame;
- all of the programme stars had the same motion component *p_k* across the slits at time *t_k*;
- the instantaneous field of view was properly centred for each observed star;

• the instantaneous field of view sensitivity profile $\Psi(\varrho)$ was perfectly known;

the resulting statistics would still have been sufficient. The basic formulae for correcting an observation j for the influence of a brighter observation l would have been the following:

$$\hat{a}_{1j} = b_{1j} - f b_{1l}$$

$$\hat{a}_{2j} = \overline{(\hat{b}_{2j} - f \hat{b}_{2l})^2 + (\hat{b}_{3j} - f \hat{b}_{3l})^2}, \quad \hat{a}_{3j} = \arctan -\frac{\hat{b}_{3j} - f \hat{b}_{3l}}{\hat{b}_{2j} - f \hat{b}_{2l}}$$

$$\hat{a}_{4j} = \overline{(\hat{b}_{4j} - f \hat{b}_{4l})^2 + (\hat{b}_{5j} - f \hat{b}_{5l})^2}, \quad \hat{a}_{5j} = \arctan -\frac{\hat{b}_{5j} - f \hat{b}_{5l}}{\hat{b}_{4j} - f \hat{b}_{4l}}$$
[5.48]

where $f = \Psi(\rho_{jl})$ is the instantaneous field of view attenuation factor depending on the angular separation of the two pointings.

In practice the above corrections would have been seriously affected by calibration uncertainties in the instantaneous field of view profile, which increased in relative terms as the value of f decreased. Since the veiling-glare corrections were proportional to $\Psi(\varrho)$, the quantity:

$$q = \frac{\mathrm{d} \ln \Psi(\varrho)}{\mathrm{d} \varrho} \, \sigma_{\varrho}$$

indicates the relative precision of the corrections, given an estimate of the standard deviation σ_{ϱ} , including uncertainties in instantaneous field of view positioning and star distance. Since $d \ln \Psi(\varrho)/d\varrho < 0.05 \ \mathrm{arcsec}^{-1}$ for $\varrho > 20 \ \mathrm{arcsec}$, the veiling-glare correction could be effective also with σ_{ϱ} around 2 arcsec.

The procedure adopted by FAST made the correction only when necessary and directly on the statistics ($\hat{\mathbf{a}}$, \mathbf{A}), which required more complex formulae than those presented above. The corrected phases \hat{a}_3 and \hat{a}_5 and the covariance matrix \mathbf{A} were then used to estimate the reference phase g_0 and its variance σ_0^2 according to:

$$\hat{g}_0 = \hat{a}_3 + w(\hat{a}_5 - \hat{a}_3), \quad \sigma_0^2 = (1 - w)^2 A_{33} + w^2 A_{55} + 2(1 - w) w A_{35}$$
 [5.49]

where *w* was defined by Equation 5.6, and A_{ik} are elements of the estimated covariance **A**.

5.9. Optical Transfer Function Calibration and Three-Parameter Solution

The transit of a double or multiple component object consisted of the superposition of two or more signals as described by Equation 5.2 or 5.4. This changes the values observed for (μ, ν) or (β_4, β_5) . For single stars, these values were a function of position in the field of view and of the colour of the star and were described by the optical transfer function. By calibrating the optical transfer function, it became possible to test the hypothesis that the transit was due to a single star only.

Calibration of the Optical Transfer Function

In NDAC, the calibration of the optical transfer function was done using the values for (μ, ν) or (β_4, β_5) collected over one orbital period of the satellite. The successful



Figure 5.5. The behaviour of β_4 and β_5 as a function of position in the field at the beginning of the mission. The scan direction is indicated by the arrow, the two graphs for β_4 and β_5 refer to the two fields of view. Scales for the two fields of view are identical.

collection of data was always interrupted by the perigee passage of the satellite, where observing conditions were very poor and no ground station control was available over a period of 1 to 2 hours around perigee time. Thus, data collected during an orbit formed a natural unit for many calibrations.

The frame transit data was associated with a position on the grid (G, H) at mid-frame time, and with the colour index for the star observed. The calibration values obtained for stars not known or found to be double were modelled with a two-dimensional third-order polynomial in position, first and second order in colour index, and cross-terms between colour and position on the grid, 14 parameters in total. The calibrations were done independently for the two fields of view. Of the parameters, the positional dependence



Figure 5.6. Evolution with time of the phase difference v and the amplitude ratio μ . Data for the preceding field of view are shown by filled symbols (points in the lower parts of the graphs), for the following field of view by open symbols (points in the upper parts of the graphs). Features in the top graph around days 450 and 1020 are related to variations in the satellite's exposure factor, shown in Figure 8.3.

was the most important. An example of this dependence is shown in Figure 5.5. The calibration was time dependent, as shown in Figure 5.6, and affected by refocusing of the instrument. The colour dependence in the preceding field of view for β_4 increased during the mission by a factor 2, while for β_4 in the following field of view and for β_5 this dependence was much less a function of time. The amplitude ratio μ decreased towards redder stars, but could not be calibrated for the very red stars, as too few measurements were available. At B - V = 2 the decrease was approximately 10 per cent in the following field of view, less in the preceding field of view.

In FAST, the calibration of the optical transfer function was, as were most calibrations, performed twice. For the image dissector tube data processing, the calibration was done in Utrecht by the 'First Look' task once a week. The mean for a calibration period

(as defined in Chapter 4) was then made and used by the image dissector tube data processing as described in this chapter. In a second run, the five-parameter solutions obtained in this processing were used in a more refined analysis intended to be used by the multiple star and photometry tasks. About twenty orbits contributed to each calibration period, chosen from those giving the best results in the great-circle reduction.

The procedure was as follows. All the results of the five-parameter solution were examined and several tests performed in order to exclude known double stars, faint stars (magnitude > 11), stars with an unknown colour, and all stars for which either the ratio of the 'ac' to 'dc' photometry or the difference $a_3 - a_5$ suggested that the star was double. Additional rejections were made from the analysis of histograms of these quantities. The 'ac' intensity was computed and used to determine M_1 and M_2 from a_2 and a_4 .

Each quantity M_1 , M_2 and $a_3 - a_5$ for each field of view was expressed as a third-order polynomial in the grid coordinates G and H plus a similar polynomial multiplied by C - 0.5, where C is the colour index; this gave 20 parameters in total. The equations were solved by least-squares giving for each a weight proportional to the intensity of the star (a_1) . An *a posteriori* rms was computed as two rms residuals of the colour independent part and of the colour dependent part.

In addition, the reference intensity response was calibrated for a mesh of 19×19 points on the grid for three classes of colours and also by a polynomial of the third order in *G* and *H*, second order in colour and some mixed terms.

The Three-Parameter Solution

In NDAC, the optical transfer function was applied to the (*G*, *H*) coordinates and star colour of each frame transit to provide predicted values for β_4 and β_5 . These provided, together with the relevant elements of \mathbf{B}_{β}^{-1} , a χ^2 estimate for the likelihood of the signal being the result of one point source only:

$$\chi^{2} = \delta\beta_{4}^{2} [\mathbf{B}_{\beta}^{-1}]_{44} + 2\delta\beta_{4}\delta\beta_{5} [\mathbf{B}_{\beta}^{-1}]_{45} + \delta\beta_{5}^{2} [\mathbf{B}_{\beta}^{-1}]_{55}$$

$$[5.50]$$

where $\delta\beta_4$ is the difference between the observed and predicted value of β_4 , and similarly for $\delta\beta_5$. The χ^2 values were collected per star for the purpose of double-star recognition. This was done using a transformation of χ^2 into a variable that has a flat distribution for single point-source objects, but becomes increasingly skew for disturbed objects:

$$c = 8 \exp(-0.5\chi^2)$$
 [5.51]

where the factor 8 allowed the accumulation into eight discrete bins by taking the integer part of c as the index of the bin (from 0 to 7). A second criterion for detecting similar problem cases was derived from the photometric reductions and is described in Chapter 14.

In FAST the optical transfer function calibration allowed for the representation of the signal through the three-parameter model, Equation 5.5. For a true single star transit, the residuals $I_k(\hat{\mathbf{b}}) - I_k(\hat{\mathbf{r}})$ should be a zero-mean white noise, with the variance estimated by s^2 as in Equation 5.43. The statistical test used the Fisher ratio F_{35} :

$$F_{35} = \frac{p_{35}^2/2}{s^2/(l-5)}$$
[5.52]

where:

$$p_{35}^2 = \frac{m}{k=1} \frac{I_k(\hat{\mathbf{b}}) - I_k(\hat{\mathbf{r}})^2}{I_k(\hat{\mathbf{b}})}$$
[5.53]

was estimated from the residuals of Equation 5.5. Also, in this case, the Fisher variable $2F_{35}$ could be assumed to be asymptotically distributed as a χ^2 distribution having two degrees of freedom. The null hypothesis (single star) was rejected when $F_{35} > 3.5$. The F_{35} statistic became unreliable for stars of extreme colour, as these could not be included in the calibration model as described above.

5.10. Comparisons

Comparisons were carried out before launch using simulated data, and after launch in March 1991 using data from one orbit, covering roughly 11000 frames and 50000 individual transits. This comparison showed that differences in the reduction results between the two consortia were very small and in all cases negligible. The comparison exercise resulted in a relaxing of the instantaneous field of view pointing accuracy criterion in NDAC from 7 to 10 arcsec and a correction of the bias in the estimate of the first harmonic amplitude. Standard deviations of the differences between the phase estimates were well below the expected photon-noise level, showing that both consortia were producing results that were not significantly affected by any calibration or modelling errors: the noise remaining on the estimated parameters was primarily the result of the original photon noise on the counts, which was the same for both groups.

F. van Leeuwen, E. Canuto, F. Donati, J. Kovalevsky

6. STAR MAPPER DATA PROCESSING

The star mapper data formed the second data stream for the Hipparcos satellite. Its primary goal was the provision of measurements for monitoring the attitude of the satellite, using the transits of selected stars through two sets of four slits. The recognition of transits and determination of transit times and signal intensities is the subject of this chapter. The continuous data stream from the same detectors was analysed as the Tycho data stream, of which the processing is described in Volume 4.

6.1. The Measurement Principles

The Star Mapper Slits

Star mapper slits were situated on either side of the main grid, but only those preceding the main grid in the scanning motion were used during the mission. There were two slit groups, 4 inclined slits, followed by 4 vertical slits (as shown in Figures 6.1 and 6.2). The uneven spacing of the slits made it possible to recognise a transit in absolute sense (contrary to the regular main grid, where the transit time had an ambiguity of an integer number of slit-intervals). The average width of each slit was 0.909 arcsec in the vertical slit-group, 0.916 arcsec in the inclined slit-group upper branch, and 0.922 arcsec in the lower branch. The variations in width were less than 0.1 per cent.

The star mapper signal was acquired from all information coming through the entire grid. Thus, it mixed data from different parts of the sky, both due to the superposition of the two fields of view as well as due to the size of the area covered. The signal was split into a *B* and *V* channel, roughly equivalent to B_J and V_J channels, and referred to as B_T and V_T , the *T* standing for 'Tycho' (see also Volume 1, Section 1.3).

The Data Sampling and Extraction

The data were sampled at a rate of 600 Hz simultaneously in the two channels. At the nominal scanning rate of 168.75 arcsec s^{-1} , the grid line spacings corresponded to intervals of 40, 60 and 20 sampling periods. The extraction of samples for processing by NDAC and FAST was based on the real-time attitude determination and the apparent positions of the stars at the time of observation. This predicted transit time as well as the star identifier formed part of the star mapper record.



Figure 6.1. The star mapper slits (B) relative to the main grid (A). The scan direction, i.e. the motion of a star, is indicated by the arrow. The fiducial reference line for the main grid is indicated by 'a', for the vertical slits by 'b' and for the inclined slits by 'c'.



Figure 6.2. The reconstructed slit response for a transit through the vertical slits in the preceding field of view. The detector is the V_T channel. The origin on the horizontal axis corresponds to the fiducial reference line.

In the telemetry, samples were grouped in batches of 25, and 10 of these groups were extracted at ESOC for every star mapper transit record. In the NDAC processing, information from all 250 samples was used to optimise the detection of possible parasitic transits, and for the processing of double stars. This could also improve the distinction of data that could be associated with background measurements. In the FAST processing 200 samples were extracted based on the predicted transit time, from 91 samples before to 108 samples after the sampling closest to this transit time. Stars fainter than magnitude $B \simeq 10$ mag were not included in this selection.



Figure 6.3. The differences in 600 Hz sampling periods, between observed transit times and real-time attitude based predictions, showing the convergence of only one field of view at the start, followed by both fields of view from frame 2400 onwards. Transits through the vertical slits are indicated by squares, for the inclined slits by triangles for the upper branch, and stars for the lower branch. Filled symbols refer to the preceding field of view, open symbols to the following field of view. Thruster firings are indicated by the dotted lines.

The use of the real-time attitude determination in the extraction process meant that when the on-board attitude had not converged, the extracted photon counts could entirely miss the stellar transit signal. The detection of the star mapper transits in the provided extracts, and the comparison between the predicted transit time and the transit time observed was therefore a powerful means of checking the performance of the real-time attitude reconstruction at any time. In NDAC, graphs were made of these differences for all data received, and time-intervals with bad attitude convergence thus recognised were excluded from further reductions. The FAST consortium relied primarily on the quality flag provided by ESOC for this purpose (see Section 5.1) which was in the first year of the mission not able to recognise the convergence of one field of view from the convergence of both fields of view. In this case transits went undetected in at least one field of view. Obviously in such instances the involved transits between two consecutive thruster firings were rejected by the attitude processing, the attitude was not computed and no image dissector tube transits were processed. In addition an *a posteriori* comparison of the grid abscissae computed on ground with the pointing derived from the coil current (on-board computation) allowed poor pointing due to either bad attitude or wrong celestial coordinates of the star to be detected and these transits were removed from the photometric solution. Figure 6.3 shows an extract of a monitor plot made by NDAC.

The average time interval between star mapper transits extracted for the attitude reconstruction was set by the density of these stars on the sky ($\simeq 2 \text{ deg}^{-2}$), the span of the star mapper slits (0.667 deg) and the scan velocity of the satellite, 0.0469 deg per second. This gave per slit group and per field of view on average one transit every 16 s. The total data-stream thus consisted of approximately one transit every 4 s. This number could vary by 20 to 30 per cent depending on the inclination of the scanning circle with respect to the galactic plane. The total sky area as seen by the star mapper slits was 0.025 deg². Down to about 10 mag, with close to 10 stars per square degree, an average of 10 per cent of all transits were at least double. This number varied considerably with galactic latitude. Considering all possible transits that could be detected, originating from about 1 million stars over the entire sky, the chances of getting a completely clean transit were rapidly diminishing. However, most of the disturbing transits were too faint to cause serious problems.

The Single-Slit Response Functions

The single-slit response functions represented the normalised intensity profiles caused by a stellar image passing across a single slit at nominal scan velocity, integrated over sampling periods of 1/600 s. The profiles were different for the two slit groups, and in the case of the inclined slits, also for the upper and lower branch. They were also different for the two fields of view and for the two photometric channels. The response functions were obtained from selected transits of brighter, single stars (between magnitudes 4 and 8). Initially, at the start of the mission, data had to be reduced with pre-launch estimated response functions. From this data the first calibrated response functions were obtained. In NDAC, the final reductions were all done with a final set of evolving response functions, based on data accumulated over intervals of 2 to 3 months.

As stated above, the calibration of the response functions used data from transits of relatively bright stars. These transits, through all four slits, had been assigned a reference transit time and a relative scan velocity (see Section 6.3). The positions of the four slits as projected on the sky were known from the calibration described below. The background signal had been derived in the data reduction (see Section 6.3 and 6.4). The measured intensities, after subtracting the background signal, were assigned to bins according to their distances from the assumed slit centre. Where the wings of the intensity profiles overlapped, data were not used. Data were first accumulated for individual slits at different positions along the slits, and then added for the four slits for the different positions along the slits. Data from 30 000 to 40 000 transits were used per calibration, describing the response functions at a time resolution of 9600 Hz (i.e. 16-fold oversampling) with some 2000 contributing data points per bin. The wings were only followed to 4 arcsec from the centre of the slit, but it was clear from data for very bright stars that the wings extended well beyond this range, albeit at a low level (see also Volume 4, Section 4.2 and Figure 4.3).

The averaged data per bin were fitted with a spline function, which was subsequently rescaled to give a maximum response of 1.0. The spline functions also provided the derivatives of the response functions, which was essential for transit time estimates. The fitted response functions are in the following represented by $R_q(t - t_0)$, where t_0 is the transit time at the designated slit centre, taken to be the point halfway between $R_q = 0.5$ on the rising and descending branches of the single-slit profile. The index *q* represents one of the 16 different combinations of slit group, upper/lower branch, passband and field of view. Examples of two fitted response functions and their derivatives are shown in Figure 6.4.

The procedure described above was as applied by NDAC. The FAST procedure followed the same general principles, but differed in the following respects:

(1) in selecting the transits, only photometric standard stars were retained for normalisation. In order to avoid contamination by double or parasitic stars, all transits which showed a large deviation from the expected brightness, or failed any of the statistical tests described in Section 6.5, were rejected;



Figure 6.4. The single-slit response functions (left) and their derivatives (right) for the inclined slits (solid line) and vertical slits (dotted line), upper branch, in the preceding field of view, recorded in the B_T channel. The derivative is given as change of response per 600 Hz sampling period. At nominal scan-velocity, one sampling period corresponds to 0.28 arcsec.

Table 6.1. Calibrated distances between star mapper vertical slits compared with the ground calibration values.

Interval	arcsec	mm	ground (mm)
1 to 2	11.247	0.07638	0.07640
2 to 3	16.867	0.11455	0.11460
3 to 4	5.623	0.03819	0.03820

- (2) the time resolution of the response function was 1/32 arcsec (5400 Hz) and the table extended to ± 2.3 arcsec (half of the smallest slit distance);
- (3) the centre point was determined as in NDAC, but R_q was normalised to unity at the centre point, leading to values slightly above 1.0 at the maximum for asymmetric cases (1.005 maximum).

Some evolution of the response functions took place during the mission, possibly associated with the overall temperature of the spacecraft. There were no significant differences found for stars of different colour index, although this could not be checked for very red stars because of insufficient data.

Calibration of the Slit Spacings

The spacings of the slits were calibrated from cross-correlations between the data collected for the individual slits. This showed the offset of the actual centres of the four slits from the assumed centres. The response functions for the four slits were effectively identical. Separate calibrations were obtained for the vertical slits and the upper and lower branches of the inclined slits. The calibrated separations are shown for the vertical slits in Table 6.1. In both consortia, individual slit distances were obtained for all cases mentioned above. Similarly, the position of the inclined slits with respect to the vertical slits had to be calibrated. In NDAC this was done as an optional part of the attitude reconstruction software, which allowed also for the calibration of the slit orientations. In FAST, a full calibration was performed as described in the following sections.

Medium-Scale Distortion Corrections

The star mapper slits were created through engraving transparent lines in a nontransparent layer deposited on a piece of glass, which was spherically convex to match the focal plane curvature. The engraving was made in small areas, referred to as scanfields. After the production of the grids they were measured for irregularities. This produced the medium-scale distortion values, which together with the slit orientations and relative positions provided a clean reference frame to which star mapper transit times could be referred. In NDAC, the implementation of the medium scale distortions (being not entirely unambiguous in direction and orientation) was checked using accumulated residual transit times for star mapper transits, obtained after reconstruction of the satellite attitude. The medium scale distortions were of the order of 0.01 arcsec, with a maximum of 0.05 arcsec.

In FAST, the slit distortion, that is the difference between the theoretical positions of the mean slits and their actual positions as projected on the sky (including consequently the grid-to-field transformation) was split into two components:

(1) a slope with respect to the ideal orientation (vertical or at 45° inclination). This was essentially due to the rotation of the grid with respect to its ideal position. This linear slit model imposed a fixed intersection with the horizontal axis at the following abscissae from the centre of the grid:

 $G_v = -0.009\,370\,100$

for the vertical slit, and:

 $G_i = -0.015\ 443\ 305$

for both inclined slits, the unit being grid radians;

(2) a shift ΔG with respect to the linear slit model along the horizontal direction was defined for the 34 equal scan-fields dividing each half-grid.

During the first year of data reduction, the slope was deduced from the global grid rotation determined by the great-circle reduction software run by the 'First Look' task (see Chapter 4 and Volume 2, Section 9.2), while ΔG was taken from the measurements made on the ground, the grid-to-field transformation being assumed known. Later, a direct calibration of the slit distortion was performed.

FAST Slit Distortion Calibration

From the great-circle reduction task (Chapter 9), the following results were obtained and used for the star mapper grid calibration:

- the smoothed along-scan attitude with an accuracy of the order of a few milliarcsec, this then gave the scan velocity to significantly better than 1 mas per frame period $(T_4 = 2.133...s)$, except around thruster actuations;
- mean abscissae of stars to within a few milliarcsec accuracy in the same reference frame.

In addition, improved star positions were available for most single stars from the preliminary sphere solution so that, in the same reference frame, one could compute the apparent coordinates of the star to a precision of a few milliarcsec in both coordinates. The main uncertainty thus came from the star coordinates.

The next step was to determine the theoretical transit times on the ideal grid, using the known attitude and star coordinates. The differences with the observed transit times were transformed into the shifts ΔG . This quantity was stored together with the perpendicular grid coordinate H of the star (Section 5.4) and the slit index q. Only 'safe' data were kept: all observations made within $\pm 4T_4$ of a thruster firing were rejected, as were stars known or suspected to be double, or having unknown magnitudes or inaccurate positions and observations presenting a large ΔG . Each reference greatcircle set provided some 2000 to 4000 data points.

Twenty data sets were chosen from among the best in each calibration period of about one month (see Chapter 4). These $\simeq 50\,000$ data points were used to compute by linear regression the slope of each half-grid system. Then, the residuals of this regression were used to compute, for each scan-field, the medium-scale distortion. The resulting precision was about 0.5 arcsec to 1 arcsec for the slopes and, for ΔG , 5 mas on the vertical slits and 10 to 18 mas on the inclined slits. This precision was more than sufficient for the needs of attitude reconstruction. The calibration also provided the separation between the star mapper slit system and the main grid.

6.2. The Star Mapper Transit Signal

The star mapper photon counts N_k , collected at time t_k by either channel $c = B_T$, V_T during the passage of a star, were described as a sequence of statistically independent and Poisson-distributed counts, having the following time-varying average:

$$E[N_{k,c}] \equiv I_{k,c} = I_{b,c} + I_{s,c} \prod_{i=1}^{4} R_q(t_k - \tau_j)$$
[6.1]

where $I_{b,c}$ and $I_{s,c}$ are the intensities of the background and star, respectively, R_q is the single-slit response function, and τ_j are the transit times at the four slits. Given R_q and the observed counts, this model could be fitted to give the background and star intensities and the four slit transit times. The transit time of the whole slit group was then defined as the unweighted mean value, $\tau = \langle \tau_j \rangle$. However, this procedure would only work if all four slit transits were available. Moreover, it does not take advantage of the very precise knowledge of the slit spacings available from the calibrated slit spacings.

Evidently the time τ corresponds to the transit of the star image across an imaginary line situated at the mean coordinate, in the scan direction, of the four slit centres; this defines the 'fiducial reference line' of the slit group (Figure 6.2). Let d_j be the calibrated positions of the slit centres from this line, such that $_j d_j = 0$. Provided that the local scan velocity v_q is known from the attitude reconstruction, the slit transit times can then be referred back to the fiducial line by means the formula $\tau = \tau_j - d_j/v_q$, and the group transit time can be obtained also from an incomplete transit. Alternatively, if $\tau_j = \tau + d_j/v_q$ is substituted into Equation 6.1, the whole group transit may be fitted with only the three parameters $I_{b,c}$, $I_{s,c}$ and τ .

The calibration of the position of the fiducial reference line with respect to the main grid by FAST is described in the preceding section. In NDAC, it was obtained from a feed-back of calibration results from the great-circle reduction, which accurately related star mapper based reconstructed attitude to the fiducial reference line on the main grid (see Chapters 5, 9 and 10).

Scan Velocity

In the FAST solution, the four values τ_j were estimated, and from their mean value τ was derived. As long as all four values were available, this made the transit time determination independent of the scan-velocity derivation. When only three values were available, the scan velocity based on the real-time attitude determination was used, which even under worst conditions introduced an error less than 20 mas, negligible with respect to the requirements.

In the NDAC solution, the scan velocity was derived using gyro data (see Chapter 7), which reduced the solution to an estimate of only one transit time parameter, τ . The scan velocity had to be known for each field of view along, and perpendicular to, the scandirection. Variations with respect to the nominal scan-velocity amounted to a maximum of 3 to 4 arcsec s⁻¹, a few per cent of the nominal scan velocity of 168.75 arcsec s⁻¹. The main effect of the variations in scan-velocity was on the spacing of the slit transits in time. The effect on the width of the response function was very small.

At a minimum accuracy of 0.05 arcsec s^{-1} for the gyro-based scan velocities, the maximum error on the slit spacing for the first and last slits was of the order of 0.002 arcsec, giving maximum response estimate errors of 0.2 per cent, which were only relevant at intensity levels where the photon counts were already affected by the much more damaging effects of saturation. The accuracies obtained for the scan velocities from the gyro data were therefore in normal conditions more than sufficient for the star mapper signal processing. At times of gyro breakdown this was not always the case, and provisions could have been, but were not, made to correct scan velocities for the very brightest objects.

The effective spacings of the inclined slits were in addition affected by the rotation of the grid by $\simeq 5$ arcmin. For the lower branch they became wider by 0.17 per cent, for the upper branch narrower by the same margin. This was accommodated in the calibrated slit spacings.

6.3. Signal Recognition and Background Determination by NDAC

The star mapper counts had to be searched for the presence of one or more transit signals. For this purpose, the counts from the B_T and V_T channel were added, and combined in pairs, so that the resulting signal contained 125 samples at 300 Hz. A provisional background was subtracted from this signal to leave primarily counts related to star transits:

$$J(k/2) = I_{k-1,c} + I_{k,c} - 2\bar{I}_{b,c} , \quad k = 2, 4, 6...250$$
 [6.2]



Figure 6.5. NDAC processing of star mapper data for a transit of the double star HIP 51560/51561 through the vertical slits. The first two bars show the original counts, the third bar the cumulated signal J, and the fourth bar the filtered signal F. Between the V_T and B_T signals is indicated which samples were excluded from the calculation of the background (all samples coinciding with the black line at b). Below F are indicated the recognised transits: here the programme star transit is fainter than the companion. The arrows below indicate the predicted positions (as based on the real-time attitude determination). The lower half of the graph shows the original B-channel counts folded with the final signal fit. This fit was not made for the first 26 and last 35 samples, as undetected single transit peaks could occur there.

All negative values of J(k/2) (counts below the estimated background level) were reset to zero. This signal was subjected to a multiplicative filter. This filter consisted of four Dirac functions, spaced like the expected slit responses. With a time resolution of 300 Hz no corrections for scan-velocity variations were needed, and the spacings were fixed at 20, 30 and 10 intervals:

1 / 4

$$F(l) = \int_{n}^{1/4} J(l+n) , \quad n = -30, -10, 20, 30$$
 [6.3]

When l - 30 or l + 30 was outside the range covered by J, only three inputs were considered; when two values were outside the range, only the remaining two were used. Single peaks close to the start or the end of the 250 samples could not be detected. Such samples were also *a priori* excluded from background calculations. This reduced the signal of every star transit to one peak. An actual signal could be recognised as a cluster of F(l) values different from zero. When more than one transit was present, then each transit produced its own cluster. Due to the cumulation of the data before filtering it was possible to recognise still relatively faint signals, and under normal conditions (background typically 2 to 4 counts per sample of 1/600 s), there was no difficulty in recognizing the faintest programme stars in the star mapper data stream, $\simeq 10$ mag. Figure 6.5 shows an example of the various stages in the recognition and processing.

The individual clusters in F(l) provided first estimates of τ (as in Equation 6.1) associated with these possible transits. A special mechanism, similar to that used in

automated star counts from plate scans, was designed to separate almost overlapping peaks. Depending on the magnitude differences, separations down to 1.5 arcsec could still be handled in the reductions. Similarly, up to three additional transit signals could be present without losing the programme star signal. In such a case, the signal most closely resembling the magnitude and colour of the programme star was identified as the programme star, or, if this were ambiguous, the transit closest to the expected transit time.

When known double stars were encountered, a prediction of the expected separation of the transit signals was made, against which the observed signals were matched. This allowed star mapper processing of some 1200 Hipparcos double stars with separations more than 1.5 arcsec, resulting often in much improved coordinates, and occasionally improved separations and orientations.

Background Determination

The estimates of τ , together with the slit spacings, defined where single-slit transit signals could be expected in the original samples, and conversely, which samples were most likely to represent only a background signal. The mean and standard deviation of these background samples were determined, and checked for outliers due to spikes. When, after four iterations, the observed standard deviation was still more than 1.25 times the expected standard deviation (assuming the background counts represented a stochastic Poisson process, the expected standard deviation was rejected and replaced by the estimated background obtained from a running mean of earlier star mapper background determinations. This running mean consisted of the mean of the last successfully measured background and (up to) 9 times the previous running mean. The running mean was also used as first background estimate in the filtering described above (Equation 6.2).

Background data were accumulated independent of whether a signal was detected or not. Levels of background varied considerably, primarily due to the sensitivity of the star mapper detectors to high energy radiation from outside the satellite. The star mapper background has provided a 3.5 year record of the radiation activity inside and outside the van Allen belts during a period of high solar activity. Away from the radiation environment, background levels varied between 1200 and 3600 Hz, or 2 to 6 counts per sample interval of 1/600 s. During van Allen belt passages, however, it could increase to several hundred counts per sample, and at times of high solar activity to well over 1000 counts and occasionally to complete saturation (see also Figure 2.2).

6.4. Signal Recognition and Background Determination by FAST

Efficient estimates could be provided by the maximum likelihood principle, if all star mapper samples N_{k,B_T} and N_{k,V_T} were the realization of a Poisson stochastic process with the average as defined in Equation 6.1, i.e. a superposition of a background signal and the four peaks of the programme star signal. Owing to parasitic transits and overlapping programme star transits, as well as to spikes in the data, this was in general not true for all photon counts in a record. A detection procedure was therefore implemented to extract the sequence C of model-consistent samples. Then the model parameters could

be estimated by maximizing the logarithm of the likelihood function, restricted to the model-consistent samples:

$$\ln L(\tau_1, \tau_2, \tau_3, \tau_4, I_{b,B}, I_{b,V}, I_{s,B}, I_{s,V}) = N_{k,c} \ln I_{k,c} - I_{k,c} - \ln(N_{k,c}!)$$

$$= 0.41$$

Moreover, since the four slit transit times τ_j were sufficient statistics for the transit time τ , they in turn provided an efficient estimate for τ .

To extract the model-consistent sequence C the detection process had to separate the star mapper record into three subsets of samples:

- (1) the background samples, i.e. samples consistent with the hypothesis H_b : $I_{k,c} = I_{b,c}$ of a constant intensity;
- (3) the (single) star samples, i.e. samples consistent with the composite hypothesis H_s: $I_{k,c} = I_{b,c} + I_{s,c} \quad \stackrel{4}{}_{i=1} R_q(t_k - \tau_j);$
- (3) the samples biased by parasitic or overlapping stars, i.e. samples not consistent with any of the above models (composite hypothesis H_p).

All the different tests applied to a generic sample *m* to discriminate among the above hypothesis were based on five variables $T_j(m)$, $j = 0, 1 \dots 4$, obtained from the convolution of the cumulated counts of the B_T and V_T channels with the single-slit response function:

$$T_{j}(m) = \sum_{c=B_{T}, V_{T} \ k} N_{k,c} R_{q}(t_{k} - t_{m} - d_{j}/v_{q})$$
[6.5]

where t_m is the mean time of the sample *m*, and $d_0 = 0$. Figure 6.6 shows a record of photon counts for a 6.6 mag programme star, and the corresponding convolution $T_0(m)$. By comparing the convolution with a suitable threshold *s*, the value of which depended on the hypothesis that had to be verified, the boolean test variables $L_j(m)$ were obtained: $L_j(m) = 0$ if $T_j < s$, $L_j(m) = 1$ otherwise (see Figure 6.7).

The convolution and channel cumulation allowed the effect of photon noise on the tests to be reduced. It should also be noted that, under Poisson statistics, the convolution $T_j(m)$ tends to the log-likelihood ratio as $I_s/I_b \rightarrow 0$, and it yields a good approximation when $I_s/I_b \simeq 1$, i.e. when the star intensity is comparable to the background. This case represented the most critical situation for separating purely background samples from star-biased samples since, for the faintest programme stars included in the star mapper data stream ($\simeq 10$ mag), the star intensity was normally of the same order as the background.

The further processing of the star mapper data was partitioned into three main steps:

- (1) detection of background samples and background determination;
- (2) detection of programme star samples (if any); when a single star could be detected, a first estimate of the four slit transit times and the star intensity was obtained;
- (3) maximum likelihood estimation of the transit time and of the star intensity for both channels.

Item (1) is described hereafter, while (2) and (3) are covered, respectively, in Sections 6.5 and 6.7.



Figure 6.6. Top: a photon count record of a 6.6 mag star (HIP 47115) through the vertical slit-group, with the estimated average signal superimposed. The abscissae are in sample units (1/600 s), the ordinates in counts per sample. A spike, which occurred in the V_T channel, is indicated by an arrow. Bottom: the corresponding convolution $T_0(m)$.



Figure 6.7. A sketch of the construction of the basic test variables.

Background Detection and Estimation

Background estimation for both channels was made independently from subsequent processing (star intensities and transit time determinations). The background in the star mapper signal was defined as a Poisson-like stochastic process having constant average for a single transit but variable from one transit to another: the sky background was variable and the photomultipliers were sensitive to the radiation environment (van Allen belts). The background detection process compared two alternative hypothesis: the null-hypothesis H_b (constant background) and the alternative hypothesis H_s \cup H_p (programme or parasitic star signal).

A generic sample *m* was accepted as a background sample if $L_0(m) = 0$ and $|m - n| > n_0$ for all *n* such that $L_0(n) = 1$. The first condition, $L_0(m) = 0$, occurring when $T_0(m) < s_0$, detected the samples acceptable as background samples. The second condition, depending on the integer threshold n_0 , rejected in addition those which were likely to be biased by star signals. This refinement of the detection process aimed at reducing as far as possible the risk of attributing a star-biased sample to background and avoid its overestimation. The expression of the significance threshold s_0 was the following:

$$s_0 = \bar{T}_0 + 2.3S_0 \tag{6.6}$$

where $\overline{T}_0 = \overline{I}_b$ $R_q(t)dt$ was the average and $S_0^2 = \overline{I}_b$ $R_q^2(t)dt$ the variance of the convolution $T_0(m)$ under hypothesis H_b . Here, \overline{I}_b designates the running background estimate explained below. The numerical factor 2.3 was set by fixing the risk of rejecting a background sample. The risk was fixed to a value not particularly low, about 1 per cent, since the major concern was to minimise the risk of attributing a star-biased sample to background. This risk was further reduced by removing the background samples biased by a star transit signal. Accepting such samples as background, with the risk of not detecting faint programme stars in subsequent processing. The background samples detected in a record were then averaged to produce the (local) background estimates for the transit:

$$\hat{I}_{b,V_T} = \frac{1}{n} \sum_{k \in \mathcal{A}} N_{k,V_T}, \qquad \hat{I}_{b,B_T} = \frac{1}{n} \sum_{k \in \mathcal{A}} N_{k,B_T}$$
[6.7]

where *n* is the number of accepted background samples and A is their sequence in the record.

Since the detection test assumed known background statistics, a running estimate of the background, \bar{I}_b , was maintained during the whole data set (between 5 and 8 hours). This allowed the risk of rejecting background samples as described above to be increased, without hampering the precision of background estimates as based on the samples from successive and semi-contiguous records.

The running estimates were updated at each transit *i* from the local estimates $I_{b,B_T}(i)$ and $\hat{I}_{b,V_T}(i)$. A first-order Kalman filter was used for each channel:

$$\bar{I}_{b,c}(i) = [1 - K(i)]\bar{I}_{b,c}(i-1) + K(i)\hat{I}_{b,c}(i)$$
[6.8]

where the time varying gain was set to K(i) = n(i)/(n(i)+250), depending on the number n(i) of detected background samples, but restricted to the range $0.05 \le K(i) \le 0.5$. This range was designed to trade off between two contrasting goals: attenuating the photon



Figure 6.8. Flow chart of the background detection and estimation.



Figure 6.9. The estimated background intensity over an orbit on 10 June 1991, coinciding with a period of very high solar activity. The background levels detected at apogee (centre of the graph) were one order of magnitude higher than under normal conditions. The top curve represents the V_T channel, the lower curve the B_T channel. The van Allen belt crossings are indicated by VA, resets of the Kalman filter by arrows.

noise error (typically when the background was low and few samples could be detected), and tracking the real background variations. The processes described above are also shown in a flow diagram in Figure 6.8. An example of the running background estimate for a time interval with particularly strong background variations is shown in Figure 6.9.

Under normal conditions, i.e. when the running estimate $\bar{I}_b = \bar{I}_{b,V_T} + \bar{I}_{b,B_T}$ of the total background was assumed to be accurate and reliable, the significance threshold s_0 of the detection test was computed as a function of \bar{I}_b according to Equation 6.6. When the running estimate was likely to have lost accuracy, e.g. after a large interruption in the observations (more than 1 minute), it had to be reset. In that case, since no background estimates were available, an iterative and converging detection was implemented, providing also the initial values of the running estimates. Running estimates could fail in case of too few background samples being available, but such events were very rare.

6.5. Programme Star Detection by FAST

The Multi-Step Detection

The aim of this processing step was to detect in a star mapper record the signal, if present, of the programme star whose transits were expected to be sufficiently accurate for the attitude determination. So, the objective was to detect an isolated and unperturbed transit per record generated by the programme star—a transit of this kind was referred to as an attitude star transit. To meet this objective, an attitude star had to be detected not only with respect to the background, but very often among different star signals generated by overlapping parasitic or programme stars. This situation caused two main risks of not reaching the objective:

- (1) the multiple star risk: the risk of detecting more than one candidate star without being able to select the expected programme star. In this case, a potentially useful transit might have to be rejected to the detriment of the attitude estimation process, in particular when few transits were available or when the attitude underwent quick variations. This risk could be reduced for the transits of bright stars by isolating in the record only sufficiently bright signals;
- (2) the false transit risk: the risk of obtaining one isolated but false programme star detection and hence generating an erroneous transit time for the attitude reconstruction. This risk was particularly high in the records of faint programme stars, owing to the relatively large number of potential parasitic stars of similar magnitude. For those transits, it was preferable to increase the chance of detecting more stars, in order to accept more isolated programme stars.

Such a complex detection problem could not rely only on accepting or rejecting single samples of the convolution $T_0(m)$, but instead, it demanded a multi-step detection procedure applied to a suitable group of samples. The main steps are described below, further details are shown in the flow chart in Figure 6.10:

(1) detecting a star: any group *m* of four samples separated by the calibrated slit spacings d_j was tested for how many samples could be accepted as generated by the same star. Note that *m* indicates the sample corresponding to the fiducial reference line. The four convolutions $T_j(m)$, j = 1...4 represented the four slits at the

distances d_j relative to the reference line for sample *m*. As already explained, each one was associated with a boolean variable $L_j(m)$ indicating whether the signal was above the given threshold *s*. By adapting the threshold to the star magnitude, the test was made more or less selective, according to the objectives assigned. The four variables $L_j(m)$ were added to produce the variable H(m), having values from 0 to 4 according to the number of accepted samples. H(m) was defined only when all $T_j(m)$ were defined, i.e. when all four slit responses were contained within the 200 samples record;

- (2) detecting a single star: when *m* coincided with the transit time of a star, *H(m)* was equal to 4. Some neighbouring values of *H(m)* would also be equal to 4, in particular for a bright star. Thus, when a position *m̂* was found with *H(m̂)* = 4, the neighbouring samples *m̂* + *k*, *k* = ... 1, 0, 1, ... were checked until *H(m̂ + k)* < 3. This set of samples, *S(m̂)*, constituted the detection of a transit with provisional transit time equal to *t_{m̂}*. When still other samples were detected with *H(m)* = 4, without being contiguous to *S(m̂)*, the record was assumed to contain more than one transit and was rejected;
- (3) detecting a single, unperturbed transit: transit signals could be perturbed by spikes or closely overlapping other transit signals, in a way that could not be detected in the preceding step. To discriminate unperturbed transit signals, suitable two step χ^2 tests were applied to the convolutions $T_i(\hat{m})$;
- (4) detecting an attitude star transit: when only one transit was detected in the record, and this transit was accepted as single and unperturbed, the transit was accepted as the result of the attitude star transit and used for the attitude reconstruction process.

Some of these steps are now described in more detail.

Detecting a Stellar Transit

To detect a transit, the four tests $T_j(m) \ge s_1$ $(j = 1 \dots 4)$ were applied to each sample group *m*, resulting in the boolean variables $L_j(m)$. The tests discriminated between two hypotheses H_s^* and $H_0 \cup H_p^*$, where H_s^* is the composite hypothesis that the sample is biased by a transit signal from a star as bright as or brighter than the expected programme star (the so-called candidate stars). For each transit record, the detection threshold was computed as:

$$s_1 = \max(s_0, T^*), \quad T^* = I_b \quad R_q(t)dt + I_s \quad R_q^2(t)dt$$
 [6.9]

where s_0 is defined in Equation 6.6, T^* is the lower limit of the convolution average under hypothesis H_s^* and I_s the *a priori* estimate of the programme star intensity for channels B_T and V_T together. This threshold was designed to reduce the risk of accepting as a candidate transit a too faint, and hence probably parasitic, transit. At this stage, I_s was computed from the (uncertain) catalogue magnitude M and colour index B - V, with a safety margin applied to account for the uncertainty in these values:

$$M_0 = M + 0.5(B - V) + 0.5, \quad M_0 \le M + 1.25$$
 [6.10]

As the expected magnitude approached the highest programme magnitudes (9 to 10), the threshold s_1 was less and less effective in reducing the multiple star risk, in agreement with the objective of avoiding false transits. Since those magnitudes were comparable



Figure 6.10. Flow chart of the programme star detection.

to background, the threshold s_1 had to be more effective in separating star samples from background samples and consequently was forced to be equal to s_0 .

Perturbed Transits

Quite frequently a star-transit signal was partially perturbed by the transit of a brighter programme star or by unpredictable spikes. This could lead to an inaccurate transit time estimate. To reduce this risk, the following strategy was applied:

- (1) the four convolutions $T_j(m)$ for a transit were tested through a χ^2 test, $\chi_4^2 \ge 16$, to detect outliers. When the test was significant, the higher outlier, e.g. the convolution $T_i(m)$ corresponding to photon-counts collected during the passage of slit *i*, was dropped from the subsequent processing. Otherwise the transit was accepted as unperturbed;
- (2) the remaining three convolutions were tested again for $\chi_3^2 \ge 9$. When this test was significant too, the transit was rejected.

In the case of spikes outside the star signals, two risks had to be managed:

- the risk of accepting it as background—this was managed by the background detection process;
- (2) the risk of accepting it as part of a star signal—this risk happened to be very low, as most spikes could only yield a test variable H(m) = 1.

First Transit Time and Intensity Estimates

When a single and unperturbed transit had been detected, a first estimate of the four transit times τ_j and of the stellar intensities I_{s,V_T} and I_{s,B_T} was obtained. The four samples \hat{m}_j of the group \hat{m} detected as the maximum of the test variable H(m) in the set $S(\hat{m})$ of the single transit, were taken as a first estimate of the slit transit times τ_j . All the peak samples around \hat{m}_j were then used to provide the first estimate of the stellar intensities in both channels.

6.6. Transit Time and Intensity Determinations by NDAC

The estimation of the transit time τ and intensity I_s were linked in an iterative loop. All estimates were obtained through linear least-squares. In the first iteration step, equal weights were used. In subsequent iteration steps the weights were derived from the estimated values of the preceding preliminary solution. Effectively, this was equivalent to a joint maximum likelihood estimation of the two parameters.

Successive estimates of the transit time and its standard error were obtained from corrections to the previous values. The differences between the observed counts $N_{k,c}$ and a preliminary model fit, using the transit time $\tilde{\tau}$, were related to the required correction $\Delta \tau$ in the transit time estimate through the derivative of the single-slit response function, $R'_q = dR_q(t)/dt$ (see Figure 6.4). Excluding the noise term, Equation 6.1 gives:

$$N_{k,c} - I_{b,c} - I_{s,c} \int_{j=1}^{4} R_q(t_k - \tilde{\tau} - d_j/v_q) = -I_{s,c} \int_{j=1}^{4} R'_q(t_k - \tilde{\tau} - d_j/v_q) \Delta \tau$$
 [6.11]

Equation 6.1 was used for solving $I_{s,c}$ using estimated values of τ , while Equation 6.11 was used for obtaining corrections to $\tilde{\tau}$ using the estimated values for $I_{s,c}$. Estimating all the parameters at the same time proved hazardous due to disturbances of the star mapper signal. Spikes, possibly caused by cosmic rays, could not always be recognised before processing started, but were able to distort a solution with too many degrees of freedom. Thus, the above-described iterative approach was adopted. Solutions generally converged rapidly. Figures 6.11 and 6.12 show the astrometric and photometric precision reached with these reductions. Also shown in each case is the slope of a relation where the error is proportional to the photon noise on the signal. This does not apply to faint stars, where the relative error is larger due to the significant contribution of the background signal. From Figure 6.11 it is clear that the majority of star mapper transits



Figure 6.11. The noise levels on the transit times (σ_{τ} , in arcsec) as a function of the total signal intensity. The graph shows the results for 9400 reduced star mapper transits obtained over a period of 9 hours in the beginning of 1990. The horizontal scale presents the observed counts as approximate magnitudes.

contributing to the attitude reconstruction process had transit time errors between 0.1 and 0.01 arcsec.

The approach described above allowed for the solution of the parameters for more than one signal. This was used for double stars and for recovery of transits disturbed accidentally by another transit (usually from the other field of view). In the case of double stars the predicted separation was used to recognise the signal. Only double stars with separations larger than 1.5 arcsec were treated this way. As it was not known generally where an accidentally superimposed transit originated (which slit group, field of view, etc.,) the correction procedure was not ideal, being unable to implement the proper scan velocity and single-slit response functions for the stray transit.

6.7. Transit Time and Intensity Estimation by FAST

Transit Time Estimation

In principle, the first estimate \hat{m}_j of the transit times provided by maximizing the photon count convolutions (Equation 6.5), and having the resolution of the samplings (0.28 arcsec), could have been refined by interpolating $T_j(m)$ around the maxima. Unfortunately, convolutions are not an acceptable approximation of the log-likelihood function (Equation 6.4) for this task, since they tend to the log-likelihood function only when $I_s/I_b \rightarrow 0$. Therefore a better approximation, still independent of the star intensity, was used, with the constraint of accurately approximating the log-likelihood function at least for the faintest programme stars, i.e. $I_s/I_b \simeq 1$. The new function can be shown to be the convolution of the photon-counts with $\ln[1 + I_sR_q(t)/I_b]$ (see also Perryman *et al.* 1989, Volume III, Chapter 3). Moreover, by imposing $I_s/I_b = 1$, the approximate likelihood function is made independent of star intensity and reads:

$$\ln L(\tau_j) = \sum_{c=B,V \ k} N_{k,c} \ln[1 + R_q(t_k - \tau_j)], \quad j = 1...4$$
[6.12]

Maximizing this function with respect to τ_j , using a sufficient time resolution, provided efficient transit time estimates $\hat{\tau}_j$ for at least the faintest programme stars. Moreover, by quadratically interpolating $\ln L(\tau_j)$ around the maxima, the negative inverse of the second derivative (Hessian) could be computed, providing the variance of the transit time estimate.

A suitable χ^2 test, with a low risk (0.1 per cent) of the first kind, was then applied to the four estimated times $\hat{\tau}_j$ to detect any possible outlier which could arise due to partial overlapping by a parasitic transit. When the test was significant, the outlier was dropped from subsequent processing. The transit time τ across the fiducial reference line was then estimated from the model:

$$\hat{\tau}_j = \tau + d_j / v_q + \epsilon_j \tag{6.13}$$

where the index *j* denotes the peaks of the transit signal accepted by the detection procedures (a minimum of three), and ϵ_j their estimation error. In the normal case of four accepted measures, the estimate of the transit time was given by:

$$\hat{\tau} = \frac{1}{4} \int_{j=1}^{4} \hat{\tau}_j$$
 [6.14]

The residuals of the above estimation were calculated *a posteriori*, using accurate estimates of the scan velocities, to assess the accuracy of the transit time estimator. Under the assumption of a known scan velocity v_q , Equations 6.13 could be transformed by the following orthogonal matrix **T**:

The first row of the transformed equation yielded the transit time estimate as in Equation 6.14; the other three rows yielded three uncorrelated residuals having the same variance as the transit time estimation. By collecting such residuals, the *a posteriori* standard deviation of the transit time was assessed.

Intensity Estimation

After the transit of an attitude star had been detected, and the transit time was estimated, the log-likelihood function (Equation 6.4) could be solved for the photometric parameters through the following gradient equations (one set for each channel *c*), applied to the sequence C of model-consistent photon counts:

$$\frac{\partial \ln L(I_{s,c}, I_{b,c})}{\partial I_{s,c}} = \sum_{\substack{j \quad k \in \mathcal{C}}} \frac{N_{k,c} - I_{k,c}}{I_{k,c}} R_q(t_k - \hat{\tau}_j) = 0$$
[6.16]

and:

$$\frac{\partial \ln L(I_{s,c}, I_{b,c})}{\partial I_{b,c}} = \frac{N_{k,c} - I_{k,c}}{I_{k,c}} = 0$$
[6.17]

Since the above equations are not linear in the unknown parameters, they would have required an iterative solution. In practice, a single-step solution was used, after obtaining estimates for the weight $w_{k,c} = 1/I_{k,c}$ from the available background estimates and the first estimates of the star intensities provided by the detection step. This simplified

procedure corresponds to the weighted least-squares solution of the following linear equations:

$$N_{k,c} = I_{b,c} + I_{s,c} \qquad R_q(t_k - \hat{\tau}_j) + \epsilon_{k,c}, \quad \text{Var}(\epsilon_{k,c}) = I_{k,c}$$
[6.18]

which proved to be accurate enough for the photometric parameters.

6.8. Comparisons

Several comparison exercises were carried out in the beginning of 1991 at the Royal Greenwich Observatory, involving star mapper reduction results obtained by NDAC, FAST (main processing and 'First Look') and TDAC. At that stage there was very good consistency with transit time and intensity determinations. Since then, some changes have taken place in the software, most notably by FAST. No direct comparisons were done with results obtained after implementation of those changes, although indirectly the star mapper transit time results were checked in the attitude reconstruction comparisons, described in Chapter 7.

6.9. Star Mapper Astrometry

The reduced star mapper data in combination with the reconstructed attitude contained positional information for the observed stars. The standard errors of the positions provided by the Hipparcos Input Catalogue were 3 to 6 times larger than the possible standard error of the attitude fit. As a result, the differences between the predicted positions (based on the Input Catalogue) and the observed positions (based on the transit times and the reconstructed attitude) represented at least partially the positional errors that were present in the Input Catalogue.

In the NDAC data reductions these differences were accumulated in a 'working catalogue', containing next to *a priori* data several small information arrays, representing least-squares solutions of positional and magnitude corrections. These were updated using a Householder transformations based mechanism every time new data became available, and could be solved for the updates at any time. During the first 18 months of the mission this information was used to improve the stellar positions used by the satellite for its real-time attitude determination, leading to a considerable improvement in performance.

6.10. Star Mapper Photometry

The star mapper intensity estimates were used to derive the B_T and V_T magnitudes, which were used during the mission to improve the colour information made available through the Input Catalogue (which was in many cases only an estimated value). The intensities were calibrated with respect to a set of standard stars constructed for the Tycho photometric reductions.



Figure 6.12. The noise levels on the accumulated V_T and B_T magnitudes for constant stars.
The calibrations were carried out in relative intensity scale. The magnitudes of the calibration stars were converted to a pseudo intensity by:

$$I_c = 10^{-0.4(M-10)}$$
 [6.19]

where $M = B_T$ or V_T was the magnitude of the calibration star. The calibration used the following model:

$$I_s/I_c = a_1 + a_2 C + a_3 C^2 + a_4 z + a_5 z^2 + a_6 C z$$
[6.20]

where $C = (B_T - V_T) - 0.7$ stands for the colour of the star, and a_i are the unknown parameters. *z* is the vertical coordinate of the transit. Calibrations were done over data accumulated for an average of 2 to 3 days. They were done separately for the inclined and vertical slit groups, preceding and following fields of view and for the B_T and V_T channels, giving 8 calibrations in total. The star mapper detectors were much less affected than the image dissector tube detector by transmission loss due to radiation. This meant that longer stretches of data could be used in the reductions.

A 'running solution' was used, where the solution obtained for the preceding data interval was used as additional, down-weighted, observation equations, again using a Householder transformations based least-squares routine.

As the effective integration time for a star mapper signal was fixed, the errors on the individual photometric measurements were proportional to the inverse of the square root of the intensity. This is evident in Figure 6.12, where the $\log(\sigma_M)$ is plotted against magnitude, showing also the expected relationship. The increase in errors for faint magnitudes is due to background contributions, on the bright end due to modelling inaccuracies. The errors on the mean magnitudes are on average a factor 10 smaller. Variable stars were excluded from the diagrams.

The response in the star mapper channels decreased by an average of 3 per cent per year, a total of about 12 per cent over the mission (see also Volume 2, Figure 10.2). This compares with a decrease of around 40 per cent in the image dissector tube detector (see Chapter 14). The response in the vertical slits was sensitive to focus variations.

More details on star mapper photometry can be found in Volume 4, Chapter 8.

F. van Leeuwen, E. Canuto, F. Donati, J. Kovalevsky

7. ATTITUDE RECONSTRUCTION

The attitude reconstruction turned out to be one of the most complex and challenging data reduction tasks, not least due to the complications caused by the elliptical orbit the satellite was forced to use and the resulting torque variations. There were also disturbances caused by the radiation received while passing through the van Allen belts, both direct (increased background levels in the star mapper detectors, preventing the recognition of signals) and indirect (damage to the gyro electronics and the on-board computer). In the reconstruction of the attitude the two consortia proceeded along very different strategies, both having their advantages and disadvantages. Generally, they were both designed to achieve the accuracy required for the reduction of Hipparcos data at the great circle level. The FAST method turned out to be more stable in particular near thruster firings, while the NDAC method was better equipped to deal with the more extreme behaviour (such as torques near perigee, gyro tests and accidental hits). Comparisons showed good agreement between attitudes computed by both methods even in the most difficult cases and both produced results well within the specifications.

7.1. The Attitude Reconstruction Problem

The aim of the attitude reconstruction was the determination of the attitude of a telescope reference frame as defined by the star mapper grid calibrations. The accuracy required by the Hipparcos mission was a standard deviation of the error better than 0.1 arcsec. During further processing in the great-circle reductions the along-scan attitude was further improved to a standard deviation of a few mas, which was acceptable also for the Tycho data reductions.

The on-ground attitude reconstruction was a very complex task and it is not possible in this short chapter to present in detail all the work done by the two consortia, which adopted different attitude models and different estimation procedures. The aim of this chapter is to present in a compact way the main lines of the estimation methodology followed, the assumptions introduced, and the quality of the results obtained.

Data Available for the Reconstruction of the Attitude

The attitude reconstruction relied primarily on the star mapper transit time determinations, described in Chapter 6. It also required the timings and lengths of the thruster firings in order to recognise sudden changes in rotation rates and in order to provide boundary conditions linking data intervals between thruster firings. Other data sources available were the real-time attitude determination, providing the on-board estimates of the orientation and rotation rates of the satellite, and the gyro data, providing a constant record of the rotation rates. The gyro data and the real-time attitude determination were partly correlated. For the final attitude reconstruction results from the great-circle reductions could also be used.

The star mapper data stream, an average of one star every 4 seconds, was regularly interrupted for a period of 200 to 1000 seconds because of an occultation by the Earth or the Moon: shutters were closed when the image of the Earth or the Moon came too close to one of the fields of view. As no image dissector tube data was collected over these intervals either, there was no need for a reconstructed attitude over those intervals. Collection of gyro data continued during these intervals. Occultations were particularly long when the satellite was close to perigee.

Reconstructed attitude data was required for the processing of the image dissector tube data (see Chapter 5) and for the great-circle reductions (see Chapter 9). The required accuracy was 0.1 arcsec, and was set by great-circle reduction requirements. The accuracy of the rotation rates and the acceleration around the spin axis could influence the image dissector tube reductions, in particular for brighter stars (see Chapter 5). The attitude was also required for the Tycho data processing, which benefitted from the higher accuracy that was in fact obtained (see also Volume 4).

Principles of the Attitude Reconstruction

The attitude is given by three angles, suitably defined, which are semi-independent time functions. In the attitude reconstruction process three fundamental steps can be outlined: the attitude modelling, the attitude estimation and the hypothesis verification.

The attitude modelling is the step in which the attitude angles were defined and their variability in time was modelled by a finite set of unknown variables (degrees of freedom). Since in the most general case the above time functions presented an infinite continuous set of degrees of freedom, the attitude modelling had to introduce an approximation of the reality motivated by physical reasoning, numerical simulations and experience, in order that the modelling error remained negligible with respect to the globally required accuracy.

Assuming that the model error is negligible, an estimation procedure was developed, which allowed the number of degrees of freedom of the attitude model to be estimated from the available measurement data. The same estimation procedure, again with the assumption that the model error is negligible, computed the covariance matrix of the estimated unknown variables and then of the reconstructed attitude angles.

The goodness of the approximate mathematical model, that is the assumption that the model error was negligible with respect to the estimation error, was assessed on the basis of the measurement data by suitable statistical tests. When the acceptance of the



Figure 7.1. Flow chart of the NDAC attitude determination procedure.

above hypothesis had too high a risk, the attitude model was adapted by increasing the number of degrees of freedom.

Given the complexity of the problem, involving the estimation of continuous functions of time representing the satellite's response to a wide spectrum of quasi-random perturbations, it is not surprising that the reduction consortia proceeded along very different paths. Briefly, these can be characterised as follows:

The NDAC approach: The dynamical model adopted by NDAC and described in Section 7.2 attempted to represent all the significant torques acting on the satellite (assuming it to be a rigid body), for which the parameters were calibrated as presented in Section 8.5. This approach resulted in a good physical view of why the satellite behaved as it did, and had the great advantage of producing values of the torques which were a great help to ESOC in forecasting the attitude after the passage at the perigee and, even more important, to be the basis of ESOC observing procedures when only two gyroscopes were left. In contrast with FAST, NDAC worked differentially with respect to the nominal scanning law (Section 7.3). The attitude modelling together with the estimation procedure are given in Section 7.4. Figure 7.1 summarises the structure of the NDAC attitude determination procedure.

The FAST approach: The FAST attitude model is given in Section 7.5. It was derived from theoretical considerations supported by accurate numerical simulations based upon dynamical models. It can be noted that, at least between gas jets, this independent approach led to a model very close to the one currently used in astronomy for the rotation of natural bodies. It was proven later, that the whole model is conformable to the classical approach in Celestial Mechanics, even if gas jets occur within the representation interval. Two different models were used: Fourier series and polynomials. They are schematically represented in Figure 7.2.

The main advantage of the NDAC approach was that it permitted the origin of torques to be traced. This allowed ESOC to minimise the time of recovering the attitude after perigee passages. It has also been the basic tool with which it was possible to continue the mission when only two gyroscopes were functioning. The main advantage of the FAST approach was that it could apply to any kind of torque whatever the origin, and this within a very large domain including the recognised torques, with the exception of gas-jet actuation effects for which a specific dynamical treatment was adopted. The second advantage was that no numerical integration was necessary to determine the



Figure 7.2. Flow charts of the FAST attitude reconstruction models.

attitude and the treatment, quite simply, reduced to fitting observations to adequately chosen parameterised expressions.

In conclusion, it is important to stress that both approaches were based on dynamical considerations and that, finally, both converged to numerical descriptions which were assessed as being comparable and well within the margins set by the requirement that the attitude errors should not introduce biases in the reduction on great circles or in Tycho consortium procedures.

7.2. Physics of the Attitude of the Satellite

The satellite attitude describes the positions of the three major axes of the satellite as a function of time. It also describes the rotation rates around these three axes. The positions and rotation rates were affected by a wide range of internal and external torques. The internal torques were caused by the spinning motion of the satellite combined with velocity changes around an axis perpendicular to the main spinning axis. Torques were also caused by an interaction between the rotation axes of the gyros and the spin axis of the satellite, and became particularly noticeable during gyro test runs, when the tested gyro was spun up and spun down. In addition, there were short thruster firing torques. The external torques arose primarily from three sources: solar radiation, gravity gradient and interaction between the magnetic moment of the satellite and the Earth magnetic field. The satellite was also hit a few times by external objects, and was subject to drag, but these occured only at times very close to perigee when observations were not possible anyway. Partial heating of the outer surface during very low perigee passages seems to have played a role too.

and *t* is measured in years from that epoch. The inertia tensor was scaled to a fixed value of I_{XX} . $\frac{x \quad y \quad z}{x \quad 534.220 + 0.000t \quad 3.459 - 0.469t \quad 2.031 + 0.082t}$

Table 7.1. The inertia tensor, in kg m^2 , for the Hipparcos satellite. The reference epoch is day 70 in 1991,

	A	y	Z
X	534.220 + 0.000t	3.459 - 0.469t	2.031 + 0.082t
У	3.459 - 0.469t	590.680 + 0.209t	-0.806 + 0.005t
Z	2.031 + 0.082t	-0.806 + 0.005t	459.420 - 1.390t

The motion of the satellite is described by the Euler equation (assuming that the satellite can be considered as a rigid body):

$$\mathbf{I} \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} = \mathbf{N} - \boldsymbol{\omega} \times \mathbf{I} \boldsymbol{\omega}$$
 [7.1]

where **I** is the inertia tensor of the satellite, **N** the sum of the external torques, and $\boldsymbol{\omega}$ the angular velocity vector. These are all expressed in the satellite reference frame, i.e. with respect to the axes **x**, **y**, **z** assumed to be fixed in the satellite (Figure 7.3). The (partly calibrated) values for the inertia tensor are given in Table 7.1. Calibration was only possible for the ratios between the diagonal elements and for the relative contributions of the off-diagonal elements. All inertia tensor elements were therefore scaled to a fixed adopted value for I_{xx} , based on the post-launch ground-based estimate for this value (including the effect of the full fuel tank for the apogee boost motor). The calibrations are described in Chapter 8.

The descriptions in the following sections are based on results obtained from the reconstructed attitude as well as theoretical considerations.

Solar Radiation Torques

The solar radiation reflected and absorbed by the outer surface of the satellite caused a force working on the satellite which was not balanced with respect to its centre of mass, and therefore resulted in torques. The size of these torques depended on the orientation of the satellite with respect to the direction to the Sun, and because the satellite had a basic three-fold symmetry, the torques caused by the solar radiation were predominantly a function of 3Ω , where Ω is the angle between the direction to the Sun and the satellite *x* axis (as in Figure 7.3). The nominal scanning law (see Volume 2, Chapter 8) kept the *z* axis of the satellite within a 10 arcmin margin from a fixed distance of 43° from the direction to the Sun. The relative position of the *x* axis, given by the angle Ω , was therefore sufficient to describe these torques.

The periodicity at multiples of 3Ω applied only when the torques were seen in a reference frame fixed with respect to the solar direction. Let $[\bar{\mathbf{x}} \ \bar{\mathbf{y}} \ \mathbf{z}]$ be such a reference frame with \mathbf{z} along the satellite nominal spin axis, $\bar{\mathbf{x}}$ on the great circle through \mathbf{z} and the direction to the Sun as seen from the satellite, and $\bar{\mathbf{y}} = \mathbf{z} \times \bar{\mathbf{x}}$ completing the triad. The solar radiation torques can be described in this frame as:

$$\bar{\mathbf{N}}_{\mathrm{R}} = \begin{pmatrix} 0\\b_{0}\\0 \end{pmatrix} + \sum_{n=1}^{m} \begin{pmatrix} a_{n}\sin 3n\Omega\\b_{n}\cos 3n\Omega\\c_{n}\sin 3n\Omega \end{pmatrix}$$
[7.2]

The torques N_R as experienced in the satellite reference frame were obtained through a rotation by Ω around the *z* axis:

$$\mathbf{N}_{\mathrm{R}} = \begin{pmatrix} \cos \Omega & \sin \Omega & 0\\ -\sin \Omega & \cos \Omega & 0\\ 0 & 0 & 1 \end{pmatrix} \bar{\mathbf{N}}_{\mathrm{R}}$$
 [7.3]

As a result, the solar radiation torques around the satellite *x* and *y* axes can be represented as:

$$N_{\text{Rx}} = b_0 \sin \Omega + \frac{1}{2} \sum_{n=1}^{m} (a_n - b_n) \sin(3n - 1)\Omega + \frac{1}{2} \sum_{n=1}^{m} (a_n + b_n) \sin(3n + 1)\Omega$$
[7.4]

and:

$$N_{\text{R}y} = b_0 \cos \Omega + \frac{1}{2} \sum_{n=1}^{m} (b_n - a_n) \cos((3n - 1))\Omega + \frac{1}{2} \sum_{n=1}^{m} (a_n + b_n) \cos((3n + 1))\Omega$$
[7.5]

This defined the coefficients characteristic for torques caused by the solar radiation.

A further peculiarity of these torques was the fact that their strength varied inversely proportional to the square of the distance between the Earth and the Sun, i.e. by about ± 3.5 per cent. The solar radiation torques around the *x* and *y* axes were of the order of 2 μ Nm, which translated into accelerations around these satellite axes at a level of 2.3 mas s⁻². Around the *z* axis the torque and acceleration were about a factor 3 smaller.

The solar radiation torques disappeared during eclipses, and the transitions from sunlit to eclipse were among the most difficult to describe in the attitude reconstruction, in particular as this transition was accompanied by changes in the magnetic moment of the satellite (see below). In addition, during and shortly after a transition, sudden (small) torques occurred, possibly related to the attachments of the solar panels (see also Volume 2, Section 11.4).

Gravity Gradient Torques

The gradient of the Earth's gravitational field across the satellite caused a torque described by:

$$\mathbf{N}_{\rm G} = \frac{3GE}{I^3}\,\mathbf{\hat{r}} \times \mathbf{I}\,\mathbf{\hat{r}}$$
[7.6]

where $\hat{\mathbf{r}} = \langle \mathbf{r} \rangle$ is the unit length vector indicating the geocentric direction to the satellite's centre of mass, and $r = |\mathbf{r}|$ is the geocentric distance of the satellite (the position vector \mathbf{r} is the same as \mathbf{g}_0 in Chapter 12). *GE* is the geocentric gravitational constant (Table 12.1). $\hat{\mathbf{r}}$ was described in the satellite coordinate system $[\mathbf{x} \mathbf{y} \mathbf{z}]$ by means of the two angles ξ_e and Ω_e :

$$\hat{\mathbf{r}} = \begin{pmatrix} \sin \xi_e \cos \Omega_e \\ \sin \xi_e \sin \Omega_e \\ \cos \xi_e \end{pmatrix}$$
[7.7]

These angles were computed from the geocentric ephemeris \mathbf{r} , given in equatorial coordinates, and the components of the satellite axes, also expressed in the equatorial frame, by means of the relations:

 $\cos \xi_e = \mathbf{z}' \mathbf{\hat{r}}$

and:

$$\cos \Omega_e = \mathbf{x}' \hat{\mathbf{r}} / \sin \xi_e, \qquad \sin \Omega_e = \mathbf{y}' \hat{\mathbf{r}} / \sin \xi_e \qquad [7.9]$$

Both ξ_e and Ω_e varied with the position of the satellite in its orbit, but near apogee (t_0) the variations in ξ_e were relatively small and Ω_e varied mainly because of the spin of the satellite, $\dot{\Omega}_e \simeq \dot{\Omega}$. Substituting $\Omega_e = \Omega_{e0} + \dot{\Omega}t$ in Equation 7.7 and evaluating Equation 7.6, while considering that the inertia tensor **I** is dominated by the diagonal elements, one finds:

$$\mathbf{N}_{\rm G} = \frac{3GE}{r^3} \begin{pmatrix} (I_{ZZ} - I_{yy}) \sin 2\xi_e \sin(\Omega_{e0} + \Omega t) \\ (I_{XX} - I_{ZZ}) \sin 2\xi_e \cos(\Omega_{e0} + \dot{\Omega} t) \\ (I_{yy} - I_{XX}) \sin^2 \xi_e \sin 2(\Omega_{e0} + \dot{\Omega} t) \end{pmatrix}$$
[7.10]

Thus, around apogee the characteristic coefficients for the gravity gradient torque could be recognised by a periodicity with Ω for the *x* and *y* axes, and with 2Ω for the *z* axis, showing a phase shift with respect to the solar radiation torques.

The geostationary transfer orbit in which the satellite was stuck complicated this representation considerably. The amplitude of the torque, ξ_e , and Ω_e became complicated functions of orbital position. This resulted in a signal that was no longer periodic with the rotation period of the satellite, and which was in addition strongly variable in amplitude.

The size of the gravity gradient torque was of the order of $0.2 \,\mu$ Nm at apogee, increasing by a factor 200 towards perigee, and dominating the torques from 1.5 hours before to 1.5 hours after the perigee passage.

Magnetic Torques

During the data analysis and the calibrations described in Chapter 8, it was found that the satellite had a distinct magnetic moment of about -2.7 Am^2 directed along the *y* axis. There also appeared to be a similar size magnetic moment directed along the *z* axis, existing only during eclipses, when satellite power had switched from solar panels to batteries. The torque caused by the interaction of the magnetic moment vector $\bar{\mathbf{m}}$ and the Earth's geocentric magnetic flux density vector $\bar{\mathbf{B}}$ is given by:

$$\bar{\mathbf{N}}_{\mathrm{M}} = \bar{\mathbf{m}} \times \bar{\mathbf{B}}$$
 [7.11]

As seen from the satellite reference frame $\mathbf{\tilde{B}}$ is rotating. In the satellite reference frame the flux density vector is expressed as:

$$\mathbf{B} = \begin{pmatrix} \cos \Omega_m & \sin \Omega_m & 0\\ -\sin \Omega_m & \cos \Omega_m & 0\\ 0 & 0 & 1 \end{pmatrix} \bar{\mathbf{B}}$$
 [7.12]

where Ω_m describes the instantaneous orientation of **B** with respect to the satellite *x* axis. Within the satellite reference frame **m** was assumed constant. This gave the following relations:

$$\mathbf{N}_{\mathrm{M}} = \begin{pmatrix} m_{y}B_{z} - m_{z}B_{y} \\ m_{z}B_{x} - m_{x}B_{z} \\ m_{x}B_{y} - m_{y}B_{x} \end{pmatrix}$$
[7.13]

[7.8]

Gyro		X	У	Z
 1	а	-0.54090	0.89363	-0.00030
	b	1.102	0.675	0
	с	-5.52	9.01	0
2	а	0.52620	0.86930	0.00013
	b	0	-1.3	0
	с	10.06	0	0
3	а	1.05650	0.00057	-0.00090
	b	0	0	-1.3
	с	0	0	0
4	а	-0.00060	0.00080	1.05430
	b	-1.102	0.675	0
	с	-5.52	-9.01	0
5	а	0.00154	0.00091	1.05443
	b	-1.102	0.675	0
	с	-5.52	-9.01	0

Table 7.2. For each of the five gyros, the table gives: (a) orientation of the input axes times the scaling correction factor; (b) the angular momentum (10^{-2} Nms) ; and (c) the nominally induced torque (μ Nm).

where at apogee the torques around all three axes were approximately modulated by Ω . Away from apogee this modulation became disturbed due to the changes in \mathbf{B} , and both the amplitude and the period were varying strongly.

The description for \mathbf{B} used the 1985 coefficients of the International Geomagnetic Reference Field (IGRF) (Barraclough 1985), describing the magnetic field as a series of spherical harmonics. The most important (dipole) coefficient is proportional to r^{-3} . The magnetic torques were slightly smaller on average than the gravity gradient torques, but increased in a similar fashion with decreasing distance to the Earth. No corrections were applied for compression or stretching of the field due to interaction with the solar wind. The analysis of the data indicated, however, that such effects were probably present.

Gyro Induced Torques

The five gyros on-board the satellite had 'fixed' spin axes within the satellite reference frame (see Chapter 8). Three of the five gyros were normally active, and each of these would cause a torque on the satellite, depending on the orientation of the spin axis with respect to the satellite spin axis. The orientations of the spin axes and input axes are summarised in Table 7.2. Spinning-up a gyro in addition caused a torque around the spin axis of the gyro. The sum of the torques of the active gyros entered Equation 7.1. The gyros selected for most of the mission were 1, 2, and 4 or 5. The resulting torques from this combination were relatively small.

Thruster Torques

Thruster torques were of very short duration, and intended to change the rotation rates around the satellite axes such as to keep the satellite close to its intended scanning law. Pulses lasted between 2/75 s and 45/75 s, except close to perigee when longer pulses were permitted. The average $\Delta \omega$ caused by a pulse of one unit of 1/75 s was 0.1 arcsec s⁻¹. The thruster firing strategy changed over the mission, as described in Chapter 8 and in Volume 2, Chapter 13. Thruster firings always took place at the start of an observational frame, and would normally last for no more than 0.25 times the length of a frame. The intervals between firings varied from 200 to 1200 observational frames. Close to perigee the intervals became much shorter. Thruster firings were recorded in the telemetry with the timing and length of the firing for each axis, but information could become ambiguous or get lost due to telemetry problems. Chapter 8 describes the thruster firing calibrations.

Incidental Micrometeoroid Hits

Incidental hits due to micrometeoroids (see Volume 2, Chapter 6) had the same appearance as thruster firings, except that they were evidently not recorded in the telemetry and not tuned to the start of an observational frame. Two fairly serious hits and many minor hits were detected from examinations of the gyro read outs. They were also conspicuous in the course of the great-circle reductions (see Chapter 9).

Miscellaneous Effects

A torque possibly related to thermal radiation from the satellite after perigee passages was noted in the examinations of the accelerations of the satellite (see Chapter 8, Figure 8.16). Depending on the height of perigee, which varied between 440 and 580 km above the Earth surface, the satellite was more or less affected by drag. As the rotation period of the satellite was relatively long with respect to the time the satellite was exposed to drag, this affected the satellite in an unbalanced way. After perigee, the satellite had been warmed up on one side and started losing this excess heat through radiation, which due to its distribution over the surface, could cause a small torque. The size of this effect was at most of the order of $0.02 \ \mu$ Nm.

7.3. The Nominal Scanning Law and Real-Time Attitude Determination

The scanning of the sky by the satellite proceeded along a pre-determined nominal scanning law. This scanning law had been designed such as to optimise the coverage of the sky, while avoiding the sunlight to affect the measurements. As this was a scanning satellite, this meant that scanning circles always had to be inclined with respect to the ecliptic plane. The scanning law is for this reason described in the ecliptic coordinate system. The details of the scanning law can be found in Volume 2, Chapter 8. Here the importance is in the relation between the equatorial coordinates and the satellite's main axes. These relations are described by a series of orthogonal rotations. If we denote by $\mathbf{R}_i(\alpha)$ the rotation of a coordinate triad around axis *i* by an angle α , then for the transformation from equatorial direction cosines **e** to satellite coordinates **s** the following relation applies:

$$\mathbf{s} = \mathbf{R}_3(-\Omega)\mathbf{R}_2(\xi - \frac{\pi}{2})\mathbf{R}_1(\frac{\pi}{2} - \nu)\mathbf{R}_3(-\lambda_{\odot})\mathbf{R}_1(-\epsilon)\mathbf{e}$$
[7.14]

where ϵ is the inclination of the ecliptic plane for equinox J2000.0, and λ_{\odot} , ν , ξ and Ω are the angles in which the nominal scanning law was described. They were referred to



Figure 7.3. The heliotropic angles $(\lambda_{\odot}, \nu, \xi, \Omega)$ and the satellite axes $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ in the ecliptic reference frame. The ecliptic is indicated by 'E', the scanning circle by 'G'. The two fields of view (PFOV, FFOV) and the basic angle γ are also indicated.

as the 'heliotropic angles' and are shown in Figure 7.3. Of these, all but ξ were time dependent.

The inertial rates resulting from the nominal scanning law were accordingly given by:

$$\boldsymbol{\omega}_{n} = \begin{pmatrix} -\cos\xi\sin\nu\cos\Omega - \cos\nu\sin\Omega & \sin\xi\cos\Omega & -\sin\Omega & 0\\ \cos\xi\sin\nu\sin\Omega - \cos\nu\cos\Omega & -\sin\xi\sin\Omega & -\cos\Omega & 0\\ \sin\xi\sin\nu & \cos\xi & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{\lambda}_{\odot} \\ \dot{\nu} \\ \dot{\xi} \\ \dot{\Omega} \end{pmatrix}$$
[7.15]

where in this case $\dot{\xi} = 0$.

The real-time attitude determination was provided in the form of three small rotation (Tait-Bryan) angles relative to the nominal attitude. These Tait-Bryan angles could be added to the nominal attitude heliotropic angles to provide a new set of heliotropic angles, giving the actual rather than the nominal satellite attitude. The Tait-Bryan angles were given by Θ_1 , Θ_2 and Θ_3 . The following quantities are defined:

$$A = \cos \Omega \sin \Theta_1 \cos \Theta_2 - \sin \Omega \sin \Theta_2$$

$$B = (\cos \Omega \cos \Theta_2 - \sin \Omega \sin \Theta_1 \sin \Theta_2) \sin \xi - \cos \xi \cos \Theta_1 \sin \Theta_2$$

$$C = \sin \xi \sin \Omega \cos \Theta_1 - \cos \xi \sin \Theta_1$$

$$D = (B^2 + C^2)^{1/2}$$
[7.16]

These define the new heliotropic angles as:

$$\hat{\lambda}_{\odot} = \lambda_{\odot}$$

$$\hat{\xi} = \arcsin D$$

$$\hat{v} = v + \arcsin(A/D)$$

$$\hat{\Omega} = \Omega + \Theta_3 + \arcsin C$$
[7.17]

The real-time attitude determination operated on-board the satellite and provided a record of the satellite in Tait-Bryan angles. The process of updating the Tait-Bryan angles was based on a mixture of gyro data and star mapper transit times. The gyro data were transformed to inertial rates, then corrected for the inertial rates resulting from the nominal scanning law and then transformed into corrections to the Tait-Bryan angles. Correctly recorded star mapper transits were used to control and correct the resulting Tait-Bryan angles. The rate estimates supplied as part of the real-time attitude determination were based on the evolution of the Tait-Bryan angles and could be badly affected by a sudden change in these angles, such as caused by an update from the ground. These updates were required to assist the satellite in recovering attitude convergence after long gaps in the observations, in particular after a perigee passage.

7.4. Attitude Modelling and Estimation by NDAC

By integrating numerically the Euler equation, using the full representation of the inertia tensor and the most accurate approximations of the torques, an accurate description was obtained of the rates of the satellite. These rates would normally include an offset caused by an error in the starting point. This error was largely removed by adjusting the rates to the observed gyro rates through linear offsets. The rates could then be integrated to provide a first approximation to the actual attitude angles. This integration was also affected by an error in the starting point, as well as by minor errors in the original model. These last adjustments were corrected using star mapper transit data. By calibrating the thruster firing performance, it was possible to carry the integrations across the thruster-firing discontinuities in the velocities and acceleration. This was basically the method adopted by NDAC.

The *a priori* detection of any discontinuities in the rotation rates was very important. Most discontinuities were known: they were due to thruster firings, for which information was stored in the telemetry. Some discontinuities were, however, not recorded in the telemetry. In order to find these, the gyro rates were examined for every data set for the coincidence of thruster firings and rate discontinuities. Through these examinations thruster firings were detected for which information was lost due to telemetry problems, as well as sudden rate changes due to the satellite being hit by a micrometeoroid. Also the start and end time of gyro spin-up and spin-down events were recognised. All such events were entered into the data stream as pseudo thruster firings. The attitude solution would not cross these events but instead provide solutions up to, and starting from these instances.

The calibrations used in the NDAC attitude reconstruction made it necessary to repeat the process two to three times: the reconstructed data was used to improve the calibration results which were subsequently reused in the reconstruction. At the same time, improved reference positions for the star mapper stars were introduced. A schematic flow chart of the NDAC attitude determination procedure is given in Figure 7.1.

Integration of the Torques to Inertial Rates

The first step in the NDAC attitude reconstruction consisted of estimating, by integrating the Euler equation (Equation 7.1), the rotation rates around the three satellite axes, over intervals between thruster firings. The torques were obtained from calibrations using earlier reconstructed attitude results. This integration needed as a starting point a relatively accurate estimate of the rotation rates at the start of the interval, which was, for the first interval considered, obtained from gyro data. For subsequent intervals it was obtained by integrating across thruster firings (see below). In normal mode (no thruster firings), the accelerations were calculated in time steps of 0.71 s (at reference times -1.066 s, -0.355 s, +0.355 s with respect to the centre of an observational frame of 2.133 s). The accelerations were numerically added to the rotation rates (at reference times -0.71 s, 0.0 s, +0.71 s). All three rotation rate estimates per observational frame were stored. Experiments with shorter integration times showed that for normal conditions the time step used was sufficiently small.

The time step for the integration of the Euler equation was set by the rate of change of the cross product $\boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega}$ in Equation 7.1. This was partly accommodated by provisionally extrapolating $\boldsymbol{\omega}$ to the central time of the integration interval. The rate of change of $\boldsymbol{\omega}$ was within a range of $\pm 3 \text{ mas s}^{-2}$, which gave a change in the accelerations, resulting from the cross product, of the order of 2.5×10^{-4} mas s⁻³. The uncertainty in the accelerations resulting from the estimates of the external torques was approximately 10 times larger. Using time steps of 0.71 s the error on the velocities obtained from the integration generally accumulated to no more than about 1 to 2 arcsec s⁻¹ over a time interval between thruster firings (ranging from 300 to 2000 s), except for cases where the initial rate estimates at the start of the integration had been very uncertain due to excessive gyro noise or telemetry problems. Due to uncertainty of the reconstructed external torques, decreasing the time steps would have increased precision without improving accuracy.

Integrations were carried out in this manner until a thruster firing or an interruption was encountered, at which point the first inertial rate estimates $\boldsymbol{\omega}_t$ over an interval had been obtained, described by three estimates per observational frame.

The rates $\boldsymbol{\omega}_t$ obtained from the torque integration were adjusted to the observed (and calibrated) gyro rates by applying an offset and a linear correction with time. When available, the rate around the *z* axis was also fitted with scanning rates derived from the great-circle reduction. The gyro data were corrected for drift and orientation (see Chapter 8) and weighted according to the specific gyro noise levels. As gyros were recording rotation rates coming from all three axes, one solution was made for the three axes together. Thus, the equations solved by the method of least-squares were:

$$\mathbf{a} + (t - t_0)\mathbf{b} = \mathbf{T}(\mathbf{g} - \mathbf{g}_0) - \boldsymbol{\omega}_t + \boldsymbol{\epsilon}_g$$
[7.18]

where **g** represents the gyro observations, which were corrected for drift (\mathbf{g}_0) and transformed to rotation rates around the three satellite axes through matrix **T**, obtained from Table 7.2. The time *t* was measured in units of 100 s relative to a reference time t_0 halfway into the thruster firing interval. The errors on the gyro readings were given by ϵ_g , for which the variances were known from the gyro calibration process described in Chapter 8. These variances had been determined over an entire data set (6 to 8 hours of observations), and compared for consistency with neighbouring data sets. The square-root variances for the gyro data ranged from 0.008 arcsec s⁻¹ to 0.1 arcsec s⁻¹. With

the errors at the low end of the range the gyro data were contributing to the attitude solution, while with errors at the high end of the range, the contribution was effectively restricted to providing estimates of the star mapper scan velocities (see also Section 8.3).

The variance observed over an interval between thruster firings was used as an indication for the data being sufficiently well described by Equation 7.18. There was also the possibility of rejecting outliers in the gyro data, but this was rare under normal conditions, and had to be suppressed under bad conditions (noise bursts). When the variance observed over an interval was higher than expected, the gyro data were examined for discontinuities. Many such cases revealed instances when the satellite had been hit by a micrometeoroid, causing (mostly small) discontinuities in the rotation rates. At such discontinuities, an artificial thruster firing event was added to the data set to account for the discontinuity, which then split the solution over the interval investigated into two separate intervals.

The solution of Equation 7.18 led to $\omega_g = \omega_t + \mathbf{a} + (t - t_0)\mathbf{b}$, the estimated inertial rates based on the integrated torques and gyro data. These rates were used in the star mapper reductions (see Chapter 6) for the determination of the effective slit spacings. Using ω_g , an estimate of the rotation rates at the end of the thruster-firing interval was obtained, providing the rates at the start of the next thruster firing.

The integration of Equation 7.1 was carried across the thruster firing using information obtained from the thruster calibrations. Here, however, the time step was much shorter in order to accommodate the very rapidly changing rotation rates. Every 0.71 s interval used in the normal integration was subdivided into 160 intervals of 1/225 s, still providing the three estimates of the rotation rates per observational frame. The 'cross-talk' effect of the thrusters (described in Chapter 8 and primarily resulting from the shift in the position of the centre of mass due to the full apogee boost motor tank), was fully taken into account, as were the 'zero points'. Thus, a firing on the *z* axis would also cause small velocity changes for the *x* and *y* axes rotations. At the end of a thruster firing integration, estimates of the rotation rates for the start of the next thruster firing interval had been obtained.

The solutions for Equation 7.18 over all intervals contained in a uninterrupted stream of data (interruptions could be due to gaps in the data containing one or more thruster firings, or so-called pseudo thruster firings, representing various discontinuities in the rotation rates) were all connected through conditions describing the expected changes in the rotation rates across thruster firings relative to the corrections already applied. The final solutions for Equation 7.18 were obtained from such a chain of simultaneously solved, linked least-squares solutions, and applied to the estimates ω_t .

Integration of the Rates

Given an estimated *a priori* orientation of the satellite axes, the inertial rates could be used to evaluate those orientations as a function of time, providing the first estimate of the satellite attitude. The attitude at this stage was described with respect to the nominal attitude in the form of Tait-Bryan angles. This allowed some incorporation of data from the real-time attitude determination, which was described in the same way. If the satellite were to follow exactly the nominal scanning law, then this would result in inertial rotation rates given by $\boldsymbol{\omega}_n$ (as described in Section 7.3, Equation 7.15). These rates were, however, described in a slightly different coordinate system: the coordinates of the nominal scanning law, with respect to which the satellite was displaced by the

Tait-Bryan angles. Thus, in order to evaluate the evolution of the Tait-Bryan angles relative to the nominal attitude, the difference $\delta \boldsymbol{\omega} = \boldsymbol{\omega}_g - \mathbf{R} \boldsymbol{\omega}_n$ had to be integrated, where $\mathbf{R} = \mathbf{R}_3(\Theta_3)\mathbf{R}_2(\Theta_2)\mathbf{R}_1(\Theta_1)$. The integration to position angles needed a starting point. At the start of an interval, a real-time attitude determination position was used as such. However, the real-time attitude determination was not always converged at such a point. This was shown in the data from the examination of the star mapper reduction results (see below). In such a case the interval was rejected, and the integration started again for the next interval, until the attitude appeared to have converged. From then on the integration could proceed unaided by real-time attitude determination data.

The transformation from $\delta \boldsymbol{\omega}$ to increments of the Tait-Bryan angles Θ were obtained from:

$$\frac{\mathrm{d}\Theta}{\mathrm{d}t} = \begin{pmatrix} \cos\Theta_3/\cos\Theta_2 & -\sin\Theta_3/\cos\Theta_2 & 0\\ \sin\Theta_3 & \cos\Theta_3 & 0\\ -\cos\Theta_3\tan\Theta_2 & \sin\Theta_3\tan\Theta_2 & 1 \end{pmatrix} \delta\boldsymbol{\omega}$$
[7.19]

where the time step dt was 1/22.5 s for normal observational frames (2.133 s), and 1/225 s for the first half of an observational frame starting with a thruster firing (thruster firings never lasted for more than half an observational frame, and mostly did not last for more than 0.5 s). In the numerical integration $\delta \omega$ was calculated for the central time of the integration interval, using the three estimates of ω_g per observational frame in the form of a second order polynomial describing the estimated inertial rates at any time during the frame. For frames with thruster firings the estimates of the rates also used the lengths of the firings on each axis. Experiments with time steps of different lengths had shown that there was no significant loss of accuracy for the time steps used (the numerical accuracy over a thruster firing interval was much higher than the expected accuracy of the attitude angles).

In the first solution of the attitude, when star mapper transit times were still to be determined, the estimated Tait-Bryan angles were fitted to the real-time attitude determination Tait-Bryan angles, providing a kind of smoothed and stabilised real-time attitude estimate. These fits were done with only an offset or at most a linear time dependence in order to preserve the dynamical model. Such a fit was only needed in order to produce predicted star mapper transit times, which could be compared with observed transit times. The comparison, which was preserved in the form of a graph for data in every orbit, showed the convergence of the real-time attitude determination in both fields of view as a function of time. This allowed the detection and early elimination of bad convergence time intervals from the data (see Figure 6.3).

The results of the integration were stored in the form of heliotropic angles (see Equations 7.16 and 17), at the three reference times per frame that were earlier used for the inertial rates (see above).

Final Attitude Corrections

The preliminary attitude angles obtained this way were used together with star mapper transit times to determine the actual satellite attitude angles. The three estimates per observational frame of the heliotropic angles were represented through second order polynomials, describing the angles and their rates of change at the exact time of the transit. When star mapper transit times still had to be calculated, the rates of change of the heliotropic angles were translated into inertial rates around the satellite axes, using Equation 7.15, and subsequently into inertial rates along the scan direction and perpendicular to the scan direction for each field of view. These rates were then used

in the determination of star mapper transit times (see Chapter 6), and the heliotropic angles were recalculated for the observed transit time.

The heliotropic angles and the apparent stellar coordinates, were used to calculate the predicted coordinates (η , ζ) in the field of view. These coordinates were related to the expected coordinates, defined by the star mapper geometry. The differences between the observed and predicted value for η , $\delta\eta$, were expressed in three coordinates: a correction to the scan phase ($\delta\psi$), and corrections to the attitude angles perpendicular to the scan-direction in the preceding ($\delta\theta_p$) and following field of view ($\delta\theta_f$):

$$\delta \psi + g \,\delta \theta_{p,f} + \epsilon_{\eta} = \delta \eta \tag{7.20}$$

Here g = 0 for the vertical slit group and ± 1 for the inclined slit group of the upper and lower branch respectively, and suffix p or f applies according to the field of view of the transit. The variance of the error on the transit time, ϵ_{η} , was known from the star mapper processing.

The corrections $\delta \psi$, $\delta \theta_p$ and $\delta \theta_f$ were expressed as polynomials in time, with the order depending on the length of the interval, the amount of data available, and the demands placed by the observations. The polynomial degree was always initialised at a minimum level (usually 1 or 2), and increased when the observed variance indicated that the solution was inadequate, up to a point where it was considered that insufficient data was available for any further increments. The need for increments was determined on the basis of the observed unit weight variance. Although one solution was made for the three coordinates together, variances were calculated for each coordinate separately, and also increments of polynomial degree were decided independently. Preliminary solutions were made over intervals between thruster firings. Boundary conditions were imposed that linked these solutions across the thruster firings. These boundary conditions forced approximate, and not exact, continuity of the solutions across thruster firings, thus accommodating the uncertainties imposed by the thruster firing calibrations. The final solution was made over all intervals and boundary conditions without further iterations.

The estimation standard errors on the reconstructed attitude were calculated from the local (thruster firing interval) normal equations matrix, **A**, of the fitted attitude parameters through a convolution with the basis functions at the centre of every observational frame. These estimated errors agreed also with the requirements of the Tycho data reductions, which used the NDAC reconstructed attitude results. Thus, if $\mathbf{f}(t)$ was the vector of basis functions for one of the attitude angles in the interval considered, and the unit weight error of the solution was u, then the uncertainty in the attitude angle was given, as a function of time, by:

$$\sigma(t) = u \left[\mathbf{f}(t)' \mathbf{A}^{-1} \mathbf{f}(t) \right]^{1/2}$$
[7.21]

When the uncertainty was above 0.3 arcsec the data was flagged as 'bad'. In the error evaluation, only this last solution was relevant: all preceding steps provided the smooth background relative to which the final solution was made.

The final transformation from $\delta \psi$, $\delta \theta_p$ and $\delta \theta_f$ to corrections of the heliotropic angles was given by:

$$\begin{aligned} \widetilde{\lambda}_{\odot} &= \widehat{\lambda}_{\odot} \\ \widetilde{\xi} &= \widehat{\xi} + (\delta\phi\sin\Omega - \delta\theta\cos\Omega) \\ \widetilde{\nu} &= \widehat{\nu} - (\delta\phi\cos\Omega + \delta\theta\sin\Omega) / \sin\xi \\ \widetilde{\Omega} &= \widehat{\Omega} + \delta\psi + (\delta\phi\cos\Omega + \delta\theta\sin\Omega) \tan\xi \end{aligned}$$
[7.22]

where $\delta\phi = (\delta\theta_p - \delta\theta_f)/2\sin(\gamma/2)$, $\delta\theta = (\delta\theta_p + \delta\theta_f)/2\cos(\gamma/2)$ and γ is the basic angle between the fields of view. The incremental angles $\delta\psi$, $\delta\theta$ and $\delta\phi$ were approximately the same as those used in the attitude reconstruction by FAST.

In conclusion, the NDAC attitude was determined by using star mapper transit data to determine small corrections relative to a dynamical model for the satellite, which was supplemented with gyro observations.

Basic Angle and Star Mapper Geometry Calibration

Three components of the star mapper geometry were calibrated using the attitude reconstruction processes: the orientation of the slits with respect to the scanning circle (defined as the great circle going through the intersecting points of inclined slits in the preceding and following fields of view), the separation between the vertical and inclined slits and the basic angle between the two fields of view for the vertical slits (due to projection and distortion effects this was slightly different from the basic angle for the main grid). These calibrations were carried out using star mapper transit time residuals obtained from the attitude reconstruction solution, accumulated over 2 to 3 days. The residuals were accumulated in histograms (68 bins) as a function of vertical coordinate H of the transit. Each bin coincided with a scan-field, the basic element used in manufacturing the grid. Medians in each bin were fitted with an offset and a linear function of the H coordinate. These fits were made for the two slit groups and the two fields of view separately. The offsets measured were of the order of 10 to 30 mas, representing small basic-angle corrections and corrections for the slit-group separation. The rotations (relative to the main 5 arcmin rotation) were of the order of 0.3 to 0.7 arcsec.

The remaining residuals were accumulated in a histogram to provide the medium-scale distortion corrections. These corrections were of the order of 0.1 arcsec or less, and were further improved in the Tycho data reduction (see Volume 4, Section 7.3). The final correction, describing the position of the star mapper slits with respect to the main grid reference line, was provided by the great-circle reduction, presented in Chapter 9.

7.5. FAST Attitude Model

Assuming an attitude model with a finite number of degrees of freedom, the model error is defined as the difference between the true attitude and the attitude that could be reconstructed by using in an optimal way the degrees of freedom of the model if measurement data without any error were available. Then, according to an attitude estimation method, which was based on this model, the estimation error is defined to be the effect on the attitude reconstruction of the actual measurement errors, if the models were perfectly correct.

The model error can be evaluated by simulations assuming the true system is perfectly known, however it cannot be evaluated by experimental data corrupted by measurement errors. In this latter case the model error can be monitored by significance tests, which, assuming by hypothesis that the model is correct, gives the *a priori* probability of getting the resulting test values. When this probability is too low the hypothesis tested is usually rejected. On the other hand, the estimation error caused by the measurement errors

can be evaluated by the estimation procedure, assuming, of course, that the model error is negligible.

For these reasons it was required that the model error should be negligible with respect to the estimation error, setting its maximum limit, evaluated by simulations, to the order of several mas. It is generally possible to reduce the model error by increasing its number of degrees of freedom, but, of course, the estimation error increases with the number of degrees of freedom. The research was oriented to get the required model accuracy with the smallest possible number of degrees of freedom.

The use of the rigid body equations to describe the satellite motion was excluded because it was considered that it was too difficult to calibrate, with the required accuracy, the telescope reference frame position with respect to the inertial axes of the satellite, and to maintain such an accuracy throughout the mission. Similarly the use of torque models derived from physics was excluded because at the level of the required accuracy they would require too large a number of degrees of freedom, and above all because it is never possible to be sure that all the non-negligible causes have been taken into account. This method was nevertheless successfully implemented by NDAC (see previous section).

Particular attention was devoted to the choice of the attitude angles. In fact, the various sets of three angles are related among themselves by non-linear transformations so that their choice influences the simplicity of the attitude model, the estimation method, and accuracy.

FAST Attitude Angles

The FAST attitude representation was described relative to the reference great-circle frame (Section 11.2), defined by the position of its pole in ecliptic coordinates, $\lambda_{\rm R}$ and $\beta_{\rm R}$. This pole was selected as the position of the *z* axis in the nominal scanning law for a reference time halfway between the start and the end of the data set concerned. Relative to this reference system, three attitude angles were defined: ψ , θ and ϕ . They are related to the classical 3-2-1 Euler angles ψ_e , θ_e and ϕ_e by:

$$\phi = \phi_e, \qquad \theta = \theta_e, \qquad \psi = \psi_e - \arctan\left(\frac{\sin\theta\sin\phi}{\cos\theta + \cos\phi}\right)$$
 [7.23]

The choice of the angle ψ was motivated by the fact that it models the scanning motion better, because its rate of change is much closer to the rotation rate around the satellite *z* axis. It was in addition proven that estimation results obtained with these angles were significantly better than those obtained by keeping ψ_e .

Approximate Motion Equations

The ϕ , θ , ψ attitude angles are modelled as the outputs of the following two independent systems of differential equations, which were obtained as an approximation of the satellite dynamic equations in the case of small perturbations:

$$\dot{\omega}_z = \delta_z(t)$$

$$\dot{\psi} = \omega_z$$
[7.24]

and:

$$\begin{split} \dot{\omega}_{x} &= \delta_{x}(t) \\ \dot{\omega}_{y} &= \delta_{y}(t) \\ \dot{\phi} &= -\omega_{0}\theta + \omega_{x} \\ \dot{\theta} &= -\omega_{0}\phi + \omega_{y} \end{split}$$
 [7.25]

where ω_0 is a constant parameter equal to the nominal spin rate.

The functions $\delta_x(t)$, $\delta_y(t)$, $\delta_z(t)$ are the derivatives of the rate components ω_x , ω_y , ω_z . In a first rough approximation they model the effect of the torque acting on the satellite, but more precisely they have been introduced not only to model the torque effect but also to compensate for the approximation introduced by using the above simplified equations to model the telescope motion. It is well known that any given set of three continuous time functions $\phi(t)$, $\theta(t)$, $\psi(t)$ can be obtained as the output of the above dynamic system by the application of a suitable set of input functions $\delta_x(t)$, $\delta_y(t)$, $\delta_z(t)$ and by initial conditions. Then, in principle, the model error can be reduced as much as required by attributing a sufficient number of degrees of freedom to the input functions.

System Input Functions

The system inputs $\delta_{x,yz}(t)$ are decomposed into the sum of two terms, respectively the control inputs $u_{x,yz}(t)$ and the disturbances $d_{x,yz}(t)$. The control inputs are introduced to describe the effects of the on-board attitude control: they are assumed to be a sequence of ideal pulses (Dirac functions) of unknown amplitude applied at the central time of the gas-jet actuations. The disturbances are introduced to describe the effects produced by the perturbing torques and to keep into account, at the same time, the difference between the above simplified dynamic system and the reality. The disturbances are expressed by different mathematical models (Fourier and polynomial models) depending on the length of the time interval considered and the satellite perturbing conditions. In the case of Fourier models the disturbances are described by a Fourier series with unknown coefficients and the response of the system is computed on the basis of the above differential equations. In the case of polynomial models the system forced response to the disturbances is directly described by polynomial series. The resulting attitude model is presented in the two cases in Figure 7.2. The following models were used:

• long-term Fourier model: this was the standard model, used in normal operating conditions over a time interval corresponding to a full rotation around the satellite *z* axis, i.e. 128 min. Independently from the actual duration of the considered time interval, the disturbances are described by a Fourier series whose fundamental harmonics have the frequency ω_0 corresponding to the satellite nominal spinning rate used in the approximate equation of motion (Equation 7.25). In this case the forced solution of the differential equations has the form:

$$\phi_{d}(t) = b_{s0}t\sin\omega_{0}t + b_{c0}t\cos\omega_{0}t + \sum_{k=2}^{n} [b_{sk}\sin(k\omega_{0}t) + b_{ck}\cos(k\omega_{0}t)]$$

$$\theta_{d}(t) = c_{s0}t\sin\omega_{0}t + c_{c0}t\cos\omega_{0}t + \sum_{k=2}^{n} [c_{sk}\sin(k\omega_{0}t) + c_{ck}\cos(k\omega_{0}t)]$$

$$\psi_{d}(t) = \sum_{k=1}^{m} [a_{sk}\sin(k\omega_{0}t) + a_{ck}\cos(k\omega_{0}t)]$$
[7.26]

A different number of harmonics in the range 12 to 36 was used in modelling ψ (*m*) and in modelling θ and ϕ (*n*). The number of harmonics was determined by an adaptive method based on the computation of the Fisher-Snedecor test (Cramér, 1946). The adaptation was made by increasing the number of harmonics by 3 at a time, starting from 12. The optimal number was the maximum number for which the Fisher-Snedecor test value was still above a given threshold;

- intermediate term Fourier model: this model was used over intervals of 30 min to 115 min. The perturbing torques were modelled as above by a finite Fourier series of unknown amplitude harmonics (Equation 7.26), but without the mixed terms for ϕ and θ (describing the torque caused on ϕ through the change of θ and *vice versa*). Here the fundamental period was chosen equal to the length of the time interval, and the minimum number of harmonics at the start of the computation was determined by the interval length. They were increased by one at a time, followed by the tests described above, until a test value within a given threshold was obtained;
- short-term polynomial model: this model was used for the shortest considered time intervals, namely a period between two thruster firings (varying from a few minutes to more than half an hour, but on average about 12 to 15 minutes). The perturbing torque effect was modelled by Legendre polynomials, the degree of which was adapted to the considered case, independently for ψ and for θ and φ. The polynomial degrees were increased by one at a time until the Fisher-Snedecor test value obtained was within a given threshold;
- short-term polynomial model in penumbra: while the satellite was in penumbra, before and after an eclipse, the solar radiation torque followed a transient from high values to very low or *vice versa*. Conditions during different transients were similar but never the same. The transient was modelled as a stochastic process for which a Karhunen-Loève expansion (Papoulis 1991) was computed. This provided an ordered set of orthonormal functions, of which each one was approximated by a Legendre polynomial representation. The first 10 (ψ) and 15 (θ and ϕ) functions were stored. The order of the Karhunen-Loève expansion used was adapted to the particular case considered, starting from a minimum value determined by the number of star mapper transits available, up to the maximum number of stored functions, applying at each step the Fisher-Snedecor test.

7.6. FAST Estimation Procedure

The Different Attitude Estimation Algorithms

The uncertainty with which the attitude can be determined depends upon the precision with which star positions are known. Since the positions provided by the Input Catalogue had not the required accuracies, an iterative procedure was adopted by FAST for the Hipparcos data reduction based upon successive improvements of star positions which, in turn, were used to improve the attitude (see Chapter 4). This required the implementation of three different attitude reconstruction algorithms. **First year attitude reconstruction (OGARO):** Input Catalogue errors were taken into account modelled as uncorrelated random variables with known variance. The star coordinates were estimated together with the attitude angles.

Standard attitude reconstruction (OGAR): After the first Hipparcos catalogue construction, the errors affecting the apparent coordinates, whether they were improved or not, were no longer considered as unknowns. An equivalent standard deviation of the transit time was introduced which was obtained by the combination of errors in the star mapper transit time measurements, the star mapper calibrations, and the star coordinates. The software estimated the attitude angles only, assuming that the apparent star coordinates were perfectly known.

Iterated attitude reconstruction (OGARI): The satellite rotation around the *z* axis obtained by the great-circle reduction was used as additional input data, together with the star mapper transit times. This mode was used for the final iterations of the catalogue.

These estimation algorithms are not significantly different from the point of view of their methodology. They will be illustrated as applied to OGAR.

Principle of Attitude Estimation

The only types of data used for the attitude reconstruction were the star mapper transit times and apparent star coordinates, with their respective standard deviations, and the central time instants of thruster firings. Gyro measurements and real-time attitude data were not used, as these data were not found to be accurate enough to give a useful contribution to the attitude reconstruction accuracy both because of the gyro measurement errors and the errors of the position of their axes in the telescope frame.

Assuming the attitude model given in Section 7.5 and the availability of the above measurement data, a measurement equation was written for each star transit obtained by star mapper data processing (Chapter 6). Such equations require that the measured transit time is equal to the sum of an ideal transit time, expressed by a function of the attitude angles, star coordinates and calibrated star mapper geometry, and of the errors resulting from the star mapper data processing, the star catalogue used and the star mapper calibrations.

The attitude model was then inserted into the measurement equations, yielding a set of equations where the unknown variables were given by the degrees of freedom of the attitude model. The data was divided in time segments, always starting and ending with thruster actuations. The attitude was reconstructed for each segment in one step through the solution of a set of simultaneous equations, using the maximum likelihood estimation criterion to determine the unknown parameters by maximization of the measurement error probability. A Gaussian distribution of the errors was assumed and the solution was obtained working at a second-order level with the Gauss-Markov method. The unknown parameters of the attitude model and their covariance matrix were estimated and used to calculate the attitude angles and their standard deviations.

The rest of this section gives a more detailed account of the processes outlined above.

Measurement Equation and Estimation Procedure

For each detected star mapper transit the following data were available:

- the transit time $\hat{\tau}$ and its standard error as evaluated in the star mapper data processing. Using the instantaneous rotation rate, the standard error could be expressed as an error angle ϵ ;
- the apparent stellar coordinates (*v*, *r*) in the reference great-circle frame, with their standard deviations whenever they came from the Input Catalogue and were not improved by an intermediate sphere solution;
- the field of view index (f = +1 for preceding and f = -1 for following) and the slit-group index (g = 0 for vertical, g = +1 for upper inclined, g = -1 for lower inclined);
- the reference position of the slit group, $\eta_0(f,g)$. This position consists of $f\gamma/2$ (where γ is the basic angle) and the distance between the fiducial reference line for the main grid and the reference line for the slit group (G_v or G_i in Section 6.1);
- the calibrated grid-to-field transformation for the star mapper with its standard deviations (see Chapter 6).

On the basis of the above data the following measurement equations were written for a vertical slit transit:

$$\psi = v + \eta_0(f, g) + \Delta \eta_{fg}(\phi, \theta, r) + \epsilon_v$$
[7.27]

and for a transit through the inclined slits:

$$-\phi fg \sin \eta_0 - \theta g \cos \eta_0 = v + \eta_0 (f,g) - gr - \Delta \eta_{fg}(\phi,\theta,r) - \psi + \epsilon_i$$
[7.28]

where the attitude angles refer to the observed transit time $\hat{\tau}$. ϵ_v and ϵ_i are random variables due to the errors in transit time estimate, apparent star coordinates, and the star mapper calibration. Their standard deviations were calculated from the input data. $\Delta \eta_{fg}$ are complicated non-linear functions of ϕ , θ , r including corrections from the calibrations. The terms $\Delta \eta_{fg}$ have small derivatives with respect to the variables ϕ , θ , r in all their admissible range. This particular property allowed these equations to be considered as linear in the variables ϕ and θ , and to be updated in an iterative procedure.

When a time interval and an attitude model had been selected the attitude angles $\phi(t)$, $\theta(t)$, $\psi(t)$ were computed as functions of the degrees of freedom of the model by solving the differential equation system, Equations 7.24 and 7.25. Since the system is linear, the attitude angles were obtained from the linear measurement Equations 7.27 and 7.28, and a set of equations was obtained in the unknown variables corresponding to the degrees of freedom of the model. As already pointed out, $\Delta \eta_{fg}$ could be computed from tentative values of ϕ and θ , so that the above system of equations could be assumed linear in the unknown variables.

The unknown variables were solved according to the maximum likelihood criterion, maximizing the probability of the random variables ϵ_v and ϵ_i . Since it was known that these variables have a Gaussian distribution, the maximum likelihood criterion was implemented by iterative application of the Gauss-Markov method.

The procedure started by assuming $\theta(t) = \phi(t) = 0$, from which the first approximation of $\psi(t)$ was calculated, using only vertical slit transits. Subsequently, the inclined slit equations were considered and a first approximation of $\theta(t)$ and $\phi(t)$ was obtained. The procedure was iterated until it converged to a stable solution.

The Gauss-Markov method allowed the estimated values of the unknown parameters and an estimate of the covariance matrix to be obtained simultaneously, from which the standard deviation of the reconstructed attitude was computed for each observation frame. A simplified approach was adopted in order to reduce the computing time.

Time Segmentation and Model Selection

In normal conditions, i.e. in the absence of eclipses, occultations or other events disturbing the data, the long-term Fourier model was used. Intervals of a length close to a rotational period of the satellite were considered, starting and ending with a thruster firing. An overlap of at least one thruster firing interval between neighbouring solutions was used to avoid discontinuities in the attitude reconstruction. This was the preferred solution which was applied whenever possible, introducing occasionally large overlapping in order to allow this type of solution.

Occultation periods during which the thrusters were not fired could be included in the long-term model. When thruster firings took place during the occultation, then the data before and after the occultation could not be included in the same model. When, due to the presence of long occultations, it was not possible to select a time interval of the order of one satellite rotation (115–130 minutes), then, for intervals of a duration in the range of 30–115 minutes, the intermediate-term Fourier model was used.

When the satellite was in penumbra the short-term polynomial model in penumbra was imposed, interrupting the long- and intermediate-term models.

The short-term polynomial model was used only for single thruster firing intervals that could not be joined up with other time intervals.

Both polynomial models required a large number of degrees of freedom in consideration of the length of the time interval and they presented an estimation error larger than the one obtained by the Fourier model. Thus, they were used only in the case of penumbra or anomalous behaviour, when the application of the Fourier model could give nonnegligible model errors.

Significance Test and Anomalous Situation Detection

The residuals for all observations were computed and subjected to a χ^2 test to assess the validity of the assumptions made: the goodness of the estimated variances of the measurement errors and the goodness of the parametric model used. Since the variances of the estimated star mapper transit time errors were very well assessed, the χ^2 test was mainly used to control the goodness of the attitude model, but it also allowed the star mapper data to be automatically rejected when the residuals were grossly inconsistent with its variance (as in the case of the observation of a wrong star or very erroneous star positions).

The χ^2 test permitted also the detection of situations in which the perturbing torque acting on the satellite had an anomalous behaviour. In such situations the short-term polynomial model was imposed, but this model was so general that it was rather suitable for any torque behaviour.

Procedure	σ_{ϕ}	$\sigma_{ heta}$	σ_{ψ}
OGARO	200	100	60
OGAR	60	30	15
OGARI	30	20	2

Convergence of the Iteration Process

The improvement of the attitude at different iterative stages was essentially due to two factors:

- each iteration involving either more observations or being just a re-iteration with the same data provided improved star positions for more and more stars (OGAR and OGARI);
- the determination of the attitude in ψ by the great-circle reduction permitted the assumption that this angle is perfectly determined so that only θ and ϕ remained to be computed, allowing them to be decorrelated from ψ (OGARI). In addition (see Section 6.1) its high precision meant that remaining transit time residuals reflected primarily the star mapper geometry corrections, which could therefore be calibrated, providing another means of improving the attitude.

The improvement obtained during this iterative procedure is illustrated by Table 7.3, which gives typical orders of magnitude of the *a posteriori* standard deviations for the three angles as obtained in the mean by each of the attitude determination procedures.

7.7. Performance Comparisons

A comparison of the reconstructed attitude results obtained by FAST and NDAC was carried out on suitably selected sets of reference great circles. The aim of the comparison was the assessment of the attitude reconstruction accuracy and the level of agreement between the results obtained by the two consortia, pointing out any unjustifiable disagreements. The comparison involved also results obtained with the great-circle reduction. The comparison was carried out along the following steps.

Preparation and Synchronization of the Data

The attitude reconstruction results produced by NDAC and FAST were expressed in different variables and referred to different celestial frames, as was described in the preceding sections. The attitude results were collected in data sets, describing the satellite axes positions and rotation rates for every observational frame. The criteria for accepting and deleting data were slightly different between the consortia, and gaps of different lengths occurred in the data. The first step in the comparison exercise was

therefore to synchronize the data, to extract a common sampling time set. Such a set contained typically 12 000 samples.

Differential Rotations

The reconstructed attitude can be represented by an orthonormal matrix, describing the attitude by the direction cosines of the equatorial coordinates at J2000 of the satellite (or instrumental) axes:

$$\mathbf{A}_{j}(i) = [\mathbf{x}_{j}(i) \mathbf{y}_{j}(i) \mathbf{z}_{j}(i)]$$
[7.29]

where *i* denotes the sampling time instant and j = 1, 2 the two consortia.

The differences between the two sets of attitude data were expressed by 3-2-1 Euler angles, corresponding to the three rotations $\phi(i)$, $\theta(i)$, $\psi(i)$ through which $\mathbf{A}_2(i)$ was obtained from \mathbf{A}_1 . The rotation vector $\mathbf{r}(i) = [\phi(i) \ \theta(i) \ \psi(i)]'$ described the attitude reconstruction differences with respect to the on-board instrumental reference frame, the satellite axes. The same difference can be expressed with respect to the celestial reference frame through the transformation:

$$\mathbf{r}_c(i) = \mathbf{A}\mathbf{r}(i)$$
 [7.30]

where **A** can refer to either A_1 or A_2 without any significant difference.

The differential rotation vector $\mathbf{r}(i)$ was decomposed as the sum of systematic and random differences. Two possible sources of systematic differential rotations were considered: the definitions of the celestial and the instrumental reference frames respectively.

The instrumental reference frame was defined by the star mapper geometry calibrations. Differences between the consortia in this calibration would cause a differential rotation that is constant in the instrumental reference frame. This constant rotation was denoted by the vector:

$$\mathbf{r}_0 = \begin{pmatrix} m_\phi \\ m_\theta \\ m_\psi \end{pmatrix}$$
[7.31]

The celestial reference frame was defined by the positions of the star mapper stars as used in the attitude reconstruction. The two consortia determined and applied different corrections to the original Input Catalogue positions, resulting in slight differential rotations between the celestial reference frames used. This rotation was constant when expressed in celestial coordinates and described by the vector:

$$\mathbf{r}_{c0} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$$
 [7.32]

Thus, the differential rotation vector could be described by:

$$\mathbf{r}(i) = \mathbf{r}_0 + \mathbf{A}^{-1}(i)\mathbf{r}_{c0} + \mathbf{e}(i)$$
[7.33]

where $\mathbf{e}(i)$ denotes the residual random term.

The systematic differential rotations could be estimated in principle by applying an ordinary least-squares method, but a collinearity problem arises, not allowing a rotation of the instrumental reference frame about the body z axis to be distinguished from a rotation of the celestial reference frame. Considering the fact that the calibration of the body reference frame with respect to rotation about the z axis was verified at the level of the great-circle reduction (see Chapter 9), it was assumed in the attitude comparison

that $m_{\psi} = 0$, leaving only the variables m_{ϕ} , m_{θ} , m_x , m_y , and m_z to be estimated by a Householder procedure.

Estimation of Variances

Once the estimated values were computed it was necessary to verify if the hypothesis that they were different from zero could be accepted, or if the resulting values were to be retained as incidental effects, produced by the random error $\mathbf{e}(i)$. Actually, while a statistical model for FAST attitude reconstruction was available, this was not the case for NDAC. So, an *a posteriori* approach was applied, based on the computation of the entropy of the signal $\mathbf{r}(i)$ (Donati & Sechi 1992). The distribution function equivalent number of the signal was computed and the separator thresholds were derived and used to establish the significance of the systematic components m_{ϕ} , m_{θ} , m_x , m_y , and m_z of $\mathbf{r}(i)$ obtained by applying the least-squares method to the system of equations (Equation 7.33).

Global Evaluation of the Standard Deviations

Then, modelling $\mathbf{r}(i)$ by a second order stochastic process with a uniform distribution of energy in a suitable orthogonal basis of dimension equal to the already evaluated equivalent number of degrees of freedom, the standard deviations of these components were evaluated:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{\phi} \\ \sigma_{\theta} \\ \sigma_{\psi} \end{pmatrix}$$
 [7.34]

Denoting by σ_{1q} and σ_{2q} the standard deviations of the two attitude reconstructions to be compared, in any of the angles ($q = \phi$, θ or ψ), the following holds:

$$\sigma_q^2 = \sigma_{1q}^2 + \sigma_{2q}^2 - 2\rho\sigma_{1q}\sigma_{2q}$$
[7.35]

where ρ is the correlation coefficient between the two realisations of the attitude. The correlation is close to +1 because both are obtained from the same star mapper observations and their Poisson noise. Given σ_{1q} and σ_{2q} , this relation allowed the construction of a test of the hypothesis that both attitude reconstructions were within given limits. This had to be satisfied for an acceptable value of ρ . The test could, of course, not exclude that both reconstructions were affected by some large correlated errors.

Results of the Last Attitude Comparison

The last attitude comparison was carried out on a set of 24 orbits chosen among those having some critical problems such as spin-up of the redundant gyro, redundant gyro running, well covered eclipse, reference great-circle pole close to the ecliptic plane, strong perturbing torques.

Systematic differences were found both in the instrumental and in the celestial reference frames. The first were explained by some differences in the star mapper calibrations, and the second by differences in the celestial reference frames adopted by the consortia. These differences were taken into account by the catalogue merging procedure. Assuming that the term m_{ψ} is negligible since it was determined by the great-circle reduction, the effects in each frame were evaluated. The results are presented in Table 7.4. In

Table 7.4. Estimated values and standard deviations of the rotation components, expressed in mas.

Component	Estimate (mas)
m_{ϕ}	-12 ± 14
$m_{ heta}$	$+98\pm23$
m_X	$+15\pm 6$
m_y	$+17\pm 6$
m_Z	$-13\pm~5$

Table 7.5. Estimated standard deviations of the random terms in the differential rotation in body axes, expressed in mas. Column (a): all frames included; (b): excluding outliers, i.e. random terms exceeding $\pm 3\sigma$.

Component	Estimates (mas)	
	(a)	(b)
σ_{ϕ}	129 ± 90	82 ± 24
$\sigma_{ heta}$	81 ± 47	54 ± 14
σ_{ψ}	12 ± 10	3 ± 3

addition, Table 7.5 shows the standard deviations σ_q of the components ($q = \phi, \theta, \psi$) of the differential random rotation $\mathbf{e}(i)$ along with their estimated standard deviations σ_{σ_q} .

The results shown in Tables 7.4 and 7.5 correspond to the most difficult cases, presumably subject to the largest errors. However, in normal cases found in the great majority of great circles, they happen to be also comparable to the figures given in Table 7.3. The interpretation is that attitude reconstructions made in both consortia show a general agreement which is of the order of the requirements and is sufficient to avoid the introduction of significant differences in the results of the great-circle reduction—the primary objective of the attitude determination activities. The uncertainties in the final NDAC attitude were in addition estimated in the Tycho data reductions as being at least 7 mas along scan and 30 mas perpendicular to the scan, much better than the 100 mas expected before launch for the latter quantity.

F. van Leeuwen, F. Donati, J. Kovalevsky

8. TIMING AND CALIBRATIONS FROM THE ATTITUDE RECONSTRUCTION

The attitude reconstruction processes, described in Chapter 7, permitted the calibration of a variety of instrumental and environmental influences acting on the satellite to an accuracy that had not been previously possible. This was a result of the accuracy demanded from, and achieved by, the attitude data, in particular the star mapper transit times combined with the accurate stellar positions provided by the Hipparcos mission. It allowed precise calibration of orientations, scales, drifts and noise levels of gyro readings (used and implemented in the real-time attitude determination), thruster performance (used in the on-ground attitude reconstruction), the detector grid geometry (used in real-time and on-ground attitude reconstruction) and also the inertia tensor (used in on-ground attitude reconstruction). Further analysis of the attitude results also showed what torques were acting on the satellite, and how these torques evolved during the mission. Accurate timing of the measurements was very important for certain aspects of the mission, in particular for minor planet observations and the description of the satellite orbit; the associated calibration procedures are described.

8.1. Characteristics of the Orbit

The satellite described a perturbed elliptical orbit (the geostationary transfer orbit with increased perigee height) with average elements as given in Table 8.1. The decrease rates in semi-major axis and period were directly related to passages through the outer layers of the Earth's atmosphere, and were three to four times higher than the average value when perigee was low, and much smaller when perigee was high. Perigee height varied according to the orientation of the major axis of the orbit with respect to the direction of the Sun as seen from the Earth.

The orbital period had been chosen close to 5 rotational periods of the satellite (38400 s); 9 orbital periods covered close to 4 days. This resulted in semi-periodic patterns of ground-station visibility. In the data processing the successive apogee passages received a monotonically increasing 'orbit number' o (see Volume 1, Section 2.8), helping to relate data reduced by the two reduction consortia for comparison exercises. Successive orbits for which $o_m = \text{mod}(o, 9)$ were equal, were generally very similar in characteristics.



Table 8.1. Characteristics of some of the elements of the satellite orbit: mean value, secular rate of change (unit per year) and amplitude of periodic variations.



Figure 8.1. The orbital eccentricity (top) and orbital period (bottom) over the mission. The orbital period is given relative to the intended nominal period for the recovery mission.

Figure 8.1 shows the evolution of two of the main orbital parameters over the mission. Figure 8.2 shows characteristics of the orbit that directly affected the temperature and other aspects of the satellite: the height, longitude and local time of perigee, reflecting a period of 588.7 days between successive conjunctions of the satellite at apogee and the Sun.



Figure 8.2. The local geographic longitude for perigee passages (top), the local solar time in hours at perigee (middle) and the perigee height above the Earth's surface (bottom). The numbers in the top graph refer to orbit numbers modulo 9 and show the repeating pattern over 9 orbital periods (4-days) interval. The longitude of apogee falls halfway between successive perigee longitudes. The dashed line in the central graph represents mid-day.

The local longitude at perigee drifted according to the relation between the orbital period and precession rate of the satellite and the orbital period of the Earth. As the orbital period of the satellite decreased, those drifts changed. Also the precession rate varied according to the orientation of the orbit with respect to the Earth and the Sun. From Figure 8.2 it can be seen that around day 1030 that drift was almost zero. At that time the orbital period was around 38327 s, giving a precession rate for the orbit of 0°.30 per day, at a time when the perigee passage took place close to mid-day local time. The average precession rate over the mission was 0°.37 per day.



Figure 8.3. The eclipse lengths (top) and exposure factor (bottom) over the mission.

As could be expected, the lowest perigee passages took place near mid-day local time, when the satellite orbit was stretched away from the Sun. During these low passages the satellite was subjected to increased friction from the Earth's atmosphere.

In addition to the effects due to the satellite orbit around the Earth, the effects of the Earth's orbit around the Sun were also clearly noticeable. Variations of around ± 3.5 per cent in radiation received by the satellite due to the ellipticity of the orbit of the Earth, were reflected in temperature sensitive instrumentation on-board as well as in the solar radiation torques. The radiation received was further affected by the occurrences and lengths of eclipses, shown in Figure 8.3.

To describe the combined effect of eclipses and the solar radiation variations, a quantity E_x , the exposure factor, was introduced and defined as:

$$E_x = \frac{1}{d^2} \left(\frac{T_{\rm orb} - T_{\rm ecl}}{T_{\rm orb}} \right)$$
[8.1]

Time-span	Frequency	Name	Alias	Use
	921.6 kHz	Oscillator frequency		Basis for all timing
	230.4 kHz	Clock frequency		Input for OBT
0.000833 s	1200 Hz	IDT sampling period	T_1	Integration time for main detector
0.001667 s	600 Hz	SM sampling period		Integration time for SM detectors
0.006667 s	150 Hz	IDT slot	T_2	Allocation of observing time
0.013333 s	75 Hz			Thruster firing time interval unit
0.041667 s	24 Hz	Telemetry frame		Basic unit of telemetry data
0.133333 s	15/2 Hz	IDT interlacing period	T_3	Star switching cycle time
0.416667 s	2.4 Hz			Length of SM extract
1.066667 s	15/16 Hz			Gyro integration time
2.133333 s	15/32 Hz	Observational frame	T_4	IDT observations, RTAD data
10.66667 s	3/32 Hz	Telemetry format		Main unit of telemetry data
20.625 m				Time between crossing of the FOV

Table 8.2. The on-board time and the (nominal) time intervals used during the mission. Abbreviations: FOV = field of view, IDT = image dissector tube (main detector), OBT = on-board time, RTAD = real-time attitude determination, SM = star mapper.

where T_{orb} is the orbital period of the satellite, T_{ecl} the time during an orbit with the Sun eclipsed by the Earth and *d* the distance between the Earth and the Sun expressed in astronomical units. The variation of E_x , shown in Figure 8.3, was the driving force behind the long-term temperature variations in the spacecraft.

Rotation period of satellite

Orbital period of satellite

8.2. The On-Board Time

As all the main processing took place using the on-board time as reference, it was necessary to investigate the relations between the on-board and ground-station time. This section deals with the various effects that were observed to influence this relation.

Time Units

2.1333 h 10.650 h

All actions on-board the Hipparcos satellite were controlled by timings provided by the on-board time, which derived its signal from a 921.6 kHz crystal controlled oscillator in the bus controller of the central terminal unit, situated on the spacecraft platform almost halfway between the entrance pupils of the two fields of view. Table 8.2 shows the time intervals used, their relationships and the way they were referenced. The basic frequency for the Hipparcos observations was 1200 Hz, equal to the sampling time for the image dissector tube detector (T_1).

The basic frequency for the telemetry was (3/32) Hz, the time of one telemetry format, built up from 256 equal length (24 s^{-1}) telemetry frames. The extraction of star mapper counts was closely related to the telemetry organisation, and the same applied to recordings of thruster firings.

The first stages in the data reductions (star mapper processing, attitude reconstruction, image dissector tube processing, great-circle reduction) were done using the on-board time scales, while carrying along UTC time as provided by the ground stations through time tagging. UTC time was used for the calculation of ephemerides of planets, minor planets and the moons of Jupiter and Saturn and had to be reliable to 0.01 s. It was also used for the calculation of the ephemerides of the satellite, the Earth, Moon and Sun, for determining the barycentric velocity of the satellite and for describing eclipse and occultation conditions.

Satellite to Ground-Station Delays

Delay times between satellite and ground station were calculated from the satellite ephemeris at epoch of observation and the geocentric coordinates of the receiving antenna. These corrections were of the order of 0.01 s to 0.1 s, and were calculated once per telemetry format (see Table 8.2). The calculation involved the following steps:

- (1) calculate the Greenwich Mean Sidereal Time (GMST) for the time of observation;
- (2) using the GMST and the antenna coordinates, calculate the equatorial station coordinates (**a**);
- (3) using the satellite ephemeris, calculate the geocentric equatorial satellite coordinates (\mathbf{g}_0) ;
- (4) calculate the vector $\mathbf{e} = \mathbf{g}_0 \mathbf{a}$ from the antenna to the satellite;
- (5) check if the satellite was above the horizon (observations could continue with the satellite just below the horizon);
- (6) calculate the delay time from the length of **e**;
- (7) subtract the delay time from the ground-station time.

The coordinate vectors used for calculating the delay were expressed in the equatorial system of the mean equinox of date, in which the satellite ephemeris was originally provided by ESOC (see Section 12.1). The satellite ephemeris used in the data reductions, e.g. for calculating stellar aberration, was transformed into the J2000 system.

Ground-Station Delay Time Checks

The behaviour of the on-board time was checked with respect to the ground-station time for short (minutes) and long (weeks) time drifts. During these checks it became apparent that ground-station delay times were not as accurate and stable as they were claimed to be, and for some stations were affected by equipment changes. The ground-station delay time described the time interval between the signal reaching the receiver and the moment of tagging the signal. The signal tagging was tuned by using a 1-second block signal from the satellite, to which the ground station had to lock-in. Occasional lock-in errors did occur, causing an error of 1 (and once 2) seconds in the relation between ground-station time and on-board time. These errors were detected from the examination of the evolution of the differences between ground-station and on-board time, and subsequently corrected for. A total of 11 such cases were detected, one from Perth, 4 from Kourou and 6 from Goldstone.

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Figure 8.4. Relative time-delay corrections derived for the Goldstone ground station, excluding the full second corrections.

The ground-station time delay for the Goldstone receiver was rather unreliable, with sudden changes of several tens of milliseconds, an order of magnitude more than the accuracy that was initially requested. The Goldstone time delays were re-calibrated for the entire mission relative to the much more reliable Odenwald and Perth timings. Ground station change-overs during observations were used for most of these calibrations: the gap in coverage of the observations usually lasted less than a minute. In such a case the relation between on-board time and ground-station time was represented with a simple second-order polynomial over a short stretch covering data before and after the station switch. Two zero points were used in the solution, one for each station. The difference of these zero points was a measure of the relative ground-station delay correction. Comparisons between various determinations showed the most likely source of a correction. Figure 8.4 shows the corrections added by NDAC to the provided Goldstone time delay values (excluding the full second corrections).

On-Board Time Drifts

After adjusting for all the relative ground-station delay corrections, the drift of the on-board time could be determined. This was done by fitting spline functions to the differences between ground-station and on-board time over undisturbed intervals. Almost discrete shifts in these differences occurred when heater problems developed on board the spacecraft, and no solutions could be made across such data. The clock drift was given by the derivative of the spline functions. The drift turned out to consist of two short-time-scale components (the orbital period and the rotation period of the satellite) and a long-time-scale component, related to the orbit of the Earth and the relative position of the satellite orbit with respect to the positions of Sun and Earth. All these effects were caused by temperature changes. Local changes in the case of the rotation period, global changes in the case of the satellite orbit, and longer lasting changes related to the Earth's orbit and the orientation of the satellite orbit with respect to Sun and Earth. Figure 8.5 shows the evolution of the long-term changes, and a comparison with Figure 8.3 identifies beyond reasonable doubt that temperature variations (due to the exposure factor) were the main contributor to the rate variations. The rate variations in Figure 8.5 represent variations of the on-board time at apogee.



Figure 8.5. Long-term variations in the on-board time. The top graph shows the measured differences between on-board time (OBT) and ground-station time (GST) over the mission. The bottom graph shows the derivative of the top graph, the rate of change of the time difference OBT-GST, caused by the drift of the on-board time. A comparison between the bottom graph and Figure 8.3 shows that the frequency of the crystal controlled oscillator, which regulated the on-board, increased when the spacecraft cooled down.

The same relation between on-board time drift and temperature changes can also be seen in Figure 8.6 for the short-term changes, where in this case the spacecraft cools down during eclipses and heats up again outside the eclipses. On top of this variation is a modulation caused by the rotation of the satellite, which causes variations in the local exposure to sunlight. Figure 8.7 shows the effect of heating-up during a perigee passage, and how alternate orbits are more similar in behaviour than consecutive orbits. This is due to the longitudes of the perigee passages as shown in Figure 8.2: for alternate orbits the difference in longitude is much smaller than for consecutive orbits. The density of the Earth's atmosphere at high altitude varies considerably, leading to different amounts of heating up of the spacecraft for perigee passages at different longitudes. This heating up was noticed very clearly when comparing clock drift during periods of low perigee with periods of high perigee.


Figure 8.6. Short-term variations (covering four orbital periods) of the difference between on-board time (OBT) and ground-station time (GST) during a period with long eclipses. The top graph shows the time differences, the lower graph the rate of change in the time difference. The times of eclipses are indicated by the raised sections of the line at the bottom of the lower graph. Cooling down of the spacecraft during eclipses caused the clock to run faster.

The changes in on-board time drift could be as much as 1×10^{-6} over a period of less than an hour, but was generally at the level of 5×10^{-7} to 2×10^{-7} over one orbit. The maximum drift of the on-board time over one orbit was approximately 0.07 s, which took place when eclipses lasted for almost 90 minutes and the spacecraft cooled down significantly.

FAST derived a third-order polynomial to describe the relation between on-board and ground-station time over one orbit and applied only the full second corrections to the ground-station delay times, while NDAC tagged every observational frame with the corrected ground-station time, corrected for ground-station delay times, determined as described above. The study presented above showed that in both cases sufficient accuracy was obtained for relating the on-board time to Terrestrial Time (TT).

8.3. Gyro Calibrations

The reconstructed attitude provided measurements of the inertial rates of the satellite that could afterwards be compared with the gyro readings for the benefit of the real-time attitude determination and some aspects of the ground-based attitude reconstruction.



Figure 8.7. Short-term variations (covering four orbital periods) of the difference between on-board time (OBT) and ground-station time (GST) during a period of low perigee passages. The heating up of the spacecraft during perigee passages, caused the clock to run slower. Details of the variations repeated between alternating orbits, for which the change in local longitude of perigee was relatively small, as shown in Figure 8.2. The 2.13 hour rotation period of the satellite is also clearly seen.

The comparison described the gyro readings **g** as a function of the inertial rates $\boldsymbol{\omega}$ and the gyro drift \mathbf{g}_d :

$$\mathbf{g} = \mathbf{A}\boldsymbol{\omega} + \mathbf{g}_d$$
 [8.2]

where the matrix **A** takes care of the orientation and scaling of the gyro readings. Equation 8.2 was solved for each gyro independently. Noise levels on the various gyros could be very different as well as variable. In order to avoid correlations between the drift and the *z* axis component of the two gyros in the *xy* plane, the component ω_z in the solutions for the first two gyros was corrected for the mean scan velocity of the satellite.

Drifts were found to reflect short-term variations due to temperature changes during solar eclipses, and were of the order of 0.1 to 0.4 arcsec s⁻¹. Long-term variations were avoided through the use of thermostatically controlled gyro heaters. Noise levels on the gyro readings were more related to the general deterioration of the gyro electronics over the mission. The evolution of the gyro drifts over the mission is shown in Figure 8.8. The background of the drift variations is unknown. The drifts are given here as measured per gyro, rather than as measured per input axis (as was done in Figure 13.4 in Volume 2).

The gyro readings were made once every 16/15 second (half an observing frame), and were added in pairs, giving one measurement per observing frame. The readout quanta in the operational fine mode were 9×10^{-6} degrees, or 0.03 arcsec per 1.06 s. On a



Figure 8.8. Variations in the drifts of the five gyros (indicated in the graphs by their numbers) over the length of the mission. The noisy region around day 450 is associated with the very long eclipses and large temperature fluctuations around that time. As a result, the measured drift depended on the time coverage of a data set. The discontinuity in the bottom graph around day 545 is due to the change from gyro 4 to gyro 5. For the last part of the mission only two gyros were in operation. The top graph shows the effects of gyro 1 breaking up between day 1260 and day 1320. In the final part of the mission, starting from day 1390, gyro 1 had been replaced by gyro 3, for which the heater had earlier broken down, leading to the relatively large drift and sensitivity to temperature changes in the spacecraft, as shown by the offset around day 1450 to 1460, when the satellite operated in sun-pointing mode.

single reading this would produce a quantization noise of approximately 0.01 arcsec s⁻¹, and on a pair of measurements a noise of approximately 0.007 arcsec s⁻¹. The standard deviations observed for the gyro readings varied from 0.008 to 0.100 arcsec s⁻¹, with the exception of noise associated with gyro failures. This meant that the quantization noise was in some cases (in particular for gyro 2) a major noise contributor, and that the actual gyro readings in such a case could have been much better. At the noise level

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of 0.008 arcsec s⁻¹ or less the gyro data could contribute significant information to the attitude reconstruction process, in particular for the *z* axis position.

Gyro data could obviously only contribute information to the time derivative of the satellite attitude. The contributions of gyro data and the star mapper data to the attitude solution can be estimated from the total data volume W_n for a polynomial coefficient of degree *n*:

$$W_n = \sum_i \frac{t_i^{2n}}{\langle \epsilon_i^2 \rangle} + \sum_j \frac{n^2 t_j^{2n-2}}{\langle v^2 \rangle}$$

where the first part represents the contribution by the star mapper data and the second part the contribution by the gyro measurements. The variance of the noise on the star mapper transit time determinations is given by $\langle \epsilon_i^2 \rangle$, the variance of the noise on the gyro readings by $\langle v^2 \rangle$. The density of star mapper transits (see Section 6.1) is one every 8 s for the vertical slits, and one every 16 s for the inclined slits in each field of view. The typical error on a transit time was 0.05 arcsec. The possible contribution of gyro data thus depended on the length of the stretch of data considered, the number of star mapper transits available, and the noise level on the gyro readings. The number of star mapper transits available was influenced by density variations on the sky and by background level variations. When gyro noise levels were low (at a level of 0.008 arcsec s⁻¹), gyro data could contribute significant information to the attitude reconstruction process, in particular in situations where the star mapper data was reduced in quantity and/or quality.

The contributions to the standard deviations obtained from the solution of Equation 8.2 consisted of a combination of readout and repositioning noise. Correlations between errors on successive gyro readings were expected and observed, although the exact behaviour was often difficult to understand. The correlation between errors on gyro readings originated from the measuring mechanism: the input axis was to remain 'fixed' with respect to the satellite axes. Thus, a measurement of a rotation angle by a gyro was followed by a resetting of that gyro by the measured angle. It was this resetting that caused the gyro induced torque acting on the spacecraft. Errors on a reading were transmitted to the resetting, and the next reading contained both the error on the resetting and its own error on the reading. In addition, the resetting itself was affected by its own error. As a result of this, consecutive gyro readings had correlated noise, with a correlation coefficient of approximately -0.2. These correlation coefficients could change dramatically with the deterioration of the gyro electronics. When readout noise became dominant, the correlation coefficient could increase to -0.5.

In the case of gyro 3, used over the last 6 months of the mission, the situation was worse. Here, due to a failure of the gyro heater, a wave of amplitude 0.025 arcsec s⁻¹ was noticed. The frequency at which it was detected, 0.1488 Hz, could have been an alias of the real frequency with the frequency of observations, 15/32 Hz.

The gyro readings were, especially later in the mission, occasionally disturbed by socalled noise bursts. These were sudden and often dramatic increases in the noise level on the readings of one gyro without an apparent reason. Such changes in noise could not be properly accommodated in the overall calibration of the gyro noise levels, due to their erratic appearance.

During the mission all but one of the gyros broke down, the remaining one operating without a heater. The breaking down of gyro 4 early in the mission was characterised by changes in torques acting on the spacecraft, noticed in the rates measured primarily around the x axis. This could have been caused in two ways: the rotation axis of gyro 4 was changing position significantly, or by changes in the angular momentum of gyro 4. The second of these options seems the more likely one, as it would require rather large

displacement variations of the gyro 4 rotation axis to cause noticeable torque variations. Similar problems were encountered later in the mission when gyro 4 was running in parallel with gyro 5 and at later stages when gyro 5 failed.

The non-operational gyros were tested once every month. A gyro de-storage was not recorded in the telemetry and only the day at which it took place was recorded in the two-weekly operations reports. The examination of the gyro data allowed for an accurate reconstruction of the start of the spin-up, end of spin-up, start of spin-down and end of spin-down times, which were accounted for in two ways: at each of these points an artificial thruster firing was inserted, and the interval between end of spin-up and start of spin-down was recorded in a separate file for use in the torque analysis: while a redundant gyro was running, the torque on the satellite would be quite different. The intervals between start and end of spin-up or spin-down (approximately 1 minute each) were discarded: these intervals were too short with too rapidly changing rates, which made a proper reconstruction of the attitude impossible and modelling not very useful.

8.4. Thruster Firings

The nominal attitude described the planned path of the satellite's *z* axis on the sky and the associated spin velocity. It was designed so that the scanning of the sky took place in a smooth way, suppressing variations in scanning density across the sky as much as reasonably possible. The satellite was subject to a variety of torques (see Chapter 7), all of which disturbed the pointing and motion of the satellite. Through cold-gas thruster firings the satellite pointing was kept to within 10 arcmin from the nominal attitude.

The firing lengths were calculated on-board the satellite, using a very much simplified model of torques acting on the satellite, the estimated current rotation rates and error angles (real-time attitude determination, see Chapter 7), and the time span over which these error angles were to be brought back to zero. Thruster firings always took place at the start of an observational frame, and lasted an integer number of 1/75 s intervals. At the start of the mission, the minimum time was 0.05 s (4 units) and the maximum time 0.5 s (42 units). Close to perigee longer on-times were allowed. Later in the mission the minimum on-time was reduced to 2/75 s. Initially when one of the error angles was found approaching its limit, all thrusters were fired, later in the mission the z thrusters were only fired if there was the need for at least a minimum length firing of 8/75 s: zero-length z-firings are the most common during that period. A number of times during the mission, due to an update of the Tait-Bryan angles (see Section 7.3) from the ground, the offset between actual and nominal attitude was recognised to be well beyond 10 arcmin. The control software on-board the satellite was not prepared for such a situation, which would have been very rare under nominal conditions. The result of this was a repeated violation of the control condition for the nominal attitude, and minimum length firings of the thrusters in every observational frame (in alternating directions) until the satellite had returned to within the 10 arcmin margin from the nominal attitude. Such data stretches, which could last for several minutes, could not be used in the reductions.

By describing the relation between velocity changes and thruster firing lengths, boundary conditions could be included in the attitude reconstruction software, that allowed for attitude modelling across thruster firings. The relations between thruster firing lengths and velocity changes around the three axes of the satellite were calibrated by means of



Figure 8.9. The evolution of the torques produced by the cold-gas thrusters. Thrusters producing positive velocity changes are indicated with filled symbols, those producing negative changes with open symbols. The discontinuity is due to a change of gas tank, the drift is due to the emptying of the gas tanks and a slight over-compensation for loss of pressure.

on average two days of data. Using the reconstructed attitude the changes in velocity on all three axes across a thruster firing were measured. Separate calibrations were made for thrusters causing positive velocity changes and for thrusters causing negative velocity changes, 6 components in all. In a least-squares solution these changes were expressed as linear functions of the recorded on-times of the thrusters and an offset. Table 8.3 shows for each thruster the measured offsets, expressed as zero points in the on-time. Slow increases in the zero-point values were observed, as well as some increase coinciding with the gas-tank change-over. In general, the zero points were stable to within 5 per cent of 1/75 s.

Axis	+Thrust	-Thrust
X	0.65	0.90
У	0.70	0.90
Z	0.60	0.70

Table 8.3. The zero points in the thruster on-times, given in units of 1/75 s.

The linear relation between the change in rotation rate dv and the thrust t activated over an interval τ describes the acceleration caused by the thruster involved, which, using the arm-length of the thruster and the inertia tensor of the satellite, can be translated into a torque. Figure 8.9 shows the evolution of the torques produced by the 6 thrusters over the mission. The sudden change at day 900 is due to a change of gas tank. The increase in torque over the mission is due to depletion of the gas tanks, with the decreasing pressure being over-compensated. Further modulations were related to temperature variations of the spacecraft. This is most notably so for the peak around day 450, which is related to the minimum in the exposure factor shown in Figure 8.3: with the spacecraft cooled down, the pressure in the gas tanks was diminished, and automatically over-compensated to give a higher resulting torque. The thruster torque specified by the manufacturer was 20 mN.

In the nominal situation (geostationary orbit, empty apogee boost motor tank) the mass centre of the satellite would have coincided with the plane in which the *z* thrusters were situated. However, with a full tank the mass centre was shifted by 72 cm, which resulted in the *z* thrusters also causing torques on the *x* and *y* axes. The calibration values obtained for the *z* thrusters confirmed the estimate of the mass centre of the satellite by the manufacturer to within a few mm. Figure 8.10 shows the *x* and *y* components in the *z* thrusters as calibrated in comparison with the values predicted from the positions of the thrusters and the mass centre.

The noise level left after fitting the thruster firing lengths against the changes in rotation rates was an indication of the precision to which this information could be used in the reductions. The figures observed changed significantly from 0.01 arcsec s⁻¹ for the first gas tank to 0.02 arcsec s⁻¹ for the second tank. That means that after a thruster firing the uncertainty in the rotation rates is at least at a level of 0.01 to 0.02 arcsec s⁻¹, and at most at the level of 0.1 arcsec s⁻¹. The noise was independent of firing length. The increase in the noise level while using the second gas tank could be due to stronger temperature fluctuations resulting from a different position in the spacecraft.

8.5. Inertia Tensor and Torque Calibrations

Basic Model

The calibration of the inertia tensor and of the environmental torques relied on the fact that the attitude reconstruction results were available for all three axes in the form of rates, and when differentiated, in the form of accelerations. If we assume that the



Figure 8.10. The relative torques caused by the *z* thrusters on the *x* and *y* rotation rates. Nominally, a firing by the +*z* thruster, giving a velocity change of v_z , also gave a velocity change of $-0.028 v_z$ on the *x* axis, and $+0.086 v_z$ on the *y* axis (solid symbols). Similarly, a firing by the -z thruster, giving a velocity change of $-v_z$, also gave a velocity change of $-0.089 v_z$ on the *x*- and $-0.019 v_z$ on the *y* axis (open symbols).

Hipparcos satellite was a rigid body moving freely in space, then the relation between the rates and the accelerations was described by the Euler equation:

$$\mathbf{I} \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} = \mathbf{N} - \boldsymbol{\omega} \times \mathbf{I} \boldsymbol{\omega}$$
 [8.3]

where **I** is the inertia tensor, **N** the external torques and $\boldsymbol{\omega}$ the inertial rates around the satellite axes. Thus, $\boldsymbol{\omega}$ and $d\boldsymbol{\omega}/dt$ were observed, and **N** and **I** were to be calibrated. In principle one should first orthogonalize the left-hand side of Equation 8.3, but using the ground-based starting values of the inertia tensor, this orthogonalization would have very little effect (considering also that the magnitudes of the acceleration rates for the three axes are not very different). Thus, it was assumed that in a first approximation the left-hand side of Equation 8.3 could be expressed as:

$$\mathbf{I}\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} = \begin{pmatrix} I_{xx}\dot{\omega}_{x}\\ I_{yy}\dot{\omega}_{y}\\ I_{zz}\dot{\omega}_{z} \end{pmatrix}$$
[8.4]

where I_{xx} , I_{yy} and I_{zz} are the diagonal elements of the inertia tensor. The off-diagonal elements of **I** were approximately 100 times smaller than the diagonal elements (see Table 7.1). Small changes in the inertia tensor elements were most likely related to the depletion of the cold-gas tanks.

The same approximation could not be made for the second term on the right-hand side of Equation 8.3. Developing this term for the *x* coordinate gives:

$$\left[\boldsymbol{\omega} \times \mathbf{I}\,\boldsymbol{\omega}\right]_{x} = \omega_{y}\omega_{z}(I_{zz} - I_{yy}) - \omega_{z}^{2}I_{yz} - \omega_{x}\omega_{z}I_{xy} + \omega_{y}^{2}I_{yz} + \omega_{x}\omega_{y}I_{xz}$$

$$[8.5]$$

Because the diagonal elements enter as a difference, the contributions by the off-diagonal elements become relatively more significant. Also, the rotation rate around the z axis was about 100 times the average rotation rate around the x and y axes. For this reason we can ignore the last two terms in Equation 8.5, but have to account for the first three. Of these, the second varied relatively very little and was strongly correlated with a constant torque. Similar considerations for the other two axes led to the following equations:

$$\boldsymbol{\omega} \times \mathbf{I} \, \boldsymbol{\omega} \simeq \begin{pmatrix} \omega_{y} \omega_{z} (I_{zz} - I_{yy}) - \omega_{x} \omega_{z} I_{xy} \\ \omega_{z} \omega_{x} (I_{xx} - I_{zz}) + \omega_{y} \omega_{z} I_{xy} \\ \omega_{x} \omega_{y} (I_{yy} - I_{xx}) + \omega_{x} \omega_{z} I_{yz} + \omega_{y} \omega_{z} I_{xz} \end{pmatrix}$$

$$[8.6]$$

where the first term in the equation for the z axis is fully defined by the first terms for the x and y axes. As this term is also, due its small size, badly determined, it was not solved for. In the implementations it was derived from the other calibrations and implemented as such. The complete calibration equations expressed in the observed quantities thus became:

$$\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} = \begin{pmatrix} N_x/I_{xx} \\ N_y/I_{yy} \\ N_z/I_{zz} \end{pmatrix} + \begin{pmatrix} \omega_y \omega_z (I_{zz} - I_{yy})/I_{xx} - \omega_x \omega_z I_{xy}/I_{xx} \\ \omega_z \omega_x (I_{xx} - I_{zz})/I_{yy} + \omega_y \omega_z I_{xy}/I_{yy} \\ \omega_x \omega_z I_{yz}/I_{zz} + \omega_y \omega_z I_{xz}/I_{zz} \end{pmatrix}$$

$$[8.7]$$

It is clear from this equation that only the ratios of the elements of the inertia tensor can be calibrated, not the absolute values. This was no problem, as similarly the application of the inertia tensor was primarily as ratios between the elements. In practice the elements were scaled to a fixed value for I_{xx} (see Table 7.1).

Calibrations were first carried out on data collected over one orbit at a time, for the inertia tensor elements and the external torques. The gravity gradient torque N_G (see Section 7.2) was subtracted using an approximate *a priori* inertia tensor (correct to within a few per cent). Subsequently, the calibrations were repeated for the other external torques, applying the full, calibrated inertia tensor in Table 7.1:

$$\begin{pmatrix} N_x/I_{xx} \\ N_y/I_{yy} \\ N_z/I_{zz} \end{pmatrix} = \frac{\mathbf{d}\boldsymbol{\omega}}{\mathbf{d}t} + \begin{pmatrix} (\boldsymbol{\omega} \times \mathbf{I}\,\boldsymbol{\omega})_x/I_{xx} \\ (\boldsymbol{\omega} \times \mathbf{I}\,\boldsymbol{\omega})_y/I_{yy} \\ (\boldsymbol{\omega} \times \mathbf{I}\,\boldsymbol{\omega})_z/I_{zz} \end{pmatrix} - \begin{pmatrix} N_{\mathrm{G},x}/I_{xx} \\ N_{\mathrm{G},y}/I_{yy} \\ N_{\mathrm{G},z}/I_{zz} \end{pmatrix}$$
[8.8]

where N_G is given by Equation 7.6 in Section 7.2.

The coefficients which remained to be calibrated were those related to solar radiation and the magnetic moment of the satellite, as well as effects related to temporary localized heating of the outer surface of the satellite. Thus, the remaining torques were expressed as a set of harmonics, related to Equations 7.2, 7.4 and 7.5, and linked coefficients related to Equations 7.11 and 7.13. Some additional coefficients were required too, some possibly related to deformation of the Earth's magnetic field, others as yet unexplained.

Torques Related to Solar Radiation

Torques caused by solar radiation could be identified in three ways: they appeared as coefficients of specific trigonometric terms, related to the three-fold symmetry of the satellite (see Section 7.2), they disappeared during eclipses and sun-pointing mode observations, and they were subject to an annual variation resulting from the ellipticity

Table 8.4. The solar radiation related torque components as observed in the accelerations. The observed values, after having been corrected for Sun–Earth distance variations, were fitted to a linear function of time with day 800 as reference point. The coefficients (at day 800) are measured in mas s^{-2} and the derivatives in mas s^{-2} yr⁻¹.

Axis	Term	Coefficient		Derivative		
X	$\sin \Omega$	-2.28005	± 0.00031	-0.00980	± 0.00042	
X	$\sin 2\Omega$	0.59429	± 0.00012	-0.00895	$\pm \ 0.00016$	
X	$\sin 4\Omega$	-0.20310	$\pm \ 0.00008$	-0.00028	$\pm \ 0.00010$	
X	$\sin 5\Omega$	0.09826	$\pm \ 0.00008$	-0.00230	$\pm \ 0.00011$	
X	$\sin 7\Omega$	-0.01112	± 0.00009	0.00039	$\pm \ 0.00012$	
X	$\sin 8\Omega$	0.00375	± 0.00009	-0.00025	$\pm \ 0.00012$	
X	$\sin 10\Omega$	0.01143	± 0.00009	0.00218	$\pm \ 0.00012$	
X	$\sin 11\Omega$	0.00271	± 0.00009	0.00074	$\pm \ 0.00012$	
У	$\cos \Omega$	-2.16908	± 0.00024	0.01662	± 0.00032	
У	$\cos 2\Omega$	-0.64572	± 0.00011	-0.00968	± 0.00015	
У	$\cos 4\Omega$	-0.17513	$\pm \ 0.00008$	0.00319	$\pm \ 0.00011$	
У	$\cos 5\Omega$	-0.09407	± 0.00010	-0.00099	$\pm \ 0.00013$	
У	$\cos 7\Omega$	0.00948	± 0.00011	0.00152	$\pm \ 0.00014$	
У	$\cos 8\Omega$	-0.00562	$\pm \ 0.00010$	0.00000	$\pm \ 0.00013$	
У	$\cos 10\Omega$	0.01117	$\pm \ 0.00009$	0.00020	$\pm \ 0.00011$	
У	$\cos 11\Omega$	0.00205	$\pm \ 0.00008$	0.00036	$\pm \ 0.00011$	
Z	$\sin 3\Omega$	-0.77013	± 0.00022	0.00261	± 0.00029	
Z	$\sin 6\Omega$	-0.12250	$\pm \ 0.00008$	0.00337	$\pm \ 0.00011$	
Z	$\sin 9\Omega$	-0.02593	$\pm \ 0.00006$	0.00049	$\pm \ 0.00008$	
Z	$\sin 12\Omega$	-0.01035	$\pm \ 0.00006$	0.00106	$\pm \ 0.00007$	

of the Earth's orbit. In fact, the torque variations may allow a measurement of the eccentricity value of the Earth's orbit to better than a few per cent.

Figure 8.11 shows the principal solar radiation torque related coefficients before correction for the varying distance to the Sun, Figure 8.12 shows the same after correction for this effect. There were drifts observed for most solar radiation related coefficients. These drifts can probably be explained as due to changes in the structure of the outer surface of the satellite due to exposure to radiation.

During periods of sun-pointing observations, the solar radiation torques virtually disappeared. With the z axis pointing towards the Sun, the radiation forces were almost completely balanced with respect to the centre of mass. The torques around the z axis were found to be more variable at times of long eclipses, possibly related to the cooling of the outer surface of the satellite during the eclipse, and the heating up during the perigee passage. The results for the solar radiation torques, normalised to a distance of 1 AU, are summarised in Table 8.4.

Calibration of Magnetic Moments

The magnetic moments around the satellite axes were calibrated in a simultaneous solution for the observations on all three axes, using Equation 7.13. The *x* component was, however, difficult to determine as it was for most of the orbit strongly correlated with other coefficients. Figure 8.13 shows the observed values for the magnetic moments



Figure 8.11. The primary components of solar radiation torque observed in the accelerations around the three axes.

over the mission. Magnetic moments were noticed clearly around the *y* and *z* axes, at a level of -2.7 Am^2 and $+0.3 \text{ Am}^2$ respectively. The *y* axis component was very stable over the mission, the *z* axis component drifted slowly and showed more variation.

An additional z axis component, at -2.2 Am^2 , was noted during eclipses. This component appeared to be related to the change in power supply from the solar panels to the batteries, although all cabling associated with the power supplies appeared to be properly shielded according to the design drawings. This component was given a fixed value if insufficient data covering an eclipse was available.

Remaining Coefficients

Four coefficients were used in the *z* axis torque representation that had no immediate explanation in terms of solar radiation: $\cos \Omega$, $\sin \Omega$, $\cos 2\Omega$ and $\sin 2\Omega$. The first two may be related to the Earth's magnetic field, but were not removed with a magnetic moment around the *x* and *y* axes. They may possibly represent a distortion of the



Figure 8.12. As Figure 8.11, but corrected for the varying distance between the Earth and the Sun.

magnetic field: their variation resembles the period of 57 days (the spin period of the rotation axis of the satellite) and the period of 588.7 days between two successive conjunctions between the Sun and the satellite at perigee. Another indication that these coefficients represent the magnetic field distortion comes from their correlation with variation on the constant in the *x* coordinate solution, which indicates that it could be the *y* axis magnetic moment that gives rise to these coefficients. Figure 8.14 shows these three coefficients.

The other two coefficients for the *z* axis are more difficult to explain. Their variability shows a semi-regular behaviour with a period of 28 days, half the spin period of the rotation axis of the satellite. This kind of behaviour would be expected from a gravity gradient related coefficient (see Equation 7.8). The sin 2Ω , however, is also likely to be related to the solar radiation torques, as seems to be indicated by the way it changed when the satellite went into sun-pointing mode. There is also a resemblance in behaviour between these two coefficients and the cos Ω coefficient for the *x* axis.



Figure 8.13. The measurements of the magnetic moments of the satellite. The y-component is most clearly present. There is probably also a z-component during normal observing conditions (middle graph) and there appears to be a significant z-component during eclipse conditions (bottom graph). The eclipse related component could only be solved for under good conditions: converged attitude during most of an eclipse and at least two complete revolutions of the satellite outside the eclipse.

The constant for the *y* axis (Figure 8.15) showed a long-term exponential decrease, a 57 day modulation with varying amplitude, and short-term variations explained below. The exponential decrease was probably related to changes in the outer surface of the satellite due to exposure to solar radiation. The exact zero point for this term is badly determined as it depends strongly on the value used for the component I_{yz} of the inertia tensor (see Equation 8.5). The modulation is probably related to a magnetic moment on the *x* axis, although entering this as a degree of freedom in the solutions did not give a consistent result (partly due to correlations with other coefficients). The points near days 1000, 1300 and 1450, which are offset by about 0.2 mas s⁻², are related to sunpointing mode observations. Their offset indicates that the constant on the *y* axis could be solar radiation related, and representing a spinning motion driven by an insufficiently balanced torque around the *y* axis. The gap around day 750 is due to the running of



Figure 8.14. Three coefficients, showing through their variability a possible link with the magnetic moment of the satellite and a distortion of the Earth's magnetic field at high altitudes. The short-period variation is probably related to the spin period of the satellite z axis (57 days). The long-period modulation appears to be related to the period between two conjunctions of the Sun and the satellite at apogee (588.7 days).

gyro 4 in addition to gyros 1, 2 and 5. This caused additional torques on the x and y axes (see Section 8.6).

A number of coefficients show short-term variations with correlations between alternate orbits, in particular when the perigee of the satellite orbit was low. The longitude of alternate orbits changed by only 40° ; between successive orbits the change was 160° . Thus, the conditions around a perigee passage for alternate orbits were likely to be more similar than between successive orbits, in particular concerning the results of friction by the outer layers of the atmosphere. One of the finest examples is shown in Figure 8.16 for the constant torque on the *y* axis. A constant torque either originated in the spacecraft (gyro-induced) or represented a local (on the spacecraft) external and unbalanced torque. It appears that the satellite was heated up on one side during a perigee passage, with the amount of heating depending on the conditions of the outer



Figure 8.15. Long-term variations in the constant for the y coordinate, showing a kind of exponential decrease as well as a 57 day modulation.



Figure 8.16. Short-term variations in the constant for the y coordinate, showing the effect of alternating local longitudes of the perigee during a period with very low perigee height. Orbits with even numbers are indicated with a cross, odd-numbered orbits with an open square, and have been connected to show their correlation.

atmosphere of the Earth and therefore being correlated between alternate orbits. During the orbit following the perigee passage the local heat loss from the satellite produced the windmill effect.

8.6. Miscellaneous Effects

The remaining effects were of short, or even impulse, nature. They concerned hits of the satellite by external objects (two fairly major ones, causing rotation rate changes of the order of 2 arcsec s⁻¹, and 10 to 15 smaller ones); gyro de-storage, causing temporary torques during spin-up and spin-down procedures, and, during operations of the additional gyro, an offset in the constant related to the inertia tensor, the spinning rate of the satellite and the angular momentum of the additional gyro. In the case of gyro 4, the offsets observed were 2.200 mas s⁻² in the *x* coordinate, and 3.305 mas s⁻² in the *y* coordinate. Using the inertia tensor values given in Table 7.1, this gives values of $-5.70 \ \mu$ Nm and $-9.45 \ \mu$ Nm for the nominal induced torque of gyro 4, compared to the values given in Table 7.2: $-5.52 \ \mu$ Nm and $-9.01 \ \mu$ Nm, indicating that the orientation

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of the rotation axis was correctly recovered, but that the angular momentum of gyro 4 appeared to be 4 per cent higher than expected (unless the much less likely possibility that the entire inertia tensor is 4 per cent too low).

The final source of disturbances came from the reaction of the solar-panel hinges on the temperature changes when the satellite moved into an eclipse. These were noticeable only in the great-circle reduction, described in the next chapter, and caused a small saw-tooth like behaviour of the spin-phase.

8.7. Conclusions

The calibration results presented above show some of the problems encountered when trying to operate satellites with very high pointing accuracy in one-axis-stabilised mode. In the case of Hipparcos these problems were largely aggravated by the orbital conditions, but the same orbital conditions allowed the recognition of some contributions (in particular the magnetic moment) that would otherwise have been difficult to observe. Important improvements in the torque modelling would probably be obtained with the full implementation of the correct inertia tensor and a description of the deformation of the Earth's magnetic field (for which some models are available). In addition, torques as produced by the gyros were not strictly smooth, but rather like a semi-regular string of small impulses and it still needs to be investigated to what precision the current approximation as smooth torques was valid.

F. van Leeuwen

9. GREAT-CIRCLE REDUCTIONS

The great-circle reductions combined the observations obtained in a time interval of up to nine hours into a set of one-dimensional coordinates of the objects, the so-called abscissae, defined along a designated reference great circle. These were the main input to the subsequent sphere solution, in which the astrometric parameters of the stars were derived. The great-circle reductions also determined the geometrical instrument parameters, including the basic angle, transforming the observed signal phases into true angles on the sky, and the accurate along-scan attitude needed for the Tycho astrometry. This chapter describes the principles of the great-circle reductions and their practical implementation by FAST and NDAC. Great-circle results obtained by the two reduction consortia are presented. The early results were found to be affected by systematic errors, which however disappeared after several iterations of the great-circle reductions, by using improved star catalogues from previous iterations, fine tuning of the instrument description, and special treatment of star outliers. The final quality of the results was confirmed by intercomparisons between the consortia.

9.1. Introduction

In the Hipparcos great-circle reduction semi-contiguous batches of grid coordinates, each computed from image dissector tube data collected over an observational frame (see Chapter 5), were combined in so-called reference great-circle sets and processed together. A reference great-circle set contained the data collected over one orbit of the satellite and generally covered 2 to 4 revolutions, or 4 to 9 hours of data. All such data were referred to a reference frame, corresponding to a great circle chosen somewhere in the middle of the band on the celestial sphere scanned during the reference great-circle set. The abscissae of the stars contained in the reference great-circle set and an improved along-scan attitude were computed in this intermediate reference frame, together with instrument parameters, by a least-squares adjustment.

Data obtained from the Hipparcos main grid provided along-scan information only. Therefore, and because of the small inclination of the scanning circles with respect to the reference great circle, the great-circle reduction could only determine one component of the star position and spacecraft attitude in the reference great-circle frame: the star abscissa *v* along the reference great circle and the along-scan attitude ψ . The star ordinate *r* and the two transverse attitude components θ and ϕ could not be estimated. Hence they did not participate in the least-squares adjustment, but were used as obtained

from the attitude reconstruction results (see Chapter 7), using the (updated) Input Catalogue. No effort was made at this stage to estimate the proper motions and parallax: they could not be estimated due to the very limited time span of the reference greatcircle sets. The final great-circle reductions were all carried out with respect to a star catalogue based on preliminary reductions of all the Hipparcos data, providing stellar coordinates with errors that were negligible for the purpose of the great-circle reduction process.

9.2. Great-Circle Reduction

The great-circle reduction forms a geometric adjustment problem on the sphere with grid coordinates as observations. Semi-contiguous batches of up to 70 000 grid coordinates, gathered during about 2–4 revolutions, were processed together, producing one observation equation for each grid coordinate. The observation equations were solved by a least-squares adjustment with a diagonal weight matrix for the grid coordinates. The unknown parameters were roughly 1800 star abscissae, forming our prime objective, up to 15 000 along-scan attitude parameters, and some 24 instrumental parameters. In fact two types of along-scan reconstructed attitude were produced. At first a geometric along-scan attitude was estimated, consisting of one parameter for each observational frame of 2.133...s. Later the attitude was smoothed to form a continuous representation using about 500 B-splines (one every \simeq 2 minutes). Smoothing of the attitude also improved the quality of the star abscissae, although excessive smoothing could introduce systematic errors. This was verified by statistical tests; when necessary, the number of B-splines, and the location of knots, were adjusted.

Observation Equations

The geometric direction to a star in the reference great-circle frame at set mid-time was expressed by two angles v (abscissa) and r (ordinate). The determination of the abscissae v were the prime objective of the great-circle reduction. The abscissa–ordinate pair (v_i, r_i) for star number i was related to the grid coordinate G_{ik} observed for the star in frame number k, in three steps. First, (v_i, r_i) were transformed into the apparent (or 'proper') star direction at the time of observation, expressed either in the celestial reference frame (Section 12.3) or directly in the reference great-circle frame (Section 11.2), using the known orientation of the reference great circle. Secondly, the direction to the star was transformed into the instrument reference frame, yielding the field angles (η_{ik}, ζ_{ik}) or field coordinates (w_{ik}, z_{ik}) , as defined in Section 10.2. Thirdly, the field angles or field coordinates were related to the observations G_{ik} which were defined with respect to the modulating grid by means of the field-to-grid transformation, described by the instrument parameters (Section 10.2).

The transformation of the reference great-circle frame into the reference frame linked to the instrument was described by the three angles $(\psi_k, \theta_k, \phi_k)$ corresponding to the spacecraft attitude. The angles θ_k and ϕ_k were rotations around the *y* and *x* axes of the instrument, and specified the direction of the *z* axis. The angle ψ_k specified the orientation of the instrument around the *z* axis. The notation used here for these angles corresponds to that used in the FAST attitude model (Equation 7.23), but the presentation below is valid for any representation of the attitude in which (θ_k, ϕ_k) describe the 'transverse attitude' (in NDAC given by the heliotropic angles ξ and *v*; see Section 7.3) and ψ_k describes the 'along-scan attitude' (in NDAC given by the heliotropic angle Ω).

The transformation from field coordinates to grid coordinates, including corrections to the basic angle, is described in Chapter 10 (see also Volume 2, Chapter 10). It usually required 24 or more instrument parameters, which for notational convenience are collected in a vector **d** (see Section 9.6).

The relation between the grid coordinate and geometric position was written symbolically as:

$$G_{ik} = G(v_i, r_i, \psi_k, \theta_k, \phi_k, \mathbf{d}, \ldots) + \epsilon'_{ik}$$
[9.1]

with ϵ'_{ik} representing the photon noise effect on the grid coordinates. The parameters needed to correct for aberration, relativistic effects, residual proper motion and parallax are not mentioned explicitly in the equations. More details about these computations can be found in Chapter 12.

Linear observation equations, needed for the least-squares estimation, were obtained by taking the truncated Taylor expansion of Equation 9.1 in a point G_{ik}^{calc} calculated from a provisional star catalogue, star mapper attitude and instrument calibration. The linearized equation is:

$$\Delta G_{ik} = \frac{\partial G_{ik}}{\partial v_i} \Delta v_i + \frac{\partial G_{ik}}{\partial \psi_k} \Delta \psi_k + \frac{\partial G_{ik}}{\partial \mathbf{d}'} \Delta \mathbf{d} + \frac{\partial G_{ik}}{\partial r_i} \Delta r_i + \frac{\partial G_{ik}}{\partial \theta_k} \Delta \theta_k + \frac{\partial G_{ik}}{\partial \phi_k} \Delta \phi_k + \dots + O(\Delta^2) + \epsilon'_{ik}$$
[9.2]

with $\Delta G_{ik} = G_{ik}^{\text{obs}} - G_{ik}^{\text{calc}}$. The Δ -quantities on the right hand side were the unknown corrections to the provisional, or approximate, values for the parameters used in the calculation of G_{ik}^{calc} . The term $O(\Delta^2)$ represents the linearization error, which is of second order in the corrections. When the grid coordinates were expressed in angular units (radians), the partial derivatives with respect to v_i and ψ_k were close to -1 and +1 respectively. The partial derivatives in r_i , θ_k and ϕ_k were much smaller in absolute value ($\leq 10^{-2}$), but not zero. The reason is that the grid coordinates G only referred to the along-scan component, while the other component H was not measured, so that information on the star ordinate and the transverse attitude components was only available through the inclination of the scan circles with respect to the reference great circle. This inclination was at most about $\pm 1^\circ.5$ through the choice of the reference great circle close to the mean scanning direction in the data set.

Therefore, only the corrections Δv_i , $\Delta \psi_k$ and $\Delta \mathbf{d}$ were computed during the least-squares adjustment; no attempt was made to estimate Δr_i , $\Delta \theta_k$ and $\Delta \phi_k$ in the great-circle reduction. The observation equations were consequently reduced to:

$$\Delta G_{ik} = \frac{\partial G_{ik}}{\partial v_i} \Delta v_i + \frac{\partial G_{ik}}{\partial \psi_k} \Delta \psi_k + \frac{\partial G_{ik}}{\partial \mathbf{d}'} \Delta \mathbf{d} + \epsilon_{ik}$$
[9.3]

In this equation ϵ_{ik} is a general noise term including:

- (1) the photon noise error ϵ'_{ik} ;
- (2) the projection error on the reference great circle:

$$\epsilon_{ik}^{\prime\prime} = \frac{\partial G_{ik}}{\partial r_i} \Delta r_i + \frac{\partial G_{ik}}{\partial \theta_k} \Delta \theta_k + \frac{\partial G_{ik}}{\partial \phi_k} \Delta \phi_k$$
[9.4]

- (3) various modelling errors (e.g. instrument, attitude, residual proper motion and parallax);
- (4) the linearization error $O(\Delta^2)$.

The first component is the most important. At the great-circle level it could be modelled as Gaussian noise.

Least-Squares Solution

The observed grid coordinate differences ΔG_{ik} were collected in a vector **y** of length *m*, and the unknown corrections in a vector **x** of length *n*. Equation 9.3 could thus be written in matrix notation as:

$$y = A x + e$$

= A_Sx_S + A_Ax_A + A_Ix_I + e [9.5]

with the $m \times n$ design matrix **A** of partial derivatives. This system of equations was partitioned in a star part, an attitude part and an instrument part, denoted respectively by suffix *S*, *A* and *I* in Equation 9.5. The sub-matrices **A**_A and **A**_S were very sparse, each of them containing only one non-zero element per row. **A**_I on the other hand was almost completely filled.

Although Equation 9.5 had many solutions, it was not difficult to select a unique solution, $\hat{\mathbf{x}}$, namely one for which $\mathbf{A}\hat{\mathbf{x}}$ was as close as possible to the observed data \mathbf{y} . The well-known least-squares solution follows from minimising the residual sum of squares $E = \hat{\mathbf{e}}' \mathbf{Q}_y^{-1} \hat{\mathbf{e}}$, with $\hat{\mathbf{e}} = \mathbf{y} - \mathbf{A}\hat{\mathbf{x}}$ the vector of least-squares residuals, and \mathbf{Q}_y the covariance matrix of the observations \mathbf{y} . The least-squares solution $\hat{\mathbf{x}}$ was computed by solving the normal equations:

$$(\mathbf{A}'\mathbf{Q}_{V}^{-1}\mathbf{A})\,\hat{\mathbf{x}} = \mathbf{A}'\mathbf{Q}_{V}^{-1}\mathbf{y}$$
[9.6]

It deserves to be emphasized that the vector of observations **y** was, from a statistical viewpoint, a stochastic variable; consequently the least-squares estimate $\hat{\mathbf{x}}$, the residuals $\hat{\mathbf{e}}$, and other functions of these variables, such as *E*, were also stochastic. The errors in the observations were dominated by photon noise, and could therefore be assumed to be uncorrelated. Consequently a simple diagonal covariance matrix \mathbf{Q}_{y} could be used.

The least-squares solution was computed using Cholesky factorization of the square symmetric (semi-)positive definite normal matrix $\mathbf{A}' \mathbf{Q}_y^{-1} \mathbf{A}$. Once the Cholesky factor had been computed, the equations were rewritten in two triangular systems, which were solved by simple forward and backward substitution. The actual computation was organised in the following steps:

- (1) elimination of the attitude unknowns;
- (2) Cholesky factorization of the (block partitioned) normal equations, solution of the equations by forward and backward substitution, and computation of the variances;
- (3) solution of the attitude parameters, computation of the residuals to the observations, and testing of the solution.

The star part of the normal matrix was, even after elimination of the attitude parameters, very sparse (Figure 9.1). In the software only the non-zero elements of this matrix were stored and numerical operations were performed only on these elements. However,



Figure 9.1. Non-zero elements in the star part of the normal matrix (lower triangle) and envelope of the Cholesky factor (upper triangle) after elimination of the attitude parameters. The star parameters have not been re-ordered (left) or have been re-ordered by the modulo 60° algorithm (right) described by van der Marel (1988).

during the Cholesky factorization new non-zero elements were created, causing socalled fill-in of the sparse matrix. The fill-in depended on the order in which the unknowns, and hence the rows and columns of the normal matrix, were given. Before the Cholesky factorization the star parameters were consequently re-ordered in such a way as to reduce the fill-in during the factorization (Figure 9.1). The reduction in computing time allowed by the re-ordering is considerable, not only for the factorization itself, but for all computations using it, such as the calculation of variances (van der Marel 1988).

The covariance matrix $\mathbf{Q}_{\hat{x}}$ of the least-squares estimator is the inverse of the normal matrix if \mathbf{Q}_{y}^{-1} is used as weight matrix. The computation of the complete inverse would have been too time consuming and is also not very useful. However, a subset of the covariance matrix, corresponding to the non-zeroes in the Cholesky factor ('sparse inverse'), could be obtained in only twice the time of the factorization itself. The sparse inverse contained all the elements needed to perform statistical testing of the observations and to produce the proper diagnostics.

Slit Errors

The grid coordinates could be determined from the grid phase only up to an unknown integer number of slits. The slit numbers had to be computed from approximate values for the star and attitude parameters. Considering uncertainties in the *a priori* positions of 0.2 to 0.8 arcsec in the initial star catalogue, and the slit period of \simeq 1.2 arcsec, it will be obvious that there were a substantial number of slit errors, resulting in ambiguities and inconsistencies throughout the first reduction iterations. The great-circle reduction suffered only from inconsistent slit numbers. It could not recognise a situation where all the grid coordinates of a certain star had the same slit error. Therefore, the computed star abscissae could still be wrong by one or more grid steps. The slit errors in the abscissae were ultimately corrected during the sphere solution and astrometric parameter extraction (see Section 11.6).

Slit inconsistencies resulted in contradictions between the observations of one star during the great-circle reduction. Several methods were used for detection and correction of slit inconsistencies. First, inconsistencies in the linearized grid coordinates ΔG_{ik} per star were detected and corrected. This method works well when the attitude and instrument description were properly described. This method worked even better after improvement of the along-scan attitude, simultaneously by a sequential adjustment, using the grid coordinates ΔG_{ik} as observations and estimating correction to the along-scan attitude and estimating correction to the along-scan attitude and star abscissa. It was possible that some slit inconsistencies were left at this stage. Therefore, after the least-squares adjustment the results were checked and remaining slit inconsistencies were corrected and a new solution was computed.

In further iterations of the great-circle reductions, the *a priori* values were taken from the last complete sphere solution, while NDAC also used at early reduction stages star coordinates determined from star mapper observations (see Chapters 6 and 7). In the final iterations of the great-circle reductions errors in the star catalogue used as input to the process were decreased to a level that slit number inconsistencies became very rare, and easy to correct when they occurred at all.

Statistical Tests and Validation

The results of the great-circle reduction were validated by statistical tests. On the basis of the outcome of these tests, in combination with internal and external iterations, several actions were possible:

- (1) correction of slit numbers;
- (2) skipping of doubtful observations, or (in NDAC only) re-weighting of observations;
- (3) skipping stars from the normal equations solution (FAST passive stars).

Depending on the outcome of statistical tests, data was validated, i.e. accepted as sufficiently conforming to the model, or rejected (Section 9.7). In the latter case not only proper diagnostics were generated, but also a new solution without the rejected, and possibly erroneous, observations was computed. Two types of iterations were possible: external iterations of the complete great-circle reduction with an improved attitude description and star catalogue after a preliminary sphere solution and astrometric parameter extraction, and internal iterations within the great-circle reduction itself. Internal iterations mainly dealt with correction of slit inconsistencies and re-weighting of observations. Each iteration involved in principle a new least-squares adjustment. This was, of course, not a very attractive prospect. However, the burden was lightened considerably by *a priori* selection of suspected problem stars within FAST, combining internal iterations with necessary (for other reasons) external iterations, and special procedures for correcting slit inconsistencies.

The FAST great-circle reduction software did not distinguish between primary and nonprimary reference stars, as was done in the sphere solution (Section 11.4), but rather between 'active' and 'passive' stars. Grid coordinates of active stars participated in the rigorous least-squares adjustment which computed the abscissae of active stars, alongscan attitude and instrumental parameters. The passive stars were added in later, using the previously computed active star abscissae, attitude and instrumental parameters, without changing them.

In general, passive stars were 'problem' stars, stars with a high probability of erroneous measurements, or very faint stars which did not contribute much to the attitude and instrumental solution. The passive stars were selected (by the software) by static criteria,



Figure 9.2. Final number of active stars (dots), passive stars per reference great circle with 1-parameter solutions (+, the points just above the bottom of the graph) and 2-parameter solutions (\times , the points at the bottom of the graph) during the mission (FAST).

for example double star characteristics, as well as dynamically, based on the results of the statistical testing for star outliers (Equation 9.12). A few passive stars could only be computed if a linear motion—during the sets—was assumed (2-parameter solutions). As few as possible passive stars were selected during the last external iteration, to get the best possible precision for the star abscissae and attitude parameters. Therefore, during the procedure of external iterations, passive stars which turned out to have good solutions, were re-introduced as active stars during the next iteration. While, on the other hand, 'bad' active stars were treated as passive stars during the next iteration. On the average 300–400 passive stars were selected during the first treatment. In the final iteration this number was reduced to about 100. Figure 9.2 gives the final number of active and passive stars per reference great circle during the mission.

In the NDAC great-circle reductions data for identified 'problem' stars were not passed through this process, but were side-tracked to the special double-star processing (Chapter 13); thus all the stars retained for the great-circle reductions were effectively treated as 'active' stars.

9.3. Attitude Smoothing

The attitude of the Hipparcos spacecraft was, except for small vibrations (jitter) following thruster firings, a smooth function of time. Thus the along-scan attitude, which was initially computed once per observing frame of 2.133...s, could be further improved by introducing relations between the attitude values of neighbouring frames. In fact, an additional adjustment of the along-scan attitude, the so-called smoothing step, was carried out using a model for the attitude which required relatively few parameters.

The improvement of the attitude led also to improved star abscissae, and hence to an improved final star catalogue (Figure 9.3).

For attitude smoothing an additional equation was added to the observation equations for the geometric solution (Equation 9.5):

$$\mathbf{x}_{\mathrm{A}} = \mathbf{B}\mathbf{x}_{\mathrm{B}}$$
 [9.7]

The smoothed attitude could be expressed in a smaller number of parameters \mathbf{x}_B than the frame-by-frame attitude \mathbf{x}_A which was computed in the geometric solution step. The observation equations for the smoothed solution were:

$$\mathbf{y} = \mathbf{A}_{\mathrm{A}}\mathbf{B}\mathbf{x}_{\mathrm{B}} + \mathbf{A}_{\mathrm{S}}\mathbf{x}_{\mathrm{S}} + \mathbf{A}_{\mathrm{I}}\mathbf{x}_{\mathrm{I}} + \mathbf{e}$$
 [9.8]

The equations were again partitioned in an attitude, star, and instrument part, but now the star and attitude unknowns had changed roles: the stars were eliminated first, and the attitude unknowns—now much fewer than in the geometric mode—were reordered using the modulo 360° ordering (van der Marel 1988). In fact, the smoothed solution was computed as an update to the geometric solution. It was not necessary to re-compute the instrument parameters. They were already determined very well in the geometric solution. This was the approach in FAST. In NDAC a slightly different procedure was used: first Equation 9.5 was solved to give the geometrical attitude \mathbf{x}_{A} . This was then inserted in Equation 9.7, which was solved by least-squares to give the parameters \mathbf{x}_{B} of the smoothed attitude. These, in turn, were inserted into Equation 9.8, together with \mathbf{x}_{I} from the geometrical solution, and the resulting system was finally solved for the star parameters \mathbf{x}_{S} .

In the great-circle reductions the smoothed attitude was modelled by cubic B-splines. In general splines consist of polynomial segments, of fixed degree, joined end to end with continuity in a limited number of derivatives at the joints, the so-called knots. Actually, the B-spline series is a linear combination of shifted base functions or B-splines. It could represent the attitude at the milliarcsec level by choosing the right knots. This was performed automatically in the software. Thruster actuations were modelled as instantaneous impulses, which was justified in view of the relatively short duration of the pulses, which resulted in a discontinuity in the first derivative of the B-spline series at the thruster actuation time.

Smoothing of the attitude effectively increased the longitudinal field of view, since more stars were connected directly. Especially more bright stars were now linked directly to each other, and not only by chains of measurements between fainter stars (Lacroute 1983). Smoothing had, therefore, two favourable effects: it led to an overall increase in precision for the astrometric parameters and it permitted a more liberal observing strategy.

9.4. Rank Deficiency and Minimum Norm Solution

The observations in the great-circle reduction were invariant under a simultaneous shift of all the star abscissae v_i and all the along-scan attitude parameters ψ_k . This follows from the fact that the first two (dominant) terms in Equation 9.2 have practically equal and opposite coefficients. In practice this corresponded to an unknown zero point for the abscissae. The consequence was that the design and normal matrices in the great-circle reduction did not have full rank. Under normal circumstances the rank deficiency was



Figure 9.3. Square root of the star variance versus the star abscissa for a base star solution (top), minimum norm solution (middle), and for the minimum norm solution after attitude smoothing (bottom). Data from 21 May 1990 10:00–17:20 (day 506).

Great-Circle Reductions



Figure 9.4. Averaged auto-covariance function of the star abscissa (based on simulated data for the geometric solution).

one. During the great-circle reduction the rank deficiency was provisionally eliminated by forcing the abscissa correction of one star, the so-called 'base star', to zero. This was equivalent to skipping the corresponding column in **A** and the corresponding row and column in the normal matrix. The base star was usually a bright star close to one of the scan circle nodes. This remedy for the rank deficiency was very attractive for its simplicity, but it resulted in a variance of zero for the base star (Figure 9.3). It had the same effect as adding the constraint equation $\mathbf{c'x} = \mathbf{0}$ to the system, with **c** a vector of length *n* with all zeroes, except the element corresponding to the base star.

The choice of a particular base star was arbitrary, but it affected the solution and covariance matrix of the great-circle abscissae. For the sphere solution it did not matter which base star was chosen, if the full covariance matrix is used, because the unknown zero point was estimated anyhow. However, only the variances were taken into account for the weighting in the sphere solution, and, in this case, an arbitrary one of them was zero. This was not very satisfactory. Therefore, the great-circle solution and its covariance matrix were transformed into a minimum norm solution. The sum of the corrections to the star abscissa in the minimum norm solution were zero, the covariance matrix had minimum trace (minimum variance), there were no zero variances and off-diagonal elements in the minimum norm covariance matrix were smaller (Figure 9.3). Therefore, the minimum norm variances were preferred instead of the base star variances.

The minimum norm solution was computed from the base star solution by what is known in geodesy as an S-transform (Baarda 1973, Teunissen 1985). Again, the abscissae were only shifted, but from the original covariances the column and row average were subtracted, and the overall average was added. This operation was coded very efficiently during the Cholesky factorization. In fact, a constraint equation $\mathbf{c'x} = \mathbf{0}$ was added to the equations, with \mathbf{c} an *n*-vector with all ones, such that $\mathbf{Ac} = \mathbf{0}$, namely \mathbf{c} is a basis for the null space of \mathbf{A} .

The covariance matrix of the minimum norm solution was almost a cyclic matrix. A cyclic covariance matrix is fully described by a single covariance function. Figure 9.4 gives the averaged auto-covariance function of the star abscissae obtained from simulations for the nominal mission. The positive correlation between stars separated by a

basic angle (58°) and multiples are of course an effect of the two fields of view of the telescope. The basic angle was chosen not to be a fraction of 360° (see also Volume 2, Chapter 1, Figure 1.2). If the basic angle had been a fraction of 360° (for example 60°) the peaks would have been amplified. The value of 58° for the basic angle was the result of a study (in the mission design phase) on the great-circle rigidity. The correlation is larger for smaller data sets (which was the case in the revised mission). It was obvious that the correlations could not be neglected in the sphere solution without some loss of precision.

9.5. Accuracy of the Great-Circle Solution

The accuracy of the great-circle reduction depended first of all on the quality of the grid coordinates computed from the image dissector tube data. The standard error in the grid coordinates was dominated by the photon noise of the individual samples. The photon noise was Poisson distributed, but since each grid coordinate was computed from many samples one could assume, according to the central limit theorem, that the grid coordinates had a normal distribution and were uncorrelated with respect to each other. Thus the covariance matrix for the grid coordinates \mathbf{Q}_y , computed by the phase estimation task, was a simple diagonal covariance matrix. Other errors, like veiling-glare, projection and other modelling errors, which were smaller, were not represented by \mathbf{Q}_y or by the covariance matrix $\mathbf{Q}_{\hat{x}}$ of the least-squares estimator. Therefore, the accuracy of the great-circle reduction could not be described by only the variances. Analysis of the residuals $\hat{\mathbf{e}}$ of the least-squares estimation by statistical tests, given in Section 9.7, was the other, very important, part of the accuracy description.

Both parts of the accuracy description were verified by tests on simulated data (van der Marel *et al.* 1989). Simulated data offered the possibility to study the error in the estimator, an advantage not available with real data. However, intercomparison of the results between FAST and NDAC gave another indication of the accuracy of the results. This was the third part of the accuracy description given in Section 9.8, and it was a very worthwhile one. In fact, creating the possibility of this kind of comparisons had been a major reason for assigning two consortia to the data reduction tasks.

Variance of the Star Abscissae

The variances of the star abscissae followed simply from the inverse of the normal matrix. The star variances were separated into three components:

- (1) the variance σ_{obs}^2 when only photon noise is taken into account, assuming a perfect attitude and instrument;
- (2) the influence of the attitude determination σ_{att}^2 ;
- (3) the influence of the determination of the instrumental parameters σ_{ins}^2 .

The variance of the star abscissae after adjustment was:

$$\sigma_{\text{star}}^2 = \sigma_{\text{obs}}^2 + \sigma_{\text{att}}^2 + \sigma_{\text{ins}}^2$$
[9.9]

The σ_{obs}^2 of a star was computed from the cumulated *a priori* observation weights of this star and σ_{ins}^2 was the difference of the computed star variances with and without solving

В	n_B	$\sigma_{ m obs}$	σ_{ins}	$\sigma_{ m att}$	$\sigma_{\rm star}$	$\sigma_{\rm att}$	$\sigma_{\rm star}$
(mag)				(geor	(geometric)		othed)
3-4	1	0.17	0.36	2.00	2.03	1.30	1.36
4–5	5	0.40	0.33	2.06	2.12	1.33	1.43
5-6	26	0.83	0.33	2.37	2.53	1.47	1.72
6-7	96	1.07	0.29	2.08	2.36	1.32	1.72
7-8	245	1.70	0.30	2.16	2.76	1.31	2.16
8-9	552	2.77	0.31	2.21	3.56	1.34	3.09
9-10	423	3.53	0.30	2.49	4.33	1.37	3.80
10-11	111	4.11	0.30	2.49	4.81	1.38	4.35
11-12	28	6.02	0.31	3.55	6.99	1.68	6.25
12-13	6	6.63	0.33	3.09	7.32	1.79	6.88
all	1493	3.01	0.30	2.34	3.82	1.36	3.31

Table 9.1. Square root of the mean variances in milliarcsec per magnitude class (data from 21 May 1990 10:00–17:20 = day 506). The table shows the contributions of the observational errors (σ_{obs}), the instrument (σ_{ins}) and the attitude (σ_{att}) to the total standard error of the star abscissa (σ_{star}) for the geometric and smoothed solutions.

for instrumental parameters. Finally σ_{att}^2 was a derived quantity, computed from the above mentioned variances.

In Table 9.1 the square root of the average of the minimum norm variance per magnitude class is given for the data set of Figure 9.3. The error σ_{obs} , and therefore σ_{star} , were clearly magnitude dependent: σ_{obs} varies between 0.1 mas for very bright stars and 3.2 mas for the 10 mag, and was even larger for 12–13 mag stars. The influence of the attitude and influence of the instrument were more or less the same for each magnitude class. The influence of the instrumental parameters (0.3 milliarcsec) was very small compared to the influence of the attitude. This value was very sensitive to the length of the reference great-circle set (Figure 9.13). It was a little larger than expected because the reference great-circle sets in the revised mission were shorter. The improvement brought by attitude smoothing is striking. The influence of the attitude was reduced very significantly (2.4 milliarcsec for the geometric solution and 1.4 milliarcsec in the smoothed solution), resulting in better star variances (Figure 9.3). The improvement affected the brighter stars in particular (Figure 9.5). The error in the fainter stars was still dominated by photon noise.

Attitude Smoothing

Figure 9.6 gives the variances of the attitude parameters for the example of Table 9.1. The differences between the geometric and smoothed attitude are shown in Figure 9.7. The influence of the attitude, σ_{att} in Equation 9.9, was reduced considerably by smoothing. The improvement was a function of the number of attitude parameters needed to represent the attitude. In Table 9.1 the mean standard error of the star abscissae was given for the optimum number of B-splines. In the unrealistic, but informative case of a perfectly known along-scan attitude, the standard error of the star abscissae is equal to σ_{obs} (neglecting the instrument).

The optimum number of B-splines was initially calculated by simulation experiments with the great-circle reduction software (van der Marel 1985). In Figure 9.8 the mean



Figure 9.5. Star abscissae improvement by attitude smoothing (data from 21 May 1990 = day 506).



Figure 9.6. Square root variance of the geometric (dots) and smoothed (circles, lower accumulation of symbols) attitude. Vertical lines are drawn at thruster actuation times. The time is given in units of $T_4 = 2.133...s$ (data from 21 May 1990 = day 506).

standard error (measurement induced error), the modelling error in the smoothed attitude, the rms error in the estimated attitude (estimation error) and the unit weight variance (Equation 9.10) are plotted versus the number of attitude parameters. The rms error in the estimated attitude and star abscissa reached a minimum at some point. With a smaller number of B-splines the modelling error became significant, for a larger number the inherent smoothness was not sufficiently exploited.

The rms errors in the estimated star abscissa could not be determined with real data. The only information available to determine the optimum number of B-splines with real data are statistical tests based on the test statistics in Equation 9.10–9.12, and on visual



Figure 9.7. Differences between the geometric and smoothed attitude (data from 21 May 1990 = day 506).



Figure 9.8. Attitude improvement by smoothing as function of the number of attitude parameters per circle (based on simulations).

inspection of the differences between the geometric and smoothed attitude for selected great-circle sets (Figure 9.7).

Figure 9.9 gives the square root of the average variance of the active star abscissae and the along-scan attitude during the mission. The improvement for the along-scan attitude by the attitude smoothing is striking. There is quite a significant number of great circles with a larger average standard error for the star abscissae and along-scan attitude. This is mostly for short great-circle sets, which had some difficulty in estimating the instrument parameters. This is also visible in the top plot of Figure 9.9, where the dots give the square root of the average of σ_{ins}^2 of Equation 9.9.



Figure 9.9. Average standard error (square root of the average variance) in star abscissae after smoothing during the mission (top, crosses), influence of the solution of instrument parameters (top, dots), average standard error of along-scan attitude parameters (bottom) after smoothing (dots) and geometric (crosses) solutions during the mission.

Projection Error

The projection error on the reference great circle, $\epsilon_{ik}^{\prime\prime}$ of Equation 9.4, depended on the size of the catalogue error Δr_i and star mapper attitude error $\Delta \theta_k$ and $\Delta \phi_k$. During the first treatment these quantities could be rather large due to the quality of existing star catalogues. Therefore, it was necessary to iterate the great-circle reduction after the sphere solution, making better values for r_i available. The attitude reconstruction was repeated too, resulting in better values for θ_k and ϕ_k . After at most two iterations the error in r_i was 2–4 mas and could be neglected, but the error in θ_k , ϕ_k remained of

Table 9.2. Predicted projection errors (in milliarcsec) during first treatment (assuming catalogue errors of $\sigma_{\rm S} = 1.5$ arcsec) and in the iterations (assuming $\sigma_{\rm A} = 0.1$ arcsec and negligible $\sigma_{\rm S}$).

	First treatment		Iteration	
	rms	max	rms	max
Field coordinate	12.0	72.3	0.9	6.3
Attitude	6.0	72.3	0.8	6.3
Star	10.1	72.3	0.6	3.3



Figure 9.10. Projection error in the star abscissae. The nodes are near 70° and 250°

the order of 50–100 mas, due to star mapper photon noise, and could not be improved further.

In Table 9.2 the projection error effect on the field coordinates, star abscissa and alongscan attitude are given for a typical reference great circle. The results are from analytic formulae, and were confirmed by extensive simulations (van der Marel 1988). The projection error depended also on the size of the partial derivatives. The projection errors were large when the inclination of the scanning direction with respect to the reference great circle was large. Therefore the projection errors were large near the nodes of the scan circles. However, the projection error effect on the attitude and star parameters averaged out at locations with a uniform scanning. This happened exactly on the nodes of the scan circles. Therefore maximum projection errors were expected near the nodes, but not on the nodes. Figure 9.10 is a scatter diagram of the projection error effect on the star abscissae for a typical first treatment. The predicted maxima near the nodes are clearly visible.



Figure 9.11. Medium-scale residuals of a third degree polynomial representation of the non-chromatic field-to-grid transformation. The contours range from +1 mas (solid line) to -0.5 mas (broken, dotted line). See also Figures 10.14 to 10.16.

9.6. Instrument Parameters

The basic angle distortion and large-scale field-to-grid transformation is the third group of parameters which was estimated during the great-circle reduction. The field-to-grid transformation was modelled by a polynomial in several variables. The polynomial degree was 3 or 4 for the non-chromatic terms and 1 for the chromatic terms. Figure 9.11 shows a grid map of the mean residuals over the first 30 months of reduced data, where third or fourth degree polynomials were adopted to describe the non-chromatic part of the field-to-grid transformation.

A new set of instrument parameters was normally estimated for each great-circle reduction. However, since the highest order parameters were assumed not to change with time, these parameters were fixed at their average values. The choice and time evolution of the instrument parameters is described in more detail in Chapter 10 and in Volume 2, Chapter 10. Here some of the implementation aspects are described briefly.

The choice for a power series was a little arbitrary. At the time of implementation there were no numerical or functional reasons for choosing different types of functions. However, there were signs from the real data that some refinements were necessary. A disadvantage of power series was certainly the large correlation between some of the estimated parameters. In any case, not all of the instrumental parameters can be estimated equally well. The so-called 'constant term' (g_{00} in the NDAC notation), and the 'constant chromaticity' (c_{00}), could not be estimated at all from a single reference great-circle set. They were omitted from the great-circle equations and determined during the sphere solution (see Chapters 11 and 16).

In FAST, it was shown that the variances of stars and attitude determinations could be improved when less instrumental parameters were taken as unknown and replaced by fixed pre-determined values. These were obtained by computing the mean of the coefficients determined by the great-circle reduction during earlier iterations for every calibration period as defined in Section 4.2. In addition, the variations of the basic angle were modelled by a linear function of time. The analysis of the instrumental parameters showed that a fourth-order polynomial gave a significantly better representation of the field-to-grid transformation as shown in Figure 9.11, and that the third and fourth degree terms were very stable throughout the mission, but were rather strongly correlated with certain first and second degree terms.

From these considerations, the following scheme was adopted by FAST. While the first treatment of data was performed using third-order polynomials, the next iteration was performed with fourth-order formulae. Then, in further iterations, the calibrated third and fourth degree coefficients as well as the chromatic terms were considered as known, reducing by 11 the number of instrumental unknowns. For short reference great circles (1 or 2 rotations), the terms proportional to *y*, *xy* and y^2 (where *x* is along the scan and *y* normal to it) were difficult to estimate because the inclination was small, the risk being large variances for star parameters and even a singular system of equations. They were then taken from the calibration file. In case of even shorter data sets, no instrumental parameters were computed except the coefficients for *x* and x^2 ; all the others, including the basic angle, being taken from the calibration. Finally, when thermal disturbances occurred (see Chapter 2), the basic angle was not stable and its variations were represented by a linear function of time.

Figure 9.12 gives the number of instrument parameters which were solved by the FAST consortium during the final iterations as a function of the length of the great-circle set. Also plotted are the average standard error of the stars and the influence the instrument parameter estimation had on the standard error of the stars. In shorter great-circle sets less instrument parameters were solved than in longer sets. For the longer sets, which had the power to estimate the more difficult instrument parameters, fewer instrument parameters were replaced by values taken from the calibration. Even despite this strategy, the effect of the instrument parameter solution on the standard error of the star abscissae was more pronounced for the shorter reference great-circle sets.

9.7. Analysis of the Least-Squares Residuals

The least-squares adjustment can be interpreted as the orthogonal projection of the *m* dimensional vector of observations $\mathbf{y} \in \mathbb{R}^m$ onto the vector $\hat{\mathbf{y}} = \mathbf{A}\hat{\mathbf{x}}$ in the *n* dimensional linear manifold spanned by the columns of \mathbf{A} (the range space). The metric of \mathbb{R}^m is defined by the weight matrix \mathbf{Q}_y^{-1} of the observations (which is in fact a metric tensor). The adjusted observations $\hat{\mathbf{y}}$ and least-squares residuals $\hat{\mathbf{e}}$ are orthogonal, so the residual sum of squares $E = \hat{\mathbf{e}}' \mathbf{Q}_y^{-1} \hat{\mathbf{e}}$ is a minimum. Moreover, the residual sum of squares has a χ^2 distribution with m - n degrees of freedom if \mathbf{y} has a normal distribution with m - n and ∞ degrees of freedom, or unit weight variance:

$$F = \frac{E}{m-n} = \frac{1}{m-n} \sum_{i} \sum_{k} \frac{e_{ik}^2}{\sigma_{ik}^2} \simeq F(m-n,\infty)$$
[9.10]



Figure 9.12. Number of instrument parameters solved during the final iterations versus the length of the great-circle sets (top), and the average standard error of the stars (dots) and the influence of the instrument σ_{ins} (+) as a function of the length of the great-circle sets (bottom).

with e_{ik} the least-squares residual of star *i* in frame *k* and σ_{ik} the standard error of the observation. The expected value of *F* is one. The null hypothesis H₀ that the model of Equations 9.5 and 9.7 and covariance matrix \mathbf{Q}_y were correct, and there were no outliers, was verified by hypothesis testing. The test was:

reject
$$H_0$$
 if $F > F_{\alpha}(m - n, \infty)$ [9.11]

with $F_{\alpha}(m-n,\infty)$ the critical value for the test with level of significance α , the probability that the test was rejected wrongly if H₀ was true.

Figure 9.13 gives the value of *F* during the mission. The values for *F* were larger for smoothing. Almost every value for the smoothed solution exceeded the expected value of 1 significantly, resulting in a rejection of the test in Equation 9.11. For a typical reference great-circle set the critical value was $F_{0.001}(20\,000,\infty) = 1.03$. The rejections were a result of the modelling error in the attitude. The B-spline series was only able to



Figure 9.13. Unit weight variance *F* (top) and rms error of the residuals of the grid coordinates (bottom) during the mission (crosses refer to the geometric solution, dots to smoothing).

describe the attitude up to the 1 milliarcsec level. Taking these into account, the tests were almost always accepted. Despite all this the F test values were close to one. This meant that the variances of Table 9.1 were representative. For this particular data set the F test values were 1.048 and 1.076 respectively for the first treatment, and 0.9688 and 0.9974 for the final iteration given in Figure 9.13.

The power of the test in Equation 9.11 was not very good. A few small errors in the observations did necessarily lead to a rejection of this test. Neither did it provide an indication of what problems caused rejection. Fortunately other more powerful tests
could be used to identify specific problems. The quality of the adjusted star abscissae was checked by a Fisher test statistic similar to Equation 9.10:

$$F_{i} = \frac{1}{s_{i}} \sum_{k} \frac{e_{ik}^{2}}{\sigma_{ik}^{2}} \quad \text{with} \quad s_{i} = \sum_{k} \frac{\sigma_{e_{ik}}^{2}}{\sigma_{ik}^{2}}$$
[9.12]

with s_i the degree of freedom, or redundancy, of star *i*, computed from the variance of the least-squares residuals $\sigma_{e_{ik}}^2$. F_i has a Fisher distribution with s_i and ∞ degrees of freedom, which could be used in the test of Equation 9.11. Also used was F_i/F instead of F_i . F_i/F has a Fisher distribution with s_i and m-n degrees of freedom. This test was indicative of modelling problems related to specific stars, e.g. single stars which turned out to be double, veiling-glare, etc.

The star-by-star test was the main instrument for the selection of FAST active and passive stars. The static criteria which were applied during the first treatment were gradually replaced by the results of this test. After every external iteration the results of the star-by-star test from all great-circle sets participating in the iteration were collected. Those stars which had many rejections during the great-circle treatment were then selected as passive stars for all great-circle sets in the next iteration. Passive stars which had very few rejections, were selected as active stars for the next iteration. In addition to the global list of active and passive stars, another list was maintained in which stars were made passive for specific great-circle sets. In this way, an occasional star outlier could be accommodated. During the first treatment about 300 to 400 passive stars per great-circle set were selected using static criteria, and up to 2 per cent of *a priori* unsuspected active stars were flagged. In the final iteration, about 100 passive stars per set were left (Figure 9.2), and there were no serious rejections of the test of Equation 9.12.

Statistical tests similar to Equation 9.12 were derived to specifically check the frameby-frame attitude and B-spline smoothing by altering the summation in Equation 9.12. The test on the B-spline intervals was indicative of the modelling error in the smoothed attitude caused by insufficient B-spline parameters. This test was used within FAST to build a list of intervals which needed additional B-spline parameters, or should be excluded from the reductions completely, for instance because the satellite had been hit by small particles, or for other reasons. Some periods had so many new B-splines that effectively a frame-by-frame attitude representation was used. Although this procedure was automated by FAST to some extent, manual intervention was necessary on several occasions. The advantage of using a list was that this work did not have to be repeated in subsequent iterations.

The summation in the test of Equation 9.12 could be restricted to a single observation, which resulted in the test statistics with standard normal distribution:

$$\bar{e}_{ik} = \frac{e_{ik}}{\sigma_{e_{ik}}} \sim N(0, 1)$$
[9.13]

A grid coordinate error was suspected if $|\bar{e}_{ik}| > N_{\alpha}(0, 1)$. Using this test the grid coordinates were inspected one by one. This procedure is a common technique in geodesy and is known as 'data snooping' (Baarda 1968). The major problem with these techniques was a lack of robustness caused by smearing and masking effects. Smearing was caused by the correlation between the least-squares residuals. A single outlier in the data could result in the rejection of several data snooping hypotheses. Similarly, a large outlier may mask smaller outliers, which could only be found after the large outlier had been removed. Therefore, whatever the procedure for detection and correction was, it had to be iterated: i.e. the most evident cases were tackled first, then a new solution



Figure 9.14. Intercomparison of the FAST and NDAC abscissae after attitude smoothing (data from 14 May 1990 0:04–5:08 = day 499).

was computed and the residuals, or testing variates, were inspected again. The process converged if in later iterations more and more subtle cases were recognised as errors. This procedure can be automated. This was for instance implemented by Kok (1985) in his iterated data snooping procedure, or by Eeg (1986) in his iteratively re-weighted least-squares, which was the method used by NDAC.

9.8. Intercomparisons

Several identical sets were reduced by NDAC and FAST for comparison purposes. Figure 9.14 shows the difference in abscissae for a typical comparison set halfway during the reductions, just after the input catalogue had been improved for the first time. Passive stars, which had differences up to several hundred milliarcsec, were removed from Figure 9.14. These differences were caused by the different treatment of double stars in the two consortia. In the FAST great-circle reduction the weighed mean of the first and second harmonic of the grid phase were used as the observation. In NDAC only the first harmonic was used (see Chapter 5). This affected the double stars in particular.

The comparison set shown in Figure 9.14 is one of 15 comparison sets—with specific difficulties—which were tested during the mission. The difficulty with the set in Figure 9.14 was an eclipse, but this had no adverse effects in this case. Actually, this comparison set was an example of a normal set. Other comparison sets sometimes

Table 9.3. Summary of projection error differences resulting from the use of different star catalogues for the
NDAC great-circle reduction. All values are given in milliarcsec. The rms errors for the two catalogues were
270 milliarcsec (Hipparcos Input Catalogue) and 185 milliarcsec (partially improved working catalogue).

	Abscissa			Instrument
	rms	min	max	(corner)
Set 1 (all stars)	2.68	-40	+25	0.3
Set 1 (active stars)	1.56	-25	+8	0.5
Set 2 (all stars)	2.94	-70	+10	0.4

had much larger differences, with large systematic effects. Often a very significant 6th harmonic was found, which was due to the basic-angle design, one of the periods for which the great circle was not very robust. This 6th harmonic could be triggered by almost anything, for example a serious outlier in the data, or short great-circle sets in combination with estimating too many instrument parameters. Also in good comparison sets this harmonic was present, as can be observed from the averaged auto-covariance function in Figure 9.14. This led to experiments in the sphere solution (Chapters 11 and 16) whereby it was tried to estimate a 6th harmonic for each great-circle set.

The rms difference for good comparison sets was usually 3–5 milliarcsec, with maximum errors up to several tens of milliarcsec. The rms differences seemed to be too large considering that both consortia had reduced the same data. The rms difference was of the same order as the standard error of the star abscissae. Also, the correlation between the abscissa differences, given in Figure 9.14, was very similar to the correlation function of the great-circle abscissae in Figure 9.4 (computed from simulated data). In fact, two completely independent measurements would have resulted in differences which were not much larger. This requires some explanation. The first reason was that the consortia did not really use the same data because NDAC did not use the second harmonic of the grid phase. This mainly affected the double and multiple stars. There was a variable bias between the first and second harmonic which also affected the single stars and especially the calibration of the instrument (Schrijver & van der Marel 1992).

The second reason was that the projection errors were not the same because different working catalogues and star mapper attitudes had been used. The rms difference in the star catalogue used for this particular example was 0.1–0.3 arcsec. The effect was illustrated by comparing two runs of the same consortium on the same data with different catalogues, but otherwise completely identical. The results are given in Table 9.3. An error in a single catalogue position would also affect the other stars due to the smearing effect of the least-squares estimation. The covariance function was, therefore, similar to the covariance function of the abscissae.

The third reason was that different sets of observations and stars participated in the actual least-squares adjustment. In FAST the so-called passive stars were fitted in later without affecting the attitude. Outliers in the observations were treated differently. In NDAC the great-circle reduction was iterated several times with some of the observations re-weighted. FAST used data snooping for the grid coordinates and variance tests for the stars. When an active star was rejected by the statistical tests it was made passive in the next external iteration. The effect of using passive stars was studied by two runs on the same data with different sets of stars. The results are given in Table 9.4. A fourth reason is that in the attitude smoothing the number of B-splines and the location of their

	Abscissa			Instrument
	rms	min	max	(corner)
NDAC	2.32	-42	+10	0.4
FAST (geometric)	1.99	-10	+9	0.3
FAST (smoothing)	1.56	-5	+5	-

Table 9.4. Effect on the abscissa differences when different sets of stars were selected (FAST active star set versus all stars). All values are given in millarcsec.

Table 9.5. Normalised standard errors for great-circle reductions with weighted phase, first harmonic and second harmonic only.

	Weighted	First only	Second only
Geometric solution	1.019	1.017	1.030
Smoothed solution	1.043	1.040	1.043



Figure 9.15. Difference between the FAST and NDAC instrument description (first harmonic).



Figure 9.16. Difference between the instrument for the first harmonic and second harmonic.

knots were different. Also, the estimation procedures in FAST and NDAC were not the same.

The instrument parameters agreed up to 1.5 mas in the corners of the field of view when FAST used the first harmonic of the grid phase. The only significant differences, i.e. more than two times the standard error, were in the terms g_{01} and g_{30} (+1.4 and -0.94 mas, respectively, at the upper left corner of the field of view, see also Figure 9.15). When FAST used the weighted phase, which was their standard approach, the agreement was lost between the FAST and NDAC instrument parameters. This did not mean that the weighted phase is worse, it only meant that the field-to-grid transformation was different. In fact, test runs with only the second harmonic showed that it is good data (Table 9.5). The difference in instrument parameters for the first and second harmonic are shown in Figure 9.16. The differences in the corners were 13 mas.

9.9. Conclusions

In the previous sections results of the great-circle reductions have been given based on simulations, comparisons, or the software's internal accuracy description. The accuracy of the software's internal accuracy description has been verified in every possible way. A final test was the next reduction step: the sphere solution. Here the abscissae from the great-circle reduction with their minimum-norm standard error were used as observations. Statistical tests, like those in Equation 9.12, were used to verify the abscissae observations and the stochastic model. Considering the fact that at the level of the sphere solution abscissae from the same great-circle set were assumed to be uncorrelated, which they were not (Figure 9.4), it turned out that the great-circle software gave a very fair description of the accuracy.

H. van der Marel, F. van Leeuwen, J. Kovalevsky, C. Petersen

10. EVOLUTION OF INSTRUMENT PARAMETERS

The geometrical instrument parameters describe the transformation from ideal, angular coordinates in the fields of view to the observable 'grid coordinates' connected with the slit pattern on the main grid. The instrument parameters were determined as part of the great-circle reductions, and while they are not of any direct astronomical interest, they are highly relevant for understanding the behaviour of the Hipparcos instrument. The transformation models used by the FAST and NDAC consortia are specified in this chapter and the resulting instrument parameters are shown as functions of time.

10.1. Introduction

The purpose of the great-circle reductions (Chapter 9) was to compute the coordinates of stars along designated reference great circles, the so-called star abscissae. In doing so, the geometrical distortions of the Hipparcos main field of view had to be determined as part of the least-squares solutions for the star coordinates. These distortions are described by the geometrical instrument parameters. While not of any direct astronomical interest, the instrument parameters and their temporal evolution are highly relevant for understanding the Hipparcos instrument.

This chapter provides a relatively complete documentation of the geometrical instrument parameters as determined by the FAST and NDAC consortia. The main results are given in the form of diagrams showing the temporal evolution of each parameter over the whole mission. Since the consortia used different models for the field distortions, different sets of parameters were used and their intercomparison is not always a simple matter (see also Section 9.8). Section 10.2 gives the approximate relations between the two sets of parameters, but the two representations are not strictly comparable, because of the different procedures and conventions adopted. In particular it should be noted that the grid coordinate, as defined by FAST, was obtained from a weighted mean of the phases of the first and second harmonics of the image dissector tube signal, while in NDAC only the first harmonic was used. The phase difference between the harmonics was a function both of time and position in the fields, as illustrated in Figures 5.5 and 5.6.

The geometrical instrument parameters are here presented without any discussion of the actual values and their evolution. An attempt at a physical interpretation of some of the FAST and NDAC instrument parameters is however made in Volume 2, Chapter 10.



Figure 10.1. Definition of the grid coordinate G (= number of slits, reckoned from a conventional point near the grid centre) and the approximate orientations of the axes for the field angles (η, ζ) , the FAST field coordinates (x, y), and the NDAC field coordinates (w, z). The sense of $+\eta$, +x and -w is given by the apparent motion of stellar images in the field; the perpendicular axes are directed such that the image of the Sun would be in the hemisphere $-\zeta$, +y or +z.

The instrument parameters represent the large-scale part of the transformation from field angles to grid coordinates. A complete description of the grid pattern requires also a medium- and a small-scale component. The medium-scale component is discussed in Section 10.3. The small-scale component was, in the data analysis, treated as noise.

10.2. Geometrical Instrument Parameters

The light modulation produced by the grid allowed the instants when the star image was exactly centred on one of the 2688 slits of the main grid to be determined. The term 'centred' can be loosely understood as meaning 'maximum intensity', but the precise meaning depends on the fitting of a Fourier model of the intensity variations (Section 5.2) and the subsequent definition of the 'reference phase', which, as already mentioned, differed slightly between FAST and NDAC. In practice this led to differences of the order of 10 mas between the two representations; these differences are not further considered here. Conceptually, therefore, the only directly observable 'coordinate' was a quantity *G* which may be defined as G = 0 at the designated central slit, and incrementing by one unit for each slit in the direction of the motion of the stellar image (Figure 10.1). This continuous grid coordinate was related to the true angles as projected on the sky, or the field coordinates, by means of the so-called field-to-grid transformation. (The FAST grid coordinate *G* in Section 5.4 is similar to the *G* defined here, only multiplied by the nominal grid period.)

Field Coordinates

Inevitably NDAC and FAST chose different conventions for expressing the angular coordinates in the field. Both may be defined in terms of a third set of field coordinates, the so-called field angles (η , ζ). The field angles are spherical coordinates with $\zeta = 0$ representing the viewing plane through the two sky projections of the star mapper apex and with $\eta = 0$ at the geometrical centre of the grid, i.e. halfway between the 1344th

and 1345th slits, as reckoned from the edge of the main grid where the stellar images entered.

The FAST field coordinates (*x*, *y*) are similar to (η , ζ) except that the *y* axis is in the opposite direction and the origin is taken to be at the 1344th slit; thus:

$$\begin{aligned} x &= \eta + \frac{1}{2}s \\ y &= -\zeta \end{aligned} \tag{10.1}$$

where $s \simeq 1.208$ arcsec is the grid period. The NDAC field coordinates (*w*, *z*) are the direction cosines:

$$w = -\cos\zeta\sin\eta$$
 [10.2]

$$z = -\sin\zeta$$

The transformation from FAST to NDAC coordinates is, therefore:

$$x = -\arcsin\frac{W}{\sqrt{1 - z^2}} + \frac{1}{2}s$$

$$y = \arcsin z$$
[10.3]

with the reverse transformation:

$$w = -\cos y \sin(x - \frac{1}{2}s)$$

$$z = \sin y$$
[10.4]

The shift by a half grid step between the NDAC and FAST origins is of no consequence for the presentation of the instrument parameters, and the term $\frac{1}{2}s$ is subsequently dropped.

Field-to-Grid Transformation

The complete field-to-grid transformation can be written in the form:

$$G = G_{\text{ref}}(\eta, \zeta) + \Delta G(\eta, \zeta, f, C, t) + \delta G(\eta, \zeta) + \delta g$$
[10.5]

where G_{ref} is a fixed reference model for the field-to-grid transformation, ΔG is the large-scale distortion relative to the reference model, δG the medium-scale distortion and δg the small-scale distortion. (η, ζ) was replaced by (x, y) and (w, z) in the detailed representations of FAST and NDAC. The large-scale distortion was given by a polynomial model and included terms depending on the field index (f = +1 for the preceding field of view and f = -1 for the following field of view), the colour parameter C = (B - V) - 0.5 or (V - I) - 0.5, and time.

The time dependence was usually taken care of by the independent solution of the instrument parameters for each reference great circle, i.e. about twice per day. In the FAST reductions of some great circles explicit time-dependent terms were included. In the final iteration this concerned 57 great circles. The results in a few cases where the basic angle showed significant variation are given in Volume 2, Section 12.4.

The medium-scale distortion was generally a fixed matrix of corrections which was derived either empirically, by mapping the residuals of many great-circle reductions, or from laboratory measurements of the grid (Section 10.3). The small-scale distortion described the irregularities of the individual slits and was treated as noise in the data reductions.

Reference Model

The reference model differed between the consortia. G_{ref} was in NDAC taken to be the nominal field-to-grid transformation for the nominal values of the grid step ($s_0 = 1.208$ arcsec exactly) and basic angle ($\gamma_0 = 58^{\circ} 00' 30''$ exactly). Since the slits were nominally parallel and equidistant in an orthographic projection onto the tangent plane of the curved grid, the nominal relation was:

$$G_{\text{ref N}} = -S_0 W \qquad [10.6]$$

where $S_0 = 170749.01$ slits/rad is the nominal scale corresponding to a grid step of exactly 1.208 arcsec. FAST adopted the same nominal scale at the grid centre, but defined the reference grid coordinate to be proportional to the angle *x* rather than to the direction cosine *w*:

$$G_{\text{ref F}} = S_0 x \tag{10.7}$$

As a consequence the FAST large-scale distortion contained components attributable to the nominal grid pattern. These components can be derived from Equation 10.4 by means of a series expansion of the trigonometric functions. Since |x|, |y| < 0.01, terms of order $O(x^5)$ are smaller than 0.02 mas and can be neglected; hence:

$$G_{\text{ref N}} = G_{\text{ref F}} - \frac{1}{6}S_0 x^3 - \frac{1}{2}S_0 x y^2$$
[10.8]

Large-Scale Distortion Models

Both consortia used polynomials in the field coordinates to model the large-scale distortion ΔG in Equation 10.5. This choice was motivated by optical calculations, showing that perturbations of the nominal instrument produced distortions which were accurately represented by low-order polynomials (e.g. Bertani *et al.* 1986).

The polynomials were expressed in terms of the normalized field coordinates $\bar{w} = w/q$, $\bar{z} = z/q$, $\bar{x} = x/q$, $\bar{y} = y/q$, where $q = \sin 0^{\circ}.45$ or $q = 0^{\circ}.45$, respectively, is the approximate extension of the field of view in either direction from the origin. The quantity q, regarded as a unit for the field coordinates, is also denoted 'hfov' (half field-of-view). The polynomial coefficients thus give the distortion produced by the term at the corner of the field of view (i.e. $\bar{w} = \bar{z} = 1$), and are conveniently expressed in mas/hfov^{*n*}, where *n* is the degree of the distortion term.

The NDAC representation of the large-scale distortion was:

$$G = G_{\text{ref N}} + s^{-1} \sum_{\substack{0 \le i \\ 0 \le j \\ 0 \le l+j \le 4}} (g_{ij} \pm h_{ij}) \bar{w}^{i} \bar{z}^{j} + s^{-1} \sum_{\substack{0 \le i \\ 0 \le j \\ 0 \le l+j \le 1}} (c_{ij} \pm d_{ij}) \bar{w}^{i} \bar{z}^{j} C$$
[10.9]

where, in the final reductions, C = (V - I) - 0.5. Upper and lower signs refer to the preceding (f = +1) and following (f = -1) field of view, respectively. The factor s^{-1} takes into account that the sums give the distortion in angular measure (e.g. milliarcsec), while G is measured in grid steps. In Equation 10.9 the terms containing g_{00} and c_{00} were excluded because they cannot be estimated in the great-circle reductions. g_{00} represents the origin of the field coordinate w and was set to zero by definition. c_{00} represents the so-called 'constant chromaticity' that was instead estimated in the sphere solution (see Section 16.3). The model thus contained 34 instrument parameters. The quartic

terms (i + j = 4) were partially kept constant and are not discussed in the following. The temporal evolution of the remaining 24 parameters are shown in Figures 10.2 to 10.7.

The FAST representation may be written:

$$G = G_{\text{ref F}} - s^{-1} \sum_{\substack{0 \le j \\ 0 \le j \\ 1 \le i+j \le 4}} a_{ij}^{p/f} \bar{x}^i \bar{y}^j - s^{-1} \sum_{\substack{0 \le j \\ 0 \le j \\ i+j = 1}} b_{ij}^{p/f} \bar{x}^i \bar{y}^j C \pm \frac{1}{2} s^{-1} (\Delta \gamma_0 + \Delta \gamma_1 C)$$
[10.10]

where, again, C = (V-I)-0.5 and the upper/lower sign refers to the preceding/following field of view. For the field distortion, a separate set of coefficients is used in each field of view, namely a_{ij}^p and b_{ij}^p in the preceding field, a_{ij}^f and b_{ij}^f in the following. $\Delta \gamma_0$ is the correction to the reference value of the basic angle; $\Delta \gamma_1$ is the chromatic variation of the basic angle. The total number of parameters is 34, of which the first 24 are displayed in Figures 10.8 to 10.13. The cubic and quartic terms were fixed at their values determined in earlier iterations, and the quartic terms are not displayed. In some great-circle reductions, considered too short to give better estimates, some of the lower-order parameters were also fixed (see Section 9.6).

Relations Between the NDAC and FAST Instrument Parameters

As previously mentioned, no strict relation exists between the NDAC and FAST instrument parameters due to the different treatments of the signal harmonics. Disregarding this difficulty, the approximate relations can be established by equating the *G* in Equations 10.9 and 10.10. Using Equation 10.8 and, in the polynomials, the approximations $\bar{w} \simeq -\bar{x}$ and $\bar{z} \simeq \bar{y}$, the relations become:

$$a_{ij}^{p/f} = (-1)^{i+1} (g_{ij} \pm h_{ij}) + \begin{cases} \frac{1}{6} S_0 sq^3 & \text{if } (i, j) = (3, 0) \\ \frac{1}{2} S_0 sq^3 & \text{if } (i, j) = (1, 2) \\ 0 & \text{otherwise} \end{cases}$$
[10.11]

The additional terms for (i, j) = (3, 0) and (1, 2) represent the nominal distortion of the grid and amount to 16.65 mas/hfov³ and 49.96 mas/hfov³, respectively. Relations analogous to Equation 10.11 are found among the chromatic terms. For the zero-order terms the relations $\Delta \gamma_0 = 2h_{00}$ and $\Delta \gamma_1 = 2d_{00}$ are found. Table 10.1 provides the explicit relations for all the parameters displayed in Figures 10.2 to 10.13. Results of an actual comparison for a particular great circle are shown in Figures 9.15 and 9.16.

10.3. Medium-Scale Distortion

The medium-scale distortion, represented in Equation 10.5 by the term $\delta G(\eta, \zeta)$, was in the FAST reductions applied as *a priori* corrections $\delta p = s\delta G$ to the relative modulation phases of the image dissector tube samples for each interlacing period ($T_3 = 0.133...s$), as described in Section 5.4. Thus, this term was in principle eliminated before the great-circle reductions and should not appear in the FAST field-to-grid transformation. The corrections were derived from the laboratory measurements displayed in the lower-left panel of Figure 10.15. The measurements gave a mean displacement for each scan field, thus providing a correction matrix of 168×46 values for the whole main grid.

In the NDAC reductions, the modulation phases were not corrected by the laboratory measurements. Instead, the residuals of the great-circle reductions (one per star and

observation frame of $T_4 = 2.133...$ s) were accumulated in maps of 18×46 areas covering each field of view. The resolution in the scanning direction of 0.9/18 = 0.05 corresponded to the motion of stellar images in half an observational frame and thus provided a sufficient oversampling of the irregularities smeared by the distribution of samples in the observational frame. The binning of residuals in the perpendicular direction coincided with the division into scan fields.

Residual maps were calculated for the preceding and following fields and separately for each great-circle reduction, and later averaged over longer periods of time. Figures 10.14 and 10.15 show the mean residuals from the provisional processing of the first year of satellite data. Each pair of maps represents some three to five months of observations. The pattern of residuals is remarkably stable, and also fairly similar in the two fields of view, indicating that much of the details are due to the irregularities of the grid. This is also supported by a comparison with the laboratory measurements, after convolution with the observational frame and subtraction of a fourth-degree polynomial (Figure 10.15, lower-right panel). Although the overall resemblance to the residual maps is not striking, there are many detailed similarities validating the laboratory measurements. The residual and laboratory maps are also compared, after averaging in the zdirection, in Figure 10.16. Although the laboratory measurements were smeared by the width of the observational frame, their amplitude appears to be substantially larger than the variations of the residuals. This could indicate that an additional smearing mechanism was at work in the great-circle reductions, or that the medium-scale distortions at frame level were underestimated by the mapping process.

After the provisional processing of the first year of data, the resulting mean residual maps were adopted as fixed corrections to the grid coordinates taken as input to the great-circle reductions. These corrections were applied to all the subsequent great-circle reductions in NDAC, including the reprocessing of the first year of data.

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FAST parameters in terms	s of NDAC parameters	Unit	Meaning
$\Delta \gamma_0 = 2h_{00}$		mas	basic angle
$a_{10}^{p} = g_{10} + h_{10}$ $a_{01}^{p} = -g_{01} - h_{01}$	$a_{10}^{\rm f} = g_{10} - h_{10}$ $a_{01}^{\rm f} = -g_{01} + h_{01}$	mas/hfov mas/hfov	scale rotation
$a_{20}^{p} = -g_{20} - h_{20}$ $a_{11}^{p} = g_{11} + h_{11}$ $a_{02}^{p} = -g_{02} - h_{02}$	$a_{20}^{f} = -g_{20} + h_{20}$ $a_{11}^{f} = g_{11} - h_{11}$ $a_{02}^{f} = -g_{02} + h_{02}$	mas/hfov ² mas/hfov ² mas/hfov ²	tilt "
$a_{30}^{p} = g_{30} + h_{30} + 16.65$ $a_{21}^{p} = -g_{21} - h_{21}$ $a_{12}^{p} = g_{12} + h_{12} + 49.96$ $a_{03}^{p} = -g_{03} - h_{03}$	$a_{30}^{f} = g_{30} - h_{30} + 16.65$ $a_{21}^{f} = -g_{21} + h_{21}$ $a_{12}^{f} = g_{12} - h_{12} + 49.96$ $a_{03}^{f} = -g_{03} + h_{03}$	mas/hfov ³ mas/hfov ³ mas/hfov ³ mas/hfov ³	cubic distortion (including nominal) "
$\Delta \gamma_1 = 2d_{00}$ $b_{10}^{\rm p} = c_{10} + d_{10}$ $b_{01}^{\rm p} = -c_{01} - d_{01}$	$b_{10}^{f} = c_{10} - d_{10}$ $b_{01}^{f} = -c_{01} + d_{01}$	mas/mag mas/hfov/mag mas/hfov/mag	chrom. basic angle chrom. scale chrom. rotation

 Table 10.1. Approximate relations between the FAST and NDAC instrument parameters.



Figure 10.2. Evolution of the NDAC instrument parameters h_{00} , g_{10} , h_{10} and g_{01} .



Figure 10.3. Evolution of the NDAC instrument parameters h_{01} , g_{20} , h_{20} and g_{11} .



Figure 10.4. Evolution of the NDAC instrument parameters h_{11} , g_{02} , h_{02} and d_{00} .



Figure 10.5. Evolution of the NDAC instrument parameters c_{10} , d_{10} , c_{01} and d_{01} .



Figure 10.6. Evolution of the NDAC instrument parameters g_{30} , h_{30} , g_{21} and h_{21} .



Figure 10.7. Evolution of the NDAC instrument parameters g_{12} , h_{12} , g_{03} and h_{03} .



Figure 10.8. Evolution of the FAST instrument parameters $\Delta \gamma_0$ [bang], a_{10}^p [x-prec], a_{10}^f [x-foll], a_{01}^p [y-prec].



Figure 10.9. Evolution of the FAST instrument parameters a_{01}^{f} [y-foll], a_{20}^{p} [x^{2} -prec], a_{20}^{f} [x^{2} -foll] and a_{11}^{p} [xy-prec].



Figure 10.10. Evolution of the FAST instrument parameters a_{11}^{f} [xy-foll], a_{02}^{p} [y^{2} -prec], a_{02}^{f} [y^{2} -foll] and $\Delta \gamma_{1}$ [bang(V-I)].



Figure 10.11. Evolution of the FAST instrument parameters b_{10}^p [x(V-I)-prec], b_{10}^f [x(V-I)-foll], b_{01}^p [y(V-I)-prec] and b_{01}^f [y(V-I)-foll].



Figure 10.12. Evolution of the FAST instrument parameters a_{30}^p [x^3 -prec], a_{30}^f [x^3 -foll], a_{21}^p [x^2y -prec] and a_{21}^f [x^2y -foll].



Figure 10.13. Evolution of the FAST instrument parameters a_{12}^p [xy^2 -prec], a_{12}^f [xy^2 -foll], a_{03}^p [y^3 -prec] and a_{03}^f [y^3 -foll].



Figure 10.14. Maps of the mean residuals of the provisional NDAC great-circle reductions for the first year of data. The mean values of $G_{obs} - G_{calc}$ are shown as function of the position in the preceding field of view (PFOV, to the left) and following field of view (FFOV, to the right). The area reproduced corresponds exactly to the area of the main grid $(0^\circ. 9 \times 0^\circ. 9)$. The orientation of the maps is the same as in Figure 10.1, and is such that the rotation of the satellite caused star images to move from left to right, and the image of the Sun would be far up in the maps. The two upper maps are for orbits 1 to 415, the two lower maps are for orbits 416 to 685.



Figure 10.15. The two upper maps show the mean residuals of the provisional NDAC great-circle reductions for orbits 686 to 915 in the preceding and following fields of view. The lower maps show the results of laboratory measurements of the medium-scale distortion of the main grid. To the left the measurements of the individual scan fields are displayed; to the right the same data smoothed, in the scanning direction, by a moving average of 6 arcmin width, corresponding to the motion in one observational frame. The scan field indices ($i_G = 1 \dots 168$, $i_H = 1 \dots 46$) are indicated next to the lower-left map.



Figure 10.16. Comparison of the medium-scale distortion derived from the great-circle reductions (solid curves) and the laboratory measurements (dotted curve). This diagram shows the same data as Figures 10.14 and 10.15, but averaged over the z coordinate or scan field index i_H . The two solid curves are for the preceding and following fields. A moving average of 6 arcmin width was applied to the laboratory data in the scanning direction (w or scan field index i_G) to simulate the smearing effect of the observational frame.

11. SPHERE SOLUTION

The sphere solution combined the star abscissae obtained in the great-circle reductions (Chapter 9) into the positions, parallaxes and proper motions of the stars, expressed in a globally coherent coordinate system. It consisted of two processes: (1) the determination of the abscissa zero points of all the reference great circles, which was the sphere solution proper; and (2) the determination of the astrometric parameters of individual objects. While the first process required a simultaneous least-squares solution of a large number of stars, which must all be consistent with the single-star model, the second process could be performed sequentially using several different models as appropriate for each object. In this chapter the basic observation equation is derived and the numerical methods of solution used by FAST and NDAC are outlined. The final section of the chapter deals with the 'rank deficiency problem' and reports some numerical experiments to study this problem.

11.1. Introduction

The purpose of the sphere solution was to calculate, from the abscissae determined by the great-circle reductions, the astrometric parameters of the stars: both components of position, both components of the proper motion, and the parallax. This chapter provides a general formulation of this process. In practice the successive sphere solutions performed by the FAST and NDAC consortia differed in many details, especially concerning the use of 'global' parameters for the modelling of instrument chromaticity and the harmonic components of the abscissa errors; these detailed aspects as well as the numerical characteristics of the successive solutions are covered in Chapter 16.

In order to take advantage of the symmetry of the nominal scanning law with respect to the ecliptic, all computations in the FAST Consortium were made in ecliptic coordinates. In the NDAC Consortium, equatorial coordinates were used throughout. This difference is immaterial for a general exposition of the sphere solution and largely disappears when vector algebra is used in its formulation. When a reference to the celestial coordinates is nevertheless needed, ecliptic coordinates (λ , β) will be used, and the ecliptic is taken to be the fundamental plane. To obtain the corresponding equations and conventions according to NDAC it is only necessary to substitute (α , δ) and the equator. The generic celestial triad [**x y z**] may thus be taken to mean either the ecliptic or equatorial triad. The transformation between these two systems is completely defined by the value of the obliquity of the ecliptic (ϵ), for which the IAU (1976) value at epoch J2000 was adopted (see Table 12.1). The great-circle reductions determined the one-dimensional coordinates, or abscissae, of the stars along a number of different reference great circles (j). In an absolute sense, the abscissa is defined as the angle v, as seen from the designated pole of the reference great circle, from the ascending node of the reference great circle on the ecliptic to the topocentric coordinate direction of the object (Figure 11.1). It should be noted that the abscissa, being defined in terms of the coordinate direction of the object, is not affected by gravitational light deflection and stellar aberration; these effects, whose computation does not require an accurate astrometric knowledge of the object, were removed in the great-circle reductions.

In principle, therefore, the astrometric parameters of a given star *i* can be computed on the basis of a geometrical model of its motion, using the abscissa values v_{ji} as 'observations'. The only additional data required are the times of observation (t_{ji}) , the corresponding reference great-circle poles (\mathbf{R}_j), and the barycentric locations of the satellite (\mathbf{b}_j). This process, known as the 'determination of astrometric parameters', can clearly be made on a star-by-star basis. However, it requires that the abscissae are actually available in the form described above, i.e. as the absolute angles from the ecliptic to the object, as measured on a number of great circles.

In reality the abscissae obtained in the great-circle reductions do not satisfy this condition. The main problem is the arbitrary origin of the abscissae introduced in each great-circle reduction. This means that the abscissae on a given reference great circle are measured, not from the ecliptic, but from some other, in principle unknown origin. Consequently a set of corrections c_j need to be added in order to convert the abscissae into the absolute quantities required for the determination of astrometric parameters. These corrections can only be determined by simultaneously considering a large number of stars, and explicitly using the circumstance that the same correction applies to *all* the abscissae on the same reference great circle. However, even in this process, the corrections c_j can only be determined in such a way that the corrected abscissae express the angles from a certain fundamental plane, which need not be exactly the ecliptic, nor even fixed with respect to the ecliptic; thus a basic indeterminacy of the celestial reference frame remains after the sphere solution.

The need to determine the abscissa origins is however not the only reason for doing a 'sphere solution' in which the measurements of a large number of stars scattered over the whole sphere are considered in a single solution. There are other, more subtle effects causing systematic shifts in the abscissae which cannot be eliminated in the great-circle reductions, but may be determined in the sphere solution, due to the additional constraints introduced by the stellar astrometric model. These effects include the component of instrument chromaticity that is constant in both fields of view, representing a colour dependence of the zero points c_j . Furthermore, some harmonic components of the abscissa error, notably the sixth harmonic, are more accurately estimated in the sphere solution than in the great-circle reductions. By including such additional unknowns in the sphere solution, their effects on the 'observed' abscissae are eliminated and will not propagate into the subsequent determination of astrometric parameters in the form of colour or position dependent systematic errors.

Since the sphere solution is constrained by the stellar astrometric model describing the coordinate direction in terms of the five astrometric parameters, it is important that this model actually applies to all the stars considered jointly in the sphere solution. Resolved



Figure 11.1. The (nominal) abscissa v is defined as the angle, as seen from the pole **R** of the reference great circle, from the ascending node **P** on the fundamental plane (EE = equator or ecliptic) to the coordinate direction of the object, **u**. The vector triad [**P Q R**] defines the great-circle reference frame.

double stars, astrometric binaries showing curved motion, and other peculiar objects, require more complex models and should therefore not be used in this process.

What is here called the sphere solution can accordingly be divided into two successive processes: (1) the sphere solution proper, which primarily aims at the accurate determination of the abscissa zero point corrections c_j by means of a joint least-squares solution for a carefully selected subset of the Hipparcos stars (known as the 'primary reference stars'); and (2) the application of these corrections to all the abscissae and the subsequent determination of the astrometric parameters on a star-by-star basis—this being no longer restricted to the primary reference stars but applicable to all objects.

These processes are equivalent to the second and third steps of the so-called 'three-step method' outlined in Section 4.1. In the FAST Consortium they were executed as two separate tasks, while in NDAC they were integrated into a single task. One advantage of the FAST approach is that the second task can be made very flexible and include a variety of object models in addition to the standard five-parameter single-star model. In the NDAC Consortium all stars for which the standard model was not adopted were treated by special off-line software, sometimes completely side-stepping the three-step method, as in the case of resolved double and multiple stars (Chapter 13).

In their mathematical formulation the two steps—the sphere solution proper and the determination of astrometric parameters—are intimately connected and it is convenient to present them together. For the sake of brevity, the indices j (for the reference great circles) and i (for the stars) are suppressed where not explicitly needed.

11.2. The Reference Great-Circle Frame

The abscissae and ordinates used in the great-circle reductions are spherical coordinates, analogous to the right ascension and declination, defined with respect to a coordinate triad \mathcal{R} which may be called the reference great-circle frame. Nominally the great-circle frame is uniquely defined by the celestial coordinates (λ_R , β_R) of the reference great-circle pole and the fundamental celestial plane (ecliptic or equator). Formally, it may be represented by the vector triad $\mathcal{R} = [\mathbf{P} \mathbf{Q} \mathbf{R}]$, where:

$$\mathbf{R} = \mathbf{x} \cos \beta_{\mathrm{R}} \cos \lambda_{\mathrm{R}} + \mathbf{y} \cos \beta_{\mathrm{R}} \sin \lambda_{\mathrm{R}} + \mathbf{z} \sin \beta_{\mathrm{R}}$$
$$\mathbf{P} = \langle \mathbf{z} \times \mathbf{R} \rangle$$
$$\mathbf{Q} = \mathbf{R} \times \mathbf{P}$$
[11.1]

The topocentric coordinate direction of the star can be expressed in the great-circle frame as:

$$\mathbf{u} = \mathcal{R} \begin{pmatrix} \cos r \cos v \\ \cos r \sin v \\ \sin r \end{pmatrix}$$
[11.2]

where (v, r) are the abscissa and ordinate of the star (Figure 11.1). The topocentric coordinate direction of a star may be computed from its astrometric parameters as described in Volume 1, Section 1.2.8; given the pole of the reference great circle, the abscissa is then obtained by means of Equations 11.1 and 11.2. It is the purpose of the sphere solution to compare this calculated abscissa with the observed abscissa resulting from the great-circle reduction, in order to improve the astrometric parameters.

11.3. Observation Equation

The observation equation expresses the difference between the observed and calculated abscissa, $\Delta v_{ji} = v_{ji}^{\text{obs}} - v_{ji}^{\text{calc}}$, in terms of the different sources of error. The observation equation is in reality the same for the sphere solution proper and for the determination of the astrometric parameters; the processes differ in how the different terms are treated in the solution of the equations. Presently six kinds of error terms are considered:

- errors in the astrometric parameters;
- orientation errors in the reference great-circle frame;
- other ('local') errors on the great-circle level;
- global errors;
- grid-step errors;
- random noise.

These are discussed in subsequent subsections.

Errors in the Astrometric Parameters

The standard model of stellar motion (Volume 1, Section 1.2.8) gives the topocentric coordinate direction at time *t* as:

$$\mathbf{u} = \langle \mathbf{r}(1+\zeta t) + \mathbf{p}\mu_{\lambda*}t + \mathbf{q}\mu_{\beta}t - \mathbf{b}\pi/A \rangle$$
[11.3]

where:

 \mathbf{r} = the barycentric direction of the star;

 $\mathbf{p} = \langle \mathbf{z} \times \mathbf{r} \rangle$ = the direction of $+\lambda$ at the star;

 $\mathbf{q} = \mathbf{r} \times \mathbf{p}$ = the direction of $+\beta$ at the star;

 $(\mu_{\lambda*}, \mu_{\beta})$ = the components of the proper motion;

$$\pi$$
 = the parallax;

b = the barycentric position of Hipparcos at time *t*;

A = the astronomical unit;

 $\zeta = V_{\rm R}\pi/A$, where $V_{\rm R}$ is the radial velocity of the star.

[**p** q r] is the normal triad at **r** relative to the ecliptic coordinate system. All quantities except **b** refer to the epoch t = 0 (J1991.25). **b**, *A* and $V_{\rm R}$ are regarded as known; other quantities are uniquely defined by the five astrometric parameters λ , β , π , $\mu_{\lambda*}$, μ_{β} , since:

$$\mathbf{r} = \mathbf{x}\cos\beta\cos\lambda + \mathbf{y}\cos\beta\sin\lambda + \mathbf{z}\sin\beta \qquad [11.4]$$

The determination of the astrometric parameters proceeds by successive differential corrections to a set of initial values. To compute the effects of small changes in the astrometric parameters it is then acceptable to ignore ζ and the normalisation brackets in Equation 11.3, yielding:

$$\Delta \mathbf{u} = \mathbf{p}(\Delta \lambda * + t \Delta \mu_{\lambda *}) + \mathbf{q}(\Delta \beta + t \Delta \mu_{\beta}) - \mathbf{b} A^{-1} \Delta \pi$$
[11.5]

On the other hand, Equation 11.2 gives:

$$\Delta \mathbf{u} = \mathbf{m} \Delta \mathbf{v} + \mathbf{n} \Delta \mathbf{r}$$
[11.6]

where $\Delta v = \Delta v \cos r$ and:

$$\mathbf{m} = \langle \mathbf{R} \times \mathbf{u} \rangle, \qquad \mathbf{n} = \mathbf{u} \times \mathbf{m}$$
 [11.7]

are the unit vectors in the directions of +v and +r, respectively. [**m n u**] is the normal triad at **u** relative to \mathcal{R} . Equating $\Delta \mathbf{u}$ in Equations 11.5 and 11.6, and invoking scalar multiplication by **m** and **n**, gives:

$$\Delta v_* = \mathbf{m}' \mathbf{p} (\Delta \lambda_* + t \Delta \mu_{\lambda_*}) + \mathbf{m}' \mathbf{q} (\Delta \beta + t \Delta \mu_{\beta}) - \mathbf{m}' \mathbf{b} A^{-1} \Delta \pi$$
[11.8a]

$$\Delta r = \mathbf{n}' \mathbf{p} (\Delta \lambda * + t \Delta \mu_{\lambda *}) + \mathbf{n}' \mathbf{q} (\Delta \beta + t \Delta \mu_{\beta}) - \mathbf{n}' \mathbf{b} A^{-1} \Delta \pi$$
[11.8b]

Equation 11.8b is not used. After multiplication by $\sec r$, Equation 11.8a gives the relevant terms in the observation equation, or:

$$v^{\rm obs} - v^{\rm calc} = \dots + \mathbf{d}' \Delta \mathbf{a}$$
 [11.9]

where $\Delta \mathbf{a} = (\Delta \lambda^*, \Delta \beta, \Delta \pi, \Delta \mu_{\lambda^*}, \Delta \mu_{\beta})'$ is the column matrix of differential corrections and **d** is the column matrix of dependencies:

$$d_{1} = \mathbf{m}' \mathbf{p} \sec r$$

$$d_{2} = \mathbf{m}' \mathbf{q} \sec r$$

$$d_{3} = \mathbf{m}' \mathbf{b} A^{-1} \sec r$$

$$d_{4} = \mathbf{m}' \mathbf{p} t \sec r$$

$$d_{5} = \mathbf{m}' \mathbf{q} t \sec r$$
[11.10]

Orientation Errors in the Reference Great-Circle Frame

Section 11.2 defined the nominal reference great-circle frame \mathcal{R} , having its pole precisely at the nominal coordinates (λ_{R} , β_{R}) and the abscissa origin (**P**) exactly at the intersection with the ecliptic. Because of the arbitrary abscissa zero point adopted in the great-circle reduction, and because of errors in the attitude angles and stellar coordinates used as input to the great-circle reduction, the object was in reality 'observed' with respect to a slightly different triad $\tilde{\mathcal{R}} = [\tilde{\mathbf{P}} \tilde{\mathbf{Q}} \tilde{\mathbf{R}}]$, which shall be called the actual great-circle frame. The topocentric coordinate direction of the star can be expressed in this frame as:

$$\mathbf{u} = \tilde{\mathcal{R}} \begin{pmatrix} \cos \tilde{r} \cos \tilde{v} \\ \cos \tilde{r} \sin \tilde{v} \\ \sin \tilde{r} \end{pmatrix}$$
[11.11]

where (\tilde{v}, \tilde{r}) are the abscissa and ordinate in the nominal great-circle frame. The direction cosines in Equations 11.2 and 11.11 are related through the matrix equation:

$$\tilde{\mathcal{R}}'\mathbf{u} = (\tilde{\mathcal{R}}'\mathcal{R})\,\mathcal{R}'\mathbf{u} \qquad [11.12]$$

where $\tilde{\mathcal{R}}'\mathcal{R}$ is a 3 × 3 orthogonal matrix.

The relation between the nominal and actual great-circle frames can be represented by a vector $\boldsymbol{\theta}$ (unique for each great-circle reduction) such that a triad initially aligned with \mathcal{R} will become aligned with $\mathcal{\tilde{R}}$ after rotation through the angle $\theta = |\boldsymbol{\theta}|$ about the unit vector $\langle \boldsymbol{\theta} \rangle$. In the small-angle approximation, neglecting terms of order θ^2 , this can be written:

$$\tilde{\mathcal{R}} = \mathcal{R} + \boldsymbol{\theta} \times \mathcal{R}$$
[11.13]

and the transformation matrix in Equation 11.12 becomes:

$$\tilde{\mathcal{R}}'\mathcal{R} = \mathbf{I} + (\boldsymbol{\theta} \times \mathcal{R})'\mathcal{R} = \begin{pmatrix} 1 & \theta_{\mathrm{R}} & -\theta_{\mathrm{Q}} \\ -\theta_{\mathrm{R}} & 1 & \theta_{\mathrm{P}} \\ \theta_{\mathrm{Q}} & -\theta_{\mathrm{P}} & 1 \end{pmatrix}$$
[11.14]

Here, **I** is the 3×3 identity matrix and θ_P , θ_Q , θ_R are the components of $\boldsymbol{\theta}$ in either great-circle frame.

Inserting Equation 11.14 in 11.12 and expanding to first order in the small angles gives:

$$\tilde{v} = v + (\theta_{\rm P} \cos v + \theta_{\rm Q} \sin v) \tan r - \theta_{\rm R}$$
 [11.15a]

$$\tilde{r} = r - \theta_{\rm P} \sin v + \theta_{\rm Q} \cos v \qquad [11.15b]$$

At this point two simplifications are introduced:

- (1) since the ordinate was not estimated in the great-circle reduction, Equation 11.15b need not be considered;
- (2) since $|r| \leq 2$ degrees, due to the limited time interval of the great-circle reduction and the choice of the reference great circle close to the mean scanning plane during that interval, the components θ_P and θ_Q contribute much less than θ_R to the difference between the nominal and actual abscissa in Equation 11.15a, and are ignored.

(1) implies a small loss of information, but involves no approximation compared with Equation 11.15; in contrast, (2) causes an approximation error in the abscissae which could amount to a few milliarcsec (since θ_P and θ_Q may be of the order of the accuracy of the transverse attitude, or 0.1 arcsec). It was assumed that the outer iteration loop

of the main Hipparcos reductions—involving the attitude determination, great-circle reductions, and the sphere solution—eliminates at least the systematic part of these errors. The consequences of this approximation are further discussed in Section 11.7.

As a result of (1) and (2), Equation 11.15 simplifies to $\tilde{v} = v - \theta_R$ and θ_R can be identified with the zero-point correction c_j that must be added to the observed abscissa (in the actual great-circle frame) in order to be compared with the calculated abscissa (in the nominal frame). The corresponding term in the observation equation is, therefore:

$$v^{\text{obs}} - v^{\text{calc}} = \dots - c_j \qquad [11.16]$$

Local Errors on the Great-Circle Level

Apart from the orientation errors of the great-circle frame, the abscissae may be subject to various distortions and systematic displacements, which vary from one great-circle reduction to the next. This kind of 'local' error was not originally foreseen in the Hipparcos data reductions, and are therefore not described in the pre-launch documentation (Perryman *et al.* 1989 Volume III). Experiments with the real data, in particular FAST/NDAC comparisons made at the great-circle level and the analysis of residuals from several provisional sphere solutions, clearly demonstrated that such effects existed. The most important one seemed to be a periodic error in the abscissa, with a period of 60°, and with essentially random amplitudes and phases in the different great-circle reductions. The source of this could simply be the relatively low rigidity of the great-circle reductions to the sixth harmonic of the abscissae, due to the proximity of the basic angle (58°) to the period of that harmonic. The 'local' sixth harmonic may be introduced into the observation equation in the form of the following two terms:

$$v^{\text{obs}} - v^{\text{calc}} = \dots + C_i \cos 6(v - v_{\odot}) + S_i \sin 6(v - v_{\odot})$$
[11.17]

where v_{\odot} is the abscissa of the Sun, which for historical reasons was taken as the origin for the phase of the harmonic errors.

Additional local errors, especially depending on colour, were also detected and taken into account in some of the sphere solutions (see Section 16.3).

Global Errors

Global parameters Γ_k , $k = 1 \dots N_{\Gamma}$ were primarily introduced in order to take into account instrumental effects which could not be resolved at the level of the great-circle reductions. In the various sphere solutions they varied in kind and number, up to $N_{\Gamma} \simeq 20$, as the physical significance and mathematical form of the effects were explored.

By far the most important instrumental effect requiring global treatment was the socalled 'constant chromaticity'. In the Hipparcos nomenclature, this was the average value of the displacement of the image of a star of given colour index with respect to the image of a star of colour B - V = 0.5 mag. The displacement was measured in the direction of scanning, and the average taken over both fields of view. Assuming that the displacement was proportional to the difference in colour index, the relevant term in the observation equation was:

$$v^{\rm obs} - v^{\rm calc} = \dots + (B - V - 0.5)\Gamma_{\rm chrom}$$
 [11.18]

It was however found that the chromaticity varied (linearly) with time, requiring one more global parameter for its representation, and that the variation with colour index was perhaps not linear, requiring yet another parameter. The actual parameters used by FAST and NDAC in their successive sphere solutions are described in Chapter 16.

Another kind of global instrumental effect was foreseen as a consequence of the varying thermal impact on the payload. Under the nominal scanning law the solar illumination varied periodically with the spin phase relative to the Sun, i.e. the heliotropic angle Ω (see Figure 7.3). Consequently it was assumed that systematic thermal variations could be modelled as a periodic function in Ω . Systematic errors in the abscissae caused by such variations must be periodic in $v - v_{\odot}$, if v_{\odot} is the abscissa of the Sun. This reasoning lead to the introduction of global parameters with harmonic coefficients $\cos n(v-v_{\odot})$ ($n = 1 \dots 6$) and $\sin n(v-v_{\odot})$ ($n = 2 \dots 6$). The term containing $\sin(v-v_{\odot})$ was rejected *a priori*, as it would have a very strong correlation with the parallax zero point. Subsequently it was found that none of these global harmonic parameters attained significant amplitudes. They were abandoned in the later NDAC solutions; in the final FAST solution their amplitudes were below 0.01 mas (Table 16.3). The sixth harmonic was however found to be an important local error, i.e. with independent coefficients for each great circle, as discussed in the previous subsection.

Some sphere solutions included a global parameter representing a correction to the general-relativistic value of the gravitational light deflection in the heliocentric metric. This parameter was introduced because Hipparcos offered the first opportunity to measure the deflection accurately, for optical wavelengths, at large angles from the Sun. According to General Relativity, for an object at infinity, the projection of the deflection onto the reference great circle, or the difference in abscissa between the natural direction and the coordinate direction to the star, is given by:

$$\Delta v_{\rm GR} = \frac{2GS}{h_0 c^2} \frac{\mathbf{u}_{\odot}' \langle \mathbf{u} \times \mathbf{R} \rangle}{1 - \mathbf{u}_{\odot}' \mathbf{u}}$$
[11.19]

where *GS* is the heliocentric gravitational constant (Table 12.1), h_0 the distance from the Sun to the observer, and \mathbf{u}_{\odot} the coordinate direction towards the Sun; the latter two are computed from the heliocentric position of the observer, $\mathbf{h}_0 = \mathbf{b}_0 - \mathbf{b}_S$, as $h_0 = |\mathbf{h}_0|$ and $\mathbf{u}_{\odot} = -\langle \mathbf{h}_0 \rangle$. The global parameter may be defined in terms of the PPN parameter γ as $\Gamma_{\text{GR}} = \gamma - 1$, in which case the relevant coefficient in the observation equation is $\Delta v_{\text{GR}}/2$. A slightly different definition was used by NDAC (see Equation 16.10).

Irrespective of the choice and precise definition of global parameters, the corresponding terms in the observation equation can be expressed as:

$$v^{\rm obs} - v^{\rm calc} = \dots + \mathbf{g}' \Gamma$$
 [11.20]

where $\Gamma = (\Gamma_1, ..., \Gamma_{N_{\Gamma}})'$ is the column matrix of global parameters and **g** is the column matrix of coefficients.

Grid-Step Errors

The abscissa resulting from the great-circle reduction was sometimes wrong by a small multiple of the grid step, due to the 360° phase ambiguity of the signal produced by the modulating grid. In the observation equation the presence of grid-step errors is accounted for by the term:

$$v^{\rm obs} - v^{\rm calc} = \dots + ns \tag{[11.21]}$$
where *n* is a small integer (usually n = 0) and s = 1.2074 arcsec is the adopted mean value of the grid step.

Random Noise

The observation equation is completed by adding a noise term η representing the random part of the observational errors resulting from the great-circle reductions. This was assumed to be centred (expected value $E(\eta) = 0$), essentially Gaussian (although outliers were expected and had to be accommodated by the solution method), and of a standard deviation σ_v which was basically known from the great-circle reduction. Furthermore, the noise was assumed to be uncorrelated. This is known to be false, in general, for a pair of abscissae obtained in the same great-circle reduction (see Figures 16.36–16.37), but it is a reasonable assumption for the different abscissae of a given star obtained in different orbits.

The abscissa standard errors, σ_v , were estimated as part of the great-circle reductions. However, it was empirically found that these estimates in general required corrections, either in the form of a multiplicative factor, an added variance, or a combination of both; and which were often found to be functions of magnitude, colour and time. Such corrections were derived from the unit-weight variance of the residuals of the sphere solution, first considered individually by the data reduction consortia (see Sections 11.5 and 11.6), and finally as part of the merging of the consortia results (Chapter 17).

Complete Observation Equation

The abscissa zero point correction c_j and the other (local) errors on the great-circle level (e.g. C_j , S_j) may be brought together in a single unknown column matrix \mathbf{c}_j for each great-circle reduction, containing $1 \le n_c \le 3$ elements. The corresponding coefficient matrix, also of length n_c , is denoted \mathbf{e}_{ji} . In the simplest case of $n_c = 1$, the only element in \mathbf{c}_i is c_j , and the coefficient matrix is $\mathbf{e}_{ji} = (-1)$.

Combining the error terms gives the complete observation equation:

$$\mathbf{d}'_{ji}\Delta\mathbf{a}_i + \mathbf{e}'_{ji}\mathbf{c}_j + \mathbf{g}'_{ji}\Gamma + n_{ji}s + \eta_{ji} = v_{ji}^{\text{obs}} - v_{ji}^{\text{calc}}$$

$$[11.22]$$

where the calculated (nominal) abscissa, obtained through Equations 11.2 and 11.3, is uniquely a function of the time associated with the observation (t_{ji}) , the nominal pole of the reference great circle (\mathbf{R}_{j}), and the assumed astrometric parameters of the star (\mathbf{a}_{i}).

11.4. The Sphere Solution Proper

Primary Reference Stars

The sphere solution proper aims at a direct solution of the system of observation equations, Equation 11.22, the main objective being the estimation of the abscissa zero points (c_j) and the global parameters (Γ) . As already explained, this objective was achieved using only a subset of all the observation equations, corresponding to the 'primary

reference stars'. The selection of primary reference stars was guided by the following considerations.

In the right-hand side of Equation 11.22, all terms are less than a few arcseconds, or $\simeq 10^{-5}$ rad. Linearisation errors were therefore of the order of 10^{-10} rad $\simeq 0.02$ mas, and could be neglected. However, the presence of the grid-step term $n_{ji}s$ still made the system of observation equations highly non-linear and unsuitable for direct solution by standard (least-squares) methods. It was therefore necessary to restrict the sphere solution proper to objects with good *a priori* positions, for which $n_{ji} = 0$ could be assumed with a high degree of confidence.

The standard model of stellar motion, Equation 11.3, is only valid for stars which, from the viewpoint of the Hipparcos observations, could be regarded as point objects with uniform motion. This excludes well-resolved binaries and multiple stars, for which the abscissa derived from the phases of the detector signal is a complicated function of the geometry of the system, the relative intensity of the components, and the direction of scanning. It also excludes close binaries, where the photocentre shows a non-negligible acceleration due to the orbital motion of the system. Known double and multiple stars of such characteristics were therefore excluded *a priori*.

The choice of primary reference stars for the FAST sphere solutions was essentially made *a priori* according to these criteria. It was also attempted to use only photometrically constant stars with good coverage. Within these restrictions it was, furthermore, desirable to have an even distribution over the whole celestial sphere, preferably with at least one primary reference star per square degree. This lead to the use of approximately 72 000 primary reference stars in the final iterations of the sphere solutions. In NDAC, a first choice was made according to the above considerations of duplicity and possible grid-step errors, and further stars were rejected while setting up the observation equations, on the basis of the residuals with respect to the previous iteration of the sphere solution. This resulted in some 50 000 primary stars in the early sphere solutions, increasing to about 78 000 in the final sphere solution.

General Problem

Due to the selection of primary reference stars, the grid-step term can be disregarded for the sphere solution proper. The remaining unknowns fall into three groups depending on their different scope of validity:

for each primary reference star (*i*): $\Delta \mathbf{a}_i$

for each great-circle frame (j): \mathbf{c}_j

for each observation (*ji*): Γ

The structure of the observation equations, and hence the methods of solution, are strongly influenced by this categorisation.

Before solving the equations, it was necessary to equalise their statistical weights. This was done by dividing each equation by $\sigma_{\nu_{ji}}$, the actual standard error of the observation (empirically corrected as described in Section 11.6). The resulting equations can be written in matrix form as:

$$\mathbf{A}\,\Delta\mathbf{a} + \mathbf{C}\,\mathbf{c} + \mathbf{G}\,\Gamma + \boldsymbol{\eta} = \Delta\mathbf{v} \qquad [11.23]$$

where $\Delta \mathbf{a}$, \mathbf{c} and Γ are column matrices with the three kinds of unknowns; they are of length $5N_p$, n_cN_c and N_{Γ} , respectively. **A**, **C** and **G** are the corresponding design matrices, obtained from the submatrices \mathbf{d}'_{ji} , \mathbf{e}'_{ji} and \mathbf{g}'_{ji} in Equation 11.22 after division by $\sigma_{v_{ji}}$. $\Delta \mathbf{v}$ is the column matrix of abscissa differences (observed minus calculated, and normalised to unit weight), and $\boldsymbol{\eta}$ is a column matrix of noise with assumed covariance $E(\boldsymbol{\eta}\boldsymbol{\eta}') = \mathbf{I}$. The number of rows in **A**, **C**, **G**, $\boldsymbol{\eta}$ and $\Delta \mathbf{v}$ is equal to M_p , the number of observations (abscissae) for the primary reference stars.

The general problem of the sphere solution proper was to find the vectors Δa , **c** and Γ which minimised the Euclidean (L2) norm of the residuals, or:

$$\min \|\mathbf{A}\,\Delta\mathbf{a} + \mathbf{C}\,\mathbf{c} + \mathbf{G}\,\Gamma - \Delta\mathbf{v}\|_2 \qquad [11.24]$$

The size of the problem can be appreciated by considering the number of unknowns and equations in the final sphere solutions (Table 11.1). The matrix (**A C G**), known as the design matrix of the least-squares problem, was however very sparse: in each row, only five elements in **A**, one to three elements in **C** and N_{Γ} elements in **G** were, by design, different from zero. The filling factor was, therefore, $(5+n_c+N_{\Gamma})/M_p \simeq 8 \times 10^{-6}$ for solution F37.3 and $\simeq 4 \times 10^{-6}$ for solution N37.5 (see Table 16.1 for details of the sphere solution nomenclature). The structure of the design matrix is illustrated in Figure 11.2.

A feature of the sphere solution problem is that the reference frame for the astrometric parameters and the abscissa zero points remains unspecified by the observations. This should in principle result in a six-fold singularity of the system of observation equations, corresponding to the six degrees of freedom of the reference frame. In reality it was found that Equation 11.23 was not singular; this problem is further discussed in Section 11.7. Nevertheless, in the practical implementation of the sphere solution it was necessary to consider the theoretical rank deficiency especially for the calculation of the variances.

Implementation in FAST

Two basic algorithms were developed in FAST to perform the sphere solution. Before the launch of the satellite, a working solution was tested and fully implemented into an operational software by a team of the University of Bologna (Galligani *et al.* 1989). This software used the iterative algorithm LSQR based on the Lanczos method, which was chosen after various trials and adapted to solve the large-scale system of the sphere solution. It met at that time the stringent requirements set by the computer resources in the mid-eighties. While it gave satisfactory solutions, it had two major drawbacks:

- 1. it was to be used as a 'black box' algorithm and lacked the necessary flexibility required during the processing of real data to cope with new modelling, the need to make an *a priori* selection of observations, and to produce various statistics;
- 2. a reliable estimate of the covariance matrix was very difficult to achieve and depended on the iteration scheme adopted.

To overcome these shortcomings, in particular in view of getting a good estimate of the internal precision of the solution parameters, a second method was developed at CERGA (Frœschlé 1992). This method, based on a block iteration scheme, proved to be very flexible and was easily adapted to a changing environment, as the knowledge of the true properties of the data became more refined with time. The LSQR software was run extensively in parallel during the development phase of this alternative method and helped to speed up the tuning of the new software.



Figure 11.2. Schematic illustration of the structure of the design matrix (**ACG**) for a case with $N_p = 8$ primary reference stars ($5N_p = 40$ astrometric parameters), $N_c = 9$ reference great circles (each with $n_c = 1$ unknown, namely the abscissa zero point), $N_{\Gamma} = 3$ global parameters, and $M_p = 41$ observations (abscissae) referring to the primary reference stars. The black areas are the non-zero elements of the design matrix. In the upper diagram (a) the observations are ordered by the great-circle number (i.e. more-or-less chronologically); in the lower diagram (b) by the star number. Actual numbers M_p , N_p , N_c and N_{Γ} are given in Table 11.1.

Table 11.1. Number of equations and unknowns in the final sphere solutions F37.3 (FAST) and N37.5 (NDAC). Only data corresponding to the primary reference stars are considered. See Chapter 16 for further details on these solutions.

Solution	F37.3	N37.5	
Number of equations, M_p	2 091 926	2 451 483	
Number of unknowns:			
astrometric parameters, $5N_p$	362 455	390565	
great-circle zero points, N_c	2 281	2 3 2 6	
other local parameters, $(n_c - 1)N_c$	4562	-	
global parameters, N_{Γ}	8	3	
total number, $5N_p + n_c N_c + N_{\Gamma}$	369 306	392 894	

In linearised form the observation equations are written:

$$\mathbf{C}\,\delta\mathbf{c} + \mathbf{A}\,\delta\mathbf{a} + \mathbf{G}\,\delta\Gamma + \boldsymbol{\eta} = \delta\mathbf{v} \qquad [11.25]$$

where $\delta \mathbf{c}$, $\delta \mathbf{a}$, $\delta \Gamma$ are differential corrections to the local circle parameters, the astrometric parameters, and the global parameters, respectively. This is the order in which the unknowns are solved by the block iteration scheme; hence the exchange of the first two terms compared to Equation 11.23. The harmonic coefficients C_j and S_j (Equation 11.17) were included among the circle parameters, so the lengths of the correction vectors were $3N_c$, $5N_p$ and N_{Γ} .

The block iteration scheme operated on (parts of) the normal equations obtained by the least-squares method:

$$\begin{pmatrix} \mathbf{C'C} & \mathbf{C'A} & \mathbf{C'G} \\ \mathbf{A'C} & \mathbf{A'A} & \mathbf{A'G} \\ \mathbf{G'C} & \mathbf{G'A} & \mathbf{G'G} \end{pmatrix} \begin{pmatrix} \delta \mathbf{c} \\ \delta \mathbf{a} \\ \delta \Gamma \end{pmatrix} = \begin{pmatrix} \mathbf{C'} \delta \mathbf{v} \\ \mathbf{A'} \delta \mathbf{v} \\ \mathbf{G'} \delta \mathbf{v} \end{pmatrix}$$
[11.26]

where the dimension of the normal matrix is $N \times N$ with $N = 3N_c + 5N_p + N_{\Gamma} \simeq 370\,000$. No direct and general method of resolution could be reasonably envisioned for a system of this size. The way out was to take advantage of the block structure of **A** and of the fact that **C** is a sparse matrix. If there were only the astrometric unknowns the problem would reduce to solving as many 5×5 linear systems as there are stars, which is an easy task. The block decomposition attempts to solve more-or-less independently the unknowns related to the stars and those linked to the more general parameters. This leads to a very natural iterative design, but has the drawback of disregarding the cross-correlations between the astrometry and the general parameters.

In a first approximation one considers that the corrections $\delta \mathbf{a}$ and $\delta \Gamma$ are negligibly small. The harmonic terms $C_j = \mathbf{c}_{j2}$ and $S_j = \mathbf{c}_{j3}$ are also neglected in this approximation. Then the matrix \mathbf{C} is sorted according to the great-circle index and the correction to the abscissa origin $\delta \mathbf{c}_{j1}$ is simply the average of the δv_{ji} for that reference great circle. Denote by \mathcal{I}_j the set of primary reference stars observed with respect to great circle j, and let N_j be the number of such stars. The zero order solution is then given by:

$$\delta \mathbf{c}_{j1}^{(0)} = N_j^{-1} \sum_{i \in \mathcal{I}_j} \delta v_{ji}, \quad \delta \mathbf{c}_{j2}^{(0)} = \mathbf{0}, \quad \delta \mathbf{c}_{j3}^{(0)} = \mathbf{0}, \quad j = 1 \dots N_c$$

$$\delta \mathbf{a}^{(0)} = \mathbf{0}$$

$$\delta \Gamma^{(0)} = \mathbf{0}$$
[11.27]

where the sum is taken over the stars i included in great-circle reduction j. The corrections to the astrometric parameters are then computed, star by star, resulting in the approximation:

$$\delta \mathbf{c}^{(1)} = \delta \mathbf{c}^{(0)}$$

$$\delta \mathbf{a}_{i}^{(1)} = (\mathbf{A}_{i}'\mathbf{A}_{i})^{-1}\mathbf{A}_{i}' \left[\delta \mathbf{v}_{i} - \mathbf{C}_{i} \delta \mathbf{c}^{(0)} \right], \quad i = 1 \dots N_{p} \quad [11.28]$$

$$\delta \mathbf{\Gamma}^{(1)} = \mathbf{0}$$

where A_i and C_i are the blocks of A and C associated with the star *i*, and δv_i is the corresponding observations. Then:

$$\delta \mathbf{c}^{(2)} = \delta \mathbf{c}^{(1)}$$

$$\delta \mathbf{a}^{(2)} = \delta \mathbf{a}^{(1)}$$

$$\delta \Gamma^{(2)} = (\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}' \left[\delta \mathbf{v} - \mathbf{C} \,\delta \mathbf{c}^{(1)} - \mathbf{A} \,\delta \mathbf{a}^{(1)} \right]$$
[11.29]

Equations 11.27–11.29 were iterated until convergence. The other local parameters (C_j and S_j) were introduced from the second iteration. There were two stopping criteria tested at every step: (1) a normalised χ^2 based on the residuals left at every observation, and (2) the variation from one iteration to the next of the corrections to the origins.

The sphere solution in FAST was kept completely free to rotate and no attempt was made to remove the rank deficiency (see Section 11.7 for a discussion). Various experiments were made at intermediate stages to constrain the system by fixing the position and proper motion of ' $1\frac{1}{2}$ star', e.g. the longitude and latitude of one star and the latitude of a second, thus removing the theoretical six degrees of freedom. But the linear system of the sphere solution was in fact not singular but only mildly ill-conditioned and the constraints brought no decisive advantage. In addition the variance-covariance matrix of the astrometric parameters was to be recomputed later with an independent software and it was not a major concern during the sphere solution proper to obtain realistic variances.

Implementation in NDAC

The solution to the general problem of Equation 11.24 was implemented in NDAC by way of the normal equations. Only one local parameter was used for each great circle $(n_c = 1)$, so the complete normal equations matrix system had $5N_p + N_c + N_{\Gamma} \simeq 400\,000$ unknowns. This was reduced to a manageable size of $N_c + N_{\Gamma} \simeq 2300$ by eliminating the astrometric parameters in parallel with the accumulation of the normal equations for the remaining parameters. This required that the observation equations were ordered according to the star numbers as in Figure 11.2b. Since the abscissae were received from the great-circle reductions in the order in which those reductions had been made, a first part of the sphere solution consisted of the sorting of all the abscissa data according to the star numbers.

For the subsequent formulation there is no need to distinguish between the abscissa zero points and the global parameters, as they were treated together as a single column matrix **b** with $N_b = N_c + N_{\Gamma}$ rows. In order to eliminate outliers the calculation of all the unknowns was actually made by a sequence of differential corrections $\delta \mathbf{a}$, $\delta \mathbf{b}$ to the initial values. Introducing the matrix $\mathbf{B} = (\mathbf{C} \mathbf{G})$ of dimension $M_p \times N_b$ the observation equations for the corrections are written:

$$\mathbf{A}\,\delta\mathbf{a} + \mathbf{B}\,\delta\mathbf{b} + \boldsymbol{\eta} = \delta\mathbf{v} \tag{11.30}$$

and the full system of normal equations is:

$$\begin{pmatrix} \mathbf{A}'\mathbf{A} & \mathbf{A}'\mathbf{B} \\ \mathbf{B}'\mathbf{A} & \mathbf{B}'\mathbf{B} \end{pmatrix} \begin{pmatrix} \delta \mathbf{a} \\ \delta \mathbf{b} \end{pmatrix} = \begin{pmatrix} \mathbf{A}' \delta \mathbf{v} \\ \mathbf{B}' \delta \mathbf{v} \end{pmatrix}$$
[11.31]

Elimination of the stellar unknowns, δa , gives the following two systems:

$$\begin{bmatrix} \mathbf{B}'\mathbf{B} - \mathbf{B}'\mathbf{A}(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\mathbf{B} \end{bmatrix} \delta \mathbf{b} = \mathbf{B}'\delta \mathbf{v} - \mathbf{B}'\mathbf{A}(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\delta \mathbf{v} \qquad [11.32a]$$

$$(\mathbf{A}'\mathbf{A})\delta\mathbf{a} = \mathbf{A}'\delta\mathbf{v} - \mathbf{A}'\mathbf{B}\delta\mathbf{b}$$
 [11.32b]

The $5N_p \times 5N_p$ matrix **A**'**A** is block-diagonal, i.e. zero everywhere except for the N_p blocks of size 5×5 along the diagonal. It is therefore a straightforward process to compute the two vectors:

$$\widetilde{\delta \mathbf{a}} = (\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\delta\mathbf{v}$$
[11.33]

and:

$$\widetilde{\delta \mathbf{v}} = \delta \mathbf{v} - \mathbf{A}' \widetilde{\delta \mathbf{a}}$$
[11.34]

whereupon Equation 11.32a can be written as:

$$\left[\mathbf{B}'\mathbf{B} - \mathbf{B}'\mathbf{A}(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\mathbf{B}\right]\delta\mathbf{b} = \mathbf{B}'\widetilde{\delta\mathbf{v}}$$
[11.35]

The symmetric matrix on the left-hand side is of size $N_b \times N_b$ and practically filled, since almost any pair of reference great circles shared at least one primary reference star.

Once the observations had been ordered according to the star numbers, Equation 11.33 was used to compute provisional corrections to the astrometric parameters, after which Equation 11.34 gave the corresponding provisional abscissa residuals. This was done for one star at a time, while sequentially reading the sorted data into computer memory. Concurrently with this process, Equation 11.35 was accumulated. This system was complete when all the stars had been processed, and $\delta \mathbf{b}$ could then be solved by means of the Cholesky algorithm. After updating of the abscissa zero points and global parameters, the process started again with new provisional corrections to the astrometric parameters. This iteration ended when the correction vector $\delta \mathbf{b}$ was negligible: typically the updates to c_j were then less than 10^{-3} mas. At that time the astrometric parameters had also reached their final values, as can be seen by comparing Equations 11.32b and 11.33.

It should be noted that the above process is not an iterative solution of the normal equations (Equation 11.31) but a direct solution through rigorous elimination of $\delta \mathbf{a}$. The iteration scheme was primarily needed to handle outliers among the abscissa data. The 'pre-adjustment' of the astrometric parameters, by means of the provisional updates $\delta \mathbf{\tilde{a}}$, had some additional advantages:

- pre-adjustment was not restricted to the primary reference stars, but was in fact made for as many stars as possible, thus eliminating the need for a separate process for the determination of the astrometric parameters;
- the final decision whether to accept a star as a primary reference star could be made immediately after the pre-adjustment, partly based on an examination of the (provisional) residuals $\delta \mathbf{v}$. In practice only stars with very clean residuals were accepted as primary reference stars, and the corresponding data were then added to the normals for $\delta \mathbf{b}$;
- for non-primary reference stars, the pre-adjustment stage was a convenient place to detect and correct grid-step errors, as described in Section 11.6.

The abscissa residuals after convergence were statistically analysed in a number of ways, in particular as functions of colour, magnitude, and the abscissa difference with respect to the Sun, $v - v_{\odot}$. This revealed a number of systematic patterns, in particular the sixth harmonic in $v - v_{\odot}$, with apparently independent and random coefficients (of a few milliarcsec) in the different great-circle reductions, and the chromatic effects discussed in Section 16.3. These effects were treated in an *ad hoc* manner. For the sixth harmonic and the chromatic variation, the relevant coefficients were determined from the residuals of the penultimate solution (N37.4, see Section 16.3) and subtracted from the right-hand sides of the observation equations of the final sphere solution. In a sense this resembles the block iteration scheme adopted by FAST, but it was only used for those parameters that were not included in the formal observation equations.

The system of normal equations for $\delta \mathbf{b}$ was found to have a condition number $\kappa \simeq 2300$. Thus it could be solved without adding any constraint (such as fixing the position and proper motion for ' $1\frac{1}{2}$ star') with a moderate loss of numerical precision. In fact, most of this loss corresponded to the random selection of one particular solution from the manifold of solutions consistent with the observation equations, and was not accompanied by a corresponding deterioration of the reference frame. It did however result in large formal variances for the abscissa zero points and strong correlations between them, artifacts of the (almost) undefined state of rotation with respect to an external coordinate system. This problem was eliminated by projecting the solution onto the subspace which is complementary to the theoretical null space, and transforming the covariance matrix accordingly. This is practically equivalent to a minimum-norm solution and to using the pseudo-inverse for the covariances.

The minimum-norm solution was implemented as part of the Cholesky algorithm for the solution of Equation 11.35. Let **F** be the (upper-diagonal) Cholesky factor of the normal equations matrix, so that the direct solution is $\delta \mathbf{b} = \mathbf{F}^{-1}(\mathbf{F}^{-1})'\mathbf{B}'\delta\mathbf{\tilde{v}}$ with formal covariance $\mathbf{V} = \mathbf{F}^{-1}(\mathbf{F}^{-1})'$. Furthermore let **N** be an $N_b \times 6$ matrix containing, in the six columns, a set of vectors spanning the theoretical null space. According to Equation 11.54 these are most easily constructed by taking, as the elements in row *j*, the six components of \mathbf{R}_j and $\mathbf{R}_j t_j$, where *j* is the great-circle number. A set of orthonormal vectors $\mathbf{\hat{N}}$ can be computed e.g. by the Modified Gram-Schmidt algorithm. The minimum-norm solution is then obtained by the transformation:

$$\delta \mathbf{\hat{b}} = \delta \mathbf{b} - \mathbf{\hat{N}}' \delta \mathbf{b}$$
 [11.36]

and the covariance of the transformed vector is:

$$\hat{\mathbf{V}} = (\mathbf{I} - \hat{\mathbf{N}}')\mathbf{V}(\mathbf{I} - \hat{\mathbf{N}}) = \left[\mathbf{F}^{-1} - \hat{\mathbf{N}}'\mathbf{F}^{-1}\right] \left[\mathbf{F}^{-1} - \hat{\mathbf{N}}'\mathbf{F}^{-1}\right]'$$
[11.37]

It is seen that the inverted Cholesky factor must simply be transformed exactly like the solution vector, before the covariance matrix is formed. In practice only the diagonal elements of $\hat{\mathbf{V}}$ were computed. The standard errors of the abscissa zero points were typically about 0.1 mas.

11.5. Determination of Astrometric Parameters in NDAC

General Problem

The sphere solution proper determined the abscissa zero points c_j and global parameters Γ by elimination of the astrometric parameters from the basic observation equation (Equation 11.22). Shifting to the right-hand side the terms thus determined gives:

$$\mathbf{d}'_{ji}\Delta\mathbf{a}_i + n_{ji}s + \eta_{ji} = v_{ji}^{\text{obs}} - v_{ji}^{\text{calc}} - \mathbf{e}'_{ji}\mathbf{c}_j - \mathbf{g}'_{ji}\Gamma$$
[11.38]

In contrast to the original system, this can be solved directly for one star at a time, requiring only a very small system of equations to be handled at a time. However, there are still many complications to be considered, in particular the possible grid-step errors $(n_{ji} \neq 0)$, deviations from the standard astrometric model (Equation 11.3) for some stars, and the existence of outliers caused, for example, by the superposition of chance stars in the instantaneous field of view from the other viewing direction.

Implementation in NDAC

In NDAC the determination of the astrometric parameters was integrated with the sphere solution, as described in the previous section, for all stars except those treated by the special double-star process described in Chapter 13. Other cases where the standard stellar model was not applicable, principally the astrometric binaries requiring quadratic, cubic or orbital solutions for the motion of the photocentre, were not systematically investigated but the NDAC data were used for such solutions as part of the merging process (Chapter 17).

An attempt to eliminate grid-step errors was made as soon as more than one observation of the star had been rejected, or if the goodness-of-fit for the star exceeded a given threshold. As a first attempt, the integers n_{ii} were chosen in such a way that:

$$|v_{ji}^{\text{obs}} - v_{ji}^{\text{calc}}| \le s/2$$
 [11.39]

for all the observations of this star. If the residuals were still not acceptable, a systematic search was made to determine the correct set of integers n_{ji} . The initial coordinates of the star were modified in steps of about 0.5 arcsec, new integers determined according to Equation 11.39, and the residuals and goodness-of-fit were again computed. This process was repeated until a satisfactory fit was obtained, or until the modified coordinates were too far away from the initial position. Usually the search was limited to an area of only a few arcsec radius, because of the high risk of finding spurious fits at larger distances.

The complete expression for the variance-covariance matrix associated with the astrometric parameters of star number *i*, obtained from Equation 11.32b, is:

$$\mathbf{V}_i = (\mathbf{A}_i' \mathbf{A}_i)^{-1} + \mathbf{S}_i \mathbf{S}_i'$$
[11.40]

where A_i is the submatrix of **A** referring to the star, and:

$$\mathbf{S}_{i} = (\mathbf{A}_{i}^{\prime}\mathbf{A}_{i})^{-1}\mathbf{A}_{i}\mathbf{B}(\mathbf{F}^{-1} - \hat{\mathbf{N}}\mathbf{F}^{-1})$$
[11.41]

The term $\mathbf{S}_i \mathbf{S}'_i$ was, for practical reasons, neglected. This is tantamount to neglecting the variance contributed by the abscissa zero points through the second term in Equation 11.32b. This was believed to be an acceptable approximation in view of the rather small (~ 0.1 mas) errors on the abscissa zero points, compared to the typical abscissa standard errors (~ 3 mas).

11.6. Determination of Astrometric Parameters in FAST

The sphere solution in FAST was intended to produce the parameters required to define the system, in such a way that every abscissa could be brought into a fully consistent reference frame. The only remaining degrees of freedom were the six parameters needed for the time dependent rotation, to be determined by the link to the extragalactic reference frame. The astrometric parameters resulting from the sphere solution were considered as a by-product of this process and not as final for these stars. In any case, an independent software had to be written for the determination of the astrometric parameters of the non-primary reference stars, which were not part of the sphere solution. This software needed to be more flexible than the corresponding one in the sphere solution in order to handle all the difficult cases, the double and multiple stars, and to cope with the grid-step errors very common for stars with poor initial positions or proper motions. This led, at an early stage of the definition of the FAST organisation, to the identification of the astrometric parameter determination as a task by itself, independent of the sphere solution and to be designed to produce the astrometric solutions for all the stars.

Environment and Main Goals

The sphere solution in the FAST processing ended up with a file containing the corrections to be applied to each origin, one per circle, so that the resulting network of circles determined a consistent reference frame on the sphere. Then all the abscissae, of the primary reference stars as well as all the other stars and solar system objects, were referred to the new origins and corrected for the general parameters. The corrected abscissae for star *i* were:

$$\delta \tilde{\mathbf{v}}_i = \delta \mathbf{v}_i - \mathbf{C}_i \delta \mathbf{c} - \mathbf{G} \,\delta \Gamma \qquad [11.42]$$

In the normal case of a single star following the standard model, the least-squares problem for the determination of the five astrometric parameters was:

$$\min \|\mathbf{A}_i \, \delta \mathbf{a}_i - \delta \tilde{\mathbf{v}}_i \|_2 \qquad [11.43]$$

which is to be solved once for each star. The software for the astrometric parameter determination included a number of tests and specialised algorithms for the weighting of the observations, the recognition of outliers, and the correction of grid-step errors. It also allowed a number of alternative models to be tested in addition to the standard one with only the five astrometric parameters λ , β , π , $\mu_{\lambda*}$, μ_{β} , such as introducing an accelerated motion, or solving for the astrometric parameters of the centre of mass of double stars with known orbits.

For the double and multiple stars the abscissae were corrected for the duplicity effect as explained in Section 13.3, i.e. in such a way that the modified abscissae referred to the primary or to the photocentre of the binary, depending on the separation. The solution for the astrometric parameters of the primary or photocentre then proceeded in the same way as for the single stars.

Weighting Scheme

One of the most important aspects of the least-squares solution for the five astrometric parameters was the scaling of the variances resulting from the great-circle reductions. Each observation equation was initially weighted by $w_{ji} = 1/\sigma_{V_{ji}}^2$, where, as before, *j* stands for the circle and *i* for the star, and $\sigma_{V_{ji}}$ was the standard deviation of the abscissa estimated by the great-circle reduction. In the FAST treatment several changes were brought to these standard deviations in order to scale the observation equations correctly.

For a given weighting scheme, the unit-weight variance for a particular star i was computed as:

$$u_i^2 = \frac{1}{M_i - 5} \sum_{j \in \mathcal{J}_i} w_{ji} (\tilde{v}_{ji}^{\text{obs}} - v_{ji}^{\text{calc}})^2$$
[11.44]

where \mathcal{J}_i is the set of reference great circles in which the star was included and M_i is the number of such circles; \tilde{v}_{ji}^{obs} is the observed abscissa, corrected as in Equation 11.42. The unit-weight variance should follow the distribution of the normalized chi-square variable $\chi^2_{M_i-5}/(M_i-5)$ with unit mean. The sample distribution of u_i^2 was studied for various subsets of single and multiple stars as a function of magnitude and colours and led to a rather complex weighting system with $w_{ji} = 1/\sigma^2_{ji}$ depending on whether the star was single or double. For the stars processed as single, σ_{ji} was computed as:

$$\sigma_{ji} = (0.86 + 0.0084 \, Hp) (\sigma_{v_{ji}}^2 + \sigma_m^2)^{1/2}$$
[11.45]

where the additional standard deviation depending on the magnitude *Hp* was given by:

$$\sigma_m = \begin{cases} 1.561 \ (1+0.0978x+0.0217x^2+0.0048x^3+0.0011x^4) \text{ mas} & \text{if } Hp < 11.5\\ 5 \text{ mas} & \text{otherwise}\\ [11.46] \end{cases}$$

with:

$$\mathbf{x} = 10^{(Hp-8)/5} - 1$$
 [11.47]

This scheme was also used for weighting the equations of the primary reference stars in the sphere solution. For the double stars the corresponding expression was:

$$\sigma_{ji} = (0.86 + 0.028 Hp)(\sigma_{v_{ii}}^2 + \sigma_c^2)^{1/2}$$
[11.48]

where σ_c was the standard deviation of the correction applied to the abscissa in order to move the reference point to the primary (for separations $\rho > 0.35$ arcsec) or to the photocentre (for $\rho < 0.35$ arcsec) of the double star (see Chapter 13).

Filtering of Outliers

There were essentially three modes for selecting or rejecting the observations:

1. the great-circle reduction provided several flags for every star observed in a circle to report on problems with the solution. The flagging was based on the statistical analysis of the residuals and most problems were connected to grid-step inconsistencies in the circle adjustment of the grid-abscissae. Out of nearly 3×10^6 abscissae, this led to the rejection of 22 000 observations, or about 0.7 per cent of the total;

- 2. from the study of the residuals of the abscissae it was possible to locate outliers with residuals larger than three times the standard deviation σ_{ji} . Then a new solution was computed until no more observations were rejected. For any star the fraction of rejected observations was kept below 30 per cent. The most general situation was no rejection at all (90.2 per cent of the stars) or only one rejection (7.6 per cent of the stars); only in 2.2 per cent of the cases were there two or more outliers. On the average there were just above three outliers per great-circle reduction, but this number was subject to considerable variation because the rejections were quite often concentrated on a few bad circles with problems of attitude convergence;
- 3. a manual mode with an *a priori* rejection of great circles based on a look-up table, mainly for the purpose of comparison or to study the influence of a particular configuration. The look-up table was specific for each star to be tested, and the software could be run only for a preselected set of stars.

Correction of Grid-Step Errors

An algorithm to recognise and correct grid-step errors was devised by Bastian (1985). Its implementation worked smoothly, and was of constant use in the preliminary versions of the software. Its efficiency was however limited to circumstances when the proportion of great circles to be corrected was small and $|n_{ji}| \le 2$ or 3. This was clearly unsatisfactory for many double stars and for the few hundred single stars with large errors in the Input Catalogue.

An alternative algorithm was therefore implemented. This searched for solutions at distances as large as 20 arcsec from the reference position. The software was a specialised version of the algorithm used to determine the relative astrometry of double stars in the FAST processing (Chapter 13). Indeed, the double star algorithm is, to an essential part, a robust grid-step error solver. In Equations 13.19–13.21 for the relative astrometry of double stars, the abscissa difference $\delta \tilde{v}_{ji}$ was substituted for the projected phase difference between the secondary and primary components. The solution for the 'double star parameters' $X = \rho \sin \theta$ and $Y = \rho \cos \theta$ then provided the desired update of the reference position.

Adding a parameter for the parallax, straightforward modifications led to a new method for the astrometric parameter determination, which were much less sophisticated than the nominal method, but very useful for producing a solution within a few milliarcsec of the true position, whatever the starting value. All the stars were therefore first solved with this alternative method, and the results then became the starting points for the actual astrometric parameter determination, in which there was no longer any grid-step problem.

Practical Implementation

All the user-defined settings, combined with the possibility of running the program on a star by star basis, enhanced considerably the flexibility of the astrometric parameter software compared to the extreme rigidity of the sphere solution and proved to be decisive in the solution of all the non-trivial cases.

Two versions of a software originally developed at the Astronomisches Rechen-Institut in Heidelberg (Walter *et al.* 1985, Lenhardt *et al.* 1991) were implemented and run at

two places. The evolution of the two versions was not fully parallel and the differences noticed in the results from time to time had to be carefully investigated. Eventually all the stars were processed on a single computer to produce the final FAST solution. Many intermediate cross-checks between CERGA and ARI helped make the final result very reliable. Also during this final step, frequent comparisons were made with the astrometric parameters of the primary reference stars computed during the sphere solution proper, which proved very useful for the understanding of the whole process.

11.7. Rank Deficiency and Convergence Properties

As mentioned in Section 11.1, the zero point corrections c_j were determined in such a way that the corrected abscissae defined a globally consistent reference frame, but the observations themselves did not define any specific reference frame. This means that if a particular solution **a**, **c**, Γ to the least-squares problem of Equation 11.24 was found, then there existed an infinite number of (slightly) different solutions **a** + δ **a**, **c** + δ **c**, Γ + $\delta\Gamma$, for which the norm remained at the minimum. As a consequence the least-squares equations were expected to have a rank deficiency corresponding to the six degrees of freedom of the reference frame (Betti & Sansò 1983).

Contrary to this expectation it was found, already in the early simulations of the Hipparcos data reductions, that the equations for the sphere solution were in fact only weakly ill-conditioned (Lindegren & Söderhjelm 1985). This problem of the (absence of) rank deficiency was discussed at length in the Hipparcos literature (e.g. van Daalen, Bucciarelli & Lattanzi 1986). The conclusion has been that the non-singularity is due to the splitting of the overall problem into different steps, during which different parts of the unknowns of the problem were considered to be fixed. In this section the problem is re-analysed in the framework of the present formulation of the general problem, and the results of numerical experiments towards a more rigorous global solution of the astrometric parameters are described.

Analysis of the Rank Deficiency

For simplicity the global parameters are excluded from the present discussion, as they are not expected to contribute in any significant way to the question of the rank deficiency. Furthermore, only one great-circle parameter was considered, i.e. c_j or θ_{R_j} . (Clearly the addition of more unknowns, such as the global parameters, cannot render the problem less ill-conditioned, and could therefore not be the source of the non-singularity of the actual equations.) The expected rank deficiency would consequently lead to the existence of non-zero vectors $\delta \mathbf{a}$ and $\delta \mathbf{c}$ satisfying:

$$\mathbf{A}\,\delta\mathbf{a} + \mathbf{C}\,\delta\mathbf{c} = \mathbf{0} \tag{11.49}$$

For a particular observation this can be written:

$$\mathbf{d}'\delta\mathbf{a} - \delta\theta_{\mathrm{R}} = 0 \qquad [11.50]$$

where $\delta \mathbf{a}$ now refers to the one star in question.

This manifold of valid solutions to the least-squares problem corresponds to a set *S* of reference frames differing from each other by a time-dependent orientation vector $\boldsymbol{\varepsilon}(t)$. Since the objective is to study the effects of small variations in the unknowns, it will be

assumed that the orientation differences are small; thus, only first-order terms in the small quantities $\boldsymbol{\varepsilon}$, $\delta \mathbf{a}$, and $\delta \boldsymbol{\theta}$ are retained. Only a linear variation of $\boldsymbol{\varepsilon}$ with time can be absorbed by the proper motion components of the astrometric parameters; the time dependence must therefore be of the form:

$$\boldsymbol{\varepsilon}(t) = \boldsymbol{\varepsilon}_0 + \boldsymbol{\omega} t \qquad [11.51]$$

The six degrees of freedom correspond to the components of ε_0 and ω .

Let $[\mathbf{x} \mathbf{y} \mathbf{z}]$ be an arbitrary reference frame in the set *S*. Any other reference frame in *S* can be written as $[\mathbf{x}+\delta \mathbf{x} \ \mathbf{y}+\delta \mathbf{y} \ \mathbf{z}+\delta \mathbf{z}]$, where $\delta \mathbf{x} = \boldsymbol{\epsilon} \times \mathbf{x}$ etc. Since the direction to the star is independent of the reference frame, $\delta \mathbf{r} = \mathbf{0}$ and Equation 11.4 gives:

$$\mathbf{p}\,\delta\lambda * + \mathbf{q}\,\delta\beta = \mathbf{r} \times \boldsymbol{\varepsilon}$$
[11.52]

Inserting this into Equation 11.8a and multiplying by $\sec r$ gives for the first term in Equation 11.50:

$$\mathbf{d}' \delta \mathbf{a} = \mathbf{m}' [\mathbf{r} \times \boldsymbol{\varepsilon}] \sec r = (\mathbf{m} \times \mathbf{r})' \boldsymbol{\varepsilon} \sec r$$
$$\simeq [(\mathbf{R} \times \mathbf{u}) \times \mathbf{u}]' \boldsymbol{\varepsilon} \sec^2 r \qquad [11.53]$$

where, in the last step, Equation 11.7 was used with $|\mathbf{R} \times \mathbf{u}| = \cos r$ and $\mathbf{r} \simeq \mathbf{u}$ to first order in the small angles. For the second term in Equation 11.50 it is noted, from Equation 11.14, that $\theta_{\rm R} = \mathbf{\tilde{P}}'\mathbf{Q}$; thus:

$$\delta \theta_{\rm R} = \mathbf{\tilde{P}}' \, \delta \mathbf{Q} = \mathbf{\tilde{P}}' \, (\boldsymbol{\varepsilon} \times \mathbf{Q}) = (\mathbf{Q} \times \mathbf{\tilde{P}})' \, \boldsymbol{\varepsilon}$$

$$\simeq -\mathbf{R}' \, \boldsymbol{\varepsilon} \qquad [11.54]$$

Here, again, the small-angle approximation was invoked for the last step. It should be noted that (λ_R, β_R) are interpreted as invariants, so that $\delta \mathcal{R} = \boldsymbol{\varepsilon} \times \mathcal{R}$; on the other hand, $\tilde{\mathcal{R}}$ is effectively fixed by the great-circle reductions and therefore unaffected by $\boldsymbol{\varepsilon}$.

Combining Equations 11.53 and 11.54 gives:

$$\mathbf{d}' \delta \mathbf{a} - \delta \theta_{\mathrm{R}} = \left[(\mathbf{R} \times \mathbf{u}) \times \mathbf{u} + \mathbf{R} \cos^2 r \right]' \boldsymbol{\varepsilon} \sec^2 r$$
$$= \left[\mathbf{u} \mathbf{u}' \mathbf{R} - \mathbf{R} \sin^2 r \right]' \boldsymbol{\varepsilon} \sec^2 r$$
$$= (\mathbf{u} - \mathbf{R} \mathbf{R}' \mathbf{u})' \boldsymbol{\varepsilon} \tan r \sec r$$
$$= (\mathbf{m} \times \mathbf{R})' \boldsymbol{\varepsilon} \tan r \qquad [11.55]$$

where the vector triple product $[(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{b}\mathbf{c}'\mathbf{a} - \mathbf{a}\mathbf{b}'\mathbf{c}]$ was applied twice, and $\mathbf{R}'\mathbf{u} = \sin r$ was also used.

It is seen that Equation 11.50 is not strictly satisfied by the variations $\delta \mathbf{a}$, $\delta \theta_{\rm R}$ produced by a small rotation of the reference frame. In Equation 11.55 the right-hand side is of the order of tan *r* times the terms on the left-hand side. The condition number of the observation equations, instead of being infinite, should therefore be of the order of $|\tan r|^{-1} \simeq 10^2$, and the condition number of the normal equations should be $\kappa \simeq 10^4$. This is in fair agreement with what was found in the actual solutions (Section 11.4).

As suggested by previous studies, the reason for the non-singularity can be traced back to the approximation made in connection with Equation 11.15a, namely that the terms containing θ_P and θ_Q were neglected. Since $\delta \theta_P = -\mathbf{P}' \boldsymbol{\epsilon}$ and $\delta \theta_Q = -\mathbf{Q}' \boldsymbol{\epsilon}$, the neglected terms amount to:

$$(\delta\theta_{\rm P}\cos v + \delta\theta_{\rm Q}\sin v)\tan r = -(\mathbf{P}\cos v + \mathbf{Q}\sin v)'\boldsymbol{\varepsilon}\tan r$$
$$= -(\mathbf{m}\times\mathbf{R})'\boldsymbol{\varepsilon}\tan r \qquad [11.56]$$



Figure 11.3. Eigenvalues for two small-scale simulations of the sphere solution, using 20 stars with 60 astrometric unknowns (positions and parallaxes). Open circles: only the abscissa zero point was estimated for each reference great circle. Filled circles: all three orientation parameters in $\boldsymbol{\theta}$ were estimated for each great circle. The expected rank deficiency of three shows up only in the latter case.

exactly cancelling the previously found inequality. The inclusion of the two additional unknowns θ_P and θ_Q for each great-circle frame should therefore in principle provide the expected rank deficiency; in reality it should at least drastically increase the condition number of the design matrix.

Numerical Experiments

One of the first numerical studies of the rank deficiency problem was performed by S. Söderhjelm in 1983. The observations of only 20 stars were simulated, assuming the nominal scanning law but with a 30° field of view. The positions and parallaxes were included as unknowns, together with one (θ_R) or three (θ) orientation parameters for each reference great circle. In this case the orientation parameters, rather than the astrometric parameters, were eliminated from the full normal equations, leading to reduced systems with $3N_p = 60$ unknowns. The eigenvalues of these systems are shown in Figure 11.3. The use of a single orientation parameter per great circle gave a rather well-conditioned system (open circles; condition number $\kappa \simeq 35$) while elimination of all three orientation parameters gave a very distinct jump from the 57th to the 58th ranked eigenvalue (filled circles; condition number $\kappa \simeq 5 \times 10^6$. This latter behaviour was exactly as expected for a well-posed least-squares problem with a rank deficiency of three, considering that single-precision arithmetics (four-byte reals) was used.

The sphere solutions performed by both reduction consortia used the formulation of Sections 11.3 and 11.4, including the approximation leading to the non-singularity of the least-squares problem. In a sense this was tantamount to injecting *a priori* positional information into the observation equations, forcing the poles of the actual great-circle frame to coincide with the nominal poles. As a consequence of this approach, the consistency of the final Hipparcos reference frame could in principle be spoiled by overconstraining (Lattanzi, Bucciarelli & Bernacca 1990). The external iteration



Figure 11.4. Residual systematic differences, estimated by the method of infinitely overlapping circles, between the modified solution (solving also for the longitudes of the reference great-circle poles) and the standard FAST sphere solution. The top panels show differences in ecliptic longitude (solid lines) and latitude (dotted), the bottom panels show the parallax differences.

scheme adopted by the consortia (Section 16.2) was supposed to take care of this problem. However, it was not obvious that this procedure converged to a reference frame completely free of the distortion possibly introduced by the overconstraining; nor was it clear whether the relatively few iterations actually performed were sufficient for convergence.

A study of the convergence properties of the Hipparcos sphere solution, performed by B. Bucciarelli, M.G. Lattanzi and M. Frœschlé, compared the 37-month standard FAST solution before the last iteration with the corresponding results obtained by introducing the poles of the reference great circles as additional unknowns. Because of the low estimability of the adjustment to the latitude of the pole ($\Delta\beta_R$), the actual experiment was carried out with only one additional unknown per great circle, i.e. the adjustment to the longitude of the pole ($\Delta\lambda_R = \mathbf{z}' \boldsymbol{\theta}$). Its coefficient in the modified condition equation reads:

$$e_{\lambda_{\rm R}} = [\sin v \cos(\lambda_{\rm R} - \lambda) + \cos v \sin \beta_{\rm R} \sin(\lambda_{\rm R} - \lambda)] \cos \beta \qquad [11.57]$$

where (λ, β) is the geometric position of the star.

Before comparing the modified solution to the FAST standard solution, a small rotation was applied to bring the former onto the system defined by the standard solution. The estimated rotation parameters were:

$$\varepsilon_x = -0.3499 \pm 0.0002 \text{ mas}$$

 $\varepsilon_y = -0.2445 \pm 0.0002 \text{ mas}$ [11.58]
 $\varepsilon_z = +0.0680 \pm 0.0002 \text{ mas}$

The catalogue-wide rms of the positional differences were $\simeq 0.3$ mas and $\simeq 0.04$ mas in λ before and after the rigid rotation, respectively; analogously, for β the rms differences were $\simeq 0.2$ mas and $\simeq 0.03$ mas.

The method of infinitely overlapping circles (see Section 16.6) was utilised to evaluate residual systematic differences in the astrometric parameters. As both the modified and standard solutions were based on the same subset of 45 035 primary reference stars, the radius of the small circles was increased to $R = 3^{\circ}$. This resulted in an average of 30 stars per circle. Figure 11.4 shows the computed systematic differences in position and parallax as functions of ecliptic longitude and latitude; a similar behaviour was observed for the proper motion differences. The systematic differences, at this resolution, are typically on the level of 0.01 to 0.02 mas. These results show that the two solutions are practically identical and support the conclusion that the external iterative scheme adopted by the consortia has been adequate to completely recover the errors in the *a priori* determined coordinates of the poles of the reference great circles.

Evidently this experiment cannot address possible distortions introduced earlier in the reduction procedure. To this end, a new reduction, which directly solves for the attitude parameters along with the astrometric parameters, would be required.

L. Lindegren, M. Fræschlé, F. Mignard

12. EPHEMERIDES, TIMING, AND CALCULATION OF CELESTIAL DIRECTIONS

The interpretation of the observations in terms of astrometric parameters of stars or the motions of solar system objects required the use of auxiliary information in the form of ephemerides of the satellite and other bodies; the use of a single, uniform time scale that could be related to celestial phenomena; and the use of precise mathematical models for the calculation of celestial directions. This chapter describes the implementation of these utilities by the reduction consortia, and summarises the results of tests to compare the implementations.

12.1. Ephemerides

The ephemerides used in the Hipparcos data reductions describe the time-dependent relative positions of five points in space: the solar system barycentre, the Earth, the Sun, the observer (in this case the Hipparcos satellite) and the observed object, which may be a solar-system object or a star. In the latter case, the astrometric parameters of the star may be regarded as defining the ephemeris of its barycentric motion. The vector relations between the five points are illustrated in Figure 12.1. Each vector is a function of time T, for which the Terrestrial Time (TT) was used throughout; the periodic differences (of up to ± 1.6 mas) with respect to a barycentric coordinate time scale were thus neglected in the calculations. For the computation of occultations, an approximate ephemeris of the Moon was also needed.

Earth and Moon

The ephemeris of the Earth, $\mathbf{b}_{\rm E}(T)$, was supplied by J. Chapront, G. Francou and B. Morando from the Bureau des Longitudes in Paris. It consisted of the ephemeris of the barycentre of the Earth–Moon system with respect to the solar system barycentre on one hand, and of an ephemeris of the Earth relative to the Earth–Moon barycentre on the other. Regarding the former, a compact representation of the position and velocity vectors was derived from the best analytical theories to the required precision of 10 km in position and 0.05 m s⁻¹ in velocity, in the form of a Fourier-Poisson expansion over intervals of 400 days each.

The motion of the Earth with respect to the Earth–Moon barycentre was given by Fourier expansions based on the planetary and lunar theories developed at the Bureau



Figure 12.1. Vector relations between the solar system barycentre (B), the Sun (S), the Earth (E), the observer (O) and the point under observation (P). Barycentric, heliocentric, geocentric and topocentric vectors are denoted **b**, **h**, **g**, and **s**, respectively, with indices as appropriate for the different bodies.

des Longitudes. There were 22 harmonics for the ecliptic X and Y coordinates and their time derivatives, and eight for the ecliptic Z and dZ/dT. An approximate position of the Moon, used to calculate occultations, was derived from the same data. The ecliptic Earth ephemeris was converted to equatorial coordinates using the IAU value for the obliquity of the ecliptic at J2000, see Table 12.1.

A tabulation of the barycentric equatorial position and velocity components of the Earth, as calculated from the subroutines used in the reductions, is given in Table 12.2.

It can be noted that the reduction consortia interpreted this ephemeris as applying to the provisional reference frame of their respective solution, or actually the solution of the previous iteration; no attempt was made to take into account the difference between that frame and the actual reference frame of the ephemeris (nominally the dynamical system for the equinox of J2000). Thus, for the final FAST and NDAC solutions, the ephemeris was assumed to refer to the reference frames of F37.1 and N37.1, respectively. The orientations of these frames relative to the Hipparcos Catalogue (or the ICRS system) are given in Table 16.8.

Sun

The ephemeris of the Sun, $\mathbf{b}_{\rm S}(T)$, was needed for calculations of the gravitational light deflection (Section 12.3) and for eclipse calculations. A simple elliptical motion was assumed by FAST, while NDAC used a polynomial fit to the barycentric ephemeris of the Sun derived by Clemence (1953), but modified to modern values for the masses of Pluto and Saturn (respectively 1/130 000 000 and 1/3498.5 of the solar mass).

Minor Planets and Other Solar System Objects

The observing programme included some 60 minor planets (of which 48 were actually observed), two moons of Saturn and one moon of Jupiter. Geocentric ephemerides for these objects, $\mathbf{g}(T)$, were supplied by the Bureau des Longitudes in Paris. For the minor planets the data (ecliptic longitude and latitude, distance and magnitude) were given in the form of Chebyshev polynomials of order 8; for the moons as trigonometric series for the positions relative to the ephemerides of the parent planets, which were

given as Chebyshev polynomials of order 10. The distances to the moons were taken equal to the distance of the parent planet. The geocentric position vector resulting from these calculations included a correction for planetary aberration. A transformation to equatorial coordinates was made using the standard value for the obliquity of the ecliptic (Table 12.1).

Satellite Ephemeris

The geocentric ephemeris of the Hipparcos satellite, $\mathbf{g}_0(T)$, was supplied by ESOC in the form of auxiliary Keplerian elements (describing a reference orbit) and 10 Chebyshev coefficients for each of the components in position and velocity, describing the deviations from the reference orbit over a given interval of time. Additionally, the full geocentric position and velocity vectors at a given reference time were supplied. Usually two to four intervals of such data were given per orbital period. Accuracies to which the various orbital parameters could be determined are given in Volume 2, Chapter 6.

To compute the position and velocity vectors at an arbitrary instant, the Keplerian elements were used to compute the reference vectors, to which were then added the corrections evaluated from the Chebyshev polynomials. The resulting data referred to the mean equator and equinox of the date. For subsequent combination with the Earth ephemeris the data were transformed to the equinox of J2000 by means of standard formulae for the precession (Table 5.1–5.2 in Murray 1983).

In the NDAC great-circle reductions, the satellite ephemeris was completely expressed as Chebyshev polynomials for the fraction of the orbit during which science data had been successfully obtained. The order of this representation (up to 10) depended on the length of the interval under consideration.

A comparison of the geocentric velocity components was made in 1991 by H. Schrijver using the FAST 'First Look' facility at SRON (Utrecht), and L. Lindegren using results of the NDAC great-circle reduction software. For three different observational frames in orbit 79, separated by 191 min and 1.5 min, the absolute differences in the computed equatorial components were found to be 0.043, 0.030 and 0.031 m s⁻¹, respectively. The maximum difference corresponds to 0.03 mas in stellar aberration. The result of this comparison was thus considered satisfactory.

12.2. Timing of the Observational Data

All observations on board Hipparcos, including the gyro readings, must be put on a single, continuous time scale with a well-defined relationship to the time scales used to describe the celestial phenomena. The requirement on the absolute timing of the data were derived from the most rapid variations in the calculated proper directions. For instance, the maximum acceleration of the satellite (at ~ 10 000 km altitude) was about 1.5 m s⁻², producing a maximum rate of change in stellar aberration of 1.0 mas s⁻¹. Thus, for the calculation of aberration the timing had to be accurate to better than ± 100 ms in order not to introduce an error exceeding 0.1 mas. The most severe requirement stemmed however from the motions of minor planets, where the maximum geocentric angular velocity was approximately 20 mas s⁻¹, leading to a requirement of ± 5 ms or better in the absolute timing.

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The time scale used in all the astrometric reductions was Terrestrial Time, TT (equivalent to the former time scales TDT and ET). This is a continuous time scale with a simple relation to atomic time and suitable for describing celestial phenomena when an absolute accuracy of a few milliseconds is sufficient.

As described in Chapter 8, the timing of the data collected by the Hipparcos satellite was a combination of on-board computer regulation (driven by the on-board clock), and everything that happened between the emission of the data by the satellite and the time-tagging at the ground-station. These latter effects are fully described in Chapter 8; only the on-board timing is considered here, and only to the extent as noticed by the data reduction groups.

The scientific data and all auxiliary data were collected on-board in telemetry formats covering 32/3 s, or five observational frames. Most of the data accumulated did not, however, coincide exactly with the boundaries of the telemetry format: all gyro data (supplied 10 times per format) were shifted by 1.120 s, meaning that the first gyro data in a format referred in time to the last gyro integration interval of the preceding format. Similarly, the main detector (image dissector tube) and star mapper data were shifted, but in the case of the main detector data most of this shift was removed in the handling of the data at ESOC. The only shifts left in the data were those introduced by the on-board computer, which were of the order of 0.2 to 0.5 ms. The attitude reconstruction was represented such that the values for the angles and rates derived for an observational frame referred to the actual mid-time of the observational frame defined by the main detector samples, thus incorporating the 0.5 ms shift of the image dissector tube data.

A detailed comparison of timing calculations was made in 1991 between the Utrecht 'First look' reduction and special calculations by L. Lindegren, using NDAC data. The mid-times of two frames in orbit 429 and 696 were considered, and the calculations included the conversion of the time tag of the telemetry format from UTC to TT, application of the internal delay of the ground station, and of the propagation delay from satellite to antenna. The computed frame mid-times, expressed in TT at the satellite, differed by $-13 \,\mu$ s in one frame and by $+1 \,\mu$ s in the other. A further comparison with the frame mid-times calculated at RGO for the NDAC routine processing gave differences of $+1.3 \,\mu$ s and $-0.7 \,\mu$ s, but in all cases the results were considered satisfactory (see also Section 8.2 for further comments about some uncertainties in the ground-station delay times and the variations in the on-board clock, which were dealt with in different ways by the two consortia).

12.3. Coordinates for Stars and Solar System Objects

Following the terminology of Murray (1983), the proper direction to an object, as measured by a moving observer, is obtained by three successive transformations:

(1) the first is a translation of space-time coordinates from the adopted reference point (at the solar system barycentre) and epoch to the observer at the time of observation. In general this involves the space-time coordinates (in a given metric) of two specific events: the emission of light at the object, and its reception at the observer. For the observation of a solar system object these events are described by the ephemerides of the object and observer. For a stellar observation the transformation corresponds to the application of parallax and proper motion. In either case the result is the 'coordinate

direction' to the object $(\bar{\mathbf{u}})$, expressed in the adopted metric. It should be noted that, in the context of General Relativity, the coordinate direction is a mathematical concept devoid of physical meaning, as it depends completely on the choice of metric. For the Hipparcos reductions, isotropic coordinates were used, usually assuming a spherically symmetric, heliocentric metric; the ephemerides described in the previous section were assumed to be expressed in such coordinates;

(2) the photon track from object to observer, when expressed in isotropic coordinates, is a hyperbola (if only light bending from the Sun is taken into account). The second transformation gives the direction of the photon track, as the light reaches the observer, relative to the 'natural frame' of the observer. This frame is a locally flat (Euclidean) coordinate system at rest with respect to the barycentre. The resulting 'natural direction' to the object ($\hat{\mathbf{u}}$) is what would be measured by a hypothetical stationary observer located at the same position as the real observer at the epoch of observation. The transformation from the coordinate direction to the natural direction corresponds to the application of gravitational light deflection, and again depends on the adopted metric. The resulting direction is however an observable entity, and therefore independent of the metric;

(3) the actual observer is moving relative to the natural frame, and the actually observed direction, or the 'proper direction' (\mathbf{u}) , is obtained by a Lorentz transformation to the co-moving 'proper frame'. This corresponds to the application of stellar aberration.

These transformations are described below in the precise form that was used by NDAC. Equivalent representations were used by FAST and are described by Walter *et al.* (1986). All calculations were carried out on the equatorial direction cosines of the celestial coordinates, i.e. on unit vectors of the form:

$$\mathbf{u} = \begin{pmatrix} \cos \delta \cos \alpha \\ \cos \delta \sin \alpha \\ \sin \delta \end{pmatrix}$$
[12.1]

Isotropic Coordinate Direction

The catalogue data for stars refer to a specific epoch T_0 , equal to J1991.25 for the final catalogue (J1990.0 and other epochs were also used in the reductions). The barycentric coordinate direction to a star at this epoch is:

$$\mathbf{u}_{\rm B}(T_0) = \begin{pmatrix} \cos \delta_0 \cos \alpha_0 \\ \cos \delta_0 \sin \alpha_0 \\ \sin \delta_0 \end{pmatrix}$$
[12.2]

At the time of observation *T*, with the observer at barycentric coordinates $\mathbf{b}_0(T)$, the isotropic coordinate direction to the star is given by:

$$\bar{\mathbf{u}}(T) = \left\langle \mathbf{u}_{\mathrm{B}}(T_{0}) - \pi \mathbf{b}_{0}(T)A^{-1} + (T - T_{0}) \begin{pmatrix} -\sin\alpha_{0} & -\sin\delta_{0}\cos\alpha_{0} \\ \cos\alpha_{0} & -\sin\delta_{0}\sin\alpha_{0} \\ 0 & \cos\delta_{0} \end{pmatrix} \begin{pmatrix} \mu_{\alpha*} \\ \mu_{\delta} \end{pmatrix} \right\rangle$$
[12.3]

where π is the stellar parallax, $\mu_{\alpha*} = \mu_{\alpha} \cos \delta$, μ_{δ} the proper motion components, and A the astronomical unit (Table 12.1). In cases when parallax or proper motion information was not available, as in the initial reductions using the Hipparcos Input Catalogue, these values were assumed zero.

For a solar system object with geocentric ephemeris $\mathbf{g}(T)$, the isotropic coordinate direction is given by:

$$\bar{\mathbf{u}}(T) = \langle \mathbf{g}(T) - \mathbf{g}_0(T) \rangle$$
[12.4]

Since planetary aberration was included in the geocentric ephemerides of solar system objects, the relevant argument of **g** is the time of observation, *T*, and not the time of light emission, $T - |\mathbf{g} - \mathbf{g}_0|c^{-1}$.

Natural Direction

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The transformation from the isotropic coordinate direction to the natural direction took into account the light bending by the Sun and (in NDAC only) by the Earth; for solar system objects, their finite distances from Hipparcos were taken into account. The relevant transformation in the heliocentric isotropic metric is given by Equation 2.5.5 in Murray (1983). Adding an extra term for the deflection by the Earth (which may amount to a few tenths of a milliarcsec) and re-writing for computational efficiency (see Section 12.4) gives the following formula for the natural direction:

$$\hat{\mathbf{u}} = \left\langle \bar{\mathbf{u}} + \mathbf{h}_0 \frac{2GS}{c^2 h_0 (q_{\rm S} h_0 + \bar{\mathbf{u}}' \mathbf{h}_0)} + \mathbf{g}_0 \frac{2GE}{c^2 g_0 (q_{\rm E} g_0 + \bar{\mathbf{u}}' \mathbf{g}_0)} \right\rangle$$
[12.5]

where \mathbf{h}_0 and \mathbf{g}_0 are the heliocentric and geocentric positions of Hipparcos, and h_0 , g_0 the corresponding distances. The heliocentric and geocentric gravitational constants are given by *GS* and *GE* respectively (Table 12.1). The scalars q_S and q_E were set to one for stars, while for objects in the solar system they were computed according to the distance *s* from Hipparcos:

$$q_{\rm S} = |\mathbf{\bar{u}} + \mathbf{h}_0 s^{-1}| + h_0 s^{-1}$$

$$q_{\rm E} = |\mathbf{\bar{u}} + \mathbf{g}_0 s^{-1}| + g_0 s^{-1}$$
[12.6]

Proper Direction

The proper direction to a star or solar system object was obtained by a Lorentz transformation depending on the velocity **V** of the observer; see e.g. Equation 2.5.8 in Murray (1983). The velocity was taken to be the sum of the geocentric velocity, d**g**₀/d*T*, computed from the satellite ephemeris, and the barycentric velocity of the Earth, d**b**_E/d*T*, computed from the Earth ephemeris. With $e = (c^2 - V^2)^{1/2}$ the proper direction can be written (see Section 12.4):

$$\mathbf{u} = \left\langle \hat{\mathbf{u}} + \mathbf{V} \left[1 + \mathbf{V}' \hat{\mathbf{u}} (c+e)^{-1} \right] e^{-1} \right\rangle$$
[12.7]

Comparison of FAST, NDAC, and TDAC Calculations

The participants in this comparison were requested to provide all proper directions computed by them for a few stars in a given orbit, i.e. for all frames in all transits for the main mission reduction, and for all the star mapper transits for the Tycho reduction. The selected orbit was number 79 (9–10 December 1989), and the selected stars were HIP 7680, 7708, 67186, and 67362.

Data were received in February–April 1992 from the NDAC main data reduction (supplied by C. Petersen), the TDAC reduction (supplied by U. Bastian), and the FAST

main reduction at CERGA (supplied by J. Kovalevsky). Also available were the results from the Utrecht 'First Look' reduction (SRON).

The data were put on a common frame numbering system, using the timing data provided by the participants, and the position data were transformed to a common coordinate system. The equatorial or ecliptic system would be natural, but presented a practical problem: in the CERGA and SRON data reductions, only the component parallel to the reference great circle, influencing directly the abscissae, was computed with full precision. For the perpendicular component, the change in position during the transit was not computed, nor the contribution from gravitational deflection. This leads to errors not greater than 10 mas in the ordinate. So a useful comparison was only possible in the spherical coordinates (abscissa, ordinate) defined relative to the reference great circle used by CERGA for this orbit. The proper directions were therefore transformed into this coordinate system.

The Tycho data were only compared with NDAC, and the differences could be expressed directly in the equatorial angles, as both sources provided the proper directions to full accuracy in both coordinates. The TDAC data were given for the star mapper transit times, which fell beyond the interval covered by the main mission data for the same field transit. Linear extrapolation of the main mission data to the star mapper transit times was therefore used to enable comparison with the Tycho data.

Each participant used their own catalogue of astrometric parameters for computing the proper directions. These catalogues were all different, reflecting the various stages of catalogue updating or iteration at the different establishments. The components of these differences in abscissa and ordinate were computed and subtracted from the comparisons. The results are summarised in Table 12.3 for the main mission comparison (CERGA, NDAC, SRON) and in Table 12.4 for the TDAC/NDAC comparison.

The comparison showed good agreement between the proper directions computed by the four participants, with the exception of the known inaccuracy in the transverse great-circle direction for CERGA and SRON. Especially the average NDAC-CERGA differences in abscissa, of the highest relevance for the actual reductions, were gratifyingly small (< 0.05 mas). At the same time, this result validated the computation of the orbital velocity of the satellite, which was responsible for most of the evolution of the apparent positions observed during the great-circle set.

12.4. Formulae for Gravitational Deflection and Aberration

Equations 12.5–12.7 deviate somewhat from the standard formulae given, for example, in the *Explanatory Supplement to the Astronomical Almanac* (Seidelmann 1992). For completeness, they are here derived from the corresponding expressions given in Murray (1983).

Natural Direction (Gravitational Deflection)

Equation 2.5.5 in Murray (1983) gives the natural direction in terms of the isotropic, heliocentric coordinates of the object and observer. With **h** and **h**₀ denoting the heliocentric positions of the object and Hipparcos (at distances *h* and h_0 from the Sun), the

coordinate direction to the object is $\mathbf{\bar{u}} = \mathbf{s}s^{-1}$, where $\mathbf{s} = \mathbf{h} - \mathbf{h}_0$. Murray's Equation 2.5.5 can then be written:

$$\hat{\mathbf{u}} = \bar{\mathbf{u}} + \frac{2GS}{c^2 h_0} (hh_0 + \mathbf{h}' \mathbf{h}_0)^{-1} (\mathbf{h} \times \mathbf{h}_0) \times \bar{\mathbf{u}}$$
[12.8]

Substituting $\mathbf{h} = \mathbf{h}_0 + \mathbf{\bar{u}}s$ gives:

$$\hat{\mathbf{u}} = \bar{\mathbf{u}} + \frac{2GS}{c^2 h_0 (qh_0 + \bar{\mathbf{u}}'\mathbf{h}_0)} (\bar{\mathbf{u}} \times \mathbf{h}_0) \times \bar{\mathbf{u}}$$
[12.9]

where $q = (h + h_0)/s = |\mathbf{\bar{u}} + \mathbf{h}_0 s^{-1}| + h_0 s^{-1}$. The vector $(\mathbf{\bar{u}} \times \mathbf{h}_0) \times \mathbf{\bar{u}} = \mathbf{h}_0 - \mathbf{\bar{u}}\mathbf{\bar{u}}'\mathbf{h}_0$ is the projection of \mathbf{h}_0 in the plane normal to $\mathbf{\bar{u}}$. An equivalent form of Equation 12.9, to first order in the factor $2GS/c^2s$, is therefore:

$$\hat{\mathbf{u}} = \left\langle \, \bar{\mathbf{u}} + \frac{2GS}{c^2 h_0 (qh_0 + \bar{\mathbf{u}}' \mathbf{h}_0)} \mathbf{h}_0 \, \right\rangle \tag{12.10}$$

Equation 12.5 was obtained from this form by adding another term representing the deflection by the Earth.

Proper Direction (Stellar Aberration)

Equation 2.5.8 in Murray (1983) gives the following transformation from the natural direction $\hat{\mathbf{u}}$ to the proper direction \mathbf{u} :

$$\mathbf{u} = \frac{[\mathbf{U} + (\beta - 1)\mathbf{v}\mathbf{v}'](\hat{\mathbf{u}} + c^{-1}\mathbf{V})}{\beta(1 + c^{-1}\hat{\mathbf{u}}'\mathbf{V})}$$
[12.11]

U is the unit tensor, $\beta = (1 - V^2/c^2)^{-1/2}$ and $\mathbf{v} = \langle \mathbf{V} \rangle = \mathbf{V}V^{-1}$, where **V** is the velocity of the observer in the natural frame. The denominator in this equation is just a normalising factor making **u** a unit vector. An equivalent form is therefore:

$$\mathbf{u} = \left\langle \hat{\mathbf{u}} + \frac{1}{c} \mathbf{V} + (\beta - 1) \mathbf{v} \mathbf{v}' \hat{\mathbf{u}} + \frac{\beta - 1}{c} \mathbf{V} \right\rangle$$
$$= \left\langle \hat{\mathbf{u}} + \frac{\beta}{c} \mathbf{V} + (\beta - 1) \mathbf{v} \mathbf{v}' \hat{\mathbf{u}} \right\rangle$$
[12.12]

since $\mathbf{vv'V} = \mathbf{V}$. For numerical accuracy the factor $(\beta - 1)$, which is of order $(V/c)^2$, should be written in a form which avoids taking the difference of two nearly equal numbers. Introducing $e = (c^2 - V^2)^{1/2}$ gives $\beta = c/e$ and:

$$(\beta - 1)\mathbf{v}\mathbf{v}' = \frac{c - e}{e}\mathbf{v}\mathbf{v}' = \frac{c^2 - e^2}{e(c + e)}\mathbf{v}\mathbf{v}' = \frac{V^2}{e(c + e)}\mathbf{v}\mathbf{v}' = \frac{1}{e(c + e)}\mathbf{V}\mathbf{v}'$$
[12.13]

Inserting this into Equation 12.12 gives the aberration formula in the form implemented by NDAC, i.e. Equation 12.7.

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Constant	Meaning	Value	Unit
с	Speed of light	299 792 458	m s ⁻¹
ϵ	Obliquity of ecliptic (J2000.0)	23° 26' 21.448''	
A	Astronomical unit	$1.495978701 \times 10^{11}$	m
GS	Heliocentric gravitational constant	$1.327\ 124\ 38\times 10^{20}$	$m^{3}s^{-2}$
GE	Geocentric gravitational constant	3.986005×10^{14}	$m^3 s^{-2}$

Table 12.1. Fundamental constants used in the coordinate calculations.

Table 12.2. A tabulation of the Earth ephemeris used in the reductions, at intervals of 10 days over the mission. *T* is the time in days from JD 2 440 000.0(TT). *X*, *Y*, *Z* are the barycentric equatorial coordinates of the Earth in km. The time derivatives give the barycentric velocity components in m s⁻¹. The reference system is the mean equator and equinox of J2000.

Т	X	Y	Z	dX/dT	dY/dT	dZ/dT
7800.0	+148 357 230	+17858435	+7740978	-4 340 289	+27 006 475	+11 710 082
7810.0	+142430117	+40807475	+17690769	-9345.037	+25981.084	+11263.454
7820.0	+132286866	+62538103	+27 112 230	-14088.914	$+24\ 202.245$	+10494.289
7830.0	+118180524	+82412426	+35729900	-18490.126	+21676.876	+9398.543
7840.0	+100496735	+99792639	+43264714	-22337.008	+18451.134	+7998.782
7850.0	+79771704	+114141069	+49485893	-25 529.354	+14677.488	+6364.766
7860.0	+56592734	+125009555	$+54\ 198\ 420$	-27987.382	+10405.628	+4510.779
7870.0	+31667938	+132020751	+57237269	-29555.217	+5785.373	+2507.810
7880.0	+5774488	+134962007	+58512945	-30238.496	+1000.475	+434.460
7890.0	-20316030	+133725584	+57976493	-29992.966	-3861.158	-1675.574
7900.0	-45771911	+128328148	+55635417	-28779.737	-8 591.348	-3724.810
7910.0	-69805070	+118962234	+51575137	-26714.560	-13038.323	-5652.977
7920.0	-91692277	+105906361	+45913574	-23811.880	-17104.555	-7417.693
7930.0	-110735936	+89573280	+38831561	-20166.429	-20596.142	-8929.190
7940.0	-126380730	+70499363	+30561995	-15959.997	-23448.356	-10167.146
7950.0	-138 171 168	+49258803	+21351515	-11259.773	-25588.406	-11 095.610
7960.0	-145750953	+26524143	+11494492	-6258.045	-26901.425	-11662.873
7970.0	-148949966	+2998879	+1294552	-1 131.510	-27424.461	-11 891.661
7980.0	-147695299	-20629468	-8951132	+4026.301	-27127.049	-11761.883
7990.0	-142050756	-43636197	-18925742	+8991.844	-26003.389	-11273.844
8000.0	-132245224	-65358483	-28344425	+13651.112	-24 163.193	-10 477.951
8010.0	-118574862	-85186112	-36941795	+17909.061	-21616.311	-9371.706
8020.0	-101472152	-102535520	-44463462	+21576.873	-18458.839	-8003.311
8030.0	-81468107	-116946860	-50712665	+24623.183	-14825.097	-6429.014
8040.0	-59124581	-128026080	-55516337	+26964.488	-10757.019	-4662.900
8050.0	-35109793	-135459596	-58738959	+28494.080	-6420.298	-2784.338
8060.0	-10108138	-139077867	-60308671	+29250.809	-1933.605	-838.943
8070.0	+15194245	-138774851	-60176742	+29170.845	+2632.871	+1142.574
8080.0	+40057155	-134562658	-58350505	+28252.521	+7088.607	+3072.307
8090.0	+63798900	-126581233	-54890675	+26579.069	+11352.343	+4922.386

Table 12.2. (Continued).

Т	X	Y	Ζ	dX/dT	dY/dT	dZ/dT
8100.0	+85 756 907	-115 026 886	-49 880 178	+24 120.588	+15 327.328	+6 646.094
8110.0	+105283302	-100229023	-43464706	+20978.678	+18845.667	+8169.741
8120.0	+121842972	-82602855	-35822695	+17256.585	+21870.223	+9483.374
8130.0	+134939519	-62612062	-27154469	+12980.005	$+24\ 288.787$	+10530.877
8140.0	+144166076	-40837230	-17714259	+8329.699	+25998.369	+11271.291
8150.0	+149256863	-17886770	-7 763 237	+3410.677	+27005.717	+11710.190
8160.0	+150008266	+5595999	+2418607	-1680.048	+27212.701	+11797.965
8170.0	+146368346	+28903249	+12523202	-6725.473	+26611.644	+11537.894
8180.0	+138424723	+51365015	+22262708	-11634.283	+25251.192	+10949.107
8190.0	+126349147	+72304171	+31341146	-16248.782	+23089.754	+10009.933
8200.0	+110489942	+91061509	+39473275	-20373.448	+20225.844	+8769.958
8210.0	+91298683	+107077962	+46418294	-23948.886	+16744.488	+7260.089
8220.0	+69315205	+119830181	+51946583	-26803.691	+12693.260	+5502.232
8230.0	+45223723	+128902071	+55879810	-28826.905	+8257.635	+3581.341
8240.0	+19740740	+134017693	+58098207	-30010.703	+3542.191	+1535.126
8250.0	-6357215	+134979946	+58514398	-30237.037	-1320.044	- 573.178
8260.0	-32237247	+131759277	+57118355	-29525.043	-6111.978	-2648.998
8270.0	-57122429	+124462921	+53954730	-27926.074	-10740.416	-4658.027
8280.0	-80231740	+113296608	+49112356	-25424.842	-15032.619	-6517.873
8290.0	-100840262	+98635933	$+42\ 756\ 484$	-22169.469	-18816.245	-8157.761
8300.0	-118349405	+80940705	+35083668	-18252.767	-22040.940	-9557.849
8310.0	-132214681	+60754173	+26330749	-13765.299	-24556.352	-10646.386
8320.0	-142038310	+38732354	+16783218	-8928.813	-26295.797	-11 401.429
8330.0	-147569835	+15536252	+6725043	-3841.726	-27262.550	-11821.545
8340.0	-148648921	-8128956	-3535581	+1336.569	-27373.298	-11867.371
8350.0	-145298572	-31534580	-13683440	+6392.449	-26681.743	-11 569.403
8360.0	-137657769	-54016616	-23432038	+11251.638	-25230.499	-10939.806
8370.0	-125963812	-74910751	-32490694	+15735.107	-23018.200	-9979.269
8380.0	-110616827	-93616238	-40601171	+19704.413	-20187.243	-8753.903
8390.0	-92073143	-109632912	-47546253	+23118.059	-16796.139	-7281.989
8400.0	-70876147	-122497430	-53123244	+25819.795	-12919.844	-5601.453
8410.0	-47670759	-131871695	-57188355	+27775.122	-8734.996	-3788.468
8420.0	-23098703	-137519036	-59637031	+28968.432	-4300.502	-1863.486
8430.0	+2134509	-139269173	-60395237	+29300.023	+248.515	+107.329
8440.0	+27302867	-137103618	-59457163	+28831.358	$+4\ 752.681$	+2059.887
8450.0	+51726456	-131084090	-56846810	+27564.573	+9156.077	+3971.150
8460.0	+74696530	-121361504	$-52\ 631\ 136$	+25483.543	+13289.575	+5761.081
8470.0	+95566050	-108228680	-46937834	$+22\ 716.625$	+17045.571	+7390.496
8480.0	+113759001	-92029008	-39913267	+19283.827	+20368.964	+8832.359
8490.0	+128719443	-73206186	-31752480	+15267.487	+23095.599	+10012.492
8500.0	+140020910	-52302703	-22689683	+10828.388	$+25\ 186.436$	+10920.891
8510.0	+147320330	-29880070	-12967022	+6012.807	$+26\ 589.561$	+11528.647
8520.0	+150353454	-6588572	-2869148	+992.927	$+27\ 194.239$	+11789.702
8530.0	+149024751	+16892384	+7311664	-4071.455	+27035.105	+11722.852
8540.0	+143324817	+39898177	+17286815	-9100.608	+26077.504	+11305.856
8550.0	+133374933	+61719208	+26747008	-13873.176	$+24\ 308.222$	+10538.945
8560.0	+119462652	+81700782	+35411061	-18265.108	+21831.478	+9466.613
8570.0	+101950256	+99239546	+43015326	-22172.007	+18652.508	+8086.103
8580.0	+81349265	+113759444	+49310129	-25392.139	+14874.203	+6449.372
8590.0	+58284891	+124816410	$+54\ 104\ 944$	-27873.953	+10650.351	+4618.188

Table 1	2.2.	(Continued)
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Т	X	Y	Ζ	dX/dT	dY/dT	dZ/dT
8600.0	+33423912	+132049230	+57240318	-29 520.315	+6040.855	+2 617.950
8610.0	+7547257	+135200297	+58606317	-30223.404	+1239.997	+538.624
8620.0	-18546133	+134184993	+58166798	-30029.030	-3592.284	-1557.825
8630.0	-44068206	+129011773	+55922959	-28888.245	-8352.287	-3622.205
8640.0	-68199485	+119837477	+51945514	-26830.948	-12820.207	-5557.236
8650.0	-90209937	+106974125	+46368576	-23989.873	-16886.615	-7322.524
8660.0	-109432022	+90805123	+39357291	-20386.570	-20439.119	-8862.092
8670.0	-125262447	+71853573	+31141079	-16176.897	-23310.222	-10105.724
8680.0	-137262031	+50724816	+21979952	-11529.203	-25479.876	-11048.582
8690.0	-145080682	+28051311	+12 148 827	-6525.942	-26863.952	-11646.967
8700.0	-148501427	+4550786	+1960401	-1389.945	$-27\ 399.489$	-11879.451
8710.0	-147484435	-19072576	-8282736	+3739.335	-27152.253	-11773.510
8720.0	-142073628	-42131273	-18280348	+8747.065	-26085.895	-11309.053
8730.0	-132469575	-63926276	-27729547	+13416.836	-24251.852	-10515.522
8740.0	-119005090	-83851696	-36 369 503	+17678.118	-21 762.463	-9 436.123
8750.0	-102072865	-101345526	-43953818	+21411.863	-18631.545	-8076.953
8760.0	-82204073	-115903546	-50265733	+24467.439	-15000.327	-6504.836
8770.0	-59986298	-127155058	-55144752	+26844.652	-10982.183	-4761.361
8780.0	-36 041 162	-134784257	-58451718	+28441.480	-6636.487	-2876.672
8790.0	-11 080 616	-138 585 358	-60 100 224	+29 207.017	-2 151.048	- 933.817
8800.0	+14204317	-138485336	-60057112	+29188.408	+2387.717	+1036.167
8810.0	+39110530	-134469949	-58315331	+28319.406	+6881.390	+2983.540
8820.0	+62912359	-126663547	-54931445	+26656.315	+11 144.827	+4831.253
8830.0	+84965033	-115292234	-50000888	+24268.960	+15 125.301	+6559.223
8840.0	+104 634 311	-100 643 397	-43 649 094	+21 147.289	+18 697.280	+8 105.881
8850.0	+121340756	-83141819	-36061639	+17438.267	$+21\ 724.263$	+9418.838
8860.0	+134621727	-63266083	-27443364	+13220.696	$+24\ 183.632$	+10486.522
8870.0	+144054532	-41554021	-18029681	+8558.925	+25947.402	+11249.001
8880.0	+149 343 923	-18644464	-8 097 170	+3654.397	+26 962.757	+11 690.842
8890.0	+150 322 985	+4 825 459	+2079615	-1410.381	+27 235.603	+11809.123
8900.0	+146899575	+28175443	$+12\ 202\ 984$	-6494.247	+26675.305	+11564.719
8910.0	+139 156 166	+50695285	+21967017	-11 391.622	+25330.504	+10983.737
8920.0	+127 289 553	+71 733 218	+31 089 181	-16024.224	+23 239.327	+10075.597
8930.0	+111 598 056	+90631785	+39 282 316	-20 203.361	+20 392.175	+8 841.003
8940.0	+92 553 535	+106/95/27	+46 291 112	-23 //7.067	+16 932.958	+7 342.982
8950.0	+70698914	+119736511	+51 902 120	-26 689.652	+12 932.861	+5606.350
8960.0	+46676994	+129013224	+55 923 587	-28767.722	+8 485.363	+3679.198
8970.0	+21 242 749	+134 330 125	+58 229 732	-29 964.724	+3792.747	+1 645.398
8980.0	-4847726	+135 520 693	+58 / 45 638	-30270.118	-1054.798	- 458.493
8990.0	-30779383	+132513157	+5/441364	-29 596.585	-5 883.382	-2 550.042
9000.0	-55732174	+125 418 207	+54 366 088	-28 025.351	-10 497.032	-4 551.012
9010.0	-78 961 203	+114 456 098	+49 612 556	-25 598.979	-14 815.787	-6424.572
9020.0	-99729713	+99 957 921	+43 326 761	-22 353.763	-18 648.133	-8084.066
9030.0	-11/406885	+82 409 179	+35 / 18 691	-184/2.096	-218/3.158	-9484.184
9040.0	-131 486 498	+02 346 025	+21019140	-14031.872	-24 447.531	-10 599.995
9050.0	-141 533 615	+40 393 426	+17 501 776	-9176.208	-26 230.366	-11 371.529
9060.0	-147 288 604	+17 250 811	+7 467 842	-4120.697	-27 214.353	-11 800.423
9070.0	-148617442	-6 402 958	-2788206	+1051.827	-27 397.918	-11 878.435
9080.0	-145 493 784	-29845973	-12 951 501	+6145.538	-26 / 30.569	-11 588.870
9090.0	-1380/3830	-32370074	-22 /21 100	+10 980.237	-20 309.808	-109/4.590
9100.0	-120 399 628	-133/09/4	-31 823 046	+15 506.337	-23 102.430	-10041.396

Table 12.3. Comparison of proper directions computed by CERGA, NDAC and SRON for the main mission reductions. All comparisons refer to orbit number 79. The third and fourth columns give the number of observational frames compared, and the time span of the comparison in minutes. Subsequent columns give the average, rms and extreme values of the differences in abscissa and ordinate (the rms value being the dispersion about the average difference).

Star	Sources	ources N Span Abscissa differences (ma					(mas)	Ordinate differences (mas)			
(HIP)	compared		(min)	ave	rms	min	max	ave	rms	min	max
	CERGA-SRON	59	277	+0.14	0.069	+0.02	+0.28	-0.12	0.134	-0.31	+0.07
7680	NDAC-CERGA	51	277	+0.04	0.076	-0.10	+0.19	-1.13	0.685	-2.07	+0.76
	SRON-NDAC	51	277	-0.19	0.073	-0.31	-0.11	+1.22	0.641	-0.56	+2.11
	CERGA-SRON	59	277	+0.13	0.067	+0.01	+0.29	-0.07	0.102	-0.20	+0.08
7708	NDAC-CERGA	56	277	+0.04	0.081	-0.12	+0.21	-0.92	0.712	-1.93	+0.83
	SRON-NDAC	56	277	-0.17	0.086	-0.32	-0.07	+0.99	0.675	-0.66	+2.07
	CERGA-SRON	69	384	+0.00	0.057	-0.09	+0.10	-0.16	0.155	-0.42	+0.02
67186	NDAC-CERGA	69	384	+0.03	0.061	-0.07	+0.14	-5.69	0.937	-6.68	-2.29
	SRON-NDAC	78	405	-0.03	0.041	-0.10	+0.05	+5.71	1.066	+1.92	+7.11
	CERGA-SRON	64	384	+0.09	0.088	-0.07	+0.30	-0.07	0.114	-0.23	+0.09
67362	NDAC-CERGA	54	384	+0.01	0.061	-0.10	+0.14	-4.41	0.996	-5.55	-1.06
	SRON-NDAC	63	405	-0.07	0.079	-0.25	+0.04	+4.34	1.117	+1.02	+5.85

Table 12.4. Comparison of proper directions computed by TDAC and NDAC. All comparisons were made for orbit number 79. The third and fourth columns give the number of star mapper transits used in the comparison, and the time span of the comparison in minutes. Subsequent columns give the average, rms and extreme values of the differences in abscissa and ordinate (the rms value being the dispersion about the average difference).

Star (HIP)	Sources compared	N	Span (min)	Absc ave	issa dif rms	fference min	es (mas) max	Ordi ave	nate di rms	fference min	es (mas) max
. ,	I I		· /				-				
7680	TDAC-NDAC	10	256	-0.0	0.07	-0.1	+0.1	-0.0	0.09	-0.1	+0.1
7708	TDAC-NDAC	10	256	-0.0	0.11	-0.1	+0.2	+0.0	0.08	-0.1	+0.1
67186	TDAC-NDAC	12	277	+0.1	0.09	+0.0	+0.2	-0.0	0.08	-0.1	+0.1
67362	TDAC-NDAC	8	149	+0.1	0.07	+0.0	+0.2	-0.1	0.05	-0.1	+0.0

13. DOUBLE AND MULTIPLE STAR TREATMENT

The Hipparcos Catalogue comprises a significant amount of data related specifically to double and multiple star systems, the analysis of which was a major challenge to the data reduction consortia. The processing of these systems differed in many ways from the methods set up for the single stars and in no other place in the data analysis did FAST and NDAC specialise more in their approach to the problem. As a consequence this chapter is arranged in dedicated sections for each consortium. The text aims at providing an overview of the algorithms developed and implemented in the context of double and multiple star analysis, demonstrating the capabilities and the limitations of these procedures, and summarising the main statistics of the solutions.

13.1. Introduction

In the preceding chapters, the observation schemes and data reductions for single Hipparcos stars have been presented. As is well known, however, a majority of all stars are in reality double or multiple. Fortunately (because they constituted a major complication of the data reductions), only a reasonably small subset with rather long periods and not too unequal masses appeared non-single when observed by Hipparcos. Roughly, doubles with a magnitude difference smaller than 4 mag and a separation above 0.1 arcsec were resolved by Hipparcos, that is, separate astrometric parameters could be obtained for each component. Some few, but interesting, short-period ($P \leq 5$ yr) systems were detected at even smaller separation from their moving photocentres. For all these 'effectively non-single' objects, special reduction procedures were used, in parallel with or complementary to the standard reductions.

Although specialised and complex data reductions were needed, the discovery and measurement of double and multiple objects was soon recognised as a major scientific by-product of the Hipparcos project. Apart from the obvious fact that accurate parallaxes for orbital systems provide one of the few direct means of determining stellar masses, there are also very important gains from obtaining a more uniform and complete statistical sampling of the visual binary population close to the Sun. The importance of double-star observations with Hipparcos was realised at an early stage in the planning of the mission (Lindegren 1979), and certain instrument parameters, in particular the period of the main grid, were therefore optimised with the efficient detection of double and multiple star systems in mind. Some 9000 of the 12 000 doubles and multiples in the Hipparcos Catalogue (i.e. the output catalogue) were listed already in the Hipparcos Input Catalogue. Most of them had separations below 10 arcsec, and could be observed with a single pointing of the instantaneous field of view. Some systems with separations above 10 arcsec had two or more individual entries in the Hipparcos Input Catalogue, and the components were then pointed at individually but reduced together after the end of the mission. Apart from 3000 reasonably certain new doubles discovered by Hipparcos, there are also several thousand suspected cases, for which trustworthy elements could not be determined.

13.2. Double Star Detection

As stated above, many of the observable doubles and multiples were known and flagged in the Hipparcos Input Catalogue. For the unknown ones, there were two main discovery modes: firstly, at the individual frame level, i.e. directly from the properties of the signal recorded on the grid; and secondly, in the combined astrometric parameter determination.

In this latter case the image on the focal plane could not be distinguished from that of a point source and the signal was analyzed in the same way as that of a single star, with the basic assumption that the absolute motion was rectilinear. This assumption failed when the object was a binary star with a period comparable to or less than a few times the mission length, in which case the 'photocentre' had a sinusoidal motion on the sky superimposed on the linear motion of the centre of mass. The detection in this instance was of the same nature as that of astrometric binaries on photographic plates. Detection at the individual frame level, which is described in this section, was by far the most important.

Main Criteria

During the standard Hipparcos reductions, a five-parameter Fourier model was fitted to the photon counts from the image dissector tube. This model is valid for single stars as well as for double and multiple objects. For single stars, certain relations exist between the five parameters, which could be calibrated from the vast majority of effectively single objects. For manifestly non-single objects these relations do not generally hold, and this property was used to define statistical criteria for the detection of such objects.

Two equivalent mathematical forms of the five-parameter model were of constant use during the reductions and are given here for the sake of completeness. The first form describes the photon count rate A_k in terms of the parameters β_1 to β_5 (see Table 5.1):

$$A_k = \beta_1 + \beta_2 [\cos(p_k + \beta_3) + \beta_4 \cos 2(p_k + \beta_3) + \beta_5 \sin 2(p_k + \beta_3)]$$
[13.1]

where p_k is a known reference phase for each sample *k*. This is actually the form chosen by NDAC for the representation of the signal. The second form, which is more directly related to the physical model of the signal, reads:

$$A_{k} = I + B + IM\cos(p_{k} + \phi) + IN\cos(2p_{k} + \psi)$$
[13.2]

in which I + B, IM, IN, ϕ and ψ may be regarded as the parameters. The relationship between the two representations is readily obtained as:

$$\beta_{1} = I + B$$

$$\beta_{2} = IM$$

$$\beta_{3} = \phi$$

$$\beta_{4} = \frac{N}{M} \cos(2\phi - \psi)$$

$$\beta_{5} = \frac{N}{M} \sin(2\phi - \psi)$$
[13.3]

These parameters were obtained at a rate of one set of five parameters per observational frame of 32/15 s, for each transit of the object across one of the two fields of view. In the second form, I is the total intensity of the source and B the unmodulated stellar background and dark current. M and N are the modulation coefficients of the first and second harmonic respectively, and ϕ and ψ the corresponding phases. Typically, for a single star, one has $M \simeq 0.72$ and $N \simeq 0.25$ and the two phases are related by $\psi \simeq 2\phi$. The equality would be strict for an instrument without optical distortion and fully achromatic. Any departure from this latter rule due to instrument imperfections was calibrated twice a day as a function of colour and position in the fields, and the phases were corrected accordingly. Thus, for the subsequent analysis it can be assumed that $\psi = 2\phi$ holds for a single star, or for the individual components of a double or multiple star. Similarly, the background count rate B was obtained from the photometric calibrations, so that the stellar intensity I could be regarded as known from the observations.

In this chapter quantities related to the different components of a double or multiple star will be distinguished by indices i = 1, 2, ..., n, where *n* is the number of components. In order to avoid as much as possible the need for double indices, notations in this chapter differ slightly from those used in other chapters, notably Chapter 5. Thus, the modulation coefficients of the first and second harmonics are denoted *M*, *N* (corresponding to M_1, M_2 in Chapter 5), and their phases are also denoted by distinct letters ϕ and ψ (corresponding to g_1 and $g_1 + g_2$ in Chapter 5). Similarly, *I* and *B* are used for the stellar and background count rates (instead of I_s and I_b), and A_k for the total modulated signal (instead of I_k).

The modulated count rate A_k , and consequently the parameters β_1 , β_2 , I and B, are normally measured in counts per sample interval. They were obtained, as described in Chapter 5, by fitting the signal models to the observed photon counts N_k , using a Poissonian model for the latter. The expected number of counts in each sample is $E[N_k] = A_k$. In some places I or B may be expressed in Hz, or counts per second. The two units differ by a factor equal to the sample interval $T_1 = 1/1200$ s. For instance, the NDAC rectified parameters (Section 13.7) are put on a scale where magnitude Hp = 0 corresponds to exactly I = 6200 counts per sample, or I = 7.44 MHz.

As a result of the linearity of the Hipparcos detector, when two or more star images were simultaneously on the sensitive part of the detector their contributions added linearly and the resulting signal had the same form as Equations 13.1 or 2. In the case of a double star the signals of the individual components are written:

$$A_{k,1} = I_1 + I_1 M_1 \cos(p_k + \phi_1) + I_1 N_1 \cos(2p_k + 2\phi_1)$$
[13.4]

$$A_{k,2} = I_2 + I_2 M_2 \cos(p_k + \phi_2) + I_2 N_2 \cos(2p_k + 2\phi_2)$$
[13.5]

and the total, observed signal becomes:

$$A_k = A_{k,1} + A_{k,2} + B$$
 [13.6]

Expanding and comparing with Equation 13.2 leads to the following relations between the observed parameters and the parameters of the stellar components:

$$I = I_1 + I_2$$
 [13.7]

$$IM\cos\phi = I_1 M_1 \cos\phi_1 + I_2 M_2 \cos\phi_2$$
 [13.8]

$$IM\sin\phi = I_1 M_1 \sin\phi_1 + I_2 M_2 \sin\phi_2$$
 [13.9]

$$IN\cos\psi = I_1 N_1 \cos 2\phi_1 + I_2 N_2 \cos 2\phi_2$$
 [13.10]

$$IN\sin\psi = I_1 N_1 \sin 2\phi_1 + I_2 N_2 \sin 2\phi_2$$
 [13.11]

Naturally the modulation coefficients *M* and *N* are no longer equal to their single-star values, and $2\phi - \psi \simeq 0$ no longer holds true for a multiple source.

The recognition that the observed signal departs significantly from the expected signal of a single star is the basis for the double star detection at the frame level, and various algorithms and statistical tests based on this circumstance were implemented by the reduction teams. Because there are five signal parameters, while a single star is fully characterised by two parameters (corresponding to intensity and one-dimensional position), it is possible to build exactly three basically independent detection criteria from a comparison of the observed parameters with the expected single-star values. All three criteria, in slightly different forms, were used by both reduction consortia, and their main properties are explained in the following paragraphs and in Figures 13.1–13.3.

The first two criteria can be described in terms of the signal parameters β_4 and β_5 in Equation 13.1. From Equation 13.3 it is seen that for a single star, after elimination of the calibrated instrumental variations, one should have $\beta_4 = \tilde{N}/\tilde{M}$ and $\beta_5 = 0$, where \tilde{M} ($\simeq 0.72$) and \tilde{N} ($\simeq 0.25$) are the calibrated modulation coefficients for single stars. Both \tilde{M} and \tilde{N} depend significantly on the colour of the star, but by a happy circumstance their ratio, $\tilde{R} = \tilde{N}/\tilde{M}$, is rather insensitive to the colour. Thus the observed parameters β_4 and β_5 can be used, separately or jointly, to detect duplicity. For instance, in NDAC their deviations from the single-star values were combined with their covariances to form a goodness-of-fit statistic, which for true singles should have a χ^2 distribution with two degrees of freedom.

The sensitivity of β_4 and β_5 to different situations can be evaluated analytically. Assuming that the same modulation ratio applies to each component separately, irrespective of their colours (i.e., $N_1/M_1 = N_2/M_2 = \tilde{R}$), it is found that the normalised parameters β_4/\tilde{R} and β_5/\tilde{R} for the total signal depend only on the intensity ratio between the components, $r = I_2/I_1 = 10^{-0.4\Delta m}$ (where $\Delta m = Hp_2 - Hp_1$ is the magnitude difference), and the phase difference between the components, $\Delta \phi = \phi_2 - \phi_1$. From Equations 13.3 and 13.7–13.11 the following expressions are derived:

$$\frac{\beta_4}{\widetilde{R}} = \frac{1 + (r + r^2)(2\cos\Delta\phi + \cos2\Delta\phi) + r^3}{(1 + 2r\cos\Delta\phi + r^2)^{3/2}} \equiv F_c(r, \Delta\phi)$$
[13.12]

$$\frac{\beta_5}{\widetilde{R}} = \frac{(r-r^2)(2\sin\Delta\phi - \sin2\Delta\phi)}{(1+2r\cos\Delta\phi + r^2)^{3/2}} \equiv F_s(r,\Delta\phi)$$
[13.13]

The single star limit, obtained for small *r* (large Δm) or small $\Delta \phi$, is $F_c = 1$ and $F_s = 0$.

Figure 13.1 shows the functions F_c and F_s as functions of the projected separation (or phase difference $\Delta \phi$) and magnitude difference Δm . It can be noted that the determination of the two parameters β_4 and β_5 to some extent complement each other, since F_s is most sensitive to projected separations in intervals where F_c is relatively insensitive. However, neither parameter is sensitive to projected separations close to zero (modulo



Figure 13.1. Variation of the functions F_c and F_s with the projected separation and magnitude difference of a double star. The projected separation is measured here by the phase difference $\Delta \phi = \phi_2 - \phi_1$ between the signals of the individual stellar components; $\Delta \phi$ runs from 0 to 360° over a grid period (s = 1.2074 arcsec), see Equation 13.15. The single star limit is $F_c = 1$ and $F_s = 0$.



Figure 13.2. Variation of the function -2.5 log F_t (equal to the expected value of $Hp_{ac} - Hp_{dc}$) with the projected separation and magnitude difference of a double star. The projected separation is measured by the phase difference $\Delta \phi = \phi_2 - \phi_1$ between the signals of the individual stellar components (see Figure 13.1).

the grid period): in that case the signal cannot be distinguished from that of a single star. The typical measurement noise on β_4/\tilde{R} and β_5/\tilde{R} is of the order of 0.1 to 0.2 for a single transit, which indicates a detection limit of about 0.2 arcsec in projected separation and 2.5 mag in magnitude difference. Thanks to the accumulation of many transits in various geometric orientations over the mission, the actual limits were closer to 0.1 arcsec and 3.5 mag.

A third, independent criterion was constructed from a comparison of β_1 and β_2 . Apart from the background term (which may be subtracted), β_1 represents the total intensity of the object, while β_2 represents the amplitude of the modulated intensity. For single stars the two quantities are proportional, $\beta_2 = \widetilde{M}\beta_1$, and the factor \widetilde{M} could thus be calibrated as functions of time, position in the field of view, and colour index. The normalised ratio, $\beta_2/(\widetilde{M}\beta_1)$ is then close to unity for a single star. For a composite object or an extended source like a minor planet with a sizeable diameter, the degree of modulation invariably decreases, leading to a normalised ratio < 1 (Chapter 15). For historical reasons this was usually expressed in magnitude units as $-2.5 \log[\beta_2/(\widetilde{M}\beta_1)]$, which



Figure 13.3. Observed difference $H_{pac} - H_{pdc}$ between the magnitude scales based on the unmodulated signal β_1 (H_{pdc}) and the modulated signal β_2 (H_{pac}), plotted for the first 20 000 stars as function of the magnitude H_{pdc} . The detected double and multiple stars are those significantly above the line $H_{pac} - H_{pdc} = 0$.

corresponds to the magnitude difference $Hp_{ac} - Hp_{dc}$ of the Hipparcos Photometric Catalogue (see also Chapter 14).

For a double star with intensity ratio *r* and phase difference $\Delta \phi$ the following expression is obtained:

$$\frac{\beta_2}{\widetilde{M}\beta_1} = \frac{(1 + 2r\cos\Delta\phi + r^2)^{1/2}}{1 + r} \equiv F_t(r, \Delta\phi)$$
[13.14]

The quantity $-2.5 \log F_t$ (= E[$Hp_{ac} - Hp_{dc}$]) is shown in Figure 13.2. It can be seen that it is always positive, implying that the testing for duplicity is unilateral. The observed distribution of $Hp_{ac} - Hp_{dc}$ for the first 20 000 Hipparcos entries is shown in Figure 13.3 as a function of the star magnitude. The central and most populated distribution is associated with single stars while the scattered population with $Hp_{ac} - Hp_{dc}$ significantly positive corresponds to the stars detected as non-single by Hipparcos. The width of the negative distribution provided a reliable estimate of the acceptance region for the null hypothesis.

The Hipparcos Double Stars

There were several detection limits which restricted the capability of the instrument and data reductions to recognise that a light source was not point-like. When a star was detected as non-single it was categorised as double or multiple, otherwise it was considered as single within the Hipparcos data analysis, although it may in reality be double.

Regarding the small separations, diffraction sets the limit at $\rho > 0.10$ to 0.12 arcsec. Double stars with smaller separation went unnoticed by Hipparcos and were processed in the same way as the single stars without impairing the astrometric solution, except for short-period astrometric binaries. The separation limit depends however also on the magnitude difference of the components: detection close to the diffraction limit was only achieved for small magnitude differences (see Equations 13.40–13.41).


Figure 13.4. Normalised attenuation profile of the instantaneous field of view, $\Psi(\varrho)$, as function of the distance ϱ from the centre of the instantaneous field of view. The attenuation is expressed in magnitudes. Inset: close-up of the upper left region.

As for the large separations, only companions within some 20 to 30 arcsec from the primary could be detected, depending on the magnitude difference. This instrumental limitation followed from the use of an image dissector tube with a sensitive 'instantaneous field of view' having a nominal diameter of 38 arcsec. Actually, the sensitivity decreases gradually from the centre of the instantaneous field of view, causing an attenuation of stellar images not properly centred. For instance, a star 20 arcsec off the centre of the instantaneous field of view would appear about 1.5 mag too faint. A plot of the attenuation profile $\Psi(\varrho)$, as adopted in the data reductions and expressed in magnitudes, is shown in Figure 13.4. If the detector was pointed at the primary of a double star system, the secondary (at separation ϱ) looked fainter by $\Psi(\varrho)$ magnitudes, and the probability of its detection became correspondingly smaller. For well-resolved binaries the detection limit in magnitude difference is about $\Delta m \simeq 4$ mag, including the attenuation effect. Pairs with $\varrho \geq 25$ arcsec thus generated signals hardly discernible from those of single stars, and were consequently treated as single.

Both reduction teams followed the principles laid out above, however with significant differences in the implementation depending on the local organisation of the data flow. The details of the calibrations were defined independently in each consortium and eventually resulted in different detection levels according to separation and magnitude difference. It must be stressed that the number of accepted or suspected double stars depends strongly on the thresholds set in the various tests. Depending on the adopted criteria, anywhere between 10 000 and 20 000 objects could have been accepted as non-single. At the end a fairly conservative policy was adopted: only double stars detected by both consortia and with astrometric and photometric solutions in good agreement were categorised as reliable solutions; other cases were flagged as less reliable solutions, or simply as 'suspected non-singles' without providing a double-star solution. A summary of the number of solved or detected non-single stars is given in Table 13.1. Of the 18 800 entries solved as non-single by at least one of the consortia, only about 13 200 were finally included in Part C of the Double and Multiple Systems Annex. All the remaining entries (some 8150) either had other kinds of double-star solutions (Part G,

		FAST			
		Solved	Detected	Undetected	Total
NDAC	Solved	12710	1030	2250	15990
	Detected	500	210	2190	2900
	Undetected	2310	160	-	2470
	Total	15520	1400	4440	21360

Table 13.1. Number of entries solved as double stars by each consortium, and the number of unsolved but possibly double systems, divided into detected cases and those only detected by the other consortium.

O or V) or were flagged as 'suspected non-singles' in the main Hipparcos Catalogue (Field H61).

13.3. The Astrometric and Photometric Solution: FAST Method

In the FAST processing of double and multiple stars, a relative astrometric and photometric solution was first built. This was then used as an input to the main algorithm of the absolute astrometry, i.e. the same algorithm as used for the single stars. The required input consisted mainly of the corrections to be applied to the great-circle abscissae as if the point observed on the grid had been the primary star or, for a close pair, the photocentre. This section therefore describes separately the processing for the relative astrometry, the relative photometry, and the absolute astrometry.

The FAST method contrasts with the NDAC method where the double and multiple stars were processed in a chain fully independent of the single star processing, by fitting the absolute astrometry and photometry directly to the parameters derived from Equation 13.2.

The Relative Astrometry of Double Stars

From the fitted five-parameter model, Equation 13.2, it was possible to solve, in each field transit, for the projected difference in phase between the secondary and primary component, $\Delta \phi = \phi_2 - \phi_1$. The geometry of the problem is sketched in Figure 13.5. In Equations 13.8–13.11 the left-hand sides were known from the observations at the frame level and were calibrated to account for the inhomogeneity of the sensitive surface. The right-hand sides comprise four unknowns: the two intensities and the two phases. The ratio I_2/I_1 is directly related to the magnitude difference, and the phase difference $\phi_2 - \phi_1$ is linked to the projected separation in the scanning direction, $\Delta G = \rho \cos(\gamma - \theta)$, according to:

$$\Delta \phi = 2\pi \operatorname{mod}(\Delta G/s, 1)$$
[13.15]

where s = 1.2074 arcsec is the angular size of the grid period (or grid step).

For every transit Equations 13.8–13.11 were solved for the intensity ratio and the phase difference $\Delta \phi$ by a numerical method based on an adapted Newton-Raphson algorithm for non-linear systems. To avoid statistical bias, several transits were combined together,



Figure 13.5. Geometry of the observation of a binary star with separation ϱ and position angle θ as seen from the outside the celestial sphere. The angular coordinate along the scanning circle is denoted *G*. The signal observed depends only on the projected phase difference $\Delta \phi$ between the phase of the secondary (at G_s) and that of the primary (at G_p). The orientation of the scanning circle is indicated by γ .

provided the geometry (angle γ in Figure 13.5) did not change too much over the timespan involved in this combination. The number of transits combined depended also on the separation of the double star. For wide pairs, the combination was always limited to the two consecutive transits, respectively in the preceding and following fields. For a separation of the order of 0.3 arcsec the precision of the phase difference in this step was in the range of 10 to 50 mas. It was also noted that for small projected separations, it happened that the sign of the phase difference remained largely undetermined, while its magnitude was correct. This is equivalent to saying that in such a configuration the position angle of the binary had a 180° ambiguity. In some sequences of observations, when the orientation of the binary on the sky remained more or less parallel to the grid slits for many transits, the projected separation could remain very small. Consequently there was a risk of having inconsistencies in the set of phase differences. A special treatment was implemented in the final step of the relative astrometry solution to identify these transits and reverse the sign of the phase differences $\Delta \phi$.

The relationship, Equation 13.15, between the projected phase difference $(\Delta \phi)$ and the projected angular separation (ΔG) indicates that the detector signal only provides information on ΔG to within an integral number of grid periods and that the possible ambiguity for separations larger than the grid period had to be resolved at a later stage of the processing.

At the end of the above steps all the projected phase differences, $\Delta \phi_j$ (j = 1, ..., m), were known together with the orientations γ_j of the scans on which they were obtained. The final step consisted of finding the relative astrometry, ρ and θ , that accounted for the observed $\Delta \phi_j$. In the *j*th observation the projected separation on the Hipparcos grid was:

$$\Delta G_i = \rho \cos(\gamma_i - \theta)$$
 [13.16]

In terms of the rectangular coordinates $X = \rho \sin \theta$ and $Y = \rho \cos \theta$ of the secondary with respect to the primary, the projected separation is also written:

$$\Delta G_j = X \sin \gamma_j + Y \cos \gamma_j \qquad [13.17]$$

Subsequently it is convenient to use, instead of the angles γ_j , the components of the main spatial frequency of the grid along the *X* and *Y* axes:

$$\alpha_j = (2\pi/s) \sin \gamma_j$$

$$\beta_j = (2\pi/s) \cos \gamma_j$$
[13.18]

From Equations 13.15 and 13.17 the set of *m* observation equations can now be written:

$$\Delta \phi_j = \operatorname{mod}(\alpha_j X + \beta_j Y, 2\pi)$$
[13.19]

in which the unknowns are the relative coordinates of the binary (*X* and *Y*), and the observational data are the phase differences $\Delta \phi_i$ derived from the signal parameters.

A special minimisation procedure was developed to find the values of X and Y which made the norm of the vector of the residuals as small as possible. For this purpose it is easier to use the following two equations, which taken together are equivalent to Equation 13.19:

$$\cos \Delta \phi_j = \cos(\alpha_j X + \beta_j Y)$$

$$\sin \Delta \phi_j = \sin(\alpha_j X + \beta_j Y)$$
[13.20]

This formulation avoids the use of the discontinuous modulus function, which is awkward in minimisation algorithms, and allowed the grid-step factors to be automatically managed, without trial and error. A maximum likelihood estimator, based on an assumed von Mises distribution of the angular quantities $\Delta \phi_j$, led to searching the minimum in the (*X*, *Y*) space of the function:

$$U(X, Y) = -\frac{\sum_{j=1}^{m} \kappa_j \cos(\alpha_j X + \beta_j Y - \Delta \phi_j)}{\sum_{j=1}^{m} \kappa_j}$$
[13.21]

which would have been exactly -1 for a perfect fit. The weight of each observation was $\kappa_j \simeq 1/\sigma_j^2$.

The problem was thus reduced to a classical one of numerical analysis, but with some very specific peculiarities:

- U(X, Y) has many minima in the *X*, *Y* plane, more or less evenly distributed on a lattice with period of the order of the grid step in each direction;
- because of the noise and the possible outliers among the $\Delta \phi_j$ it happened that the deepest minimum was not necessarily the point closest to the true parameters of the binary stars;
- for large separations it proved quite important to have a starting point in order to limit the exploration of the *X*, *Y* domain to a region of a few arcsec around this point. This saved computing resources and limited the risk of finding a false minimum far away from the true solution;
- because of the sign indeterminacy of the $\Delta \phi_j$ for the small separations and the small projected phase differences, the search was always done first for a solution with a 180° ambiguity for the orientation of the binary, that is to say without distinguishing between the double stars of relative coordinates *X*, *Y* and -X, -Y. This was done by minimising the function:

$$V(X, Y) = \frac{1}{n} \sum_{j=1}^{m} \left[\frac{\cos(\alpha_j X + \beta_j Y) - \cos \Delta \phi_j}{\sigma_j} \right]^2$$
[13.22]

which has the desired symmetry, V(-X, -Y) = V(X, Y). Then the phase signs were adjusted by looking at the residuals on every transit for both solutions. Usually

one was much better than the other and the phase $\Delta \phi_j$ was changed to $-\Delta \phi_j$ when necessary;

- the second-order expansion of U(X, Y) in the vicinity of the minimum yielded the covariance matrix of the parameters, and the value of U(X, Y) at the minimum was directly related to the unit weight variance with the quadratic mean of the residuals equal to $2(1 + U_{\min})$;
- for stars with significant orbital motion a linear model was introduced, with relative coordinates expressed as $X + \dot{X}t$ and $Y + \dot{Y}t$; the minimum was searched in the four-dimensional space (X, Y, \dot{X}, \dot{Y}) .

More details about the actual implementation and the numerous settings of the software can be found in Mignard (1992).

As a result of the grid-step ambiguity, several solutions were often found which fitted the observations equally well, with no real possibility of discriminating between them. In such cases, all the acceptable solutions were saved in a special file and examined individually, using several indicators supplied by the minimisation routine to help choose one solution. Ground-based measurements of the separations, when available, were also used to resolve ambiguities. For this purpose a data base was set up at the Observatory of Torino to collect the most up-to-date information on double and multiple systems. In addition, all publications of recent speckle observations were cross-checked with the Hipparcos Input Catalogue to determine the relative astrometry of a number of close binaries at the mid-epoch of the space observations.

It must be stressed however, that for the unambiguous doubles, i.e. for systems with magnitude difference $\Delta m \leq 2.5$ mag and separation $\varrho \geq 0.2$ arcsec, the Hipparcos determination of the relative astrometry is usually very reliable, even without good *a priori* starting values. For those unambiguous cases, the main advantage of having ground-based separation values, was to make the search for a solution faster, without influencing the solution itself. For separations larger than a few grid steps, a blind search, while feasible, would have been too costly and unsafe without reasonable starting values for the separation and position angle. Hopefully, the data in the Hipparcos Input Catalogue were sufficiently reliable for this category of doubles.

The Photometric Solution

The relative astrometry and photometry were considered as two different processes in the FAST data reduction. This had the advantage of making the processing easier to handle and probably more robust, but with the drawback of neglecting important correlations between astrometry and photometry. This is particularly important for the very small separations where the errors in separation and magnitude difference become strongly correlated.

The preceding steps ended up with the relative coordinates X, Y of the components of the double star. For every grid transit (j = 1, ..., m) the scanning direction (γ_j) was also known, so the projected phase differences $\Delta \phi_j$ could be computed according to Equation 13.19. On the other hand, the normalised signal parameters β_4/\tilde{R} and β_5/\tilde{R} were also known from the analysis of the detector signal. The latter are related to the phase differences and the intensity ratio through the functions F_c and F_s defined in

Equations 13.12 and 13.13. This allowed the intensity ratio r that gave the best fit to the 2m observation equations to be determined, by minimising the function:

$$\chi(r) = \sum_{j=1}^{m} \omega\left(\frac{(\beta_4/\widetilde{R})_j - F_c(r,\Delta\phi_j)}{\sigma_{c,j}}\right) + \sum_{j=1}^{m} \omega\left(\frac{(\beta_5/\widetilde{R})_j - F_s(r,\Delta\phi_j)}{\sigma_{s,j}}\right)$$
[13.23]

where $\sigma_{c,j}$ and $\sigma_{s,j}$ are the standard errors of the normalised signal parameters. $\omega(x)$ is an influence function chosen to make the statistical procedure both reasonably efficient and robust, i.e. protected against outliers. The classical least–squares method corresponds to the choice $\omega(x) = x^2$, and would have led in the present case to a one-dimensional non-linear least-squares estimate of *r*. It is well known that the least-squares method is very sensitive to outliers, or the contamination by observations with a more extended error distribution than the Gaussian. Several influence functions were tried before selecting:

$$\omega(x) = \log(1 + x^2/a^2)$$
 [13.24]

associated with the Cauchy distribution. The scale factor a = 1.64 was adopted as a compromise between robustness (small *a*) and good asymptotic efficiency (large *a*).

Extensive simulation software was written to test the procedures on virtually any kind of double star and to investigate the bias of this estimator. Not surprisingly there was a bias both for very small separations and for large magnitude differences. A large lookup table, containing the bias correction as a function of the double star parameters and signal-to-noise ratio, was built and incorporated into the software to supplement the estimator. The minimum of $\chi(r)$ was then obtained with the 'golden section search' (Press *et al.* 1992) after the minimum was bracketed by a discrete search.

The final step in the photometric solution was to account for the attenuation profile, $\Psi(\varrho)$, of the instantaneous field of view (Figure 13.4). In the simplest case when the detector was pointed at the primary, and the secondary was consequently at a distance ϱ from the centre of the instantaneous field of view, the corrected (true) magnitude difference was obtained as:

$$\Delta m = \Delta m' - \Psi(\varrho) \tag{13.25}$$

where $\Delta m' = -2.5 \log r$ is the apparent magnitude difference computed from the measured intensity ratio. This formula is easily generalised to the situation when the pointing was at the secondary, or at an intermediate point between the two components, as was the case for some binaries with known separations around 10 arcsec.

While more than 22 000 Hipparcos entries were recognised as probably non-single objects, there were only about 16 800 for which a solution for the relative astrometry could be derived with some reliability. The magnitude differences were then computed as described above for all these objects. The final catalogue includes a significantly smaller number of solutions since a rather conservative policy was adopted for the merging of the FAST and NDAC solutions (see Section 13.7).

Results of the FAST Solutions

Two different software packages were written and implemented, one by a group in CERGA and the second by teams in Istituto di Astrofisica, CNR, in Frascati and in Osservatorio Astronomico di Torino, which focused primarily on known double stars. The FAST solution is a merge of these two solutions, adopting either solution when only one was available and some weighted mean when the two groups produced similar



Figure 13.6. The rating of the double star solutions in FAST, from 0 for the poor solutions to 10 for the most reliable. The distribution in the left diagram is for the systems known to be double before the Hipparcos mission while the diagram on the right refers to the double stars discovered by Hipparcos.

solutions. When two discrepant solutions were obtained various criteria based on the ground-based separation and position angle or on the standard errors were used to select the one considered as most likely correct.

The FAST solution for the relative astrometry and photometry consisted of 16 634 entries with separation, position angle and magnitude difference together with their standard errors. Because of the two-pointing systems these solutions accounted for about 15 200 different systems: 9500 known systems and slightly more than 5500 new systems. The separation between the two groups cannot be made more precise because it would have required a careful investigation of the existing data base with the problem of cross-identification of systems and components. This has been done at the end for the final Hipparcos Catalogue. There was an additional set of 6000 entries suspected of being non-single but with no satisfactory double star solutions.

A quality factor was computed to grade each solution between 0 and 10, taking into account various factors such as the detection level, the standard errors, the number of alternate solutions, whether the convergence toward the solution was direct or tortuous, the number of phase ambiguities, etc. Solutions with a mark close to the maximum are highly reliable while a mark below or equal to 3 indicated that the quality of the solution may be questioned. The distribution of this rating is shown in Figure 13.6 for the known double stars on the left and those newly discovered by Hipparcos on the right. The two distributions differ markedly, reflecting the higher than average difficulty encountered in solving the new double stars, which were of smaller separation in general. A significant fraction of the new double stars falls in the category of low rating and this constitutes the typical set with separations in disagreement with the NDAC solution. They were eventually rejected during the merging phase (Section 13.7).

The main results obtained by FAST are shown in Figure 13.7 for the systems whose duplicity was known from ground-based observations before the Hipparcos mission and in Figure 13.8 for the new doubles stars detected and solved by FAST. One must stress that these plots refer to the FAST processing, before the final selection of double and multiple solutions based on the comparison between the FAST and NDAC solution (Section 13.7). The distribution of the separations extends up to more than 25 arcsec for the population of known double stars with a large fraction of systems with $\rho \simeq 1$ arcsec. The separations for the new double stars is strikingly different with a maximum of



Figure 13.7. Distribution of the separations (left) in the FAST solution for the systems known to be double before the Hipparcos mission. The distribution of the mean error is shown on the right for the same systems.



Figure 13.8. Distribution of the separations (left) in the FAST solution for the systems discovered to be double from the Hipparcos observations. The distribution of the mean error is shown on the right for the same systems.

the distribution at about 0.3 arcsec and very few systems above 2 arcsec. For these difficult cases, it was usual to end up with at least two solutions, one with a small separation, $\rho \leq 0.3$ arcsec, and another one or two grid steps larger. For such a system statistics of the projected phase difference were computed over the set of transits where a positive detection had occurred. If the true separation of the double star was larger than a few grid steps, the set of phase differences was expected to be more of less uniformly distributed between 0° and 360°, while for true separations less than a grid step one expected a cluster of small values. The consistency between the observed statistics and the fitted separations eventually helped choose between alternate solutions corresponding to separations differing by one or several grid steps.

In Figures 13.7–13.8, the right panels show the distribution of the mean error on the relative position of the secondary respectively for the population of known and new double stars. Not surprisingly the first population gives a median in the error of the order of 5 mas while for the new double stars, with a much lower signal-to-noise ratio, the distribution is wider with a median value of about 20 mas.

The Multiple Stars

A dedicated treatment has been applied by FAST to process systems with more than two components and to obtain their relative astrometry and photometry. The absolute astrometry was then derived in the same way as for either double or multiple systems and is considered in this section.

The problem of multiple star treatment is more difficult than that of double stars because, with the same amount of information collected at the level of the image dissector tube data reduction, at least three additional parameters per component must be determined: separation, position angle and magnitude difference. This must be done while the same difficulties already described for double stars (in particular grid-step errors and the indeterminacies occurring when two components are close and with similar brightness) may exist for any pair within the multiple system.

There is no criterion that could be derived from the Hipparcos data that would permit the recognition of specifically multiple systems. Actually, the same criteria used to detect double stars (Section 13.2) apply just as well to systems with more than two components, so that multiple systems are to be found among the stars recognized as non-single by the double star detection software.

Because of the larger number of unknown parameters to be determined, classes of systems for which realistic solutions could be obtained are more restricted than in the case of double stars. Not only the observability conditions stated for double stars must be satisfied by any pair of components, but they must be even more stringent: each component must provide sufficient information to be clearly recognized and separated from any other. And the larger the number of components is, the more restricted become these conditions and, assuming a given number of observations, the less chances exist to obtain a solution.

For these reasons, in FAST, only triple systems were considered and experience showed that a minimum of 25 observations (well distributed over the scanning directions) were necessary to obtain a good solution.

Selection of systems: In the absence of specific criteria to recognize multiple systems, the choice of stars to be tested with the triple star reduction software was made as follows:

- first, all stars in the Hipparcos Input Catalogue which were known to be multiple with three components obeying the double-star observability conditions for the three possible pairs. In many cases, when two of them had a small separation (< 0.25 arcsec), a double star solution in which the closest pair of stars was replaced by their photocentre was found to be more reliable;
- double stars for which no relative solution could be found without rejecting a large number of observations or for which the absolute astrometric solution was obtained with a bad goodness-of-fit or which necessitated many rejections of observations.

In FAST, two completely different methods were used:

- (1) the scanning angle functions method, created and implemented in CERGA, Grasse;
- (2) the global fitting method, developed at the Osservatorio Astronomico di Torino, Italy.

In addition, an application to multiple star systems of the CLEAN imaging method was studied, but not applied because the relative astrometry approach adopted in FAST for double and multiple star reduction, did not permit the definition of a common reference frame for all the observations. This was actually possible for NDAC which performed



Figure 13.9. Projections of the positions of the components of a triple system on the tangent plane and on the reference great circle whose position angle is γ .

the reduction simultaneously with absolute astrometry so that a single reference was available for all the observations.

In the case of FAST methods, an *a priori* approximate solution was necessary to start converging toward the solution. The two methods described below differ in particular by the way that such an approximate solution is obtained.

Scanning angle functions method: The observed signal of a triple star could be represented by Equation 13.2 in which it was assumed that ψ has been corrected by the calibrated phase shift which exists for single stars as already explained in Section 13.2. The relations with the parameters of the components are extensions of Equations 13.7–13.11 with three terms rather than two. Let us consider a system of local coordinates XY with their origin at one of the components of the systems, S_1 (Figure 13.9). One considers the two vectors $\overline{S_1S_3}$ and $\overline{S_3S_2}$ whose coordinates are respectively (X_2, Y_2) and (X_1, Y_1) .

The projections on the reference great circle, which were proportional to phases, modulo 2π , were such that for the *j*th observation we have:

$$(\phi_3 - \phi_1)_j = \text{mod}(\alpha_j X_2 + \beta_j Y_2, \ 2\pi)$$
[13.26]

$$(\phi_2 - \phi_3)_j = \operatorname{mod}(\alpha_j X_1 + \beta_j Y_1, 2\pi)$$
 [13.27]

where α_j , β_j are given by Equation 13.18. In addition, the intensities were normalised with respect to the total intensity, so that the actual intensities were determined at the very end, using the global calibrated intensity provided by the photometric treatment as:

$$J_k = \frac{\sqrt{2} I_k}{I_1 + I_2 + I_3}, \qquad k = 1, 2, 3$$
[13.28]

with

$$J_1 + J_2 + J_3 = \sqrt{2}$$
 [13.29]

With these assumptions, namely using only phase differences and normalised intensities, only three relations between the parameters were left. Any three combinations of Equations 13.8–13.11 could be used. The choice was to take:

$$F = 1 - (M/M)^{2}$$

$$G = 1 - (N/\widetilde{N})^{2}$$

$$H = 2\sqrt{2} (M/\widetilde{M})^{2} (N/\widetilde{N}) \sin(2\phi - \psi)$$
[13.30]

where *F*, *G* and *H* are functions of X_1 , X_2 , Y_1 , Y_2 , J_1 , J_2 , J_3 and γ , while \widetilde{M} and \widetilde{N} were as before the mean calibrated values of the modulation coefficients.

Values of the three functions were computed for each group of transits corresponding to a given orbit and reference great circle. Equations 13.30 are strongly non-linear and it was not possible to solve them by least-squares unless an approximate solution was available. In a first instance, each function *F*, *G* and *H* was represented by a Fourier series of γ , developed to the 10th or higher order depending on the number of observations. Then, two angles γ_1 and γ_2 were chosen and the seven Equations 13.26–13.30 were solved for the seven unknowns. The solution was obtained by successive approximations using 5th degree polynomial expansions of the equations. The coefficients of these expansions were computed analytically as functions of the seven parameters.

The choice of γ_1 and γ_2 was arbitrary, although many simulations have shown which combinations were the most efficient. Actually 12 such combinations, depending or not on the distribution of the angles γ were programmed and the solution was computed by a programme for the resolution of non–linear equations. Some combinations were rejected by the programme as impossible to solve. Others gave solutions which were used as starting point to the next step.

This next step consisted of studying how any of the solutions obtained changed when its parameters were slightly modified. This was a means of searching for a minimum of the root-mean-square residuals in the vicinity of the solution in the space of parameters. This space was not systematically scanned, which would have taken too much computer time, but tested in ten pre-determined directions. The goodness-of-fit of these new solutions and of the original one were compared, and the one that showed the best fit was retained.

At the end of this step, one had up to twelve solutions, not necessarily all different. The goodness-of-fit values were compared and the solution with the best fit was retained. Then, a new iteration was started, using the parameters of this solution as the starting point of the first step for the choice of γ_1 and γ_2 . The iteration was stopped when the last retained solution did not improve the quality of the fit by at least 1 per cent.

Next the various solutions obtained for each couple (γ_1 , γ_2) were compared by direct inspection. The one giving the best fit and the smaller number of rejections was kept. Observations which were systematically rejected at the 2.5σ level by the majority of solutions were suppressed and a new round of iterations was started.

If after one, or sometimes two, rounds more than 12 per cent of observations were rejected, or if there were no solutions with a goodness-of-fit smaller in absolute value than 4, it was considered that there was no solution possible for this star. Otherwise, the final best solution (sometimes two if the goodness-of-fit values were similar) were kept for the final test using absolute astrometry determination.

Global fitting method: This method is a classical iterative least-squares procedure which was applied in Torino and Frascati for the double star treatment and was extended for multiple stars. As in the case of the preceding method, one observation provides three equations of condition, chosen to be:

$$A = IM$$

$$C = IN \cos(2\phi - \psi) \qquad [13.31]$$

$$D = IN \sin(2\phi - \psi)$$

The usual assumption was that the system does not change with time, though the method could easily be extended by adding first or even second derivatives of the rectangular local coordinates X_k , Y_k (k = 2, 3) of the components with respect to the primary. In addition to these four unknowns, the method considered all the products I_jM_j and I_jN_j (j = 1, 2, 3) for each of the three components considered as independent.

The main idea of the method was to use a classical least-squares method to solve linearised equations of condition. The problem was that equations that link A, C and D to the actual star parameters are highly non-linear and one can start such a procedure only when one has in hand a set of parameters that represent already a reliable solution. So, the first approach of the method is a 'pre-global fitting' approach, starting from system parameters provided by ground-based observations and found in the Osservatorio Astronomico di Torino data base.

The equations were simplified taking only the six unknowns I_jM_j and I_jN_j and the Levenberg-Marquardt method for non-linear systems of equations. The idea was to define a χ^2 merit function and determine the parameters by its minimisation. This was successively done using the steepest descent method combined with the inverse Hessian method. This was iterated until the merit function showed instability around its minimum.

Then the global fitting which assumed that it is possible to linearise the equations with respect to the unknowns was applied. The linearisation was made by the Newton-Raphson technique, expanding Equations 13.31 by a Taylor series up to the first order:

$$A = A(\mathbf{X}_0) + \sum_{k=1}^{10} \left. \delta X_k \left. \frac{\partial A}{\partial X_k} \right|_{\mathbf{X} = \mathbf{X}_0}$$
[13.32]

where **X** is the vector of parameters and δX_k their increments from the previously estimated approximate solution **X**₀. Equations 13.32 were solved by an iterative scheme with singular value decomposition algorithm, which is particularly well adapted to this type of problem for its numerical properties and stability.

The Absolute Astrometry of Double Stars

As mentioned earlier, the overall principles applied by FAST consisted in first obtaining the relative parameters of a double or multiple star (separation(s), position angle(s), and magnitude difference(s)), and then correcting each great-circle abscissa for the systematic offset between the, still unknown, signal phase of the primary and the observed phase of the whole system. After these two steps, all the relative parameters ρ , θ and Δm were known for all the double stars. For a 'fixed solution' these parameters were sufficient to characterise the double star over the whole mission. In contrast, when a significant relative motion of the components had been detected, two additional parameters had been determined, namely the first time derivatives of the relative rectangular coordinates, \dot{X} and \dot{Y} ('linear solution'). The variances of each of the above parameters were also available and were used to propagate the random errors to the corrected abscissae.

In the FAST processing of the great-circle abscissae all the stars were processed in exactly the same way, whether they were single or not. This meant that, in every observation frame, the grid coordinate (*G*) was computed from the phases of the five-parameter model of Equation 13.2, as if the star were single. Actually the signal phases only defined the grid coordinate modulo the grid step *s*, and the integral number of grid steps had to be computed from the *a priori* position given in the Hipparcos Input Catalogue, even though it could refer to a point not directly linked to the phase of the signal. Equations 13.8–13.11 provide the relationship between the observed phases (ϕ , ψ) and the phases of the primary (ϕ_1) and secondary (ϕ_2). To get the grid coordinate of the primary, the observed value had to be shifted by $\phi_1 - \phi$, and a corresponding corrected abscissa of the primary could still be wrong by an integral number of grid steps. This was accounted for in the astrometric processing by a mesh search dedicated to the double stars, since the number of grid steps was in general much larger for these objects than for single stars.

Equation 13.16 links the geometric parameters of the double star and the scanning direction to the projected separation on the reference great circle. Using Equation 13.15 the phase difference $\Delta \phi = \phi_2 - \phi_1$ between the secondary and primary was computed, while the intensity ratio *r* was obtained from the magnitude difference. The following equations were then used to translate the observed phases ϕ and ψ into the phase of the primary (ϕ_1):

$$\tan(\phi - \phi_1) = \frac{r \sin \Delta \phi}{1 + r \cos \Delta \phi}$$
[13.33]

$$\tan(\psi - 2\phi_1) = \frac{r\sin 2\Delta\phi}{1 + r\cos 2\Delta\phi}$$
[13.34]

From this, corrected great-circle abscissae could be computed for every great circle on which the double star had been observed. The corrected abscissae are statistically equivalent to what would have been obtained if the primary star had been observed without perturbation from the secondary.

The same kind of correction was applied to the secondary or to the photocentre of the system, with straightforward modifications. Likewise for an orbital double star, an ephemeris was computed and the instantaneous separation and position angle were used to determine the abscissa shift needed to refer the observations to the centre of mass. (The same could in principle have been done for double stars with a variable component, had an ephemeris of Δm been available.) By adding further components, it was also easy to generalise the above method to triple stars, and this was done for the absolute astrometry of these systems.

Regarding the propagation of the error estimates, this was easily done for a particular reference great circle by computing the derivatives of the phase shift with respect to the separation, position angle and magnitude difference and then adding the variances to the variance of the initial abscissa. However, there was a small difficulty here, in that the corrected abscissae could be strongly correlated (because roughly the same correction—including the same error from the relative astrometry—would apply to all great circles with roughly the same scanning angle γ). Neglecting this correlation would have resulted in an underestimation of the variances of the final astrometric parameters,

and the comparison of the astrometric variances of the primary and secondary would also have disagreed with the variances of the relative solution. To rectify this, an average correlation coefficient was computed by comparing the variance of $\phi - \phi_1$ with the variance of the great-circle abscissae; this correlation was then used to estimate the final variances of the astrometric parameters.

Extensive validations were needed to get everything working properly. Two independent solution programs were written for the astrometric parameters of the double stars. Although the routines for the abscissa shifts were identical, the determination of the position, parallax and proper motion from the corrected abscissae was based on different methods. Once the principles had been validated, one of the two programs was specialised into a systematic search for the position in a mesh extending to a distance as large as 30 arcsec from the *a priori* position. The candidate solutions were then used as starting values for the main routine, from which all the FAST solutions were eventually derived.

As for the absolute astrometry of triple stars, there was an additional check of the relative solutions carried out at this level. Several solutions for the relative astrometry were usually obtained by one or both methods described in the previous section. All these solutions were tried for the absolute astrometry and the results were compared with the results obtained from a double star solution or even sometimes from a single star model. A triple star solution was retained if it improved the goodness-of-fit and/or reduced the number of rejections. Also when two triple star solutions were available, the one which gave the best fit was finally retained. In total, out of more than 350 systems tested by any of the methods, 170 were accepted by at least one of the triple star programs, but after passing the absolute astrometry test, only 102 solutions were selected to be merged with the NDAC solutions.

13.4. The Astrometric and Photometric Solution: NDAC Method

The key idea for the NDAC double-star treatment was established by L. Lindegren in 1985. Instead of making the doubles behave like the single stars, by correcting the abscissae, the known or suspected non-singles were passed through a completely independent chain of reductions. In this way, the standard reductions were freed from possible contamination, and the double-star reductions could be made on a case-by-case basis. This required a certain time-lag, in that the main reduction had to be completed first, and the calibrations obtained were subsequently used to derive calibrated observations for the non-single objects.

These calibrated observations were collected into self-contained 'case history files', each one containing all the observations for a given object. All subsequent reductions for that object, whether it was modelled as a single, double or multiple star, needed only the case history file as input for the model fitting. Normally, a standard double-star model was first tried, and if that failed, more refined models could be tried. This general strategy proved to be very useful, not least in enabling new solutions to be rapidly computed for individual objects identified as problematic during the later stages of the reductions. The case history files have been archived, in a slightly transformed version, as the Transit Data File described in Volume 1 (Section 2.9), and are thus available for future re-examination of the solutions for about a quarter of all the Hipparcos Catalogue entries. Similar files have been also archived within the FAST Consortium. The above strategy separated naturally the NDAC double-star reductions in two main tasks. First, all the case history files were generated, using the best calibration data available. This had to be done in a single batch by going through all the observations in chronological sequence. Then, the actual solution searches could be performed, for one system at a time. Since several reductions were carried out in sequence, using a successively larger amount of observational material (the 12-month, 18-month, 30-month, and 37-month solutions), there were equally many (or more) separate generations of the case history files, and corresponding intermediate double-star solutions.

Generation of Case History Files

The task of obtaining the case history files from the actual satellite data was a large and complex one, and the final double-star results depended critically on its correctness. There were many details and problems to be solved in the process, and only an overview can be presented here.

The input to the generation of case history files had two large and many smaller components. The largest data set consisted of the original five-parameter fits to the image dissector tube counts (β_1 to β_5 in Equation 13.1), together with their information matrices (i.e. the inverse covariance matrices). These parameters, available from the image dissector tube processing carried out at the Royal Greenwich Observatory in Cambridge (Chapter 5), had to be calibrated geometrically and photometrically before being assembled in the case history files. Most importantly, the signal phases (β_3) had to be put on an absolute scale, so that $\beta_3 = 0$ corresponded to a certain point on the sky, with known astrometric parameters. A fundamental input for this was the frame-by-frame spin phases, i.e. the along-scan attitude angle as a function of time, derived in the greatcircle reductions performed at Copenhagen University Observatory (Chapter 9). These two inputs were each a couple of gigabytes in size. Smaller but equally important inputs were the geometric instrument parameters, determined once per orbit (\simeq 10.7 hour interval) as part of the great-circle reductions, the time-dependent photometric calibration parameters from the photometric processing (Chapter 14), and the abscissa zero points from the sphere solution (Chapter 11).

Ideally, it would have been useful to have a case history file for every entry in the Hipparcos Input Catalogue. Because of the large amounts of data involved, this was not practical, and an important part of the process was to select the subset of known and suspected non-singles for which the case history files should be derived. This subset evolved over the successive generations of case history files, taking into account also the detections reported by the FAST consortium, so that in the end it could be assumed with some confidence that only stars that could be passed as single in the main reduction chain had not been included. Known doubles and multiples were extracted from the Hipparcos Input Catalogue and from later compilations, and were automatically included in the subset. From the different statistical criteria described in Section 13.2, a similar number of suspected non-single objects was also included. As an important check, case history files were also derived for several thousand *bona fide* single stars, enabling more direct comparisons with the ordinary single-star reductions.

The main steps of the generation of the case history files can then be summarised as follows:

- 1. a special version of the Hipparcos Input Catalogue was created with some key data for each entry. For two- and three-pointing systems, a primary was designated, becoming the identification for the whole system. The photocentre of the system was calculated from the available Hipparcos Input Catalogue data, and this (with an assumed parallax and proper motion, also taken from the Hipparcos Input Catalogue) defined the reference point on the sky to which all observations of the system were referred;
- 2. a dedicated process was used to select the systems that should have their case history files derived. In its final version this list included about 13 500 known double or multiple systems, 14 400 suspected doubles from the NDAC criteria, and 5000 *bona fide* single stars. 4400 more objects were added as being suspected by FAST, or in previous NDAC solutions;
- 3. the tapes with original signal parameters (β_j) for all stars were then read through, to extract the data related to the objects selected for the case history files;
- 4. most of the calibration data were then collected in a single direct-access file, with one record per orbit or great-circle reduction. This included the geometric instrument parameters, specifying the large-scale field-to-grid transformations, the abscissa zero points from the sphere solution, the photometric calibration parameters, and estimates of the background count rate (*B* in Equation 13.2) for every $\simeq 2$ min of time. The calibration file contained also the geocentric velocity and position of the satellite used for correcting stellar aberration and calculating the parallax factors;
- 5. the main calculations were made by a single program. Using the attitude spin phases and the geometric and photometric calibrations, the signal parameters β_j for each field transit of an object were transformed to the calibrated parameters b_j described below, referring to an accurately specified scanning geometry;
- 6. at this stage the output was still in chronological order, with the transits of a given system scattered in the file. This was then sorted according to the system identifiers, resulting in one case history file per system.

In order to refer the observed phases to the adopted reference point, the detailed geometrical model of the field-to-grid transformation was needed, as well as the attitude angles and in particular the spin phases (along-scan attitude). As already mentioned, these data came from the great-circle reductions, but with the spin phases corrected for the abscissa zero points determined in the sphere solution. The sphere solution supplied also the time-varying chromaticity and some subtle phase shifts depending on the sixth harmonic of the satellite spin angle. Care was needed to apply correctly the first- and second-order relativistic stellar aberration (using the known motion of the satellite) and the gravitational light deflection due to the Sun. The photometric sensitivity variations over the field of view and over time were large (several tens of per cent), and again care had to be exercised to derive values calibrated at the millimagnitude level (see Chapter 14). The whole chain was accepted as correct only after passing the stringent test of giving astrometric data for the *bona fide* single stars in almost perfect agreement with the (independent) standard astrometric solution.

All calibrations, especially for the image dissector tube sensitivity, depend on colour, which gives two kinds of problems. On the one hand, the mean colour assumed in the reductions often differs from the more accurate values now available. This can be

rather simply corrected, however, using some auxiliary output in the case history file. On the other hand, there is a more fundamental problem for single-pointing doubles with two components of unequal colour: the mean of the calibrations is not necessarily equal to the calibration based on a mean colour. When the individual colours and intensities are known, a re-derivation of case history files would in principle be possible, but the amount of work involved makes this very impractical. Also, for most of the close Hipparcos doubles, the individual colours are not known, and the differential colour problem had to be left unsolved.

Interpretation of Calibrated Data

For a given object, the case history file contained the calibrated signal parameters for all the field transits of the object collected over the mission. By calibrated it was meant that all known variations of the instrument response and attitude had been taken into account, so that the parameters could be interpreted directly in terms of absolute quantities such as the astrometric parameters (in the reference frame of the corresponding NDAC sphere solution) and the magnitude *Hp*.

For each transit of the object across the field of view, the case history file gave the Fourier coefficients b_j describing the calibrated image dissector tube counts according to the general model:

$$A_k = b_1 + b_2 \cos p_k + b_3 \sin p_k + b_4 \cos 2p_k + b_5 \sin 2p_k$$
 [13.35]

where the reference phases p_k were now defined with origin at the reference point. The changing sensitivity of the image dissector tube over the field of view and with time was taken into account, and the background count rate subtracted, so that zero magnitude (Hp = 0) corresponded to exactly $b_1 = 6200$ counts per sample. The case history file also contained the variance-covariance of the coefficients b_i .

The calibrated phases and amplitudes were defined such that a point source of unit intensity at the reference point (i.e. with the astrometric parameters adopted for the reference point) produced the detector signal:

$$A_k = 1 + \bar{M}\cos p_k + \bar{N}\cos 2p_k$$
 [13.36]

where $\overline{M} = 0.7100$ and $\overline{N} = 0.2485$ are modulation coefficients fixed by convention. This expression defined more precisely the meaning of the reference phase p_k .

A point source at some distance from the reference point produced a signal phaseshifted with respect to Equation 13.36, so that p_k had to be replaced by, say, $p_k + \phi$ in that equation. The phase shift ϕ depended on the offset (ξ , η) from the reference point, the position angle of the scan (γ in Figure 13.5), and the effective grid period (s) for the observation in question. All these dependencies were taken into account by rigorously computing the derivatives $f_x = d\phi/d\xi$ and $f_y = d\phi/d\eta$ for each transit. In doing so, the offset coordinates (ξ , η) were regarded as barycentric, which meant that the effects of parallax and stellar aberration (differential with respect to the reference point) could not be taken into account when (ξ , η) were computed for a specific source in a specific observation. To account for a difference in parallax with respect to the reference point, the derivative $f_p = d\phi/d\pi$ was also computed. The total phase shift for a point source at the barycentric position (ξ , η) and with parallax $\Delta \pi$ relative to the reference point was then computed as:

$$\phi = f_X \xi + f_y \eta + f_p \Delta \pi \qquad [13.37]$$

Naturally, ξ and η could be functions of time to describe the combined effects of the proper motion and orbital motion of the components, all reckoned relative to the (in general already moving) reference point. The case history files included the factors f_x , f_y and f_p for each transit of the object. It must be noted that f_x and f_y closely correspond to the spatial frequencies α_i and β_i of Equation 13.18.

General Solution Principle

The basis for the NDAC double and multiple star analysis was the theoretical model for the calibrated photon counts of an object with n stellar components, obtained by summing the phase shifted and scaled signals, Equation 13.36, from the individual components (i):

$$A_k = \sum_{i=1}^n I_i [1 + \bar{M} \cos(p_k + \phi_i) + \bar{N} \cos 2(p_k + \phi_i)]$$
[13.38]

Here $I_i = 6200 \times 10^{-0.4 Hp_i}$ are the component intensities (expressed in counts per sample) and ϕ_i the phases relative to the reference point, calculated from Equation 13.37. Expanding the trigonometric terms and comparing with Equation 13.35 gives the following basic relations:

$$b_{1} = \sum_{i} I_{i}$$

$$b_{2} = \bar{M} \sum_{i} I_{i} \cos \phi_{i}$$

$$b_{3} = -\bar{M} \sum_{i} I_{i} \sin \phi_{i}$$

$$b_{4} = \bar{N} \sum_{i} I_{i} \cos 2\phi_{i}$$

$$b_{5} = -\bar{N} \sum_{i} I_{i} \sin 2\phi_{i}$$

$$(13.39)$$

Writing the phases ϕ_i in terms of the component positions (ξ_i , η_i), which were functions of time suitably parametrised to account for proper motion and orbital motion, and of the parallaxes of the components relative to the reference point, $\Delta \pi_i$, Equations 13.39 provided a set of non-linear observation equations for the intensities and geometric parameters of the components, in which the coefficients b_j were the 'observations'. With five equations per transit, there were typically some 500 to 1000 observation equations for a given system.

In principle, all that was now required was to specify the object model (number of components, form of motion for each component, constant or variable intensity, etc.), to insert the relevant parameters (unknowns) in Equations 13.39, and perform a robust, non-linear least-squares estimation of all the parameters. Combined with the parameters of the reference point this gave directly the absolute astrometry and intensity of each component, including their standard errors and the full correlation matrix.

Practical Considerations

In practice several different solution programs were set up to cope with the various kinds of objects, and many detailed considerations were necessary before the final solutions could be obtained. For the vast majority of the systems, however, a standard 12parameter model was sufficient. This consisted, in its most general form (Table 13.2), of one photometric and five astrometric parameters for each of two components. For two-pointing systems, a correction to the assumed attenuation profile of the instantaneous field of view could be introduced as the 13th parameter. This general form

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Table 13.2. Astrometric and photometric parameters solved for in the NDAC double-star analysis. Three different physical models of the double star result from the use of various constraints: without any constraints the solution is of type 'I', with $u_{10} = 0$ a solution of type 'L' results, and with $u_{10} = u_{11} = u_{12} = 0$ the solution is of type 'F'. The parameter u_{13} applies only to double-pointing systems, independent of solution type.

Parameter	Description		
<i>u</i> ₁	magnitude of primary (Hp_1)		
u_2	offset (ξ_1) in α of primary from reference point		
u ₃	offset (η_1) in δ of primary from reference point		
u_4	offset ($\Delta \pi_1$) in parallax of primary from reference point		
<i>u</i> 5	offset $(\dot{\xi}_1)$ in $\mu_{\alpha*}$ of primary from reference point		
u_6	offset $(\dot{\eta}_1)$ in μ_{δ} of primary from reference point		
<i>u</i> 7	magnitude difference $(\Delta m = Hp_2 - Hp_1)$		
<i>u</i> 8	relative position in α of secondary $(\xi_2 - \xi_1 = X)$		
<i>u</i> 9	relative position in δ of secondary $(\eta_2 - \eta_1 = Y)$		
<i>u</i> ₁₀	relative parallax of secondary $(\pi_2 - \pi_1)$		
<i>u</i> ₁₁	relative proper motion in α of secondary $(\dot{\xi}_2 - \dot{\xi}_1 = \dot{X})$		
<i>u</i> ₁₂	relative proper motion in δ of secondary $(\dot{\eta}_2 - \dot{\eta}_1 = \dot{Y})$		
$[u_{13}]$	correction factor for the attenuation profile ($\simeq 1$)]		

was directly applicable to optical double stars (solution type 'I', see Field DC3 in the Double and Multiple Systems Annex), and, by constraining some of the parameters to zero, to physical binaries with approximately linear relative motion (solution type 'L') and physical binaries with fixed relative geometry (solution type 'F').

The main difficulty was to choose the initial values for the model parameters. The 1.2 arcsec grid period imposed a strict 0.3–0.5 arcsec limit for the *a priori* position error in order for the least–squares adjustment to converge towards the correct position. This *a priori* accuracy was seldom available for all components of a system, and usually a double iteration had to be used to find the relevant solution. In other words, starting with a trial set of parameters, the least-squares solution was iterated some 5–8 times. If there was no convergence, the process was repeated with another set of initial parameters. In some cases, there was convergence to several different solutions with the secondary displaced by one or more grid steps, and it was necessary to choose the one which best fitted the observations. A fundamental principle in all cases was to compare also with the best single-star solution, and only accept a double-star solution if its χ^2 fit to the observations was significantly better. Along such general lines, several different analysis programs were constructed and used as outlined below.

The 12-parameter model of Table 13.2 applied rigorously only to systems with photometrically constant components. For double stars showing significant photometric variability, the double-star parameters were determined only from a subset of the data with total magnitude close to the median value. Actually, a large effort was put into a special scheme solving for the individual magnitudes at each field transit in addition to the fixed (5+5) set of astrometric parameters. Although this is straightforward in principle, it was in practice very difficult to obtain stable and reliable solutions, and most of these results had to be discarded. For systems with n > 2 components, a set of 6n unknowns was defined in close analogy with the double-star case. Again, the first six unknowns (u_1 to u_6) were 'absolute', referring to the designated primary component, the following unknowns for the other components (u_7 to u_{6n}) were defined relative to the primary, and a final unknown u_{6n+1} took care of the profile correction, when necessary. In principle, the observation equations were derived as for the double stars, but because of the larger dimension of the system of equations and the exponential growth of the mesh searches with the number of components, multiple-star solutions were only made when reasonably accurate *a priori* parameters were available. This restriction, coupled with severe time constraints during the last stages of the data reductions, led to a relatively small number of multiplestar solutions. With a larger effort in the future, especially in trying to find new third components to known doubles, the Hipparcos data may eventually yield many more multiple-star solutions.

13.5. NDAC Implementation and Results

Most of the double-star reductions in NDAC were made by a single person, working nearly full-time on this task since 1985. In retrospect, the path towards the final results was winding and tortuous, and several interesting side-tracks had to be left unexplored. In this section some definite milestones are recorded, before the main results are described.

Pre-Launch Simulations

As early as 1985, fairly realistic simulations of the whole double-star processing were started in Lund. An important result of these experiments was the realisation that double stars with 10 to 30 arcsec separation, although problematic, could be successfully treated, and thus had not to be excluded from the Hipparcos Input Catalogue. Another result was the demonstration that the parameters for previously unknown doubles could be found by a four-dimensional mesh search over the primary and secondary positions. Also, a first solution program for multiple stars was written and found to work for up to five components. Thus, already in 1986 many of the principles applied later were already decided on, and much of the later work can be described as an implementation of these principles, taking into account all the practical details and difficulties. A summary of the assumptions included in the simulations was presented at the INCA Colloquium in Sitges (Söderhjelm 1988). This included an outline of the practical steps needed to construct the 'case history files'.

Subsequent pre-launch simulations focused on the development of the two main solution programs (for known doubles, and for seeking the companions of suspected doubles). From a series of large-scale simulations the detection capabilities of Hipparcos could be specified in detail, as a function of the double-star parameters. It was found, empirically, that the 'difficulty' in resolving a double star is mainly governed by a combination of the magnitude difference (Δm) and separation (ρ), which was expressed by the parameter D (sometimes called Δm_{eff}):

$$D = \begin{cases} \Delta m - 20 \ \log_{10}(\rho/0.138) & \rho < 0.10 \ \text{arcsec} \\ \Delta m - 5.5 \ \log_{10}(\rho/0.32) & 0.10 < \rho < 0.32 \\ \Delta m & \text{otherwise} \end{cases}$$
[13.40]



Figure 13.10. Normalised differences, $\overline{\Delta a}$, between the double star and sphere solution results for the astrometric parameters of components of two-pointing double stars. The differences (divided by their estimated mean errors) in the five parameters are plotted against the 'observed' magnitude difference $\Delta m' = \Delta m + \Psi(\varrho)$ being the sum of the true magnitude difference on the sky (negative for a secondary component) and the attenuation when the instantaneous field of view was pointed at the other component.

Typical diagrams showing a discovery limit for new doubles around D = 3.5 to 4 are shown in Perryman *et al.* 1989 (Volume III, Chapter 13). In 1993, the definition of D was changed to be in better agreement with the real solutions, giving instead:

$$D = \begin{cases} \Delta m - 34 \ \log_{10}(\rho/0.112) & \rho < 0.10 \ \text{arcsec} \\ \Delta m - 2.4 \ \log_{10}(\rho/0.50) & 0.10 < \rho < 0.50 \\ \Delta m & \text{otherwise} \end{cases}$$
[13.41]

First Reductions

During the spring of 1991, a small fraction of the actual observations from the first year of the mission was released to the reduction consortia in order to allow full-scale testing of the reduction procedures. In May 1991 the first solution for a 'new' double-star (HIC 927) was obtained. Within a month, the general validity of the NDAC approach had been demonstrated on the one hand by the good agreement between the single-star absolute astrometry and the independent sphere-solution results, and on the other hand by the agreement of the double-star relative astrometry with existing ground-based data. The very preliminary results for some 1000 known and 300 new doubles (Söderhjelm *et al.* 1992; Söderhjelm & Lindegren 1992) rather exceeded the expectations, and the reduction work could be continued with confidence.

In the course of the routine processing in NDAC, a new set of case history files was generated after each major sphere solution. Thus, successive versions using 12, 18 and 30 months of observations were generated during 1992–1994. In parallel, the solution programs were considerably developed and improved. As data accumulated and the detection criteria became more powerful, more objects could also be added to the list of known or suspected non-single stars, for which case history files had to be generated.

Crucially important for the validation of the absolute astrometry was a continual comparison of the double-star results with the sphere solution results for some objects which were allowed to go through both reduction chains (bona fide single stars, and doubles with $\rho \leq 0.3$ arcsec or $\Delta m \geq 3$ mag). Already in the 12-month reductions this comparison gave a good agreement in positions and parallaxes. The random differences were of the order of the standard deviations, and no systematic differences above the 1 mas level could be detected. The covariances were however found to be systematically different, reflecting the complicated correlations between the attitude-errors in neighbouring scans using a common set of 'star-mapper' stars. No rigorous model could describe this problem, but semi-empirical correction terms were introduced in the double-star solution to make the covariances reasonably similar. Another illuminating test was to compare the (disturbed) sphere-solution data for wide two-pointing doubles with the results of the double-star processing. An illustration from a May 1993 report on the 18-month results (Figure 13.10) shows the typical 'trumpet' shape of the differences, with less and less disturbance as the 'observed' magnitude difference increases. The double-star solution is able to correct for the 'mixing' of signals from both components, but only when $\Delta m'$ (Figure 13.10) reaches about 4 mag does the sphere solution give correct results. Starting with the 30-month reductions, the covariances were calibrated not only by comparison with the sphere solution, but also by making two independent double-star solutions for each object, each solution using alternate halves of the data. Magnitude-dependent correction factors were derived in such a way that the combined standard deviations of the two solutions were in agreement with the rms differences between them.

Final Reductions

A preliminary set of 37-month case history files was derived in September 1994, using 'extrapolated' results for the instrument calibrations and abscissa zero points. A large number of test solutions were made in order to settle, in particular, the covariance calibrations. The final result was a 'half-observation' scatter generally within 20 per cent of the calibrated standard errors, with no remaining dependence on magnitude, separation or magnitude difference. Another important test performed at this stage was to solve the known doubles without any *a priori* information on the relative parameters, using the solution programs developed for searching the components of a new double star. The expected solutions were recovered for a large majority of the systems, with the failure rate reaching 50 per cent only at $D \simeq 4.5$.

After completion of NDAC's final sphere solution (N37.5) in April 1995, a 'real' set of 37-month case history files were derived. After some (small) updating with photometric 'ageing corrections' in August 1995, these case history files provided the input for the final double and multiple star solutions in NDAC.

Aided by the many preliminary results, the final large-scale double-star reductions were made during the summer of 1995. In October, a systematic round of test solutions was attempted for about 1800 systems where the solutions by FAST and NDAC differed, using the FAST results as starting values. In many cases the original NDAC interpretation could be confirmed; in others, new solutions closer to those by FAST were obtained. A similar large-scale comparison with Hipparcos Input Catalogue data (which had so far not been allowed to influence the end results) generated another set of alternative solutions. Generally, however, the new solutions were retained only if they gave an improved fit to the observations, as measured by the χ^2 statistic.

For the multiple stars, solution programs had in principle been available for many years. Only after the completion of the double star processing could they be brought up-to-date, and some 180 multiple-star solutions were obtained in January 1996. In practice these solutions required rather good *a priori* relative parameters. Ideally, the large number of poor double-star solutions should also have been checked for a possible third component, but this was not accomplished in the present reductions.

In early 1996, the final merging of FAST and NDAC double-star data was made (Section 13.7). During this work, several grid-step errors were positively identified, and new solutions were calculated for individual cases. Similarly, several improved solutions were obtained by the help of dedicated ground-based observations communicated by R.S. Le Poole *et al.* (private communication). After the merging had been completed any further errors or omissions discovered, or any further improved solutions, were only listed for inclusion in the Notes to the Double and Multiple Systems Annex.

Overview of the NDAC Results

The double-star processing produced a total of 16 016 resolved double solutions. To this should be added the 180 solutions for triple and quadruple systems. For the double stars an internal quality rating was constructed, ranging from Q = 3 (reliable solution) to Q = 0 (marginal solution). This was based mainly on the parameter *D* in Equation 13.41 and on the improvement in fit, as measured by χ^2 , from the single- to the double-star solution. The number of solutions in these quality classes were: 8415(3), 3841(2), 2485(1), 1275(0). For 14 679 additional cases, although a double-star solution was tried, a single-star solution was found to fit the data equally well. Finally, there were about 2820 systems for which no acceptable single or double star solution could be found.

For each double star, three different solutions with different constraints ('I', 'L', 'F'; see Table 13.2) were usually available, each with a complete covariance-matrix for 12, 11 or 9 astrometric and photometric parameters. Although there was in each case a preferred solution, the final selection between them was only made in connection with the merging (Section 13.7).

The NDAC approach to the double-star processing ensured that all observations collected over the mission were taken into account in a nearly optimal way, both for the detection of duplicity and for the determination of the actual parameters (absolute and relative). There was however one crucial difficulty, related to photometric variability. As described above, the NDAC solutions were made directly from the Fourier coefficients b_j in Equation 13.39, assuming constant luminosity for the components. When one or both components were variable, a model mismatch obviously resulted, and observations with important astrometric information had to be rejected. Unfortunately, this aspect of the solution process had not been tested in the early simulations, creating unexpected large problems in the reductions for variable doubles. The extensive experiments with field transit magnitudes produced in the end no useful results, but a small number of cases were noted where the secondary appeared to be the variable component.

In the standard reductions, it is clear that variability increases the probability of spurious 'new' double-star solutions, as shown by an increased number of solutions with particular values for the separation, related to the grid period of 1.2 arcsec (Figures 13.11–13.12). The detailed cause and mechanism for this problem is not well understood, but its relation to variability is shown clearly when the material is divided according



Figure 13.11. Distribution of 'new' doubles, as obtained by NDAC, with respect to separation (ϱ) and magnitude difference (Δm). Only stars with reasonably constant total magnitude are included.



Figure 13.12. Distribution of 'new' NDAC doubles, for the subset of stars with significantly variable total magnitude. Note the concentration of (often spurious) solutions at either small ϱ and small Δm , or to 'bands' on each side of $\varrho \simeq 1.2$ arcsec or its multiples.



Figure 13.13. Distribution of the NDAC solutions for 'known' doubles in the same ϱ -interval as Figures 13.11 and 13.12. No preferred separation values are apparent.

to 'constant' or 'variable' photometry. For systems with known *a priori* parameters, there is however no similar effect, as clearly seen in Figure 13.13. (In other words, a solution in the 'band'-region may be correct, it only has an enhanced probability of being spurious). The only practical remedy for this problem has been to accept new solutions very conservatively, and for variable stars only when they are confirmed by FAST solutions. By treating the relative astrometry independently of the photometry,

13.6. NDAC/FAST Comparisons

the FAST approach is much less liable to these problems.

As described above, the methods used by NDAC and FAST for the double-star processing were radically different. The two approaches had their strong and weak points, sometimes nicely complementing each other, but a major drawback has been the difficulty of making meaningful comparisons of the results. Unlike the case of single stars, there were no intermediate results that could be compared, allowing the differences in the final results to be traced back through the various stages of the processing. Throughout the comparisons, there was always a core of well-behaved objects, for which the results were in excellent agreement, but then there were also some fraction of the objects with unexpectedly large differences, and a sizable number of cases where only one of the consortia found a solution.

The first large-scale comparisons, including hundreds of solutions, were made in 1992, using solutions based on 12 months of observations. For a majority of solutions, the agreement in both the relative positions and photometry was surprisingly good, considering the radically different methods used. The differences tended to be somewhat larger than the calculated standard deviations, and some systematic effects could be seen especially in the photometry; on the whole, the agreement was however as good as could reasonably be expected. The differences were larger for the newly detected doubles, but in many cases these could be explained by grid-step errors, which, according to simulations, should be common for such a short observation interval.

A second round of comparisons was made in 1993 with the 18-month results. Again, there were thousands of stars showing good agreement in the relative data, and an effort was made to compare also the absolute astrometry (positions, proper motions and parallaxes) between the consortia. The first such attempts failed blatantly, with typically 20 to 30 mas differences in the positions, although the calculated standard errors were ten times smaller. After debugging on each side, the situation did improve early in 1994, and from then on, 'absolute' and 'relative' comparisons have shown similar differences, after normalisation by the combined standard errors, although still significantly above unity (typically 1.5 to 2). Subsequent comparisons of the 30- and 37-month results confirmed the general correctness of the solutions, but also generated lists of several thousand objects where FAST and NDAC did not agree. Big efforts were made to improve the situation, resulting in many new or alternative solutions. However, because of the very different procedures of FAST and NDAC, few of the new solutions showed radically better agreement. In the end, the remaining problem cases had to be referred to the double-star merging.

13.7. Merging of the Results for Resolved Double and Multiple Stars

Although the double-star processing in FAST and NDAC produced two sets of data, which were not always compatible, it was decided that the Hipparcos Catalogue should contain only one solution of each system. In the majority of cases, where the FAST and NDAC solutions were in reasonable agreement, this caused no problem and the published result is then essentially a mean of the two solutions ($\simeq 11\,000$ systems). Similarly, for about 6000 systems solved by one consortium but not by the other, there was no practical difficulty in accepting the one available solution, except that some criterion was needed to decide whether that solution was sufficiently trustworthy.

The greatest problem was caused by the about 1800 cases where FAST and NDAC differed very significantly in their solutions, often by a multiple of the grid step (\simeq 1.2 arcsec) in the relative position of the secondary. Some 700 of these were in the end treated as single stars, or received stochastic solutions but for some 1100 entries various criteria had to be applied in order to select one of the solutions as being the most probable one (see below).

Apart from these difficulties, the merging of the results proceeded, after some initial experimentation, following a very simple recipe: a straight mean was adopted, giving equal weight to the two solutions. This averaging applied equally to the absolute and relative astrometry, and to the photometric parameters expressed in magnitudes. The main difficulty was to estimate the standard errors and, in particular, the covariance matrix of all the parameters in a merged solution.

Data Input to the Merging

The merging for the double stars was based on solutions received, in their quasi-final form, in December 1995. The FAST data consisted of:

- relative data for 16 634 entries. The following data were always included: X, Y, and Δm , with standard errors and the correlation coefficient ρ_{XY} , and a quality rating on a scale from 0 (poor) to 10 (highly reliable). For 5715 of the entries, relative motions were also provided in the form of the parameters \dot{X} and \dot{Y} , with standard errors and the correlation coefficient $\rho_{\dot{X}\dot{Y}}$;
- the five astrometric parameters (with complete 5×5 covariance matrix) for 15 528 entries. In 5195 cases these data refer to the photocentre of the binary (where the FAST relative astrometry gave $\varrho \leq 0.35$ arcsec); in the remaining cases they refer to the primary component, or the secondary of a two-pointing system. The number of accepted and rejected observations (abscissae) and the goodness-of-fit statistic F2 were also provided with the absolute astrometry.

1155 entries thus had relative solutions but the corresponding absolute astrometry had not been accepted (e.g. because F2 > 6). The FAST relative and absolute astrometry were expressed in the ecliptical system, and a first step was to transform them to equatorial quantities (see Volume 1, Section 1.5.3), and then from the reference frame of the final FAST sphere solution (F37.3) to the provisional H30 frame by the same rigid-body rotation as was used for the merging of the single stars (see Section 17.2).

The NDAC data consisted of 15 913 entries for which the complete solution vectors (u_1 to u_{12} in Table 13.2) were given together with full 12×12 covariances. For most of the entries, all three kinds of solution (I, L and F) were given. Statistics on the number of accepted and rejected observations (field transits) and the goodness-of-fit (F2) were also given. These data referred to the equatorial system and the reference frame of the final NDAC sphere solution (N37.5); they were consequently transformed to the provisional H30 frame by the same rigid-body rotation as in the single-star merging.

An additional minor complication was that FAST and NDAC, at this stage of the reductions, used slightly different numbering systems for the catalogue entries. This affected some 80 entries. Before merging, the lists were transformed to the final numbering system of the Hipparcos Catalogue.

The Neutral Point

The content and format of the Double and Multiple Systems Annex had been worked out and agreed upon before the merging of the double star data started. Since the most complex model of the resolved systems involving orbital motion would only include the linear terms of the relative motion, it was evident that each component could be described by the same five parameters as used for single stars. The format of the Annex was therefore modelled, as far as applicable, on the format of the main Hipparcos Catalogue. The main complication to be considered was the existence of correlations between the astrometric parameters of the different components in the same system, and between these parameters and the magnitudes of the components. The correlations are often very considerable, and essential for estimating the standard error of any quantity calculated from the component data, such as the photocentre. Consequently, the Annex should list all the correlations between the astrometric parameters estimated for a given system.

The NDAC solution method provided the full covariance matrix for each system solved, and from this the required correlations were easily calculated. The FAST method, on the other hand, gave separate solutions for the relative astrometry, the relative photometry, and the absolute astrometry, and the correlations existing between the three kinds of data were not explicitly obtained. In order to merge the results properly, and to provide complete information also in the case of FAST-only solutions, it was necessary to reconstruct at least some of the correlations implicit in the FAST data. The major point to consider was to reconcile the relative and absolute astrometry.

The practical solution to this problem was based on the observation that, for sufficiently close pairs, the absolute position of the photocentre is independent of the assumed relative parameters. For well-resolved pairs, it is similarly found that the absolute position of the primary is fairly independent of the relative parameters. By generalisation it can be inferred that there is always a neutral point in a system where the absolute and relative astrometry are minimally coupled. It was assumed that this point is always located between the primary and the photocentre, the fractional distance given by a parameter *q* in the range from 0 (primary) to 1 (photocentre). Thus, with **p** and **s** denoting the positions of the primary and secondary, and $r = 10^{-0.4\Delta m}$ the intensity ratio, the neutral point can be written:

$$\mathbf{n} = \mathbf{p} + (\mathbf{s} - \mathbf{p}) \frac{qr}{1+r}$$
[13.42]



Figure 13.14. The parameter q defines the location of the 'neutral point' as a fraction of the distance from primary (q = 0) to photocentre (q = 1). This diagram shows the determination of q versus separation (ϱ) by two completely different and independent methods: (1) the points and bars show the q-values minimising the correlation between the position of the neutral point and the relative coordinates of the components. Based on individual q-values computed from the covariance matrices of 9740 NDAC solutions, the points and bars give respectively the mean values and the $\pm 1\sigma$ ranges within each interval of separations; (2) the solid curve gives the empirical relation (Equation 13.43) minimising the positional differences between FAST and NDAC.

The quantity *q* can be computed directly from the covariance matrix provided with the NDAC solutions, and is found to depend mainly on ρ (Figure 13.14).

The neutral point was also derived empirically from a comparison of the FAST and NDAC absolute astrometry. For a set of doubles with a narrow range of separations, the *q* value was varied to minimise the mean positional difference between FAST and NDAC, $\langle |\mathbf{n}_N - \mathbf{n}_F| \rangle$. Repeating this process for a number of separation intervals, it was found that the best positional agreement between the consortia was obtained with a *q* value varying with separation roughly according to the formula:

$$q = \begin{cases} 1 & \text{for } \rho \le 0.25 \text{ arcsec} \\ 1 - (\rho - 0.25)/0.45 & \text{for } 0.25 < \rho \le 0.70 \text{ arcsec} \\ 0 & \text{for } 0.70 < \rho \end{cases}$$
[13.43]

This relation is also drawn in Figure 13.14 and shows good agreement with the mean values calculated from the NDAC covariances. Based on the neutral point defined by Equation 13.43, the FAST correlations between the astrometric parameters of the components were computed such that the standard errors supplied by FAST (after modifications described below) could be recovered both for the relative positions of the components and for the absolute position of the primary or photocentre.

The concept of a neutral point was also used in order to merge the FAST and NDAC results (Figure 13.15). The relative astrometry and photometry were first merged (i.e. averaged); later, the neutral points of the two absolute solutions were also averaged, yielding the neutral point of the merged data (\mathbf{n}_m in the figure). Finally, the merged separation, position angle and magnitude difference were applied to the merged neutral point, resulting in the merged positions of the components. Normally this somewhat



Figure 13.15. This figure illustrates how the neutral point is used to merge the relative and absolute astrometry from the two consortia (indicated by subscripts 1 and 2). The relative parameters ϱ , θ and Δm are first averaged, yielding the merged relative locations and intensities of the components (\mathbf{p}_m and \mathbf{s}_m). Then the neutral points are averaged to yield \mathbf{n}_m ; finally the absolute positions of the components are obtained by applying the relative data to the merged neutral point.

roundabout method produced virtually the same result as a direct averaging of the absolute positions of the components. However, in cases where FAST and NDAC differed significantly in Δm direct averaging could lead to unlikely results for the photocentre, which were avoided with the present method.

Discrepant Solutions

The relative astrometry and photometry were compared and merged before the (absolute) astrometric parameters were considered. The criterion for an 'acceptable' agreement included limits on the differences both in relative position and in position angle:

$$\left[(X_{\rm N} - X_{\rm F})^2 + (Y_{\rm N} - Y_{\rm F})^2 \right]^{1/2} \le 0.3 \text{ arcsec} \quad \text{and} \quad |\sin \frac{1}{2} (\theta_{\rm N} - \theta_{\rm F})| \le \sin 22^{\circ}.5 \quad [13.44]$$

No limit was set in $\Delta m_N - \Delta m_F$, except that Δm_N and Δm_F should both be non-negative. A limit of 0.3 arcsec was adopted for the relative astrometry, because this is approximately the maximum range over which it makes sense to average quantities which vary periodically with the grid step of 1.2 arcsec. The onset of non-linear effects at about this distance is, for instance, illustrated by the behaviour of the neutral point, which starts to depart from the photocentre, and hence from the linear regime, at a separation of 0.25 arcsec. The criterion on the difference in position angle is needed for close pairs ($\rho \leq 0.3$ arcsec), where the first criterion is almost always satisfied, irrespective of the relative orientations.

Among the cases rejected by the criterion above, there was an excess of cases with $\theta_{\rm N} - \theta_{\rm F} \simeq \pm 180^{\circ}$. If both $\Delta m_{\rm N}$ and $\Delta m_{\rm F}$ were small, this could be attributed to the noise in the magnitude differences causing a simultaneous reversal of Δm and θ in one of the solutions. In such cases the smaller of the Δm values was given a negative sign and the corresponding θ value changed by $\pm 180^{\circ}$. After averaging, the resulting Δm was then still non-negative. This affected some 500 pairs.

However, in some 40 other cases the position angles differed by $\simeq 180^{\circ}$ even though both Δm were strongly positive. These were almost always close pairs, and the effect was attributed to the indeterminacy of the sign of $\Delta \phi$ in the FAST reductions (see Section 13.3). Consequently, in these cases $\theta_{\rm F}$ was changed by $\pm 180^{\circ}$ while $\Delta m_{\rm F}$ was not touched.

After this 'cleaning up' of the relative astrometry and photometry, there still remained about 1400 doubles for which the FAST and NDAC solutions could not be reconciled, and for which no averaging would make sense. Several criteria were used in order to select one solution in preference to the other:

- in about 60 cases, dedicated CCD observations (Le Poole *et al.*, private communication) could definitely point out one solution as correct, and the other as incorrect;
- in about 600 cases, one solution gave a good fit in the FAST determination of astrometric parameters, while the other caused a large chi-square or many rejections. In these cases the best-fitting solution was adopted;
- a large portion of the remaining systems were individually investigated by means of the 'imaging approach' using the NDAC case history files (Perryman *et al.* 1989 Volume III, Section 14.5).

The imaging approach was found to be quite a powerful tool in the separation range of 0.2 to \simeq 8 arcsec, and for $\Delta m < 3$ mag. The method, in the preliminary version implemented for this specific purpose, was however very slow, and to go through hundreds of objects required a significant effort. The ambiguous cases were therefore investigated in order of decreasing importance, as measured by the FAST and NDAC quality ratings. For the highest ratings (i.e. the 'strong' doubles), a positive decision could almost always be reached. In some cases a third solution was found which was then passed as starting point for a revised NDAC solution. With decreasing quality ratings, the number of undecided cases grew rapidly, and the process was eventually terminated when the yield was too meagre. In the end some 100 cases were decided on the basis of the imaging approach, and a similar number were decided from the quality ratings alone. The remaining cases were not retained as solutions in Part C of the Double and Multiple Systems Annex: they were treated as single stars or given stochastic solutions.

All cases retained as valid double-star solutions, but with an alternative solution from the other consortium, were graded as 'uncertain' in the Hipparcos Catalogue (flag 'D' in Fields H61 and DC5). Some key parameters from the alternative solution are given in the Notes of the Double and Multiple Systems Annex (Volume 11). It should be noted that the agreement of the FAST and NDAC solutions does not preclude the possibility that both are wrong by a multiple of the grid step: the probability of this happening may be non-negligible especially for newly discovered doubles (Field H56 = 'H') with relatively poor solutions (Field H61 = 'C').

Systematic Differences

From the earlier comparison activities (Section 13.6) it was known that small systematic differences existed between the FAST and NDAC results, e.g. in the relative astrometry and photometry. Because the differences are small, especially in comparison with uncertainties in ground-based data, it has usually not been possible to ascertain that one set of data is more accurate than the other. The general principle has therefore been to accept an unweighted mean as the best compromise, also with regard to systematic



Figure 13.16. Comparison of magnitude differences Δm as obtained by FAST and NDAC for double stars with relative astrometry in good agreement. On the vertical axis is the difference $\Delta m_N - \Delta m_F$, on the horizontal the mean value $(\Delta m_N + \Delta m_F)/2$. The line indicates the 'mean' relation adopted in order to correct the individual consortia values before the merging.

errors. Because some solutions were taken from one consortium only, it was nevertheless necessary to determine the systematic differences and apply half the difference, with opposite signs, to each solution.

A significant systematic difference was found in Δm . Figure 13.16 is a plot of $\Delta m_N - \Delta m_F$ versus $\Delta m \equiv (\Delta m_N + \Delta m_F)/2$ for single-pointing doubles with relative astrometry in good agreement; subscripts N and F indicate the consortia. There is a considerable scatter, which is also strongly asymmetric, making it difficult to define a mean relation. The adopted relation, shown by the polygon line, corresponds to the ridge of the two-dimensional distribution. The systematic differences are small ($\leq 0.02 \text{ mag}$) for $\Delta m < 2.5 \text{ mag}$, but increases rapidly for doubles with a larger intensity ratio. There is no clear trend of this effect with separation, except for very small separations ($\rho \leq 0.3 \text{ arcsec}$): in this regime the increasing correlation between the estimation errors in ρ and Δm , combined with selection limits in both parameters, introduces statistical biases which should not be corrected.

A comparison of separations shows some systematic differences for the close binaries (Figure 13.17). Again, statistical biases related to the correlations between ρ and Δm may play a role, and it should also be remembered that $\rho = (X^2 + Y^2)^{1/2}$ in general has a positive bias depending on the random errors in X and Y. Moreover, since the FAST/NDAC averaging is made in the relative coordinates X, Y, the merged ρ does not necessarily fall between ρ_N and ρ_F . Because of the generally good agreement and the difficulty in interpreting the small differences, no systematic corrections were applied to the separations.

The position angles θ were much easier to compare than both Δm and ϱ , because the distribution of differences could be expected to be completely symmetric. Rather late in the merging a significant bias was nevertheless discovered, corresponding to a mean (or median) difference of $\theta_N - \theta_F = -315 \pm 15$ arcsec for the single-pointing



Figure 13.17. Comparison of FAST and NDAC separations for double stars with relative astrometry in good agreement. Each point gives the mean value of $\varrho_N - \varrho_F$ for systems as a function of $\varrho = (\varrho_N + \varrho_F)/2$ in an interval of 0.1 dex. The error bars give the $\pm 1\sigma$ uncertainty of the mean value.

systems (Figure 13.18). The source of this discrepancy was positively identified in one of the reduction chains, as being due to the neglected physical misalignment of the grid with respect to the beam combiner. Both the sign and size of the discrepancy agrees completely with the expected effect calculated from the instrument parameter g_{01} (a quantity which varied between -325 and -333 arcsec over the mission). In this exceptional case, the data were therefore unilaterally corrected by a fixed amount (330 arcsec), after which no significant systematic difference remained in the position angles.

No significant biases were found in any other parameter. In particular the differences in absolute astrometry, including the parallaxes, showed no systematic dependence on other parameters such as primary magnitude, colour, separation, or magnitude difference.

Random Differences

The statistical analysis of the random differences between the FAST and NDAC estimates of various quantities played a fundamental role in the merging process. From this analysis certain correction factors were derived for the standard errors of the consortia estimates, and these factors in turn affected the standard errors assigned to the merged (averaged) data. This correction process depends however rather critically on the assumed statistical correlation between the consortia estimates. The point is illustrated by the following example:

Suppose that a certain quantity (*x*) is estimated by both consortia with the same, but unknown, standard error σ . Furthermore, let ρ be the assumed statistical correlation



Figure 13.18. Comparison of FAST and NDAC position angles (before correction of the bias) for double stars with relative astrometry in good agreement. Each point gives the mean value of $\theta_N - \theta_F$ for systems with $\varrho = (\varrho_N + \varrho_F)/2$ in an interval of 0.1 dex. The error bars give the $\pm 1\sigma$ uncertainty of the mean value.

between the two estimates x_F and x_N . The expected variance of the difference $d = x_N - x_F$ is given by $\sigma_d^2 = 2(1 - \rho)\sigma^2$. Since this variance may be estimated from a homogeneous sample of differences, the standard error of the individual estimates can be calculated as $\sigma = \sigma_d / \sqrt{2(1 - \rho)}$. The merging results in the mean quantity $m = (x_N + x_F)/2$, with variance $\sigma_m^2 = \sigma^2 (1 + \rho)/2$. Consequently the standard error of the merged result may be calculated as $\sigma_m = (\sigma_d/2)\sqrt{(1 + \rho)/(1 - \rho)}$. Clearly this estimate depends strongly on the assumed ρ , especially when the latter is close to +1.

Unfortunately the correlation coefficient cannot in general be reliably estimated, except when external data of comparable (and known!) precision are available for comparison. Speckle data for the relative positions of close binaries provide a possibility, but no such analysis has been made. Only in one instance during the double-star merging was it possible to estimate a correlation between the FAST and NDAC results, without introducing too many assumptions. This concerned the relative motion of the components: \dot{X} , \dot{Y} in FAST, and u_{11} , u_{12} in NDAC. For binaries with a small parallax, no actual relative motion is expected and the observed values could then be fully attributed to measurement noise. In this case a mean correlation coefficient of $\rho = 0.3$ to 0.4 was derived. For the astrometry of single stars, correlation coefficients around 0.7 were typically found. Lower values should be expected for the double stars in view of the much greater differences in methodology for these objects. The single-star correlations may on the other hand be approached for the absolute astrometry, e.g. of the photocentres of close binaries. Thus, reasonable values for the interconsortia correlations of the double-star parameters fall in the interval from 0.3 to 0.7.

Even assuming a rather small correlation ($\simeq 0.3$), the observed scatter of differences in a diagram like Figure 13.16 clearly indicates that the consortia underestimated their standard errors, at least for the relative double-star parameters. Correction factors of 2 to 2.5 in $\sigma_{\Delta m}$, and of 1.5 to 1.6 in the standard errors of the relative positions, were required in order to reconcile the FAST and NDAC solutions within their expected differences. For the absolute astrometry the situation was much better, and the standard errors in position were even somewhat overestimated by FAST.

Along these lines, the random differences in each parameter were extensively analysed, mainly as functions of magnitude and quality ratings. A set of assumed correlations and calculated correction factors resulted, which were then systematically applied to the consortia standard errors before computing the standard errors of the merged data.

Merging of the Double-Star Data

After application of systematic corrections (in Δm only) and correction factors to the standard errors, a file of the merged (averaged) relative parameters was first produced. This was used for the definitive cross-identification with the CCDM Catalogue, resulting in the system and component designations given in the Hipparcos Catalogue. It was also used to correct the photometry of the double stars for the influence of the secondary (Volume 1, Section 1.3.2). The next step of the merging was to compute the astrometric parameters and magnitudes of each component, independently from each consortium, and hence the parameters of the neutral points and the full covariance matrices. This resulted in two files, each basically containing all the data needed for the final catalogue. The final step was then the averaging of these two files for their intersection, and the copying of the remaining data to a third file. This was directly generated in the format of the machine-readable Double and Multiple Systems Annex, Part C.

13.8. Conclusions

The double star treatment of the Hipparcos data is a significant by-product of the astrometric mission. The results collected in the various sections of the Double and Multiple Stars Annex will have far-reaching consequences on the future of astronomical research in this area, both because of the homogeneous sampling of the bright double stars, including the discovery of thousands of new or suspected non-singles, and because of the important and accurate new data given for these objects, such as the parallaxes and magnitude differences. Several ground-based programmes over the coming years will help to consolidate and extend the Hipparcos conclusions on an already sifted sample.

F. Mignard, S. Söderhjelm, J. Kovalevsky, L. Lindegren

14. PHOTOMETRIC TREATMENT

The Hipparcos main catalogue includes information related to the stellar magnitudes as derived from the photon counts recorded by the main detector (the image dissector tube) and the star mapper detectors (B_T and V_T). This chapter provides an overview of the treatment of the photometric data obtained from the reduced image dissector tube photon counts. It describes briefly the processing of the raw photon counts, the definition of the Hipparcos photometric system, the photometric reduction models and techniques used by the two consortia, and finally the results obtained in the comparisons of the final consortia results and their merging to produce the Epoch Photometry Annex and Extension. The photometry obtained from the star mapper detectors is described in Chapter 6 and in Volume 4.

14.1. Introduction

The Hipparcos mission was designed to carry out high precision astrometric measurements for some 118 000 pre-selected stars. During the preparation for the mission it became clear that the on-board detectors (the image dissector tube, and the star mapper detectors) could also be employed to analyse and monitor the intensity of the starlight. The magnitudes obtained for each programme star with the main detector would be of relatively high precision, defined in an all-sky uniform wide-band filter, referred to as Hp.

The main detector was positioned behind a modulating grid in the focal plane of the instrument. It registered the transits of selected objects through photon counts over 1/1200 s intervals. The photon counts reflected the stellar intensity, modulated by the grid. The reduction of these photon counts to a simple modulated signal is described in Chapter 5. The result of this reduction was a signal that represented the estimated values of the observed counts N_k as:

$$E[N_k] \equiv I_k = I_b + I_s \ 1 + M_1 \cos g_1 + M_2 \cos(2g_1 + 2g_2)$$
[14.1]

where I_s represents the intensity of the object, I_b the background intensity, g_1 and $g_1 + g_2$ the phase of the first and second harmonics, and M_1 and M_2 the modulation coefficients of the first and second harmonics. Two measurements for the intensity of the object could be derived from Equation 14.1: the zero level or dc component, given by $I_s + I_b$ and the amplitude of the first harmonic or ac component, given by I_sM_1 (NDAC) or a weighted average of I_sM_1 and I_sM_2 (FAST). Both I_s and $M_{1,2}$ were functions of the position in the field of view and of the colour of the object. These relations, in addition,

changed as a function of time. In order to use the dc component, it was necessary to define and subtract the background contribution I_b . The aim of the photometric calibrations was to describe these corrections and to derive from the observed dc and ac components of Equation 14.1 the best estimates of the magnitude Hp:

$$Hp = 2.5 \log I_s + \text{constant}$$
[14.2]

Equation 14.1 was always solved for data accumulated over an interval of 32/15 s, referred to as a frame transit. It took a star 9 to 10 of these intervals to cross the field of view, a duration referred to as a field transit. The photometric reductions were carried out using the frame transits, but for the final data those frame transits belonging to the same field transit were combined to provide one measurement of the stellar magnitude. A total of 13 million such magnitudes were obtained, an average of 110 per star. The typical accuracy was 0.01 mag for an 8 mag star. Data were rather unevenly distributed over the 37 months of the mission, and large variations occur in the number of measurements available (up to 385, depending mainly on ecliptic latitude). The final data, made available as the Hipparcos Epoch Photometry Annex plus Extension, represent the largest homogeneous multi-epoch all-sky photometric survey to date.

The photometric data were initially reduced using photometric standard stars selected on the basis of pre-launch ground-based information, transformed to the pre-launch estimate of the passband *Hp*. This set of standard stars was updated during the mission to incorporate new stars and reject objects found variable on the basis of the Hipparcos measurements. The new standards were recalibrated to provide an improved representation of the *Hp* passband. All data were re-reduced using 22 000 standard stars, covering a colour range -0.3 < V - I < 1.8 with a pronounced peak in the distribution for V - I = 0.6 and two smaller peaks at V - I = 0.0 and V - I = 1.0. The V - I index used here is the Cousins' colour index, also referred to as $(V - I)_C$ below.

14.2. The Photometric System

The Hipparcos passband corresponded primarily to the spectral response of a S20 photocathode combined with the transmission of the optics. A pre-launch definition was used to predict an Hp value for each programme star from the existing magnitudes and colours. After the first in-orbit calibration a revised definition was obtained and used to produce predicted Hp values for the final set of standard stars, with a precision better than 0.01 mag. This was sufficient to define the magnitude scale with a precision of 0.001 mag twice a day. The Hp band is shown in Figure 14.1, as a function of the wavelength, superimposed on some commonly used broad-band filters. One should note the marked extension of the sensitivity toward the extreme red. The numerical values for the passband are given in Section 1.3 of Volume 1.

The V_J band has more or less the same effective wavelength as Hp, so that the amplitude of $Hp - V_J$ is smaller than 0.2 mag for stars with $(B - V)_J < 1.5$. Several transformation laws were derived to link the Hipparcos photometric systems to the V_J system according to star colours. The difference $Hp - V_J$ as a function of $(V - I)_C$ was very well defined in the range $-0.4 < (V - I)_C < 3.0$, from classical photometry, with uncertainties less than 0.01 mag. The extension to redder stars was obtained from dedicated observations of a set of Mirae with a CCD chain equipped with the Cousins' *I* band and Johnson-Geneva *V* band. Thus the transformation was extended up to $(V - I)_C = 5.4$ with an uncertainty


Figure 14.1. Normalized response curve for the Hipparcos Hp passband (solid line), superimposed on (from left to right) the Johnson B_J , V_J and Cousins' R and I passbands.

V - I	Hp – V	V-I	Hp-V	V-I	Hp-V	V-I	Hp-V
50	182	.40	.082	1.30	.147	2.90	-0.065
45	164	.45	.089	1.35	.147	3.00	-0.090
40	145	.50	.097	1.40	.140	3.20	-0.155
35	126	.55	.106	1.45	.135	3.40	-0.235
30	107	.60	.115	1.50	.128	3.60	-0.325
25	088	.65	.125	1.60	.116	3.80	-0.425
20	070	.70	.133	1.70	.106	4.00	-0.535
15	053	.75	.141	1.80	.096	4.20	-0.65
10	035	.80	.147	1.90	.090	4.40	-0.76
05	018	.85	.154	2.00	.082	4.60	-0.87
.00	.002	.90	.159	2.10	.072	4.80	-0.98
.05	.018	.95	.164	2.20	.060	5.00	-1.10
.10	.036	1.00	.167	2.30	.048	5.50	-1.38
.15	.050	1.05	.168	2.40	.032	6.00	-1.66
.20	.061	1.10	.168	2.50	.018	6.50	-1.94
.25	.068	1.15	.165	2.60	.001	7.00	-2.22
.30	.072	1.20	.162	2.70	018	8.00	-2.78
.35	.076	1.25	.157	2.80	040	9.00	-3.34

Table 14.1. Relationship between $(Hp - V_J)$ and $(V - I)_C$ for types O to G5, class V to II and red giants of types G5III to M8III.

V - I	Hp – V						
0.70	.133	1.10	.138	1.50	.108	2.20	.050
0.75	.137	1.15	.133	1.55	.105	2.30	.041
0.80	.140	1.20	.129	1.60	.102	2.40	.034
0.85	.142	1.25	.125	1.70	.093	2.50	.022
0.90	.144	1.30	.122	1.80	.085	2.60	.001
0.95	.146	1.35	.119	1.90	.076	2.70	023
1.00	.145	1.40	.115	2.00	.066		
1.05	.142	1.45	.112	2.10	.058		

Table 14.2. Relationship between $(Hp - V_J)$ and $(V - I)_C$ for G, K, M dwarfs with (V - I)c > 0.70.

Table 14.3. Relationship between $(V - I)_C$ and $(B - V)_J$ for early type stars and red giants.

V-I	B-V	V-I	B-V	V - I	B-V	V-I	B-V
-0.379	-0.345	0.190	0.174	0.804	0.765	1.334	1.365
-0.299	-0.276	0.252	0.228	0.847	0.825	1.392	1.413
-0.231	-0.216	0.331	0.291	0.897	0.893	1.473	1.464
-0.168	-0.164	0.412	0.351	0.946	0.960	1.567	1.527
-0.105	-0.119	0.482	0.415	0.995	1.021	1.617	1.550
-0.050	-0.072	0.553	0.482	1.050	1.088	1.644	1.568
0.002	-0.020	0.617	0.543	1.107	1.143	1.724	1.583
0.040	0.021	0.667	0.597	1.155	1.196	1.831	1.604
0.072	0.062	0.722	0.659	1.211	1.253	1.882	1.615
0.124	0.110	0.770	0.717	1.271	1.311	2.021	1.635

Table 14.4. Relationship between $(V - I)_C$ and $(B - V)_J$ for K and M dwarfs.

V - I	B-V						
0.631	0.550	1.042	0.999	1.567	1.348	2.725	1.650
0.670	0.601	1.103	1.050	1.645	1.390	2.874	1.700
0.707	0.648	1.175	1.100	1.785	1.445	3.008	1.750
0.747	0.699	1.244	1.149	1.905	1.472	3.144	1.800
0.788	0.749	1.333	1.199	2.054	1.498	3.478	1.900
0.840	0.800	1.386	1.228	2.255	1.524	3.630	1.955
0.893	0.850	1.410	1.250	2.440	1.550	4.100	2.114
0.941	0.898	1.494	1.300	2.544	1.575	4.328	2.198
0.997	0.949	1.535	1.326	2.601	1.600	4.630	2.300

of 0.03 to 0.05 mag. The values up to $(V-I)_C = 9.0$ resulted from a linear extrapolation and covered the whole range of $(V-I)_C$ colours in the Hipparcos Catalogue.

The final relationships between $(Hp - V_J)$ and $(V - I)_C$ for O, B, A, F, and G<G5 stars with low reddening and for G5 to M8 giants are given in Table 14.1. Table 14.2 gives the relationship for K and M dwarfs. The difference between both relationships is usually less than 0.03 mag and may be disregarded for most of the applications.

For the sake of completeness, it is useful to provide links with photoelectric photometry. The colour index $(V - I)_C$ is related to the Johnson $(V - I)_J$ as follows:

$$(V - I)_J < 0.0 \implies (V - I)_C = 0.713 (V - I)_J$$

$$0.0 < (V - I)_J < 2.0 \implies (V - I)_C = 0.778 (V - I)_J$$

$$2.0 < (V - I)_J < 3.0 \implies (V - I)_C = 0.835 (V - I)_J - 0.13$$

The relationships between $(V - I)_C$ and $(B - V)_J$ are summarized in Tables 14.3–14.4 for early-type stars and red giants, and for late type stars, respectively.

14.3. The Photometric Data

The Signal

As shown in the introduction, the photometric information obtained from the reduced image dissector tube photon counts consists of a dc component, $I_s + I_b$ and an ac component, I_sM_1 (or a weighted combination of I_sM_1 and I_sM_2), obtained over a frame transit of 32/15 s. For single stars the photometric information was entirely contained in both the ac and dc components, while for multiple stars, there was no such simple separation between the amplitudes of the modulated signal and the photometric information. These stars had to be processed separately (see Chapter 13).

The response I_s for an 8 mag star with $(V - I)_C = 0.5$ mag, in the centre of the field, dropped from 6600 Hz at the start of the mission to 4000 Hz at the end. With $(V-I)_C = 2.5$ mag it dropped from 5500 Hz to 4000 Hz. These changes with time were due to the 'ageing' of the optics (see Section 14.5). The background I_b varied between 20 and 70 Hz, except during the crossing of the radiation belts when it reached higher values. As the image of a star crossed the grid over an angular distance of 0°.9 it was observed on average in 9 observational frames. After the calibration was applied to each frame, either (FAST) the median of the 9 frame magnitudes or (NDAC) a weighted mean (with a rejection of outliers) were computed to produce the Hipparcos magnitude of the field transit.

The ultimate precision, if only limited by the photon noise, would be given by:

$$\sigma^2(I_s) = \frac{I_s + I_b}{T}$$
[14.3]

where T is the time, expressed in seconds, allocated to the star during a field transit. This was of the order of 4 seconds on the average, less for brighter stars and more for fainter ones. *T* also depended on the number of other stars also competing for observing time. From this expression we can derive:

$$\sigma(Hp) \approx \frac{\sigma(I_s)}{I_s} \approx \frac{1}{\sqrt{I_s T}}$$
[14.4]

which led respectively to an internal precision of 0.005, 0.01, 0.02 mag for stars of magnitude 6, 8, 10. While a very careful calibration has allowed this precision for stars fainter than 8 mag to be reached, it was not reached for bright stars because of small systematic effects which remained uncalibrated.

The Two Magnitude Scales

As explained above, the photometric information is contained in both the unmodulated (dc) intensity $I_s + I_b$ and in the modulated (ac) amplitudes $I_sM_{1,2}$. The resulting magnitudes are designated by Hp_{dc} and Hp_{ac} . For single stars both magnitudes are identical in expectation. However, the relative error on the estimated amplitude for a sinusoidal signal is $\sqrt{2}$ times larger than the relative error on the mean level. Since the value of the modulation coefficient was $M_1 \approx 0.7$, the absolute errors on the ac components were on average twice those on the dc components.

The two magnitudes were calculated at every field transit and appear, together with their standard deviation, in the Hipparcos Epoch Photometry Annex (HEPA) and the Hipparcos Epoch Photometry Annex Extension (HEPAE). Strictly, only Hp_{dc} should be used as a realization of the Hp scale. A comparison between Hp_{ac} and Hp_{dc} can be used to test the hypothesis that the object observed is a single point source. Any deviation from this assumption, such as duplicity or extension, will show up from a comparison of the two magnitudes. Occasionally, an accidental duplicity arose when a parasitic star from the complementary field of view was mapped on the focal plane near to the programme star. In this case, Hp_{dc} appears brighter than the observations made in the other field of view (where the same superposition could not take place) and, at the same time, Hp_{ac} becomes fainter than Hp_{dc} (i.e. $Hp_{ac} > Hp_{dc}$).

The ageing corrections (see Section 14.5) applied to the ac magnitudes were less rigorously determined than the corrections for the dc magnitudes. Detailed comparisons between ac and dc magnitudes are meaningless for very red stars, as their ac magnitudes can still contain small uncorrected, colour related systematic errors.

The Timescale

In the Hipparcos Epoch Photometry Annex all the field transits of each star are listed in chronological order of barycentric Julian days: BJD – 2 440 000. This timescale is based on Terrestrial Time (TT) as explained in the general introduction to the Hipparcos Catalogue. Times were corrected to the barycentre using the barycentric Earth ephemeris (Section 12.1) to correct for light-time effects. The epoch t_E of arrival of the starlight on Earth was transformed into t_B for the arrival at the solar system barycentre by:

$$t_{\rm B} - t_{\rm E} = \mathbf{b}_{\rm E}' \mathbf{u} c^{-1}$$
 [14.5]

where $\mathbf{b}_{\rm E}$ is the barycentre–Earth vector, \mathbf{u} the unit vector in the direction of the star, and *c* the speed of light. This well known correction had a range between -5.6 and +5.6 minutes (limited by the constraints of the scanning law) and was evaluated for every field transit.

14.4. The Calibration

The Principles

The intensities $I_s + I_b$ and $I_s M_1$, $I_s M_2$ recorded at each observational frame were affected by instrumental effects, which had to be taken into account in order to obtain the stellar magnitude. The following effects were calibrated and removed:

- (1) the inhomogeneity of the sensitive surface and of the residual defects of the optics made the sensitivity of the detector variable over the field of view. This was represented mathematically in various ways;
- (2) the ageing of the optics and of the detector caused a steady decrease in the overall sensitivity, with a marked chromatic dependence. The ageing was more pronounced in the blue than in the red. This was represented by the colour-terms in the calibrations;
- (3) the sensitivity was not exactly the same in the preceding and following field of view. Solutions had to be made separately for the two fields of view.

In addition to these instrumental effects, the background contribution had to be calibrated and removed from the dc component. A calibration programme aimed at converting the observed intensity at each observational frame into Hp was run on all data collected during an orbital period. Due to a 1 to 3 hour interruption of the observations during the perigee passage, the orbital period became a useful unit for calibrations and definition of data sets. The same unit was also used to constrain the data used in the great-circle reductions.

Only measurements of standard stars were used in the calibration. These measurements were still subjected to a series of quality tests, in order to reduce the number of faulty data-points. A linear model, describing generally small corrections to an underlying model, was used to fit the data, using a standard least-squares procedure.

Two different methods were used by the FAST and NDAC consortia in processing the photometric data. In FAST the fits were done by adjusting the computed intensity of the standard stars to the observed intensity, while NDAC fitted their model in magnitude scale. The variables used in the models were somewhat similar, but the techniques employed were different enough to justify a separate presentation of the FAST and NDAC procedures.

The Basic Model Used by FAST

Let $\tilde{I} = I_s + I_b$ be the observed mean intensity on a particular observational frame of a standard star and I_{ref} the computed intensity derived from its *a priori* magnitude. With $(1 - \epsilon(G, H, C))$ representing the *a priori* model correction of the intensities as a function

of colour (C) and position in the field (G, H), the functional relationship between these quantities was given by:

$$\tilde{I}(1-\epsilon) = I_{\text{ref}} \prod_{k=1}^{13} a_k X_k + b_0 + b_1 f_1(\lambda - \lambda_{\odot}, \beta) + b_2 f_2(b)$$
[14.6]

where the X_k are listed in Table 14.5. The determination of the colour terms was given great attention because of their importance in defining the photometric system. The set of standard stars was sparse for stars redder than 1.5 mag, although the calibration had to be applied to many late-type stars included in the Hipparcos programme. In the FAST solution, to limit the risk of extrapolating the calibration formulae outside the range of validity, additional calibration red stars were included. This was done by adding single stars brighter than Hp = 9.5 mag and with colours in the range 1.6 mag < V - I < 2.2 mag, provided the *a priori* standard deviation in magnitude was less than 0.009 mag and $\sigma_{V-I} < 0.05$ mag. This was sufficient to constraint the quadratic terms up to V - I = 2.2 mag. Then, in the application of the formula, the extrapolation was limited to the linear term above this colour and up to V - I = 3 mag, preserving the continuity at each boundary.

The final three coefficients in Equation 14.6 represent the modelling of the unmodulated light, where λ , β are the ecliptic longitude and latitude of the star, λ_{\odot} the ecliptic longitude of the Sun and *b* the galactic latitude of the star. These three terms were present for the calibration of Hp_{dc} but obviously not for that of Hp_{ac} . Each calibration spanned an orbital period (a theoretical maximum of 10 hours but in practice 6 to 8 hours of data, referred to as a data set).

The coefficients (three for each field of view) were kept constant within a data set, which means that the mean background was likely to be underestimated during the crossing of the van Allen belts at either end of an orbit. The information for these coefficients was mainly derived from observations of faint stars. Extra faint stars were added to those originally included in the set of standard stars, in such a way that they did not directly influence the other coefficients. The stars added were single stars fainter than Hp = 10.7 mag and with $\sigma_{Hp} < 0.007$, V - I < 2 mag, $\sigma_{V-I} < 0.1$. Because of the structure of the observation equations this proved to be a very efficient method for determining the background terms without affecting the other terms.

The unmodulated background in the Hipparcos photometry had a typical strength ranging from 20 to 70 Hz, corresponding to a correction of 0.03 to 0.10 mag for a star of 10 mag. It was not constant over the sky and originated primarily from the following sources:

- the detector thermal noise which gave roughly a count level of 20 Hz even when no star was in the instantaneous field of view;
- the zodiacal light brought about by the diffusion of the sunlight by interplanetary dust which exhibited a strong dependence on λ − λ_☉;
- the scattered faint stars produced an unmodulated count as soon as there were more than one star in the immediate vicinity (< 30 arcsec) of the programme star, irrespective of whether the perturbing objects belonged, or did not belong, to the same viewing direction as the programme star. This effect was at a maximum when one field of view was in the galactic plane and dropped off sharply with galactic latitude;
- radiation associated with the van Allen radiation belts.

In FAST a model for the starlight based on the mean number of stars fainter than V = 12 mag as a function of the galactic latitude was considered and fitted to an analytical function $\Phi(b)$ which represented fairly well the normalized galactic luminosity profile, with $\Phi(b) = 1/[1 + 5.3 (\sin b + \sin^2 b)]$.

The actual model made use of a function $\tilde{\Phi}(b)$ that averaged out to zero over a great circle and at any time the contribution of the preceding and the following field of view were added up, so that:

$$f_2(b) = \tilde{\Phi}(b_p(b)) + \tilde{\Phi}(b_f(b))$$
 [14.7]

where b_p and b_f were respectively the galactic latitude of the preceding and following field of view.

The contribution of the zodiacal light was taken from published mappings and given a tractable analytical representation as:

$$f_{1}(\lambda - \lambda_{\odot}, \beta) = \frac{1}{[1 - \cos(\lambda - \lambda_{\odot})\cos\beta]} \frac{1}{[1 + \{1 + [1 + \cos(\lambda - \lambda_{\odot})]^{2}\}|\sin\beta|]}$$
[14.8]

and was computed for the two fields of view. The three coefficients b_0 , b_1 and b_2 of Equation 14.6 were derived as explained above for each data set, and used to compute the background contribution for each observation, using Equations 14.7 and 14.8. For an individual star, these values were essentially constant over a field transit. The background determined in this manner was then removed from the raw data. Its value appears in the Hipparcos Epoch Photometry Annex Extension together with the value used by NDAC, so that anomalous deviations of the magnitude that might originate from the background can be traced back.

The *a priori* model $\epsilon(G, H, C)$ took into account the spatial frequency of the sensitivity of the detector in the form of three fixed two-dimensional maps that were established from a large amount of data covering several weeks of observations. Each map was appropriate for a specific colour range and had a resolution of 0.02 in each direction. Thus the calibration described by the model above was in fact differential, as most of the spatial variation was accounted for by the maps.

The Basic Model Used by NDAC

In NDAC the model used was not fundamentally different from that of FAST except for the background modelling, which incorporated data acquired with the star mapper. The mathematical expression (see Equation 14.6 for FAST) was given by:

$$-2.5 \log_{10}(\tilde{I} - I_b') - Hp = \sum_{k=1}^{15} a_k Y_k$$
[14.9]

where Hp is the calibration magnitude of the standard and the meanings of Y_k are listed in Table 14.6. The colour coefficients were applied without restrictions to stars of all colours. This was known to be wrong for very red stars, but simplest to correct *a posteriori* after the full calibration of the passband had been obtained (see Section 14.5). I'_b was the modelled *a priori* background described below.

The calibrations were carried out in magnitude space. Since the accuracies of the counts were so high (generally better than 10 per cent), this procedure did not cause any bias larger than 0.001 mag in the frame transit magnitudes. Performing the calibrations in magnitude space avoided a large range of values entering the least-squares solution.

Parameter <i>k</i>	Variable X _k	Meaning
1	1	Zero point
2	G	Grid abscissa
3	Н	Grid ordinate
4	$G \times H$	
5	$P_2(G)$	2nd Legendre polynomial in G
6	$P_2(H)$	2nd Legendre polynomial in H
7	$P_4(G)$	4th Legendre polynomial in G
8	$P_4(H)$	4th Legendre polynomial in H
9	$P_2(G) \cdot P_2(H)$	
10	C = V - I - 0.65	Colour
11	$P_2(C)$	
12	$C \cdot G$	mixed term
13	$C \cdot H$	mixed term

 Table 14.5. Instrumental parameters for the photometric calibration of FAST.

Parameter	Variable	Meaning
k	Y_k	
1	1	Zero point
2	C = V - I - 0.5	Colour
3	C^2	
4	C^3	
5	G	Grid abscissa
6	Н	Grid ordinate
7	$C \cdot G$	Mixed term
8	$C \cdot H$	Mixed term
9	$10^{0.4(Hp-8)}$	Background term
10	R_2	Radial terms
11	R_3	
12	R_4	
13	R_5	
14	R_6	
15	R_7	

 Table 14.6. Instrumental parameters for the photometric calibration of NDAC.

Radial distance	0.0	0.1	0.3	0.5	0.7	0.9	1.1	1.3
Variable name	R_0	R_1	R_2	R_3	R_4	R_5	R_6	R_7

Also, since most of the influences on the intensities act as a factor, when transformed to magnitudes they become offsets, which are generally easier to reconstruct.

While no significant bias was present in the frame transit magnitudes, a small bias of order 0.01 mag was found to be present in the calibrations due to the weighting scheme that had to be used. The size of this bias was determined by Monte Carlo simulation and a correction was applied.

Two features of this model require further explanation: the radial grid dependence and the background model, I'_{b} .

The model chosen for the calibration of the grid dependence was a radial one, using linear interpolation, in which each observation equation contains terms representing the relative fractions of the two nearest radial reference points. The positions of the reference points in grid coordinate units are given in Table 14.7. It was based on an examination of residuals accumulated as a function of position in the field of view and colour, using several weeks of data.

The central location for the radial dependence was determined to be (0.03, 0.11) in grid coordinate units. This was then considered to be the origin for all radial measurements. The consistency of the central location was confirmed by analysis of maps of residuals as a function of time.

Although the model contained R_0 and R_1 points, they were not solved but set to zero. The reasons for this were that R_0 was effectively the zero point Y_1 , and a zero derivative at R = 0 was required which implied that R_1 had to be equal to R_0 . Points outside R = 1.3 were calculated as an extension of the line between 1.1 and 1.3. Thus, this radial model contributed 6 parameters to the overall calibration model.

The variation of the background as a function of time was fairly complicated. Since it generally only affected stars fainter than $Hp \simeq 9$ mag the calibration of it for each transit was difficult due to the relatively small number of standard stars that were affected. In the NDAC solution, only the offset term of the background model (Y_9 in Table 14.6) was solved for. The remaining three parts of the model were calculated from the star mapper background count and the positions on the sky of the two fields of view of the satellite. The three contributions to the background accounted for in the NDAC model were:

- (1) van Allen radiation belts (maximum 100 Hz);
- (2) zodiacal light (peak 30 Hz);
- (3) the Milky Way (peak 20 Hz).



Figure 14.2. An example of a stretch of data with poor satellite pointing. The data points show the value of the background as measured by the satellite and the line shows the independently derived NDAC background model. All three components of the model are noticeable in this diagram. The position of the Zodiacal light and Milky Way contributions are indicated respectively by the letters Z and G at the bottom of the diagram. The van Allen radiation belt contribution can be seen by comparison with the star mapper background shown in the inset diagram (the units are the same as for the main diagram).

The parameters used in this part of the model were derived mainly from stretches of data with poor satellite pointing so that the background was being sampled rather than the target stars. An example of such a stretch of data is given in Figure 14.2.

The largest contribution was caused by radiation associated with the van Allen radiation belts and was very variable from orbit to orbit. It mainly affected data at the start and end of a data set. Using data such as in Figure 14.2, a correlation was found between the radiation-induced star mapper background and the main mission background. The star mapper background was measured regularly, which allowed the radiation contribution to the image dissector tube background to be determined. If this background contribution was larger than approximately 100 Hz the frame transit was rejected.

While the contributions of zodiacal light and the Milky Way were noticeable in the star mapper data too, they did not scale in the same way as the radiation contribution. It was thus necessary to assume that these were approximately constant over time and just functions of sky position. The forms adopted were derived mainly from the data and checked against what was expected from the literature. The form of the Zodiacal light contribution to the background model was given by:

30 Hz × max 0.0,
$$1 - \frac{|\beta|}{45^{\circ}}$$
 × max 0.085, $1 - \frac{(|\lambda - \lambda_{\odot}| - 40^{\circ})}{60^{\circ}}$ [14.10]

and that for the galactic contribution by:

20 Hz ×
$$\cos \frac{l}{2} \times \frac{1}{1 + (2|b|/15^{\circ})}$$
 [14.11]

where (λ, β) and (l, b) are the ecliptic and galactic coordinates respectively of the centres of the field of view. Values were calculated for both the preceding and following fields of view.

The value given in the Hipparcos Epoch Photometry Annex Extension for each transit is the combination of all three contributions along with the background offset determined, Y_9 .

The calibration to convert the counts to Hp magnitudes was a least-squares solution using the method of Householder transformations (Bierman 1977), with weights applied to the observation equations according to the estimated errors of the observations.

The calibrations were carried out on data accumulated over approximately one orbit (10 hours). Separate solutions were carried out for both the preceding and following fields of view and for both Hp_{dc} and Hp_{ac} . The solution was calculated iteratively with an *a posteriori* four sigma filter being applied to the data to reduce the effect of outliers.

The use of the Householder transformations made the application of a running solution between the calibrations particularly easy. The prime advantage of this was to ensure the stability of the calibration and to safeguard against the effect of outliers biasing the calibration coefficients. However, this also meant that care had to be taken when it was expected that the coefficients could undergo sudden changes. This was likely to occur after a refocusing of the satellite optics or after a long period of satellite inactivity.

When such sudden changes occurred, it was necessary to reinitialize the running solution in some way. After a refocusing, only a partial reinitialization was required since only a few coefficients were affected by this, while after a long period of satellite inactivity a full reinitialization was carried out. Figure 14.3 shows the effect of the reinitializations on M_1 , the modulation coefficient of the first harmonic (see Equation 14.1), as derived from the calibration coefficient for the dc and ac components. While most discontinuities were directly related to refocusing of the optics, on one occasion (on day 755, see also Chapter 2, Table 2.1), it was due to restoring proper thermal conditions after a heater failure.

One of the effects not accounted for in the calibrations was the effect of depointing. Inaccurate satellite pointing caused the instantaneous field of view not to be positioned exactly on the target star. This would cause a small attenuation in the intensity of the star. A study was carried out on the consequence of not carrying out a correction for this effect and it was shown that it would only add an additional 0.0006 mag scatter to a field transit.

The calibrations were applied to each frame transit and these were then combined to form a field transit. For stars of extreme colour (redder than $V - I \simeq 2.0$ mag) the calibration had to be extrapolated due to the low number of calibrating standard stars



Figure 14.3. The effect of the reinitializations on the modulation coefficient of the first harmonic in the preceding field of view. The general zig-zag pattern is caused by the partial reinitializations carried out after each refocusing. The jump occurring at day 1400 was due to a full reinitialization carried out after the loss of the gyros. This plot is for a colour of V - I = 0.5 mag.

available at these colours. The errors caused by the instabilities inherent in this procedure were later removed during the ageing corrections described in the next section.

The combination of the frame transits to form a field transit was carried out in intensity space. By doing this, biases that might arise from the conversion from intensities to magnitudes were minimized. Also at this stage, outlying frame transits within a field transit were rejected at the 3σ level. If insufficient transits remained, the entire field transit was rejected. Between 1 and 2 per cent of the data was rejected in this manner. The average that was formed out of the remaining transits was a weighted mean since the accuracy of a frame transit varied according to the observing time allocated to it.

14.5. Final Corrections

This section describes corrections applied to the reduced data. These corrections came from three sources:

- (1) zero-point shifts;
- (2) passband definition, and ageing correction;
- (3) final field distortion corrections.

These corrections, although different in detail, were applied to both the FAST and NDAC reduced data after all reductions were completed. All three were derived using the accumulated results from the reductions.

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Figure 14.4. Time variation of the response in the centre of the field for 8 mag stars of two different colour indices, showing the effect of the ageing of the detector chain through the decrease in response over the mission and the difference in this decrease for stars of different colour.

Zero-point Corrections

After the calibrations had been completed, various checks were carried out on the validity of the photometry. It soon became apparent that the zero points $(X_1 \text{ and } Y_1)$ were varying by around ± 0.01 mag within an orbit. Further investigation showed that these shifts were present in the data of both consortia and that they were not a function of the field of view. Various explanations were considered and investigated, but all were rejected. Neither solar activity nor attitude determination problems matched the pattern of shifts. The temperature of the satellite did not correlate with this effect either. Problems relating to the Milky Way or bright stars were thought unlikely since they would have caused a recurring pattern which was not seen.

The overall effect of these shifts on the mission was equivalent to an additional scatter of 0.003 mag to all transits. Even though the cause of these shifts was not known, it was felt that they had to be corrected. These corrections were applied independently by FAST and NDAC as small magnitude offsets. No investigation (nor correction) was carried out on shifts in the ac component calibrations due to the noisier nature of the data.

Ageing Corrections

The main detector chain contained a large number of optical elements which were affected by radiation during the mission. The radiation resulted in chemical changes in the optical elements, leading to loss of transmission. This transmission loss was most noticeable on the blue wing of the transmission curve, as can be seen from Figure 14.4, which shows the response at the centre of the field of view for stars of different colour over the length of the mission.



Figure 14.5. The pseudo-colour C as a function of $(V - I)_C$.

Table 14.8. Definition of the pseudo-colour index $C = a + b(V - I) + c(V - I)^2 + d(V - I)^3$ for different intervals of V - I.

Interval	а	b	с	d
$V - I \le 0.85$	-0.48729	0.98554	-0.31968	0.592
$0.85 < V-I \leq 2.00$	-1.03936	2.14720	-0.416	0.0
V - I > 2.0	0.5152 n	0.592	-0.0256	0.0

The changing passband response was provisionally accommodated through introducing a $(V - I)_C$ dependence in the photometric reductions (as described in the preceding section). However, the coverage in $(V - I)_C$ by the standard stars was insufficient to describe the colour relations for very red $((V - I)_C > 2.0 \text{ mag})$ stars. In addition, most stars with very red colours tend to be variable. The *a posteriori* calibration of the *Hp* passband was described in Section 14.2. Using this calibration together with ground-based data on large amplitude red stars (observations by the American Association of Variable Star Observers and specific photoelectric measurements of primarily Mira stars), predicted values for epoch photometry of these red stars were produced. These values were compared with the observed epoch photometry produced by the two consortia, and the observed differences were translated into a new colour correction, which was applied *a posteriori* after removing the old colour correction. This colour correction reduced all the photometric data to the reference passband defined for 1 January 1992.

As a result of the chromatic corrections, the use of a wrong colour for a star in the data analysis shows up in the fully calibrated photometric data as an almost linear drift of the magnitude, with dHp/dt < 0 if the true colour is redder than was assumed in the reductions. The epoch photometry can be corrected for such errors. The procedure is as follows. Take the $(V - I)_C$ value used in the data-reductions (Field H75), and transform it into a pseudo-colour C(old) according to the rules described in Table 14.8 and shown in Figure 14.5. Do the same with the improved $(V - I)_C$ index (e.g. as available in Field H40), to obtain C(new). The corrections for the Hp magnitudes are then defined as:

$$\delta Hp = Hp(\text{old}) - Hp(\text{new}) = -F(t) \quad C(\text{old}) - C(\text{new})$$
[14.12]

	dc	d	lc	ac		
	approx	preceding	following	preceding	following	
Stand.dev.	0.0031	0.0012	0.0012	0.0017	0.0017	
const	0.0084	0.00303	0.00873	0.0416	0.0478	
$(t - t_0)$	-0.0537	-0.04022	-0.04098	-0.0431	-0.0389	
$(t - t_0)^2$		0.09000	0.07720	0.1317	0.0338	
$(t - t_0)^3$		-0.18863	-0.13772	-0.2384	-0.8277	
$(t - t_0)^4$		-0.6746	-0.6735	-1.9387	-2.9383	
$(t - t_0)^5$		0.598	0.1322	-2.9443	-3.6662	
$(t - t_0)^6$		2.018	2.052	-1.3327	-1.5601	
$(t - t_0)^7$		0.340	1.989			
$(t - t_0)^8$		-0.742	0.553			

Table 14.9. Definitions of the colour correction factors F(t) expressed as a polynomial in $t - t_0$, for dc and ac magnitudes. t and t_0 are measured in units of 1000 JD, $t_0 = 2448.6225 = 1$ January 1992.

where F(t) is a function of time and of field of view. Table 14.9 gives approximations to F(t) for dc and ac magnitudes, and the associated standard errors. These standard errors are equivalent to the expected noise on the Hp corrections for a colour correction of 1 mag. Also given is an approximate correction for the dc magnitudes, ignoring the field of view differences, which can be used without the need to access the Hipparcos Epoch Photometry Annex Extension files. As can be seen from Figure 14.5, a correction in $(V-I)_C$ for a blue star will cause a larger correction in the pseudo-colour than a similar correction for a very red star. Information on the field of view can be obtained from the Extension files, where the ac magnitudes are also found. Figure 14.6 shows the observed values of F(t) and the approximating curves. For data obtained beyond day 1300 (JD 2 448 800) it is advised not to use the ac magnitudes for detailed comparisons. The ac magnitudes were sensitive to the focal setting of the telescope, which itself was sensitive to temperature variations. Such variations were quite strong when the telescope was moved to sun-pointing mode, which occurred 3 times after JD 2 448 800.

The data presented in Tables 14.8 and 14.9 also allow in principle for a redefinition of the colour correction in case more accurate data becomes available, but such corrections, if at all justifiable, are expected to be very small.

Field Distortion Corrections

After the calibrations had been carried out an investigation was made into the variations of Hp as a function of the field coordinate H (the direction of the grid slits). A systematic variation with a peak-to-peak variation around 0.008 mag was detected in the NDAC data. Although this would only add an additional scatter of around 0.002 mag to the field transits it was felt that this correction should be made since it was straightforward and would not adversely delay the processing. The corrections made were a function of H, $(V - I)_C$, time and field of view. An example of the size of the correction is given in Figure 14.7. No correction as a function of G, the perpendicular field coordinate, was possible since a field transit consists of an average over a range of G values, i.e. the frame transits. The need for the correction is a consequence of not using a detailed mapping function in the early stages of the calibration. This was confirmed by analysing a residual array that was accumulated as a part of the NDAC diagnostics. By summing



Figure 14.6. The colour correction factor as derived for the dc (top) and ac magnitudes (bottom). The preceding field of view is indicated with open squares, the following field of view with crosses.

this array over G a similar correction could be found. This is shown as the dotted line in Figure 14.7. The diagnostic data could not be used directly since it was not detailed enough. Corrections were also carried out to the FAST data, but since some form of mapping function was applied by FAST, the corrections were smaller. No corrections were applied to the ac component.

14.6. Parasitic Transit Detections

Due to the design of the satellite it was possible for a star from the other field of view to be located by chance at almost the same position on the detector as the star being observed. This had the effect of making the star being observed appear brighter. Since



Figure 14.7. An example of the magnitude corrections applied to the NDAC data as a function of field coordinate, *H*. This is at mid-mission for the preceding field of view and a star of V - I = 0.5 mag. The dotted line shows the correction deduced from the mapping diagnostic for approximately the same parameters.

this would have had a harmful effect on the variability analysis carried out on the data, it was important that these transits were identified. In order to do this, two methods were employed.

The first method was to search the Tycho Input Catalogue for possible contaminants given an accurate position for the other field of view calculated from the attitude determination. Using this information, along with the attenuation profile of the instantaneous field of view (see Section 5.1), it was possible to determine whether a transit was contaminated. The limit chosen was a perturbation of 0.01 mag. About 100 000 transits were flagged this way. The magnitude and colour (if available) of the contaminating star was retained in the coincidence file of the Hipparcos Epoch Photometry Annex Extension. The position of the other field of view for every field transit has also been retained in the Extension so that future checks may be carried out using more complete catalogues.

Since the accurate position of the other field of view was only available to the NDAC photometric calibration process, the FAST-only transits have not had this analysis performed on them. Because of this and the limiting magnitude of the Tycho Input Catalogue being V = 10.5 mag, it was felt that some additional checks were required in order to further identify parasitic transits. This was done by investigating the difference between the ac and dc components for each transit. If a star appeared in the other field of view, in addition to making the target star appear brighter, it also caused the star to appear as a double. The limit chosen for this was 3σ . A limitation of this method was that it could only be carried out on stars thought to be single. About 70 000 transits were flagged this way. In the case of FAST, transits contaminated by the presence of programme stars from the complementary field of view were removed at an early stage of the processing.

After the corrections described in the previous section had been applied to both the FAST and NDAC photometric data, a final comparison was carried out in order to best determine the merging strategy.

The main comparison was between the median photometric value of the two consortia for each star. Care was taken that the median calculations were carried out on exactly the same set of observations since the two consortia have slightly different coverage. This is most important for the very red stars since they are variable.

As a function of magnitude (in these comparisons only the dc magnitudes were considered) the systematic differences were very small. Brighter than 10 mag the differences were less than 0.001–0.002 mag. Fainter than this, a larger systematic difference was seen, equivalent to a difference in background of under 2 Hz. This comparison is shown in Figure 14.8.

Also shown are the differences as a function of V - I. The systematic differences are less than 0.002 mag out to V - I = 3.0 mag. Redder than this the differences are probably less than 0.02 mag out to V - I = 8.0 mag. There is not much data at this very red end, and saying more about the level of difference would probably not be possible. The low level of systematic differences was expected as a consequence of the way that the ageing corrections had been carried out.

Investigations on the quoted errors of the field transits for both consortia were also carried out. Most of these consisted of detailed Monte Carlo simulations to see if the observed quoted error distribution could be reproduced. The unit weight residuals were also investigated in this analysis. The two main points arising from this work were:

- the quoted error was inaccurate and biased due to only having a small number of frame transits per field transit. This is expected to follow a Student's *t* distribution. A consequence of this is that if left uncorrected, the unit weight residuals will tend to show a large number of spurious variables;
- (2) an uncalibrated and unknown residual that affects a field transit's quoted error is present thus giving larger quoted errors than in the Monte Carlo simulations.

The final conclusion was that some form of empirical correction had to be made to the quoted errors: it was known that the quoted errors were biased from (1) and that no theoretical estimate could be made from (2). The correction that was made was a function of magnitude, quoted error and consortium. It should be noted that when the data were merged another error analysis and correction was carried out to accommodate correlations between the FAST and NDAC data.

These error estimate corrections have implications for any micro-variability analysis carried out on the data. After applying such corrections, all that could be stated is that a star is more variable than stars of similar magnitude. For example, if all stars were variable we could not detect them as such since the errors were scaled. It could be argued that the unknown residual mentioned above (2) could be due to stellar variability.



Figure 14.8. Comparison between FAST and NDAC for Hp_{dc} magnitudes as a function of magnitude and V - I. For the top two diagrams the solid line is the median of distribution and dashed line is the 1σ width. The bottom diagram shows the individual data points rather than just the distribution.



Figure 14.9. The correlations between FAST and NDAC.

The level of this would be a few per cent over 20 seconds. It is more likely that this was caused by instrumental effects.

One of the most important things that had to be investigated for the merging process was the level of correlation between the FAST and NDAC data. The results of this investigation are shown in Figure 14.9. These contour plots show the density of the scatterplots of FAST versus NDAC residuals. The residuals are calculated as the differences between each field transit value and the median value for that star and consortium. Only constant stars were used for these investigations.

As is evident from the diagrams, the correlations are very strong. The correlation coefficients range from 0.6 to 0.8, with the highest values for intermediate magnitude stars (7 mag < Hp < 9 mag). This shows that the calibrations agree very well for mid-magnitude stars. Since the same photon counts were used it is expected that any deviation from perfect correlation would occur when the calibrations differ. At the faint end these differences were background related. At the bright end the situation was slightly different and the broadness of the diagrams was caused by the increased sensitivity of the data.

In order to determine how to merge the data an investigation was carried out to see what effect altering the consortia ratio would have on the general scatter of the data. For this investigation a merged magnitude was created using the formula:

$$Hp_{\rm av} = fHp_{\rm NDAC} + (1 - f)Hp_{\rm FAST}$$
[14.13]

The average scatter was then calculated for constant stars. This was then repeated for different fractions (f). The results are shown in Figure 14.10. Since the data were



Figure 14.10. The effect of varying the consortium fraction on the observed distribution widths of constant stars.

very strongly correlated not much of a decrease was expected in the width as a function of fraction. These plots indicate that a ratio of 1:1, i.e. giving both consortia equal weight, would yield a merged result close to optimal. This ratio would also be simpler to implement than a fraction varying as a function of magnitude. It was also decided to combine the errors using equal weights and combining them in quadrature. Due to the correlations the quoted errors of the merged data had to be analysed again and rescaled. Data originating from only one consortium did not have the errors rescaled a second time. Figure 14.11 compares the observed distribution of unit weight residuals with the probability expected for residuals with unit variance. Small discrepancies remain: there are additional wings in the observed distribution, and the central part of the distribution appears to have a slightly smaller width than expected.

A variance-weighted strategy was also considered, but rejected. The reason for this being that systematic errors probably remained which were likely to be comparable in size to the random ones, thus invalidating the premise of the weighting.

An important aspect of the merging process was the quality flagging of the transits. The basic principle behind the design of the quality flag (Field HT4 in the Hipparcos Epoch Photometry Annex) was that the lower the flag value the better the quality of the data. Thus, the higher bit settings were reserved for the more significant problems with the data. It was decided against rejecting any data at the merging stage due to the difficulty in choosing the most appropriate rejection limits. The strategy that was chosen was to flag suspected transits and keep them in the annex. Table 14.10 shows the percentage of transits that were flagged for each bit setting.



Figure 14.11. The observed distribution of unit weight residuals for standard stars, compared with the expected probabilities for a unit variance Gaussian distribution.

Most of the flag bit settings had various limits associated with them. The following list describes, in brief, the criteria involved in setting the bits (see Volume 1, Section 2.5):

- (Bit 0) NDAC only data: no FAST data was passed to the merging process. This implies that either the transit was rejected by FAST at an earlier stage or that FAST did not process this transit;
- (Bit 1) FAST only data: as above, but no NDAC transit was available to be merged;
- (Bit 2) Not used;
- (Bit 3) high background: the background for this transit was above 70 Hz. For faint stars there is the possibility of additional errors being present for these transits due to the uncertainty in the background estimate. The choice of the limit value is arbitrary;
- (Bit 4) field of view contamination: the transit is likely to be disturbed by a star in either field of view. As described in detail in Section 14.6, the other field of view was checked against the Tycho Input Catalogue for possible contaminating stars and via the use of the difference between Hp_{dc} and Hp_{ac} . Also, all transits for an entry were flagged if the Hipparcos Input Catalogue indicated that the star was a two-pointing double with a separation between 5 and 35 arcsec;
- (Bit 5) FAST quality flag set: during the FAST processing if the attitude determination was classified as poor this flag was set. In NDAC such data were rejected at an earlier stage in the processing;
- (Bit 6) perturbed for other identified reason: transit occurred during a period of numerous outliers. These periods were either associated with poor attitude, not identified at an earlier stage, or were close to a shutter closing event (close to Earth occultations);
- (Bit 7) sun-pointing mode observation: the quality of observations taken during sunpointing mode were believed to be affected by the non-nominal thermal environment of the satellite and were thus flagged;
- (Bit 8) significant difference: if the FAST and NDAC Hp_{dc} values differed by more than 3σ the transit was flagged.

Bit Setting	Percentage	Description
-	84.0	Unflagged
0	4.7	NDAC data only
1	6.3	FAST data only
2	_	Not used
3	1.7	Very high background estimate
4	3.5	Possible interfering object in either field of view
5	0.6	FAST quality flag set
6	0.2	Perturbed for other identified reason
7	1.5	Observation during sun-pointing mode
8	0.4	Significant difference between FAST and NDAC data

Table 14.10. The percentage of transits that are flagged for each bit setting in Field HT4 of the Hipparcos Epoch Photometry Annex.

14.8. Properties of the Photometric Data

Measurement Errors

The average standard error per field transit in Hp_{dc} and Hp_{ac} for the merged data are given in Table 14.11. As expected, the average standard errors on Hp_{dc} are about half the size of those on Hp_{ac} . Also given in this table are the average errors on the medians for stars found to be constant (stars with Field HH12 set to C). The errors on the medians are about a factor 10 smaller than the standard errors on the observations, as was expected for an average of close to 100 accepted observations per star. For the brightest stars the data are likely to be influenced by both small scale intrinsic variability and by slight inaccuracies in instrument modelling by the reduction process. Hence they do not reach the error levels one might have expected on the basis of the estimates obtained for fainter stars. In addition, the amount of integration time spent on bright stars was much shorter than on fainter stars.

The distribution of the measurement errors for individual field transit observations is shown in Figure 14.12. The widths and the wings of the distributions are primarily due to variations in the observing time (see Section 14.7). These distributions together with the data presented in Figure 14.11 clearly show that discussions concerning variability need to consider residuals weighted by their estimated error: individual observations can be of significantly different quality. This can even be the case when comparing stars of the same magnitude in different parts of the sky. The integration time spent on a star was related to the number of nearby neighbours in the Input Catalogue and photometric data for stars in more densely populated areas will be more noisy than for stars in sparsely covered areas.

Нр	Average t	ransit error	Average error on mediar		
	dc	ac	dc	ac	
2	0.0027	0.0043	0.0004	0.0008	
3	0.0029	0.0049	0.0004	0.0006	
4	0.0036	0.0063	0.0005	0.0008	
5	0.0047	0.0091	0.0006	0.0010	
6	0.0058	0.013	0.0007	0.0013	
7	0.0080	0.019	0.0009	0.0019	
8	0.011	0.026	0.0013	0.0027	
9	0.015	0.037	0.0019	0.0039	
10	0.022	0.051	0.0028	0.0055	
11	0.033	0.072	0.0044	0.0079	
12	0.049	0.100	0.0072	0.0107	

Table 14.11. The average standard error per field transit and the average error on the median for stars found to be constant (stars with Field HH12 set to C).

The distribution of errors on the medians, as shown in Figure 14.13, summarizes the precision of the photometry presented in the main catalogue. For almost all these stars the precision of the median dc magnitude is of the order of 0.001 to 0.002 mag or better, while the errors on the ac magnitudes are two times larger. Figure 14.13 also shows an accumulation towards larger errors in the dc magnitudes due to variability, and in the ac magnitudes due to variability and duplicity.

An examination of the unit weight standard errors shows to what degree the corrections of the error estimates have been successful. Histograms of the observed unit weight standard errors for all stars with at least 30 observations are shown in Figure 14.14. It is clear that the situation is not ideal, and in particular for the brightest and faintest stars the variances are much influenced by reduction noise. Also clear in these diagrams are the cumulated peaks on the right-hand side of each diagram, representing the variable stars, and in some cases the presence of a companion.

Number of Observations and Distribution over Time

As described above, original observations spanned an integration time of 32/15 s. These frame transits occurred in groups of up to 9, representing together a field transit. Data have only been preserved at this level of field transits, of which over the length of the mission approximately 13 million were accumulated. These are presented in the Hipparcos Epoch Photometry Annex plus Extension. The field transits are referred to as observations.

The short-term distribution of observations was determined by the scanning speed and the configuration of the two fields of view. A series of observations therefore contains a regular pattern of: observation, 20 min gap, observation, 108 min gap, observation, 20 min gap and so on. The length of this sequence was determined by the position of the star on the scanning circle, relative to the instantaneous rotation axis of the plane of this scanning circle. Close to the rotation axis, the sequence could continue for more than a day, while perpendicular to it, the sequence would last for no more than 1 to 2

cycles. Sequences were further interrupted by perigee passages, when no observations could be made. Due to the geometry of the scanning law, the rotation axis of the plane of the scanning circle was always situated between ecliptic latitudes $\pm 47^{\circ}$. Observations of stars near the ecliptic poles therefore always consist of short stretches of data, except during sun-pointing periods.

The total number of observations over the mission was also very much a function of ecliptic latitude, again resulting from the scanning law. The relations are shown in Figure 14.15. Total numbers of observations varied by a factor 10, from below 40 to 385, but the higher numbers of observations (> 200) were only found for stars close to ecliptic latitude $\pm 47^{\circ}$. In the study of periodic variable stars the occurrences of data gaps are very important. Figures 14.16 and 14.17 show the distribution of the lengths of gaps and the numbers of gaps as a function of ecliptic latitudes. This distribution of data limited a reliable period search to the range 0.09 to 3 days approximately. Periods in the range 5 to 100 days were generally very poorly covered.

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Figure 14.12. Distribution of the measurement errors relative to the mean standard error per magnitude interval, using all measurements with error flag equal to 0. The diagram at the bottom right is the accumulated distribution of the other 11 diagrams.



Figure 14.13. The distribution of errors on the medians in three magnitude intervals. The distribution for the dc magnitudes is shown by the solid line, the ac magnitudes are shown by a dotted line. The accumulation on the right of each histogram represents variable stars for both ac and dc magnitudes, and in addition double stars for ac magnitudes.



Figure 14.14. Distribution of the unit weight standard errors. The diagram at the bottom right is the accumulated distribution of the other 11 diagrams.



Figure 14.15. The total number of field transits per star as function of ecliptic latitude, shown as a contour map. The first contour represents 10 stars, every next contour adds 20 stars. The highest contours represent around 200 stars. The effects of the scanning-law caused the 'discontinuity' in the distribution around 47° . The cell-size is 1 transit along the horizontal and 5° along the vertical axis.



Figure 14.16. Distributions of the lengths of gaps as a function of ecliptic latitude. Gaps longer than 1.5 days were counted for all stars observed. The feature between 50 and 60 days length was the result of gaps in the scanning due to a variety of problems with the satellite. The features for low latitudes between 85 and 140 days result directly from the scanning law. The lowest contour represents 200 gaps observed, increasing by 400 per contour until 1400, then by 600 until 2600, and then by 900 until 8900. The majority of gaps in the data are between 10 and 40 days. The cell-size is 1 day along the horizontal and 5° along the vertical axis.



Figure 14.17. Distributions of the number of gaps as a function of ecliptic latitude. Gaps longer than 1.5 days were counted for all stars observed. The contours follow the relative distributions of gaps. The first contour represents 1 per cent, the next 3 per cent, and every next contour a 2 per cent higher level. Thus, roughly 20 per cent of the data is found near the central ridge of the diagram, indicating that at high ecliptic latitudes the data was interrupted around 30 to 35 times, while at low ecliptic latitudes there were from 15 to 17 interruptions. The cell-size is 1 gap along the horizontal and 5° along the vertical axis.

15. MINOR PLANETS AND PLANETARY SATELLITES

Some 48 minor planets and three natural satellites were observed during the Hipparcos main mission with the purpose of linking the dynamical and kinematical reference systems and for dynamical and physical studies of these solar system bodies. This chapter describes specific aspects of the processing implemented for the solar system objects in order to derive the astrometric and photometric solutions. Several summary tables related to the minor planets and their observability conditions are also included in this chapter.

15.1. Introduction

The observation of minor planets and natural satellites of giant planets with Hipparcos was considered during the mission planning to be of high scientific relevance with the goal of obtaining accurate astrometric positions and investigating the relationship between the dynamical and kinematical reference systems. Some 60 minor planets were included in the preliminary program, of which 48 were repeatedly observed during the actual mission together with three planetary satellites (J II Europa, S VI Titan and S VIII Iapetus) yielding astrometric and photometric data of excellent quality. The corresponding star mapper observations are described in Volume 4, Chapter 15.

As a result of the rapid and non-linear motion of these objects over a time-span of only a few hours, the basic data treatment had to be adapted for these objects to produces one-dimensional astrometric positions on the reference great circles on which they were observed. The solar system objects were observed in the same way as stellar objects. However objects with an apparent diameter larger than 0.05 arcsec were resolved by the Hipparcos telescope and the astrometric solution refers more or less to the photocentre of the illuminated fraction of the disc, i.e. to a point varying with the phase angle. In the first section of this chapter the main properties of the grid signal pertaining to the minor planets are emphasised. Then the astrometric solution on the great circle is presented, followed by the photometric aspects of the processing.

15.2. Hipparcos Observations of an Extended Source

The signal recorded behind the Hipparcos main grid during the transit of a light source was modelled by the 'five-parameter model' introduced in Chapter 5. It was shown that

the periodic signal for a point-like source could be accurately represented by a Fourier series up to the second harmonic as:

$$S(t) = I_0 \left[1 + M_0 \cos(\omega t + \phi_0) + N_0 \cos 2(\omega t + \phi_0) \right]$$
[15.1]

where M_0 , N_0 are the calibrated modulation coefficients, and ϕ_0 is the modulation phase of the signal, corresponding to the position of the source on the grid at the reference time t = 0. ω is the time frequency of the signal. For an object of significant angular size, typically with a diameter $\rho \gtrsim 0.05$ arcsec, the modulated signal results from the integration of the point-like signal over the surface of the source. This gives a signal of the same form (Morando 1986, Lindegren 1986, Morando & Lindegren 1989):

$$S(t) = I \left[1 + M\cos(\omega t + \phi) + N\cos 2(\omega t + \psi)\right]$$
[15.2]

but now with reduced modulation coefficients M, N and (in general) shifted modulation phases ϕ , ψ depending on the brightness distribution of the apparent disc. Let $x = \pi \rho / s$ be a dimensionless variable relating the angular diameter of the planet to the grid period, s = 1.2074 arcsec. Introduce the complex function:

$$U(\mathbf{x}) = \iint I_{\sigma} \exp(-\iota \mathbf{x} \mathbf{w}' \mathbf{n}) \mu \, \mathrm{d}\sigma$$

where I_{σ} is the specific brightness of the surface element $d\sigma$, $\iota = \sqrt{-1}$, **w** the unit vector in the scanning direction (Figure 15.6), **n** the unit vector normal to the surface element, and μ the cosine of the angle of reflection. The total intensity and the degradation of the modulation coefficients can then be written:

$$I = U(0), \qquad \frac{M}{M_0} = \left| \frac{U(x)}{U(0)} \right|, \qquad \frac{N}{N_0} = \left| \frac{U(2x)}{U(0)} \right|$$
[15.3]

and the phase shifts are:

$$\phi - \phi_0 = \arg[U(x)], \qquad \psi - \psi_0 = \frac{1}{2}\arg[U(2x)]$$
 [15.4]

In interferometric terminology, U(nx)/U(0) is the complex visibility of the object, in the direction of **w**, at *n* times the spatial frequency of the grid.

The abscissa was derived from the phase ϕ of the first harmonic in NDAC, and by means of a weighted average of the two phases, $0.75\phi + 0.25\psi$, in FAST. The major consequences for an extended object are that (i) the observed position does not strictly correspond to the definition of the photocentre, and (ii) the FAST and NDAC observed positions do not strictly correspond to the same point. Introducing the functions:

$$j_1(x, \alpha) = J_1(x) + J_1(x \cos \alpha)$$

$$h_1(x, \alpha) = \boldsymbol{H}_1(x) - \boldsymbol{H}_1(x \cos \alpha)$$
[15.5]

where α is the solar phase angle, and J_1 , H_1 are the Bessel and Struve functions respectively, then for a uniformly bright sphere and a scan along the intensity equator:

$$U(x) = j_1(x, \alpha) + \iota h_1(x, \alpha)$$
 [15.6]

which yields the phase offset relative to the centre of figure. The difference between the photocentre and the position assigned from the phases, using the FAST and NDAC procedures, is shown in Figure 15.1 as a function of the apparent diameter of the planet. For a phase angle of $\alpha = 20^{\circ}$, typical for the Hipparcos observations of minor planets



Figure 15.1. Theoretical difference between the observed position and the photocentre versus apparent diameter. The curves are for a spherical object of uniform brightness viewed with a solar phase angle $\alpha = 20^{\circ}$. Solid curve: positions derived from the first harmonic only (NDAC). Dashed curve: positions derived from a weighted mean of the harmonics (FAST) — this method is practically limited to objects smaller than 0.7 arcsec.

(Figure 15.10), the differences remain well below one milliarcsec except for (1) Ceres as observed by FAST at its maximum diameter, $\rho \sim 0.7$ arcsec.

The Hipparcos magnitude of the minor planets was estimated in the same way as for the stars and is described in Chapter 14. The magnitude Hp_{dc} was directly derived from the mean intensity *I* of the signal (corrected for sky background), while Hp_{ac} was based on the amplitudes *IM* and *IN* of the modulated components of the signal. Then from Chapter 14:

$$\Delta Hp \equiv Hp_{\rm ac} - Hp_{\rm dc} \simeq -2.5 \log_{10} \frac{MM_0 + NN_0}{M_0^2 + N_0^2}$$
[15.7]

 Hp_{ac} is a biased estimator for the larger planets, depending on the apparent diameter at the time of observation, through the attenuation of the modulation measured by M/M_0 and N/N_0 .

In the approximation of a spherical object of uniform brightness at zero solar phase angle, the imaginary part of U(x) vanishes because of the azimuthal symmetry of the problem. Therefore U(x) is simply given by the Hankel transform of order zero:

$$\mathcal{H}_0[1; x] = \int_0^1 J_0(xr) r \,\mathrm{d}r$$
 [15.8]

and the attenuation in the modulation coefficients can be expressed as a function of the apparent size of the source:

$$\frac{M}{M_0} = 2 \frac{|J_1(x)|}{x}, \qquad \frac{N}{N_0} = \frac{|J_1(2x)|}{x}$$
[15.9]



Figure 15.2. Attenuation of the modulation coefficients for the first (dashed line) and second harmonics (solid line) for a spherical object of uniform brightness and of apparent diameter ρ .

These functions are plotted in Figure 15.2. There is no significant attenuation in the signal modulation up to an apparent diameter of about 0.1 arcsec for the first harmonic and 0.05 arcsec for the second harmonic.

15.3. Astrometry on the Circle

Transformation to Astrometric Directions

In the great-circle reduction a one-dimensional position was obtained, about twice per day, for the Hipparcos stars observed in that interval. This great-circle abscissa was calculated by fitting all the grid coordinates of a star collected during the great-circle interval of several hours. The main principles of this process, outlined in Chapter 9, apply also to the observations of the solar system objects. However, because of the rapid and non-linear motion of the planets, a different sampling time was adopted for these objects, leading to one great-circle abscissa for every observational frame of 32/15 s. For the final catalogue, normal positions were derived at a rate of one astrometric one-dimensional position per field-transit of the object across the instrument main grid.

The instantaneous Hipparcos observations referred to the proper direction of the planet; thus the first step of the processing consisted of transforming this direction into the coordinate direction by computing the stellar component of the aberration and the parallax introduced by the satellite's motion around the Earth. These corrections were evaluated to an accuracy better than one milliarcsec, which required the modelling of the aberration to the second order in $|\mathbf{V}|c^{-1}$, where \mathbf{V} is the barycentric velocity of Hipparcos. Likewise, the light bending by the spherical potential of the Sun was taken into account to first order in $GS/ac^2 \simeq 2$ mas where *a*, the heliocentric distance of the satellite, is close to $A \simeq 1$ AU (see Table 12.1). The deflection by the giant planets was neglected.

Most minor planets have a sizeable apparent diameter (> 0.05 arcsec) compared to the Hipparcos astrometric accuracy, and the solar phase angle effect shifts the photocentre



Figure 15.3. Coordinate direction of a minor planet. The observed satellitocentric proper direction \mathbf{u}_a is first corrected for stellar aberration and light deflection into \mathbf{u}_s . It is then transformed into the (geocentric) coordinate direction \mathbf{u}_g . The epoch of observation at the satellite is also corrected by $\Delta \tau$ into an epoch at the geocentre.

with respect to the centre of figure. As this cannot be evaluated within the required accuracy without a rather sophisticated and uncertain modelling of the light diffusion on the surface, no phase correction was applied and the direction provided by Hipparcos corresponds in the first approximation to the position on the sky of the instantaneous photocentre (see Section 15.2).

Let \mathbf{u}_a be the apparent (or 'proper') satellitocentric direction of a solar system object, and \mathbf{u}_g its geocentric astrometric (or 'coordinate') direction, with $|\mathbf{u}_a| = |\mathbf{u}_g| = 1$ (see Figure 15.3 and Chapter 12). These vectors are referred to a coordinate frame associated with a given reference great circle, the so-called reference great-circle frame (Section 11.2). The one observed quantity was the apparent abscissa (*v*) on the reference great circle, while the perpendicular coordinate (*r*) was computed from the ephemerides. The initial conditions for the ephemerides of minor planets were taken from the 'Ephemerides of minor planets for the year 1992' and were numerically integrated with a Bulirsch-Stoer integrator including the perturbations of all the major planets from Mercury to Neptune. The planetary positions were taken from the JPL solar system ephemerides DE200.

The direction \mathbf{u}_a was first transformed into a satellitocentric astrometric direction, \mathbf{u}_s , by correcting for the aberration and gravitational light bending. This was actually done as part of the same processing as applied to the stars, namely in the great-circle reductions (Chapter 9). The satellitocentric astrometric direction was then transformed into the geocentric astrometric direction, \mathbf{u}_g , by applying the parallactic correction. The relationship between the apparent direction and the direction corrected for stellar aberration (the 'natural' direction) is given by inverting Equation 12.7 to the second order in V/c:

$$\hat{\mathbf{u}}_{s} = \left\langle \mathbf{u}_{a} - \left[1 - \frac{\mathbf{V}'\mathbf{u}_{a}}{2c}\right] \frac{\mathbf{V}}{c} \right\rangle + O\left(\frac{|\mathbf{V}|}{c}\right)^{3}$$
[15.10]

where V is the barycentric velocity of the Hipparcos satellite. The barycentric velocity of the Earth was provided by a compact representation of the ephemeris BDL 82 (see



Figure 15.4. Notations for the correction of gravitational light deflection.

Chapter 12 and Chapront *et al.* 1984), while the geocentric velocity of the spacecraft was provided by the mission operations centre, ESOC. The velocity vector \mathbf{V} was computed as the sum of these two velocities after transformation to the same reference system.

The application of the gravitational light deflection for a source at finite distance from the Sun leads to the following expression for the astrometric direction (\mathbf{u}_s) in terms of the natural direction $(\hat{\mathbf{u}}_s)$:

$$\mathbf{u}_{s} = \left\langle \mathbf{u}_{a} - \mathbf{d} \, \frac{2GS}{a \, c^{2}} \, \tan \frac{\Psi}{2} \right\rangle + O\left(\frac{GS}{a \, c^{2}}\right)^{2}$$
[15.11]

with $\mathbf{d} = \langle \mathbf{u}_s \times (\mathbf{r}_0 \times \mathbf{u}_s) \rangle$ the unit vector along the impact radius, *GS* the heliocentric gravitational constant and *c* the speed of light (see Table 12.1); *a* is the heliocentric distance of the satellite (i.e. $a \simeq A$), and Ψ the heliocentric angle between the object and the satellite (see also Figure 15.4 for notations).

After these two steps \mathbf{u}_s must be transformed into the geocentric direction \mathbf{u}_g , which gives:

$$\mathbf{u}_{g} = \langle \mathbf{u}_{s} \Delta + \mathbf{r}_{sat} \rangle$$
 [15.12]

where the geocentric position of Hipparcos, \mathbf{r}_{sat} , was provided with an accuracy of $\simeq 2$ km by the satellite orbit determination performed at ESOC. Δ is the distance between Hipparcos and the minor planet, which for this correction had to be known to $\simeq 15\,000$ km, a requirement easily satisfied by the available ephemerides. The epoch of observation was also corrected for the first order light-time difference due to the geocentric orbit, $\Delta \tau = (g - \Delta)/c$, yielding the time at the geocentre, where $g = |\mathbf{u}_s \Delta + \mathbf{r}_{sat}|$ is the geocentric distance to the planet (see Figure 15.3).
Construction of a Normal Place in NDAC

A normal place was derived for every field-transit from the $\simeq 8$ consecutive abscissae measured at the frame level. In NDAC, the normal place was constructed from a fit of the observed abscissae to the linear motion of a planet in the time interval of a field crossing, about 19 s. In the first stage of the great-circle reduction, all the data were stored and a temporary file was created for every object observed on the circle. This file contained a reference time $t_{\rm R}$, a reference abscissa $v_{\rm R}$, the rate q = dv/dt, and a mean ordinate $r_{\rm R}$ for each object. For a solar system object, the ephemeris was used to compute a predicted abscissa and ordinate for the first and last frame of the transit $(v_1, r_1 and v_n, r_n)$ corresponding to the mid-frame times (t_1, t_n) . With the reference abscissa $v_{\rm R} = (v_1+v_n)/2$, the reference time $t_{\rm R} = (t_1+t_n)/2$, the abscissa rate $q = (v_n - v_1)/(t_n - t_1)$, and the mean ordinate $r_{\rm R} = (r_1 + r_n)/2$, the residual of the abscissa in the *k*th frame, at time t_k , was:

$$\delta v_k = v_k - G(v_R + (t_k - t_R)q, r_R)$$
[15.13]

where G(v, r) is the transformation to grid coordinate, including attitude and instrument modelling. The result of the great-circle reduction was a weighted mean correction $\overline{\delta v}$ to the reference abscissa, so that the output for each transit consisted of $t_{\rm R}$, $v_{\rm R} + \overline{\delta v}$ and $r_{\rm R}$. Transits were discarded when the signal was too faint to be useful, or in the case of a pointing offset exceeding 10 arcsec, making the reliability of the data questionable.

The results thus obtained in the final NDAC great-circle reductions were further corrected for the abscissa zero point errors determined in the corresponding sphere solution (N37.5; see Chapter 16). They were then transformed from the reference frame of N37.5 to the provisional system H37C realised by the merging of the final FAST and NDAC sphere solutions (see Chapter 17). This transformation was slightly different from, but practically equivalent to, the subsequent transformation to ICRS described in Section 15.4. It was applied as a correction to the abscissa:

$$v_{\rm H37C} = v_{\rm N37.5} - \boldsymbol{\varepsilon}' \mathbf{R}$$
 [15.14]

where **R** denotes the (positive) pole of the reference great circle, and ϵ is the timedependent rotation detailed in Table 15.1.

Construction of a Normal Place in FAST

Unlike NDAC the normal place in FAST was based on the median of the abscissae. A typical situation of the abscissae at the frame level used to construct the normal place is shown in Figure 15.5. After the reduction on the sphere, a dedicated file was constructed for all the solar system objects to store the abscissae at each mid-frame time. The corrections to the origins δv_0 (Chapter 16) were available separately and used to obtain the abscissae in a consistent reference frame.

Let $v_{\rm R}$ be the reference abscissa at mid-transit time and q = dv/dt based on the planet ephemeris. The observation equation for the reference abscissa was then: $v_{\rm R} = v_k - (t_k - t_{\rm R})q$. The resulting reference abscissa was eventually estimated as the median \bar{v} of the $n \simeq 8$ values $v_k - (t_k - t_{\rm R})q$ in a transit. The output for each transit consisted of $t_{\rm R}$, $\bar{v} + \delta v_0$, $r_{\rm R}$ and the standard error of the reference abscissa, given by:

$$\sigma_v^2 = \frac{\pi}{2n} \frac{1}{n-1} \sum_k \left(\frac{\sigma_0}{\sigma_k} \eta_k\right)^2 = \frac{\pi}{2n} \tilde{\sigma}_0^2 \qquad [15.15]$$



Figure 15.5. Reduction of the minor planets abscissae at the transit level. A normal place for each transit was constructed by fitting the location parameter of a linear motion whose speed was taken from the ephemeris (solid line). The normal position (circle) corresponds to an average position of the frame level positions (mean in NDAC, median in FAST). The great-circle abscissa origin on the plot is arbitrary.

where $\eta_k = v_k - v_R - (t_k - t_R)q$ are the residuals, and $\sigma_0 \eta_k / \sigma_k$ the weighted residuals. $\tilde{\sigma}_0^2$ is an estimator of the variance of a single observation. Usually there were n = 8 observations per transit.

Dubious abscissae v_k with an uncertainty greater than 150 mas were systematically discarded. In order to identify transits possibly corrupted by the presence of a parasitic star in the complementary field of view, observations with $\sigma_{\Delta Hp} > 0.3$ mag were rejected, where ΔHp is the difference between the ac- and dc-magnitudes (see Equation 15.7). Two other tests were constructed to filter out unreliable transits. Transits with estimated $\tilde{\sigma}_0 > 2\sigma_0$, or containing only one frame, were rejected. Likewise, a transit was rejected if it led to a magnitude difference between the ac- and dc-scales such that:

$$|\Delta Hp - \Delta Hp_{\text{calc}}| > 5\sigma_{\Delta Hp}$$
[15.16]

with $\Delta H p_{\text{calc}} \simeq 1.214 \rho^2 + 0.03 \rho^4$ derived from Equations 15.7 and 15.9 for an object of uniform brightness of apparent diameter ρ . The reference time was taken as the mean of the first and last used frame of a transit. The resulting positions were in the reference frame of the final FAST sphere solution, F37.3.

15.4. Astrometry Final Output

Transformation to the Tangent Plane

Each observation of a solar system object is uniquely defined by the time, the abscissa and the orientation of the circle on which the planet position was projected. It was, however, considered that a different presentation of the results would be more convenient for the users, although it was not possible to provide strictly two-dimensional positions. The Hipparcos observations of solar system objects are supplied as an observation equation relating the abscissa to a perfectly defined reference point (α_0 , δ_0) (see



Figure 15.6. Transformation to the tangent plane. See text for details.

Figure 15.6). More precisely, the published data determine the equation of the straight line v = constant in the tangent plane centred at the reference point.

To minimise the errors due to the projection on the tangent plane, the reference point was chosen in the immediate vicinity of the true, but unknown, position. The reference point has the same abscissa as the observed abscissa of the planet, v^{obs} , and a calculated ordinate r^{calc} based on the ephemeris at the reference time. The astrometric position expressed in the reference great-circle frame (RGC) and in the provisional reference frame of the Hipparcos reductions (P, representing either H37C or F37.3) are related by the transformation:

$$\mathbf{u}_{\mathrm{P}} = \mathbf{R}_{3}(\Omega)\mathbf{R}_{1}(i)\mathbf{u}_{\mathrm{RGC}}$$
[15.17]

where *i* and Ω are the inclination and the longitude of the node of the reference great circle, **R**_{*k*} represents a rotation about the *k*th axis, and:

$$\mathbf{u}_{\text{RGC}} = \begin{pmatrix} \cos r^{\text{calc}} \cos v^{\text{obs}} \\ \cos r^{\text{calc}} \sin v^{\text{obs}} \\ \sin r^{\text{calc}} \end{pmatrix}, \qquad \mathbf{u}_{\text{P}} = \begin{pmatrix} \cos \delta_0 \cos \alpha_0 \\ \cos \delta_0 \sin \alpha_0 \\ \sin \delta_0 \end{pmatrix}$$
[15.18]

The direction defined in the reference great-circle frame by $v = v^{obs}$ (= constant) is given on the tangent plane, in an indirect manner, by $\theta \in [0, 2\pi]$, the position angle of the reference great circle at the reference point. This angle, reckoned counter-clockwise on the sky from $+\delta$, was computed as the direction of the reference great circle at the point $(v, r) = (v^{obs}, 0)$ and is given by:

$$\cos \theta = \frac{\sin i \cos v}{(1 - \sin^2 i \sin^2 v)^{1/2}}, \qquad \sin \theta = \frac{\cos i}{(1 - \sin^2 i \sin^2 v)^{1/2}}$$
[15.19]

The standard error refers to the uncertainty of the abscissa in the direction \mathbf{w} parallel to the reference great circle and passing through the reference point:

$$\sigma_{V*} = \sigma_V \cos r^{\text{calc}} \qquad [15.20]$$

Transformation to International Celestial Reference System

The positions obtained so far are still referred to arbitrary intermediate frames resulting from the Hipparcos sphere solutions (i.e. F37.3 for FAST and H37C for NDAC; see

	Ori	Orientation (mas)			Spin (mas/yr)			
	ε _{0x}	ε_{0y}	ε _{0z}	_	ω_X		ωy	ωΖ
N37.5→H37C	-20.648	-33.151	+46.719		-0.584	+	0.657	+0.508
$H37C \rightarrow ICRS$	-19.1	-8.5	+20.9		-0.73	+	0.05	+0.47
F37.3→ICRS	-24.218	-27.532	+56.190		-0.590	_	0.453	+3.661

Table 15.1. Values of the rotation and spin components for the reference frames transformations. The components of the orientation refer to the epoch $T_0 = J1991.25$.

Chapter 16). The published solar system data must, however, use the same reference system as the stars, namely the International Celestial Reference System, ICRS (Chapter 18). The orientation of P (F37.3 or H37C) with respect to ICRS is given by a time dependent, small rotation $\boldsymbol{\epsilon}(t) = \boldsymbol{\epsilon}_0 + (t - T_0)\boldsymbol{\omega}$, where $T_0 = J1991.25$ is the epoch of the Hipparcos Catalogue. Thus, the final coordinates \mathbf{u}_{ICRS} , referred to the ICRS, were obtained by:

$$\mathbf{u}_{\text{ICRS}} = \begin{pmatrix} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{pmatrix} \mathbf{u}_{\text{P}}$$
[15.21]

where ε_x , ε_y , ε_z are the equatorial components of $\varepsilon(t)$ for the epoch of the observation. In principle, the transformation entails also a change of the position angle θ , but this change would always be less than 0.1 arcsec and was therefore not implemented. The values of the rotation parameters for the transformations H37C \rightarrow ICRS and F37.3 \rightarrow ICRS are listed in Table 15.1.

Comparison of FAST and NDAC Abscissae

The methods applied by FAST and NDAC to process the observations of the solar system objects were generally very similar. However they differed sufficiently in their details to prohibit a merging of the two sets of abscissae.

The positions on the grid were not derived in the same way from the signal phases which implies that the observed positions do not correspond to exactly the same physical point. This discrepancy depends mainly on the object's size, and to a lesser extent on its shape, on its brightness distribution, and on the geometry of the scanning direction relative to the visible surface. The theoretical differences between the Hipparcos position and the photocentre were shown in Figure 15.1 for a spherical object of uniform brightness, and although they are not large, they are not identical for FAST and NDAC.

The systematic phase effect was shown by Söderhjelm & Lindegren (1982) to have a non-negligible influence on the realisation of the dynamical reference frame from the Hipparcos observations of solar system objects. As noted previously, these effects cannot be predicted with sufficient accuracy to be accounted for and could have different consequences in the FAST and NDAC treatments.

For all these reasons, and in order to avoid introducing additional noise in the data, it was thought preferable to publish separately the FAST and NDAC results for all the



Figure 15.7. Comparison of FAST and NDAC abscissae from the reference points in the ICRS. The histogram represents the normalised difference with a correlation factor of 0.85 (determined from faint stars) and weighted standard deviations. The solid curve is the corresponding Gaussian of unit variance $\Delta v / \sigma_{\Delta V} \in N(0.13, 1)$, the non-normalised mean offset is $\langle \Delta v \rangle \simeq 1.2$ mas.

solar system objects. However, from the calibration of the FAST and NDAC procedures with respect to the stars, it is expected that:

$$\lim_{\rho \to 0} v_{\text{NDAC}} \simeq \lim_{\rho \to 0} v_{\text{FAST}} \simeq v_{\text{photocentre}}$$

which means that the FAST and NDAC abscissae should be very similar for the smallest planets ($\rho \ll s$). Figure 15.7 shows an histogram of the normalised differences between the FAST and NDAC abscissae. The average of the normalised differences is slightly positive, corresponding to a systematic difference between the FAST and NDAC abscissae of about 1.2 mas. However, the abscissae are not strictly referred to the same circles since they have been defined independently by each group. The projections of the planets may thus be marginally different.

Finally, for objects as large as $\simeq 0.7$ arcsec, depending on their actual brightness distribution, the second harmonic vanishes (see Equation 15.9 and Figure 15.2) and the corresponding phase ψ of Equation 15.2 becomes meaningless. A consequence of this was that only NDAC positions could be derived for the planetary satellites J2 Europa and S6 Titan.

15.5. Photometry of the Solar System Objects

FAST Reduction

The photometric reduction of the solar system objects was done only by the FAST Consortium, in parallel with that of the stars. The apparent magnitudes in the *Hp* scale are provided at a rate of one value for every field transit in exactly the same way as for the stars and in the same photometric system, using the colour B - V = 0.5 mag for all the planets (see Chapter 14).



Figure 15.8. Attenuation profile of the instantaneous field of view. Left: average profile from on ground calibration $(1 \ \mu m \simeq 0.147 \ arcsec)$. The actual in-flight profile is unknown between 100 and 500 arcsec. Right: magnitude difference ΔHp corrected for bias due to the modulation, as a function of the angular separation between the planet (Jupiter or Saturn) and its observed satellite.



Figure 15.9. Position of the solar system barycentre during the mission (solid line). Rectangular coordinates of the barycentre in the ecliptic plane.

However, for the three planetary satellites on the programme, diffusion of the light of their respective planets considerably perturbed the observations and no satisfactory solution could be reached. The average instantaneous field of view profile is given for large offsets in Figure 15.8. With a planet 7 or 8 mag brighter than the satellite and located only a few hundreds of seconds off the centre of the field of view, there is still some planetary light perturbing the signal of the satellite. The effect is hard to assess because the exact attenuation profile is not known in the periphery of the instantaneous field of view. Figure 15.8 (right) shows the difference between the two magnitudes scales Hp_{ac} and Hp_{dc} after the expected effect due to the apparent size of a satellite has been removed. The remaining differences, which should be zero, were sampled as a function of the separation between the satellite and the planet at the time of observation. This unmodelled difference reflects essentially the residual disturbing light originating from the planet. The consequence on the photometric measurement is fairly large for any satellite whatever the separation to such an extent that no reliable magnitude could be provided.



Figure 15.10. Distribution of the solar phase angle of the minor planets during the Hipparcos mission. The scanning law imposed that the observations could only occur in the vicinity of the quadratures, in contrast with the prevailing situation for ground-based observations.



Figure 15.11. Folded light curves for (471) Papagena obtained at different epochs. The magnitudes are corrected for the distance to the Earth (Δ) and the Sun (r), and for solar phase angle (α). (a) epoch $t \simeq 530$, the circled points correspond to observations made about 10 days later; (b) epoch $t \simeq 1140$, the circled points correspond to observations made about 3 days later.



Figure 15.12. Magnitude-phase (top) and magnitude-aspect (bottom) relations for minor planet (4) Vesta. The solid curves in the magnitude-aspect plot correspond to a triaxial ellipsoid model (a : b : c = 1.15 : 1.2 : 1), the dotted curve corresponds to the synthesis model of ratio (a : b : c = 1.1 : 1.2 : 1) from Magnusson et al. (1994).

To help interpret the apparent magnitudes of the minor planets, the geometric parameters at the time of observations are also provided with the solutions. This includes the distance to the solar system barycentre, the distance to the Earth, the phase angle (the angle between the satellite and the Sun as seen from the minor planet's centre of mass) and the apparent diameter, based on the IRAS Catalogue (Tedesco 1989).

The observed apparent magnitudes of minor planets being accurate to a few hundredths of a magnitude, the geocentric distance of the Hipparcos satellite can be neglected. As for the Sun, over the observation period, the solar system barycentre was always within 1.6 solar radii of the centre of the Sun, so that the offset distance barycentre-centre of the Sun may also be disregarded (see Figure 15.9).

The estimator Hp_{ac} is also provided for the sake of completeness, although it is not very useful for planets of large diameter. Transits rejected during the astrometric reduction are also discarded for the photometric output. All magnitudes are given in the Hp system and can be transformed to standard V magnitudes with the expressions given in Tables 14.1 and 14.2.

Minor Planet Brightness Variations

Due to the scanning law of the satellite, the observations of minor planets took place when the planets were close to their quadratures and were not uniformly distributed over the rotational phases of the planets. The distribution of the phase angles is shown in Figure 15.10 for all the observations of the minor planets. The mean value of the order of 20° is a consequence of the observations having been made near quadrature.

Table 15.2, based on the Asteroids II data base (Magnusson 1989) and its updated version (Magnusson *et al.* 1994), indicates what is known about the shapes and poles of the minor planets, information relevant for the interpretation of the Hipparcos epoch photometry of the minor planets. Summary statistics related to the conditions of observations of the 48 minor planets are listed in Table 15.3.

An example of a photometric analysis is shown in Figure 15.11 with the folded light curves of (471) Papagena. The curves were computed with a rotation period of 7.11 hr and the magnitudes are absolute, i.e. corrected for the varying distances from the Sun and the Earth. A correction was also applied for the phase angle to show the variation with aspect. The variation with the solar phase angle and the aspect angle is illustrated for (4) Vesta in Figure 15.12. The amplitude of the magnitude-aspect relation is smaller for objects observed at opposition.

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IAU Number and Name	Tholen class	Asteroids II data base	Solı pole	ution shape	Updated version	Solı pole	ution shape
(1) Ceres	G	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(2) Pallas	В	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(3) Juno	S	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(4) Vesta	V	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(5) Astraea	S	\checkmark					
(6) Hebe	S		V	, V		√	√
(7) Iris	S	V V	v V	V		v	v
(8) Flora	S	V V	Ň	- -	V V	v V	v V
(9) Metis	S	Ň	v v	1	Ň	v v	v v
(10) Hygiea	C	v v	v _	v _	v v	v \/	v \/
(11) Parthenone	S	v _	_	_	v _	v _	v _
(11) Victoria	S	/	/	_	/	/	/
(12) Victoria (12) Egoria	C C	V	V	_	V	V	V
(13) Egena (14) Irono	G	-	-	-	-	-	-
(14) Helle (15) Function	с c		/	-	-	/	/
(15) Europhia	S	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(16) Psyche	M	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(18) Melpomene	5	-	-	-	\checkmark	\checkmark	-
(19) Fortuna	G	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(20) Massalia	S	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(22) Kalliope	М	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(23) Thalia	S	-	-	-	\checkmark	\checkmark	\checkmark
(27) Euterpe	S	-	-	-	-	-	-
(28) Bellona	S	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(29) Amphitrite	S	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(30) Urania	S	-	-	-	-	-	-
(31) Euphrosyne	С	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(37) Fides	S	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(39) Laetitia	S	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(40) Harmonia	S	-	_	_			
(42) Isis	S	-	_	-	-	-	-
(44) Nysa	Е						
(51) Nemausa	CU	• _	v _	v _	Ň	v v	v v
(63) Ausonia	S	./	./	./	v v	v \/	v v
(88) Thisbe	CF	~	~	V	~	~	v ./
(115) Thyra	S	v _	v _	v _	~	v ./	· /
(129) Antigone	M	. /	. /	. /	V	~	V ./
(120) Anugone (102) Nausikaa	S	×	V /	V	× /	V /	V /
(192) Industrial	S S	V	V	-	V	V,	V
(190) Fillioilleia	S M	_	-	-	\checkmark	\checkmark	\checkmark
(216) Kieopatra		\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(230) Atnamantis	2	-	-	-	-	-	-
(324) Bamberga	CP	- ,	- ,	-	-	- ,	-
(349) Dembowska	R	\checkmark	\checkmark		\checkmark	\checkmark	\checkmark
(354) Eleonora	S	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(451) Patientia	CU	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(471) Papagena	S	-	-	-	-	-	-
(511) Davida	С	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(532) Herculina	S	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(704) Interamnia	F			-			

Table 15.2. Tholen's taxonomic classification (Tholen 1989) for Hipparcos minor planets. Poles and shapes (as by-products of pole determinations) solution, from Asteroids II (Magnusson 1989) and updated version (Magnusson *et al.* 1994).

Table 15.3. Hipparcos minor planets general statistics on aspect data. Minimum (min), maximum (max) and median (med) value for the observation epoch, and the apparent magnitude Hp_{dc} .

Num	Obs		Fnoch		N	Agonitud	e	Stand	lard devia	ation
(IAU)	005	(JD	-2440000.0 [da	y])	1	(Hp _{dc})	c	Start	$(\sigma_{Hp_{dc}})$	
		min	med	max	min	med	max	min	med	max
$\begin{array}{c}1\\2\\3\\4\\5\\\end{array}$	65 63 60 58 81	7920.570 8154.100 7909.230 8084.980 8094.660 8094.660	8522.710 8728.980 8391.870 8351.210 8393.780	8977.859 8934.850 9038.870 8817.159 8832.319	7.84 8.51 8.42 7.15 10.23	9.20 9.57 10.60 8.26 11.50	9.49 10.78 11.55 8.69 12.36	0.004 0.003 0.003 0.002 0.005	0.009 0.014 0.018 0.006 0.029	0.024 0.053 0.036 0.019 0.077
6 7 8 9 10 11 12	91 69 56 40 51 68 24	7870.420 7911.560 7917.310 8142.720 8023.670 8144.420 7880.400	8082.890 8560.680 8503.030 8327.859 8479.570 8315.500 8771.590	8967.080 9024.380 9035.370 8826.890 9059.700 8823.000 8820.340	8.67 8.24 9.94 9.75 10.97 10.71 10.68	$ \begin{array}{r} 10.55 \\ 9.83 \\ 10.95 \\ 10.84 \\ 11.57 \\ 11.14 \\ 11.42 \end{array} $	11.67 11.31 11.93 11.90 12.21 12.47 12.33	$\begin{array}{c} 0.001 \\ 0.001 \\ 0.006 \\ 0.005 \\ 0.006 \\ 0.006 \\ 0.006 \end{array}$	$\begin{array}{c} 0.016\\ 0.013\\ 0.022\\ 0.019\\ 0.026\\ 0.023\\ 0.022\\ \end{array}$	$\begin{array}{c} 0.048\\ 0.040\\ 0.051\\ 0.033\\ 0.077\\ 0.082\\ 0.055\end{array}$
13 14 15 16 18 19 20 22 23 27 28 28	$\begin{array}{r} 34\\ 45\\ 83\\ 49\\ 100\\ 30\\ 61\\ 63\\ 66\\ 35\\ 33\\ 74\\ \end{array}$	8053.010 7885.640 7882.620 7894.530 7900.880 8087.990 7984.580 8022.960 7876.620 7876.620 7876.620 7884.310 8513.319	8506.390 8540.890 8694.800 8559.819 8354.020 8331.940 8538.230 8675.730 8402.000 8240.649 8961.450	8739.971 8795.330 9044.960 9033.670 9059.710 8787.159 8809.290 9009.119 9022.460 8830.990 9009.640	$\begin{array}{c} 11.03\\ 10.08\\ 9.51\\ 10.22\\ 10.08\\ 11.15\\ 8.37\\ 10.76\\ 11.02\\ 10.04\\ 11.81\\ 0.86\end{array}$	$\begin{array}{c} 11.95\\ 11.47\\ 10.58\\ 11.59\\ 11.36\\ 12.05\\ 11.32\\ 11.92\\ 12.34\\ 11.41\\ 12.14\\ 10.71\\ \end{array}$	$\begin{array}{c} 12.38\\ 12.53\\ 11.60\\ 12.56\\ 12.26\\ 12.78\\ 12.29\\ 12.73\\ 12.83\\ 12.20\\ 12.63\\ 11.44\end{array}$	$\begin{array}{c} 0.011\\ 0.006\\ 0.004\\ 0.004\\ 0.006\\ 0.014\\ 0.007\\ 0.009\\ 0.009\\ 0.009\\ 0.009\\ 0.009\\ 0.001\\ 1\\ 0.002\end{array}$	$\begin{array}{c} 0.032\\ 0.023\\ 0.022\\ 0.034\\ 0.028\\ 0.038\\ 0.026\\ 0.034\\ 0.039\\ 0.022\\ 0.043\\ 0.010\\ \end{array}$	$\begin{array}{c} 0.074\\ 0.074\\ 0.067\\ 0.069\\ 0.092\\ 0.058\\ 0.115\\ 0.098\\ 0.064\\ 0.082\\ 0.064\\ 0.082\\ 0.060\\ 0.062\\ 0.060\\ 0.062\\ 0.060\\ 0.062\\ 0.060\\ 0.062\\ 0.060\\ 0.00$
$\begin{array}{c} 29\\ 30\\ 31\\ 37\\ 39\\ 40\\ 42\\ 44\\ 51\\ 63\\ 88\\ 115\\ 129\\ \end{array}$	$\begin{array}{c} 74 \\ 48 \\ 14 \\ 32 \\ 112 \\ 103 \\ 51 \\ 53 \\ 14 \\ 12 \\ 36 \\ 33 \\ 40 \end{array}$	7980.090 7819.690 7883.730 7858.510 7969.470 7973.480 8050.70 8050.070 8279.390 7960.780 7960.780 7952.230	8333.090 8289.060 7966.870 7984.820 8396.220 8708.609 8120.000 8586.140 8067.220 8422.020 8169.350 8758.230 8325.319	8752.380 8298.210 8935.730 8805.640 8805.640 8278.280 8808.840 9033.500 8945.390 8618.330 8618.330 8563.950	$\begin{array}{c} \textbf{9.86} \\ \textbf{10.49} \\ \textbf{11.06} \\ \textbf{11.25} \\ \textbf{10.49} \\ \textbf{10.68} \\ \textbf{10.26} \\ \textbf{9.97} \\ \textbf{11.64} \\ \textbf{11.51} \\ \textbf{11.26} \\ \textbf{10.98} \\ \textbf{11.51} \end{array}$	$\begin{array}{c} 10.71 \\ 12.03 \\ 11.82 \\ 11.66 \\ 11.29 \\ 12.20 \\ 11.00 \\ 11.52 \\ 11.79 \\ 11.95 \\ 12.06 \\ 12.38 \\ 12.05 \end{array}$	$11.44 \\ 12.84 \\ 12.27 \\ 13.01 \\ 12.57 \\ 12.63 \\ 12.48 \\ 12.35 \\ 12.24 \\ 12.50 \\ 12.51 \\ 12.65 \\ 12.5$	$\begin{array}{c} 0.003\\ 0.011\\ 0.021\\ 0.009\\ 0.007\\ 0.007\\ 0.007\\ 0.007\\ 0.007\\ 0.010\\ 0.010\\ 0.011\\ 0.015\\ \end{array}$	$\begin{array}{c} 0.019\\ 0.039\\ 0.030\\ 0.030\\ 0.021\\ 0.039\\ 0.022\\ 0.029\\ 0.032\\ 0.026\\ 0.041\\ 0.042\\ 0.032\\ \end{array}$	$\begin{array}{c} 0.063\\ 0.179\\ 0.042\\ 0.079\\ 0.059\\ 0.111\\ 0.067\\ 0.052\\ 0.050\\ 0.057\\ 0.063\\ 0.102\\ 0.062\end{array}$
$192 \\ 196 \\ 216 \\ 230 \\ 324 \\ 354 \\ 451 \\ 471 \\ 511 \\ 532 \\ 704$	$\begin{array}{c} 32\\ 14\\ 21\\ 35\\ 73\\ 92\\ 98\\ 29\\ 112\\ 64\\ 40\\ 82\\ \end{array}$	$\begin{array}{c} 7916.310\\ 7992.570\\ 7895.420\\ 7908.390\\ 8320.710\\ 7910.830\\ 7899.520\\ 8258.271\\ 8020.210\\ 8046.070\\ 7937.440\\ 8021.100\\ \end{array}$	$\begin{array}{c} 8739.649\\ 8139.310\\ 8311.500\\ 8287.020\\ 8325.240\\ 8176.370\\ 8404.700\\ 8632.050\\ 8494.7649\\ 8588.790\\ 8398.210\\ 8534.090\\ \end{array}$	$\begin{array}{c} 8833.640\\ 8594.159\\ 8359.040\\ 8935.980\\ 8682.310\\ 9053.119\\ 8731.630\\ 9024.800\\ 9030.040\\ 8829.300\\ 8735.530\end{array}$	$\begin{array}{c} 10.90\\ 11.58\\ 11.37\\ 11.34\\ 9.35\\ 10.28\\ 10.76\\ 11.52\\ 10.84\\ 11.18\\ 10.16\\ 11.07\\ \end{array}$	$\begin{array}{c} 11.12\\ 11.93\\ 11.87\\ 11.72\\ 12.11\\ 11.77\\ 11.59\\ 11.91\\ 11.38\\ 12.01\\ 10.91\\ 11.76\\ \end{array}$	$\begin{array}{c} 12.15\\ 12.55\\ 12.80\\ 12.44\\ 12.39\\ 12.55\\ 12.46\\ 12.85\\ 12.66\\ 12.29\\ 12.58\end{array}$	$\begin{array}{c} 0.013\\ 0.019\\ 0.008\\ 0.006\\ 0.009\\ 0.012\\ 0.016\\ 0.005\\ 0.007\\ 0.003\\ 0.008\\ \end{array}$	$\begin{array}{c} 0.034\\ 0.037\\ 0.037\\ 0.028\\ 0.028\\ 0.025\\ 0.030\\ 0.033\\ 0.032\\ 0.034\\ 0.021\\ 0.032 \end{array}$	$\begin{array}{c} 0.074\\ 0.051\\ 0.061\\ 0.066\\ 0.068\\ 0.080\\ 0.088\\ 0.070\\ 0.090\\ 0.107\\ 0.064\\ 0.082\end{array}$

Table 15.3. Hipparcos minor planets general statistics on aspect data (continued). The distances to the Sun and to the Earth, the solar phase angle, and the apparent diameter.

Num		Dist	ances		Sol	ar phase	angle	Appar	ent dian	neter
(IAU)	(Sun	[AU])	(Earth	[AU])		(α [deg])		(<pre>p [arcsec])</pre>	
	min	max	min	_max	min	med	max	min	med	.max
$ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ \end{array} $	$\begin{array}{c} 2.582\\ 2.123\\ 2.013\\ 2.205\\ 2.080\\ 1.940\\ 1.844\\ 1.901\\ 2.096\\ 3.068\\ 2.383\\ 2.076\\ \end{array}$	$\begin{array}{c} 2.979\\ 3.382\\ 3.356\\ 2.576\\ 2.842\\ 2.911\\ 2.932\\ 2.539\\ 2.674\\ 3.516\\ 2.695\\ 2.843\\ \end{array}$	$\begin{array}{c} 1.864\\ 1.733\\ 1.309\\ 1.712\\ 1.369\\ 1.207\\ 1.131\\ 1.447\\ 1.371\\ 2.564\\ 1.815\\ 1.421 \end{array}$	$\begin{array}{c} 3.554\\ 3.842\\ 3.693\\ 3.052\\ 3.008\\ 3.370\\ 3.133\\ 3.042\\ 2.819\\ 3.947\\ 3.156\\ 2.282\end{array}$	$\begin{array}{c} 14.19\\ 13.53\\ 14.20\\ 17.96\\ 15.80\\ 14.94\\ 16.56\\ 17.22\\ 19.56\\ 13.54\\ 15.78\\ 16.29\end{array}$	$\begin{array}{c} 16.09\\ 17.43\\ 16.65\\ 21.77\\ 18.02\\ 18.99\\ 23.29\\ 22.72\\ 21.35\\ 16.08\\ 17.78\\ 25.16\end{array}$	22.41 27.37 26.21 26.06 28.73 30.65 31.67 29.92 28.10 19.05 24.92 29.24	$\begin{array}{c} 0.35\\ 0.19\\ 0.09\\ 0.23\\ 0.06\\ 0.08\\ 0.09\\ 0.06\\ 0.08\\ 0.15\\ 0.07\\ 0.07\\ 0.07\\ \end{array}$	$\begin{array}{c} 0.40\\ 0.29\\ 0.12\\ 0.28\\ 0.08\\ 0.13\\ 0.12\\ 0.08\\ 0.12\\ 0.18\\ 0.11\\ 0.09 \end{array}$	$\begin{array}{c} 0.67\\ 0.41\\ 0.26\\ 0.40\\ 0.13\\ 0.22\\ 0.25\\ 0.13\\ 0.17\\ 0.23\\ 0.12\\ 0.11\\ \end{array}$
13 14 15 16 18 19 20 22 23 27 28 29	$\begin{array}{c} 2.354\\ 2.155\\ 2.174\\ 2.552\\ 1.847\\ 2.057\\ 2.066\\ 2.632\\ 2.060\\ 1.948\\ 2.402\\ 2.374\end{array}$	$\begin{array}{c} 2.794\\ 3.006\\ 3.130\\ 3.297\\ 2.785\\ 2.826\\ 2.738\\ 3.003\\ 3.210\\ 2.746\\ 2.727\\ 2.674 \end{array}$	$\begin{array}{c} 1.755\\ 1.454\\ 1.865\\ 1.901\\ 1.495\\ 1.711\\ 1.555\\ 2.071\\ 1.641\\ 1.225\\ 1.932\\ 1.648 \end{array}$	$\begin{array}{c} 2.679\\ 3.552\\ 3.555\\ 3.713\\ 3.301\\ 2.500\\ 3.141\\ 3.285\\ 3.195\\ 2.841\\ 2.498\\ 2.998 \end{array}$	18.48 14.04 13.59 14.16 15.23 16.39 17.96 15.90 13.72 17.05 19.85 17.22	21.63 22.34 15.12 16.77 20.64 23.22 23.02 17.76 17.79 27.21 23.21 21.82	24.63 27.89 25.89 22.97 27.32 28.47 28.59 22.54 28.37 29.42 24.07 24.94	$\begin{array}{c} 0.11\\ 0.06\\ 0.11\\ 0.10\\ 0.06\\ 0.12\\ 0.07\\ 0.05\\ 0.05\\ 0.05\\ 0.07\\ 0.10\\ \end{array}$	$\begin{array}{c} 0.13\\ 0.09\\ 0.16\\ 0.12\\ 0.08\\ 0.15\\ 0.08\\ 0.06\\ 0.06\\ 0.06\\ 0.07\\ 0.08\\ 0.14 \end{array}$	$\begin{array}{c} 0.17\\ 0.15\\ 0.20\\ 0.19\\ 0.14\\ 0.18\\ 0.13\\ 0.08\\ 0.09\\ 0.11\\ 0.09\\ 0.18\\ \end{array}$
$\begin{array}{c} 30\\ 31\\ 37\\ 39\\ 40\\ 42\\ 44\\ 51\\ 63\\ 88\\ 115\\ 129\\ \end{array}$	2.065 2.427 2.345 2.455 2.163 1.890 2.073 2.222 2.205 2.346 2.007 2.287	2.617 3.043 3.063 2.366 2.258 2.769 2.458 2.455 2.720 2.567 2.778	$\begin{array}{c} 1.344\\ 1.786\\ 1.592\\ 1.912\\ 1.445\\ 1.110\\ 1.375\\ 1.688\\ 1.689\\ 1.704\\ 1.339\\ 2.052\end{array}$	$\begin{array}{c} 2.419\\ 2.607\\ 2.506\\ 3.522\\ 2.945\\ 2.409\\ 2.776\\ 1.934\\ 2.163\\ 2.730\\ 2.572\\ 2.800 \end{array}$	$\begin{array}{c} 16.44\\ 17.53\\ 14.23\\ 13.62\\ 18.14\\ 20.67\\ 20.51\\ 19.98\\ 19.53\\ 17.59\\ 21.32\\ 18.21\\ \end{array}$	$\begin{array}{c} 17.87\\ 22.61\\ 19.33\\ 17.64\\ 21.83\\ 23.54\\ 23.48\\ 21.23\\ 23.52\\ 21.58\\ 23.01\\ 20.86\end{array}$	28.29 24.00 22.83 23.56 26.98 32.30 28.34 26.37 26.52 23.75 28.35 24.91	$\begin{array}{c} 0.06\\ 0.13\\ 0.06\\ 0.05\\ 0.06\\ 0.04\\ 0.11\\ 0.07\\ 0.11\\ 0.04\\ 0.06\\ \end{array}$	$\begin{array}{c} 0.08\\ 0.15\\ 0.08\\ 0.09\\ 0.06\\ 0.11\\ 0.04\\ 0.12\\ 0.08\\ 0.14\\ 0.05\\ 0.07\\ \end{array}$	$\begin{array}{c} 0.11\\ 0.19\\ 0.10\\ 0.11\\ 0.13\\ 0.07\\ 0.12\\ 0.09\\ 0.17\\ 0.09\\ 0.08\\ \end{array}$
$192 \\ 196 \\ 216 \\ 230 \\ 324 \\ 349 \\ 354 \\ 451 \\ 471 \\ 511 \\ 532 \\ 704$	2.144 3.029 2.230 2.272 1.771 2.678 2.479 2.845 2.279 2.631 2.329 2.614	$\begin{array}{c} 2.546\\ 3.103\\ 2.727\\ 2.523\\ 2.401\\ 3.074\\ 3.001\\ 3.040\\ 3.349\\ 3.351\\ 3.135\\ 3.118\\ \end{array}$	$\begin{array}{c} 1.435\\ 2.399\\ 1.861\\ 1.718\\ 0.936\\ 2.107\\ 1.946\\ 2.119\\ 1.695\\ 2.207\\ 1.906\\ 2.137\end{array}$	$\begin{array}{c} 2.083\\ 2.948\\ 2.763\\ 2.494\\ 2.970\\ 3.620\\ 3.593\\ 2.719\\ 3.155\\ 3.562\\ 3.654\\ 3.364 \end{array}$	$18.89 \\ 16.59 \\ 17.14 \\ 16.68 \\ 17.46 \\ 13.89 \\ 13.84 \\ 14.35 \\ 14.87 \\ 14.61 \\ 14.22 \\ 14.05 \\ 14.0$	$19.67 \\ 18.63 \\ 19.74 \\ 18.46 \\ 18.32 \\ 14.40 \\ 18.26 \\ 17.69 \\ 17.49 \\ 18.48 \\ 20.32 \\ 18.02$	25.90 19.24 25.94 25.26 32.60 21.29 23.16 20.29 25.28 22.05 24.74 22.04	$\begin{array}{c} 0.07\\ 0.07\\ 0.07\\ 0.06\\ 0.11\\ 0.05\\ 0.06\\ 0.12\\ 0.06\\ 0.13\\ 0.09\\ 0.14\\ \end{array}$	$\begin{array}{c} 0.10\\ 0.08\\ 0.09\\ 0.09\\ 0.11\\ 0.05\\ 0.07\\ 0.14\\ 0.10\\ 0.16\\ 0.13\\ 0.16\end{array}$	$\begin{array}{c} 0.10\\ 0.08\\ 0.10\\ 0.09\\ 0.36\\ 0.09\\ 0.11\\ 0.15\\ 0.11\\ 0.21\\ 0.17\\ 0.21\\ \end{array}$

16. SUCCESSIVE SPHERE SOLUTIONS

The final step in the main astrometric reductions was the combination of the reference great-circle data sets accumulated throughout the mission into a coherent set of astrometric parameters for stars on the whole sky. The principles of this 'sphere solution' process were described in Chapter 11. In the course of the Hipparcos mission several successive sphere solutions were made independently by the FAST and NDAC consortia, involving successively larger data sets or iterations of the main reduction chain. The completion of the solutions using 12, 18, 30 and (finally) 37 months of data provided important checkpoints for the validation of the reductions, and allowed the progress of the astrometric analysis to be followed in terms of the improving statistics of the FAST/NDAC differences. In this chapter the main features of the successive sphere solutions are summarised, results of the main comparisons are presented, and the various sphere solutions are compared with the final Hipparcos Catalogue.

16.1. Introduction

The series of sphere solutions described and compared in this chapter resulted from the incorporation of successively more observations, from iterations of the previous steps of the reduction chain (attitude determination and great-circle reductions), and from improvements of the weighting schemes and modelling of instrumental effects. The evolution of the astrometric data in these solutions, and particularly of the FAST/NDAC differences, strikingly illustrates the convergence of the two complex and rather different reduction schemes into a single, final catalogue.

The principles of the sphere solution are summarised in Chapter 11. The term is used here in a broad sense, including both the determination of the abscissa zero points of the reference great circles (the sphere solution proper) and the subsequent determination of astrometric parameters for individual stars. In the FAST reduction chain these two processes were seen as separate tasks, while in NDAC they were combined in a single task. In both cases the end result was a set of astrometric parameters, in which all the positions and proper motions were given in one and the same reference frame—albeit that frame was not the same in FAST and NDAC, and indeed changed slightly for each new sphere solution. The indefiniteness of the reference frame is inherent to the principle of Hipparcos observations, where a star was only measured relative to other stars and never linked directly to any point with *a priori* known position or proper motion. Ideally, however, the reference frames of any two sphere solutions should differ

only by a rigid-body rotation, which is expressed by six numbers (Section 16.6). After elimination of this rotation difference, the differences in position and proper motion may be analysed in terms of random, regional and global differences. In this chapter the successive sphere solutions obtained by FAST and NDAC have all been aligned with the final Hipparcos Catalogue prior to the comparisons (see Table 16.8).

In contrast with the positions and proper motions, where no 'origin' is accessible to observation by Hipparcos, the trigonometric parallaxes obtained in the reductions are in principle absolute. The comparison of parallaxes is therefore quite straightforward.

Major milestones of the astrometric reductions were reached with the completion of the FAST and NDAC sphere solutions using 12, 18, 30 and (finally) 37 months of data. The main features of these solutions are described in Sections 16.3 to 16.5 and the main results of their intercomparison are given in Section 16.6. For completeness the final Hipparcos Catalogue (HIP) and the two preliminary merged catalogues H18 and H30 are included in the comparisons.

16.2. Principles of Iterations

The astrometric reductions for the Hipparcos mission were global in the sense that the astrometric parameters—positions, parallaxes and proper motions—of a large number of stars scattered over the whole celestial sphere had to be solved together. This was necessary in order to achieve a globally consistent system of positions and proper motions, and for the determination of absolute parallaxes. It was not necessary, though, that this solution included all the objects observed with the satellite: special objects like double and multiple stars, stars showing non-linear photocentric motions, and solar system objects could be linked into the same system at later stages of the reductions. In principle this global solution should have used all the data collected throughout the mission in order to obtain the optimum estimate of each parameter. The number of essentially 'non-problematic' stars suitable for this process was about 100 000. Thus, a rigorous implementation of such a solution would have involved the simultaneous adjustment of some 500 000 stellar parameters plus many more describing the instrument and its scanning motion, using the observation frames as input for the adjustment.

At the time when the software for the Hipparcos data reductions was designed, the rigorous adjustment of such a large problem was not considered feasible. An alternative, less rigorous but practicable method was devised, usually referred to as the 'three-step' reduction procedure (Section 4.1). The main idea was to introduce an intermediate level of adjustment, where instantaneous, one-dimensional stellar coordinates along selected reference great circles were estimated; these coordinates are known as the star abscissae. The need to iterate the main astrometric reductions was a direct consequence of this simplified approach adopted by both the FAST and NDAC consortia. Its principle can be understood as follows.

The elementary Hipparcos observations were one-dimensional, measuring the location of stars in the direction *G* perpendicular to the slits of the main grid (Figure 16.1). In the great-circle reduction, these measurements were combined into estimates of the star abscissae along the reference great circle. However, because the slits generally made a small angle φ relative to the lines of constant abscissae, the observed quantity *G* depended not only on the abscissa *v* but also, to a smaller extent, on the distance of



Figure 16.1. Schematic illustration of the abscissa projection error in the great-circle reduction caused by catalogue errors. RGC = reference great circle; t = true position of star; c = star position according to the current catalogue; (v, r) = true abscissa and ordinate of the star; r' = assumed ordinate (computed from the catalogue); ss = observed location of the star (from measurement of the G coordinate); v' = inferred abscissa. The error in the inferred abscissa is given by $v' - v = -(r' - r) \sin \varphi$.

the star from the reference great circle, i.e. on the ordinate *r*. Since *r* was not estimated in the great-circle reduction, it had to be computed from the current knowledge of star positions and instrument attitude. As illustrated in Figure 16.1 the error in the ordinate, r' - r, was transferred into an error in the estimated abscissa through a multiplicative factor of about $\sin \varphi$. (More accurately, the effective factor was a mean value $\langle \sin \varphi \rangle$ over all the scans across the star considered in the great-circle reduction. This averaging greatly reduced the resulting error for many stars.)

The reference great circles were generally chosen such that $|\varphi| \leq 1^{\circ}$. Consequently the catalogue-induced abscissa errors were at most some 2 per cent of the positional errors of the catalogue used in the great-circle reduction. The rms contribution to the abscissa errors was typically about 0.7 per cent of the catalogue errors (van der Marel 1988). Similar considerations can be made for the attitude errors, where the error component normal to the scanning was propagated into the abscissae with a corresponding attenuation factor. In the subsequent sphere solution the standard errors of the astrometric parameters were normally a factor $\simeq 0.2$ smaller than the standard errors in abscissae. Thus, the total attenuation factor for the positional errors, when propagated from an initial catalogue through the attitude determination, great-circle reductions and sphere solution, was typically of the order of 0.1 to 0.2 per cent. This is to be regarded as a gross average; on specific stars the situation may have been much less favourable.

Initially, the astrometric data in the Hipparcos Input Catalogue were used to determine the satellite attitude and in the preliminary great-circle reductions. The positional uncertainty of that catalogue, at the epoch 1990.0, was typically about 0.3 arcsec (Turon *et al.* 1995), with zonal systematic errors reaching 0.2 arcsec and with individual errors up to several arcsec. The corresponding errors after a first sphere solution should therefore generally be of the order of 0.5 mas, but perhaps reaching several milliarcsec on some stars. This is not negligible compared with the level of errors expected from the photon noise, instrument modelling, etc., and the obvious remedy was iteration: the

Table 16.1. Summary of successive NDAC and FAST sphere solutions, and of the three merged catalogues H18, H30 and HIP. The initial letter in the solution designations indicates the origin: NDAC (N), FAST (F), or merged (H). The following number is the approximate number of months of observations that were used in the solution, with a decimal indicating an iterated or otherwise improved version. 'R' signifies restricted solutions, solving only for positions and parallaxes (with proper motions taken from the Hipparcos Input Catalogue). NOR is the solution based on the validation data. Subsequent columns give the approximate date of the solution; the number of stars with accepted astrometric solutions; the total number of abscissae input to the sphere solution, and the number of abscissae actually used for the accepted stars; the range and number of orbits used; and the reference epoch for the resulting astrometric parameters. Concerning the range of orbit numbers it can be noted that no data exist from orbits 2 to 47. The Julian Date for the apogee of orbit number *N* is approximately given by 2 447 835.46 + 0.4441*N* – 1.3 × 10⁻⁷*N*².

Solution	Approx. date	No. of	No. of absc.	No. of absc.	Range of	No. of	Ref.
	of creation	stars	(input)	(used)	orbits	orbits	epoch
N0R	27 May 1991	15 564	269 769	81 704	1-1032	171	1990.00
N12	28 Apr 1992	47061	1312713	667447	1-915	812	1990.00
N12R	12 May 1992	82 309	1312713	996511	1-915	812	1990.00
N18	14 Oct 1992	102 411	1979988	1792839	1-1336	1215	1990.00
N30	22 Sep 1993	103 131	3158933	2876215	1-2118	1963	1990.00
N37.1	30 Sep 1994	109698	3632162	3463106	1 - 2768	2328	1990.00
N37.5	25 Apr 1995	111255	3570685	3490792	1 - 2768	2326	1991.25
F12	30 Apr 1992	31 921	1 226 986	424 227	48-925	758	1992.00
F12R	1 Jul 1992	46 716	1226986	505 885	48-925	758	1992.00
F18	25 Oct 1992	93 781	1952958	1597519	48-1336	1184	1990.75
F18.1	23 Jun 1993	93612	1952958	1573013	48-1336	1177	1990.75
F30	27 Sep 1993	99950	3221747	2670741	48-2129	1914	1991.25
F37.1	22 Oct 1994	117246	3724992	3592389	48-2763	2269	1991.25
F37.3	13 Jun 1995	116683	3743053	3 570 708	48-2763	2281	1991.25
H18	23 Sep 1993	105 371			1-1336		1990.75
H30	11 Jan 1994	107504			1-2129		1991.25
HIP	8 Jun 1996	117955			1-2768		1991.25

star positions resulting from the first sphere solution were used for an improved attitude determination, improved great-circle reductions, and an improved sphere solution. In principle a single such iteration might be sufficient to ensure that the resulting sphere solution is completely limited by observational noise, and independent of the starting values used for the astrometric parameters. However, as is generally the case in complex iterations, some error components decay much slower than the overall variance, and at least two complete iterations were considered necessary in the case of the Hipparcos reductions. In reality these iterations were important also for many other aspects of the reductions, in particular the instrument calibrations, which depended critically on the accuracy of the star catalogue being used.

The sphere solutions discussed in this chapter are summarised in Table 16.1 and further described in subsequent sections. Some entries in the Hipparcos Catalogue are clearly unsuitable for comparison with the earlier sphere solutions, because they finally required more complex modelling than the standard five-parameter astrometric model assumed in the sphere solution. This applies in particular to well-resolved double stars, orbital binaries, variability-induced movers (VIMs), and stochastic solutions (see Volume 1, Section 2.3 for an explanation of these categories). The statistics and comparisons given below are therefore restricted to the basic subset of 101 246 entries whose data

16.3. NDAC Sphere Solutions

Overview of Iterations

The NDAC iteration scheme deviated somewhat from the principle described in the previous section. In parallel with the star mapper data processing and attitude determination performed at the Royal Greenwich Observatory, a working star catalogue was maintained and successively updated by means of the star mapper transit residuals obtained in the attitude fit (Section 6.9; see also van Leeuwen *et al.* 1992). During the first 18 months of the mission this catalogue provided far better positions than the Input Catalogue for many stars, and these were used in the NDAC great-circle reductions until superseded by the first full-scale sphere solution (N18). Compared with the FAST scheme this gave a rapid initial improvement of the data, but relatively smaller improvements by the later sphere solutions.

Subsequent processing in NDAC was based on the N18 catalogue (actually on a slightly later version including about six weeks of additional data), including a re-run of the greatcircle reductions for the first part of the mission. This resulted first in the 30-months solution N30, and, after all the mission data had been been included (approximately 37 months in all), the solution N37.1. That catalogue was then used for a third and final re-run of the attitude determination and great-circle reductions. The resulting abscissae were used in a series of sphere solutions leading up to the final NDAC solution N37.5. Versions N37.2 to N37.5 used the same abscissa data as input and differed mainly in the treatment of colour terms and in the internal weighting of data, as outlined below.

Main Characteristics of the Solutions

NOR: This was the partial sphere solution based on the so-called validation data given to the reduction consortia prior to the full-scale data distribution. The purpose of the validation data was to test the interfaces between ESOC and the reduction consortia, to enable the consortia to test their software for the first time on 'real' data, and to make cross-comparisons of the intermediate results in order to validate the satellite data. The analysis of the validation data was in NDAC carried all the way through the main reduction chain, up to the sphere solution, in spite of the very scant sky coverage. This provided a very important first check of the overall consistency of the data, as discussed by Lindegren *et al.* (1992).

N12 and N12R: Both these solutions were based on the same set of abscissae from about 12 months of data collected up to 16 December 1990. In N12 all five astrometric parameters were solved whenever possible; in N12R the proper motions were constrained to

their values in the Input Catalogue and only the positions and parallaxes were updated, which allowed many more stars to be treated. The criteria for accepting the astrometric solution on a specific star included an upper limit on the standard error in parallax, $\sigma_{\pi} \leq 4$ mas in N12 and $\sigma_{\pi} \leq 3$ mas in N12R, and on the goodness-of-fit statistic, F2 ≤ 5 (for a definition of F2, see the description of Field H30 in Volume 1). Some stars were also rejected because of unreasonably large updates in position or proper motion.

Previous sphere solutions had shown that the abscissa variances estimated in the greatcircle reductions (for the 'smoothed' solutions) were systematically too small, and that an extra variance of $(2.7 \text{ mas})^2$ had to be added to obtain a reasonable agreement with the distribution of residuals. This weight correction was used in N12 and subsequent NDAC sphere solutions, until an improved weighting scheme was introduced with N37.1 (see below).

N18: This solution was based on about 18 months of data, including observations made up to 21 June 1991. All five astrometric parameters were estimated. The acceptance criteria included limits on the standard errors in parallax ($\sigma_{\pi} \leq 4$ mas) and proper motion ($\sigma_{\mu_{\alpha*}}$, $\sigma_{\mu_{\delta}} \leq 15$ mas/yr), and on the correlations between parallax and proper motion ($|\rho_{\pi^{\alpha*}}^{\mu_{\alpha*}}|$, $|\rho_{\pi}^{\mu_{\delta}}| \leq 0.6$).

N30: Observations collected up to 2 June 1992 were included in this sphere solution, which was the first complete NDAC iteration in the sense that the attitude determination and great-circle reductions had been re-computed with a star catalogue based on a full-scale sphere solution (\simeq N18).

N37.1: The NDAC 37-months solutions include data collected up to the actual end of the scientific operations on 17 March 1993. Starting with N37.1, several improvements were made in order to obtain valid solutions for as many stars as possible, and to further reduce modelling errors. The improvements included in particular: relabelling and merging of data for some stars which for historical reasons had been observed under two different identifiers; resolution of grid-step errors remaining from previous solutions; inclusion of the sixth harmonic terms estimated for individual reference great circles; introduction of non-zero assumed radial velocity for 22 stars (see Volume 1, Table 1.2.3); and the use of V-I colours (instead of B_T-V_T) as the basis for chromaticity calibrations. The previous upper limits on acceptable astrometric standard errors and correlations were also removed and replaced by a flagging of weak solutions.

N37.5: The solution N37.1 was used for a complete iteration of the attitude determination and great-circle reductions. The resulting abscissae were used as input for N37.2 to N37.5, a series of sphere solutions in which the final treatment of chromatic effects, the sixth harmonic, and the weighting of the abscissae were fixed after considerable experimentation. Starting with N37.2, the adjustment of the astrometric parameters were directly made with respect to the epoch J1991.25(TT), thus eliminating the need for epoch transformations on the resulting catalogue. Each solution produced summary statistics of the $\simeq 3.5$ million abscissa residuals binned according to colour (V - I), magnitude (Hp), and orbit number; they were also analysed by linear regression versus $\cos 6v$ and $\sin 6v$ for each great-circle reduction. The trends in terms of biases and deviations from the expected unit weight residual were carefully studied and, when relevant, incorporated as systematic corrections to the abscissae and their standard errors in a subsequent solution. What was finally obtained (in N37.5) was therefore an internally consistent solution with an overall unit weight error of 1.000 and with no significant

trends with magnitude and colour. The re-calibration of abscissa standard errors between N37.1 and N37.5 resulted in a general decrease by about 14 per cent of the formal standard errors of the astrometric parameters, although the actual improvement of the astrometric parameters was probably very marginal (see Section 16.5).

The solution N37.5 also produced output, in the form of adjusted parameters and residuals, that was used for the final merging of the FAST and NDAC astrometry (Chapter 17) and for the production of the Hipparcos Intermediate Astrometric Data (Volume 1, Section 2.8).

Treatment of Chromaticity

Both N12 and N12R included, as the only global parameters, the unknowns Γ_{23} and Γ_{24} which model an abscissa bias varying linearly with colour index and time:

$$v^{\text{obs}} = v^{\text{calc}} + \Gamma_{23}C + \Gamma_{24}(t - 1990.5)C + \eta$$
[16.1]

Here v^{obs} is the observed abscissa, v^{calc} the abscissa calculated from all parameters except the global ones, $C = (B_T - V_T) - 0.5$, *t* is the time of the observation, and η is the abscissa noise. Colour indices $B_T - V_T$ were generally taken from the Extended Input Catalogue (Perryman *et al.* 1989 Volume II, Section 18.2), with some updates resulting from the star mapper photometric processing. The results for the chromaticity were rather similar in the two solutions:

$$\Gamma_{23} = -1.355 \pm 0.015 \text{ mas mag}^{-1} \\ \Gamma_{24} = +0.339 \pm 0.100 \text{ mas mag}^{-1} \text{ yr}^{-1}$$
 N12 [16.2]

$$\Gamma_{23} = -1.425 \pm 0.009 \text{ mas mag}^{-1} \\ \Gamma_{24} = +0.313 \pm 0.038 \text{ mas mag}^{-1} \text{ yr}^{-1}$$
 N12R [16.3]

Solution N18 contained the same chromatic terms (along with additional parameters discussed below), and the result agrees well with the earlier determinations:

$$\Gamma_{23} = -1.404 \pm 0.007 \text{ mas mag}^{-1} \\ \Gamma_{24} = +0.330 \pm 0.020 \text{ mas mag}^{-1} \text{ yr}^{-1}$$
 N18 [16.4]

From N30 onwards, the reference epoch for Γ_{23} was taken to be J1991.25 instead of J1990.5. The result for N30 was:

$$\Gamma_{23} = -1.110 \pm 0.004 \text{ mas mag}^{-1} \\ \Gamma_{24} = +0.366 \pm 0.007 \text{ mas mag}^{-1} \text{ yr}^{-1}$$
 N30 [16.5]

corresponding to the value -1.385 ± 0.007 mas/mag at J1990.5. In N37.1 the colour index V - I was used instead of $B_T - V_T$, resulting in a slight change in the numerical values:

$$\Gamma_{23} = -1.166 \pm 0.006 \text{ mas mag}^{-1} \\ \Gamma_{24} = +0.332 \pm 0.007 \text{ mas mag}^{-1} \text{ yr}^{-1}$$
 N37.1 [16.6]

Using the same abscissa input as for the 30-month solution, a special solution was made in order to investigate the dependence of the chromatic displacement on the colour index. In this solution a sixth parameter (a_6) was added for each star, while no global chromatic terms were used. With v^{calc} denoting the abscissa calculated without any chromatic term, using the normal five astrometric parameters (a_1 to a_5), the observation equation for the additional parameter was written:

$$v^{\text{obs}} = v^{\text{calc}} + \left[-1.110 + 0.366(t - 1991.25)\right]a_6$$
[16.7]

According to Equation 16.5 the coefficient in brackets equals the mean chromatic displacement per magnitude in $B_T - V_T$, assuming that the effect depends linearly on that colour index. The standard errors of the individual estimates of a_6 were typically around 0.8 mas, or 0.8 mag if the parameter is interpreted as a colour index. Mean relations between a_6 and the colour indices $B_T - V_T$ and V - I are shown in Figures 16.2–16.3. It appears that the relation to V - I is the more linear one, at least for the very red stars, which motivated the switch from $B_T - V_T$ to V - I from N37.1. However, both relations show some significant curvature and kinks in the well-determined colour interval.

It was suggested by M. Grenon that the effective wavelength might be a better independent variable for modelling the chromatic abscissa displacement, and formulae for calculating λ_{eff} as function of V - I and t were provided. A simple transformation of the V - I scale in Figure 16.3 into the effective wavelength at mid-mission indicated that λ_{eff} probably gives the best overall linear representation of the effect (Figure 16.4). In N37.5 the independent variable for the chromatic displacement was taken to be the dimensionless quantity $[\lambda_{\text{eff}}(V - I, t) - 550 \text{ nm}]/(50 \text{ nm})$, replacing the (V - I) - 0.5 used in N37.1.

Preliminary runs with all 37 months of data indicated that the chromatic behaviour of the instrument changed towards the end of the mission, and that a simple linear variation of the chromaticity with time would no longer be sufficient. Figure 16.5 shows the chromaticity obtained for the individual great-circle reductions by regression of the abscissa residuals against λ_{eff} . The roughly linear variation up to day 1170 (mid-March 1992) agrees well with the previously determined Γ_{23} and Γ_{24} , but this trend is then replaced by a rather erratic behaviour. Much of the scatter seen in this figure is actually physically significant and anomalous variations can be discerned also earlier in the mission, especially around day 490 (April–May 1990). The chromatic modelling was therefore modified to include an *a priori* correction for the individual orbits, on top of which the global parameters Γ_{23} and Γ_{24} were determined. The relevant terms in the observation equations were therefore written:

$$v^{\text{obs}} = v^{\text{calc}} + \left[Q_N + \Gamma_{23} + \Gamma_{24}(t - 1991.25)\right] \frac{\lambda_{\text{eff}} - 550 \text{ nm}}{50 \text{ nm}} + \eta$$
 [16.8]

where Q_N is the *a priori* chromaticity in orbit *N* shown in Figure 16.5. The chromatic parameters were found to be:

$$\left. \begin{array}{l} \Gamma_{23} = +0.049 \pm 0.004 \mbox{ mas} \\ \Gamma_{24} = +0.010 \pm 0.005 \mbox{ mas } \mbox{yr}^{-1} \end{array} \right\} \ \ \, \text{N37.5} \eqno(16.9) \label{eq:gamma}$$

In principle these parameters should vanish in view of the *a priori* correction of chromaticity through Q_N . The above values, being below the 0.1 mas level, were however considered acceptable.

Harmonic Terms

The harmonic terms are systematic displacements of the abscissae which are periodic functions of the abscissa difference between the star and the Sun, $v - v_{\odot}$. Consideration of possible thermally induced variations of the basic angle, related to the satellite/Sun geometry, led to the introduction of the global parameters Γ_2 to Γ_{12} , which express such a variation up to the sixth harmonic, assuming phase coherence over the whole mission (Lindegren *et al.* 1992). These parameters were included in solution NOR and N18. In N18 all eleven parameters were smaller than 0.1 mas in absolute value, although some

of them were formally significant. Special tests were also carried out: the 18 months of data were split into odd- and even-numbered reference great circles, and independent sphere solutions were calculated for the two data sets. The global harmonic parameters were found to be rather different in the two solutions, supporting the conclusion that none of them were really significant. In subsequent solutions no global harmonic term was therefore included.

While systematic variations related to the satellite/Sun geometry thus appeared to be negligible, the abscissa residuals for individual great-circle reductions often showed a pronounced pattern with a dominant period of 60° (the 'sixth harmonic'). This can be understood as an effect of the relative difficulty in estimating this particular harmonic component of the abscissae, which in turn is related to the particular value of the basic angle, $\simeq 58^{\circ}$. In different great-circle reductions the sine and cosine components of the sixth harmonic are excited by unpredictable causes and therefore result in a quasi-random distribution of phases. This explains why the global parameters Γ_{11} and Γ_{12} were small, even if the effect was large on individual great circles. The amplitude of the sixth harmonic was typically about 2 mas, but may reach 10 to 20 mas in some great-circle reductions. In the 37-month solutions the coefficients of the sixth harmonic were determined independently for each great-circle reduction by analysis of the residuals. The corresponding harmonics were then removed in the subsequent solution. This process had essentially converged before the calculation of the final solution N37.5.

Gravitational Deflection

Solutions N30, N37.1 and N37.5 included the global parameter Γ_{13} , which is a correction to the general-relativistic light deflection (Perryman *et al.* 1989 Volume III, Section 9.3). It is related to the PPN parameter γ by:

$$\gamma = 1 + \frac{Ac^2}{2GS}\Gamma_{13}$$
[16.10]

where $2GS/Ac^2 = 4.0719...$ mas is the deflection at right angles to the solar direction for an observer at one astronomical unit (*A*) from the Sun (see also Equation 11.19). The following values were obtained:

$$\begin{aligned} \gamma &= 0.971 \pm 0.006 & (N30) \\ \gamma &= 0.993 \pm 0.007 & (N37.1) \\ \gamma &= 0.992 \pm 0.005 & (N37.5) \end{aligned} \tag{16.11}$$

The reason for the rather low value of γ in N30 is not known; possibly it is related to the modelling of the sixth harmonic, which was introduced with N37.1. The other two values are not significantly different from unity, as predicted by General Relativity.



Figure 16.2. The chromatic effect studied by solving the abscissa displacement for each star (as a sixth 'astrometric' parameter, a_6) and calculating a mean value for each bin in the colour index $B_T - V_T$. The data were derived by NDAC in a special 30-month solution.



Figure 16.3. The same as Figure 16.2, but with a_6 binned according to colour index V - I.



Figure 16.4. The same as Figure 16.2, but with a₆ binned according to the effective wavelength at epoch J1991.25.



Figure 16.5. Evolution of chromaticity (Q_N) determined independently for each great-circle reduction, i.e. as a function of the orbit number, N. The data were derived by NDAC in a preliminary 37-month solution.

16.4. FAST Sphere Solutions

Overall Organisation

Unlike NDAC, the sphere solution and the determination of the astrometric parameters were considered as two different tasks in the data reduction scheme adopted by FAST (see Chapter 11).

In the first step a sphere solution was computed from the reduction on the circles to bring all the abscissae of the subset of stars referred to as the primary reference stars into the same reference system. Up to this point the abscissae had been constructed with a set of inconsistent origins on the circles. The main result of the FAST sphere solution was then a file containing the correction to be applied to each origin, one per circle, so that the resulting network of circles determined a consistent reference frame on the sphere. At the same time several global parameters were computed, such as those connected to the chromaticity and the thermal effects.

The criteria used to select the primary reference stars included the number of abscissae available and the fact that the observations were clean, i.e. the star was not detected to be, or suspected of being, double. Also, a primary reference star had to be photometrically constant, as far as this could be ascertained with the Hipparcos observations. In addition, the distribution of the stars was chosen to achieve a uniform density on the sky with at least one star per square degree. Finally after a first run, all the stars with large correlation coefficients between the astrometric parameters were excluded from the selection, on the ground that this indicated a poor time distribution of the observations.

In the second step, the abscissae of the primary reference stars and of the other programme stars were referred to the new origins and corrected for the global parameters. Then, on a star by star basis, a least-squares fit of the abscissae was made for the five astrometric parameters $\Delta\lambda_*$, $\Delta\beta$, $\Delta\pi$, $\Delta\mu_{\lambda*}$, $\Delta\mu_{\beta}$. The remaining grid-step errors were searched for in this step and removed accordingly. For the double and multiple stars a similar procedure was applied for the photocentre or the primary, according to the separation, by correcting the abscissae as explained in Chapter 13.

Iterations

After every run, corresponding to a sphere solution and an astrometric solution for all the stars, an improved astrometric catalogue was made available, at least for the stars with an accepted solution. This new version of the catalogue was virtually free of grid-step errors and much closer to the true position on the sky than the Input Catalogue. In the iterative mode, advantage was taken of the good knowledge of the along-scan attitude to improve, with the star-mapper data, the two transverse attitude angles (see Chapter 7). Then, from the improved attitude and the new reference catalogue, an update was made of the residual between the observed and computed abscissae of the reference great circles already processed before the iteration. The same reference catalogue was used also for the processing of subsequent observations not yet considered.

This procedure led to the computation of several solutions as more data were made available. The main characteristics of these solutions regarding the duration, number of stars and observations are given in Table 16.1. The various iterations over the 37 months are listed in Table 16.2.

Main Features of the Iterated Solutions

The first sphere solution was computed in November 1990 on the 300 reference great circles derived from the first six months of data. This run was used to test the two methods developed within the FAST consortium, to solve the equations of the sphere, on real data. Improved positions were obtained for about 10 000 stars.

F12 and F12R: These two solutions were constructed from a full year of data. They yielded the estimates of the origins of 758 reference great circles. The instrumental effects were represented by seven global parameters for the thermal variations and a single global term for the chromaticity. Thus, with Γ_k denoting the *k*th global parameter, the abscissa correction of the *i*th star on the *j*th circle was written:

$$v_{ji}^{\text{obs}} - v_{ji}^{\text{calc}} = \sum_{k=1}^{8} \frac{\partial v_{ji}}{\partial \Gamma_k} \Gamma_k$$
 [16.12]

with the partial derivatives:

$$\frac{\partial v_{ji}}{\partial \Gamma_k} = \cos n(v_{ji} - v_{j\odot}), \quad \text{with } n = 1, 2, 3, 6 \text{ for } k = 1, 2, 4, 6 \qquad [16.13]$$

$$\frac{\partial v_{ji}}{\partial \Gamma_k} = \sin n(v_{ji} - v_{j\odot}), \quad \text{with } n = 2, 3, 6 \text{ for } k = 3, 5, 7$$
[16.14]

for the thermal variations, and:

$$\frac{\partial v_{ji}}{\partial \Gamma_8} = (B - V)_i - 0.5$$
[16.15]

for the chromatic term. In these equations v_{ji} is the abscissa of the *i*th star and $v_{j\odot}$ that of the Sun. The values of the coefficients Γ_k found in the different iterations are given in Table 16.3. The meaning of the coefficients evolved somewhat during the processing as the instrument modelling was refined.

The version F12 of the solutions included the five astrometric parameters for 30 411 stars. However the time base of 12 months was too short an interval to expect an accurate determination of the proper motions. The run served primarily as a test of all the interfaces. A restricted solution, F12R, was computed by adjusting only the two positional parameters and the parallax, constraining the proper motions to their reference catalogue values. The median formal errors in ecliptic longitude and latitude reached respectively 2.0 and 1.7 mas, and 2.5 mas in the parallax.

F18 and F18.1: These runs were based on about 18 months of data covering the observations from the beginning of the mission until 21 June 1991. The instrument modelling was the same as for the 12 months solution. The amplitudes of the thermal terms were all less than one milliarcsec. Only the chromatic term brought a significant contribution to the abscissae with an amplitude larger than 1 mas/mag. The abscissa origins could be determined with a precision better than 0.2 mas and ranged within 5–10 mas from the arbitrary origins set by the great-circle solutions. F18.1 was the first iterated solution using both a new reference catalogue and the attitude software in iterated mode. The gain in precision for the astrometric parameters was between 6 and

9 per cent as shown in Figure 16.6, where the dashed lines refer to the solution before iteration. The activation of the attitude in iterated mode was the only significant change between F18 and F18.1.

F30: All observations up to 6 June 1992 were used to build the 30-month solution leading to 1914 reference great circles. For the first 18 months the attitude and abscissae were not recomputed but were taken from F18.1. For the remaining data a non-iterated mode was used for the attitude and F18 was the reference catalogue. A new model for the chromaticity was introduced which changed the meaning of the parameters Γ_6 , Γ_7 and Γ_8 (Table 16.3), which were now defined as:

$$\frac{\partial V_{ji}}{\partial \Gamma_6} = (V - I)_i - 0.5$$
[16.16]

$$\frac{\partial v_{ji}}{\partial \Gamma_7} = [(V - I)_i - 0.5]^2$$
[16.17]

$$\frac{\partial v_{ji}}{\partial \Gamma_8} = [(V - I)_i - 0.5](T_j - T_0)$$
[16.18]

where $T_0 = J1990.75$ was the reference epoch for these solutions. Note that the colour index used was V - I instead of B - V. In Figure 16.8, which shows the corrections to the abscissa origins as a function of time, the difference in quality between the circles in iterated mode and those in nominal mode (from day 902) is evident. The astrometric precision improved, as expected, more or less as $t^{-1/2}$ for the positions and parallax and as $t^{-3/2}$ for the components of the proper motion.

F37, F37.1, F37.3: The nominal processing of the observations acquired later than 20 April 1991 was done with the F18 reference catalogue. The 18-month abscissae and attitude were kept and a new catalogue F37 was produced at the end of this processing, with the same instrument modelling as in F30.

An iterated solution was computed with all the available data, including the observations carried out in sun-pointing mode. This resulted in 2269 reference great circles and an astrometric solution (F37.1) for all the stars, single as well as double.

The very last iteration in FAST led to the catalogue F37.3 which was used as the FAST solution for the merging (Chapter 17). The instrument modelling with the global parameters was kept unchanged. However, in addition to the origin of each great circle, a function $C_j \cos 6(v - v_{\odot}) + S_j \sin 6(v - v_{\odot})$ was determined to account for a possible systematic resonance between the basic angle and 360°. The amplitudes found in the FAST solutions were very similar to NDAC's with C_j , $S_j \simeq 2$ mas, although for a few great circles the amplitude was as large as 10 mas. The mean and rms abscissa residuals over each reference great circle are shown in Figures 16.9–16.10. The marked change in the dispersion of the residuals at about day 600 followed a modification in the time allocation strategy at the grid level.

The final precision is shown in Figure 16.7 as a function of the ecliptic longitude. For the sake of comparison, the F30 solutions are shown in dashed lines. As expected the improvement was particularly noticeable in proper motion because of the longer time base. This solution included the astrometric parameters for 16 180 double stars of which 10 220 were computed for the brighter component while for the 5960 close binaries with separation $\rho < 0.35$ arcsec the astrometric solution referred to the photocentre. The typical precisions of the solutions, for single and double stars are given in Table 16.4.

Gravitational Deflection

The FAST sphere solutions described above did not include a global parameter corresponding to a correction to the general-relativistic light deflection. The observation equations were instead corrected in accordance with General Relativity, i.e. assuming the nominal value for the PPN parameter, $\gamma = 1$. However, several special runs of the 37-month solution were made in which this parameter was treated as a global parameter (see Equation 11.19). The runs differed in the modelling of other global parameters and the selection of stars and reference great circles, but they all produced results consistent with the General Relativistic value of $\gamma = 1$ to within the standard errors of the solutions. The net result of these experiments can be summarised as:

$$\gamma = 1.000 \pm 0.004$$
 [16.19]

but with non-negligible correlations with the parallaxes and several other global parameters.

16.5. Evolution of Standard Errors

The standard errors in position were generally different in right ascension and declination: in fact, the error ellipses tended to be oriented along the ecliptic axes due to the symmetry of the scanning law with respect to the ecliptic plane. The situation was similar for the standard errors in the proper motion components. When considering the global precision of the solutions, it is convenient to neglect the anisotropy of the uncertainty and adopt the rms values

$$\sigma_{\text{pos}} = \sqrt{\frac{\sigma_{\alpha*}^2 + \sigma_{\delta}^2}{2}} \quad \text{and} \quad \sigma_{\mu} = \sqrt{\frac{\sigma_{\mu_{\alpha*}}^2 + \sigma_{\mu_{\delta}}^2}{2}} \quad [16.20]$$

as representative of the standard errors in position and proper motion for any given star. These quantities are invariant with respect to the coordinate system used. σ_{μ} should not be confused with the standard error of the modulus of the proper motion.

The evolution of the standard errors of the NDAC, FAST and merged solutions are illustrated in subsequent figures and tables. Only stars in common with the 'basic subset' defined in Section 16.2 are included in the statistics.

NDAC and FAST Solutions

The distributions of the formal standard errors of the astrometric parameters are shown in Figures 16.11–16.13 for NDAC, and in Figures 16.15–16.17 for FAST. In each diagram the distributions are compared with that of the final Hipparcos Catalogue. The 10th, 50th and 90th percentiles of the distributions are given in Tables 16.5 and 16.6. The positions refer to the mean effective catalogue epochs $\langle T_{\rm eff} \rangle$ also given in the tables; these were obtained as the median values of the individual effective epochs calculated by Equation 1.2.10 of Volume 1.

Table 16.2. History of the FAST processing with attitude in the initial mode (label 0) or with the iterated mode according to the iteration level (labels 1, 2, 3). For example the solution over 30 months was constructed with the F18 catalogue as reference data, and resulted in the catalogue F30. The first 18 months included the improved attitude using the F18 positions and the along-scan angle derived with the F18 abscissae, while the observations between 18 and 30 months were processed in the initial mode directly from the star mapper attitude. HIC = Hipparcos Input Catalogue.

Reference	Output	Months							
catalogue	catalogue	6	12	18	30	37			
HIC	F12	0	0						
HIC	F18	0	0	0					
F18	F18.1	1	1	1					
F18	F30	1	1	1	0				
F18	F37	1	1	1	0	0			
F37	F37.1	2	2	2	1	1			
F37.1	F37.3	3	3	3	2	2			

Table 16.3. Values of the global parameters in the FAST solutions. The instrument model changed betweenthe 18-month and 30-month solutions.

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Г <mark>8</mark>
	mas	mas/mag						
F12R	-0.0082	+0.0146	-0.0838	+0.2833	+0.4379	+0.4877	-0.4260	-2.1491
F18.1	+0.0270	-0.0104	-0.0293	-0.0221	-0.0363	+0.0506	-0.0286	-1.5971
	mas	mas	mas	mas	mas	mas/mag	mas/mag ²	mas/mag/yr
F30	+0.0075	+0.0004	+0.0034	-0.0031	-0.0065	-0.3906	+0.0843	+0.3764
F37.3	+0.0080	-0.0050	+0.0098	-0.0038	-0.0031	-0.3221	+0.1226	-0.0622

Table 16.4. Mean precision of the astrometric parameters for a star of magnitude Hp = 8. For double stars separate statistics are given for close systems (separation $\rho < 0.35$ arcsec), for which the astrometric parameters of the photocentre were derived, and wider systems where the solution referred to the primary component.

Parameter	Single stars	Double	stars	Unit
		photocentre	primary	
$\lambda * (= \lambda \cos \beta)$	0.8	1.0	1.8	mas
β	0.7	0.8	1.5	mas
π	1.0	1.4	2.2	mas
$\mu_{\lambda*}$	1.0	1.3	2.2	mas/yr
μ_{eta}	0.8	1.1	1.8	mas/yr



Figure 16.6. Mean precision of the FAST 18-month solutions as function of ecliptic latitude. Dashed line: noniterated solution (F18); continuous line: iterated solution (F18.1).



Figure 16.7. Mean precision of the FAST 30- and 37-month solutions as function of ecliptic latitude. Dashed line: 30-month solution (F30); continuous line: the final, iterated 37-month solution (F37.3).



Figure 16.8. Correction to the a priori origins of the circles obtained in the solution F30. The attitude of the first 18 months (up to day 902) was determined with the iterated mode while the rest used standard processing.



Figure 16.9. Mean residual of the abscissae of the final FAST solution (F37.3) as a function of time. Each data point corresponds to a single circle.



Figure 16.10. Root mean square residual of the abscissae of the final FAST solution (F37.3) as a function of time. Each data point corresponds to a single circle.

Table 16.5. Summary of formal standard errors in the successive NDAC sphere solutions, for stars in common with the 'basic subset' of the final catalogue (\simeq single or at least unproblematic stars). The number of stars included in the statistics is given in the second column. The typical range of standard errors is given in the form of the 10th and 90th percentiles, i.e. the values below which 10 and 90 per cent of the standard error fall. The typical standard error is given by the median value, or 50th percentile. σ_{π} is the standard error in parallax; σ_{pos} and σ_{μ} are the standard errors in position and proper motion, defined by Equation 16.20. The standard errors in position refer to the epoch in the first column, which is close to the mean epoch of observation for the data considered.

Solution	No. of	Standard	I	Percenti	les	Unit
Epoch	stars	error	10%	50%	90%	
NOR	13 887	σ_{π}	1.64	3.67	14.94	mas
1990.40		σ_{pos}	1.31	3.00	11.35	mas
N12	43 053	σ_{π}	1.51	2.10	3.03	mas
1990.40		σ_{pos}	1.20	1.56	2.30	mas
		σ_{μ}	3.96	5.24	7.60	mas/yr
N12R	75 919	σ_{π}	1.52	2.18	2.76	mas
1990.40		σ_{pos}	1.20	1.61	2.13	mas
N18	94 210	σ_{π}	1.34	1.98	2.79	mas
1990.70		$\sigma_{ m pos}$	1.05	1.47	2.15	mas
		σ_{μ}	2.37	3.35	5.06	mas/yr
N30	96 881	σ_{π}	1.05	1.54	2.12	mas
1991.15		σ_{pos}	0.82	1.13	1.59	mas
		σ_{μ}	1.13	1.60	2.34	mas/yr
N37.1	100 717	σ_{π}	1.00	1.49	2.09	mas
1991.25		$\sigma_{ m pos}$	0.78	1.08	1.56	mas
		σ_{μ}	0.90	1.29	1.99	mas/yr
N37.5	101 071	σ_{π}	0.85	1.27	1.92	mas
1991.25		σ_{pos}	0.64	0.93	1.42	mas
		σ_{μ}	0.74	1.11	1.81	mas/yr

The standard errors shown in these figures and tables depend on the *a priori* weights assigned to the input data. For instance, a re-evaluation of the weights between solutions N37.1 and N37.5 accounts for most of the apparent improvement between these two solutions. The actual improvement of the successive solutions may however be appreciated from the distributions of the parallax values, and in particular the tail of negative values, which resembles the distribution of true errors. These distributions are shown in Figures 16.14 and 16.18, again with the final catalogue included for comparison.

The temporal evolution of the median standard errors and fraction of negative parallaxes is summarised in Figures 16.19–16.22. The positions and parallaxes improve, as expected, roughly as $t^{-1/2}$, if *t* is the total duration of the observations, and the proper motions slightly slower than $t^{-3/2}$. Empirically, the fraction of negative parallaxes improves roughly as $t^{-1.0}$.

Solution	No. of	Standard	Ι	Percenti	les	Unit
Epoch	stars	error	10%	50%	90%	
F12	30 411	σ_{π}	3.66	6.24	10.91	mas
1990.40		$\sigma_{\rm pos}$	2.81	5.10	9.50	mas
		σ_{μ}	2.93	4.96	8.20	mas/yr
F12R	44 756	σ_{π}	1.38	2.33	3.44	mas
1990.40		σ_{pos}	1.07	1.72	2.56	mas
F18	88 922	σ_{π}	1.08	1.79	2.90	mas
1990.70		$\sigma_{ m pos}$	0.84	1.34	2.21	mas
		σ_{μ}	1.91	3.07	5.20	mas/yr
F18.1	89 040	σ_{π}	1.23	1.71	2.55	mas
1990.70		$\sigma_{\rm pos}$	0.88	1.27	1.96	mas
		σ_{μ}	2.03	2.95	4.65	mas/yr
F30	95 025	σ_{π}	0.89	1.34	1.93	mas
1991.15		$\sigma_{\rm pos}$	0.69	0.98	1.43	mas
		σ_{μ}	0.98	1.42	2.16	mas/yr
F37.1	101 222	σ_{π}	0.87	1.32	1.95	mas
1991.25		$\sigma_{\rm pos}$	0.67	0.96	1.45	mas
		σ_{μ}	0.79	1.16	1.85	mas/yr
F37.3	101 189	σ_{π}	0.85	1.30	2.02	mas
1991.25		$\sigma_{ m pos}$	0.65	0.95	1.50	mas
		σ_{μ}	0.76	1.15	1.92	mas/yr

Table 16.6. Summary of formal standard errors in the successive FAST sphere solutions, for stars in common with the 'basic subset' of the final catalogue. See Table 16.5 for further explanation.

Table 16.7. Summary of formal standard errors in the merged solutions H18, H30 and HIP (the final Hipparcos Catalogue), for stars in common with the 'basic subset' of the final catalogue. See Table 16.5 for further explanation. Detailed statistics for the whole Hipparcos Catalogue are found in Volume 1, Part 3.

Solution	No. of	Standard	Percentiles			Unit
Epoch	stars	error	10%	50%	90%	
H18	96 692	σ_{π}	1.24	1.87	2.71	mas
1990.70		$\sigma_{\rm pos}$	0.97	1.39	2.09	mas
		σ_{μ}	2.23	3.20	4.99	mas/yr
H30	100 293	σ_{π}	0.97	1.44	2.03	mas
1991.15		$\sigma_{\rm pos}$	0.76	1.06	1.51	mas
		σ_{μ}	1.06	1.51	2.26	mas/yr
HIP	101 246	σ_{π}	0.71	1.06	1.62	mas
1991.25		$\sigma_{\rm pos}$	0.53	0.77	1.19	mas
		σ_{μ}	0.61	0.91	1.49	mas/yr

Merged Solutions

Table 16.7 summarises the standard errors in the catalogues obtained by merging (combining) the FAST and NDAC sphere solutions. H18 and H30 are provisional catalogues constructed from the 18 and 30-month solutions, while HIP designates the final Hipparcos Catalogue. The merging technique used for the final catalogue is described in detail in Chapter 17; briefly, it combines the intermediate abscissa results from the two consortia into new solutions for the astrometric parameters, taking into account the estimated correlations between the two data sets. The resulting standard errors reflect the improvement expected from the combination of data which are only partially correlated.

In contrast to the elaborate merging of the final data, H18 and H30 were constructed by simple averaging of the astrometric parameters and covariance matrices. Equal weight was given to the contributing solutions: N18 and F18.1 for H18, and N30 and F30 for H30. Data for stars found in only one of the contributing solutions were directly copied to the merged catalogues, which therefore contain the union of the entries solved by NDAC and FAST. The following transformations were made before the merging:

- the FAST results, including the covariance matrices of the astrometric parameters, were transformed from ecliptic to equatorial coordinates (Volume 1, Section 1.5);
- the NDAC results, including the covariance matrices, were transformed to the epochs adopted for the merged catalogues (J1990.75 for H18, J1991.25 for H30—the FAST solutions already referred to these epochs);
- each contributing solution was transformed to a common reference frame by application of suitable corrections to the orientation and spin of their coordinate systems. Since the final, extragalactic reference frame (Chapter 18) was not yet available, the common reference frame was chosen to be approximately aligned with the FK5 catalogue.

No mean result was computed when the two contributing solutions differed by more than 60 mas in position. This happened for 22 stars in N18/F18.1, and for only one star in N30/F30. After removal of these stars, no gross inconsistencies were found in parallax or proper motion.

16.6. Intercomparisons

The comparison of successive sphere solutions in terms of random and systematic differences has been one of the most important means of checking the reduction procedures and ascertaining the quality of the final catalogue. This section gives an overview of the rather extensive investigations of the various sphere solutions carried out in the course of the reductions. For uniformity, however, the computation of differences and all their analysis have been made afresh after the completion of the Hipparcos Catalogue, using a single and well-defined set of analysis tools. All comparisons are restricted to intersections with the basic subset introduced in Section 16.2.

Rotation Differences

Before comparison, all solutions were aligned with the Hipparcos Catalogue by applying the orientation and spin differences in Table 16.8. The components of the orientation



Figure 16.11. Distribution of formal standard errors in position (σ_{pos} in Equation 16.20) for the NDAC sphere solutions (NOR to N37.5) and for the Hipparcos Catalogue (HIP). The data refer to the approximate mean epoch of each solution as given in Table 16.5.



Figure 16.12. Same as Figure 16.11, but for the standard errors in proper motion (σ_{μ} in Equation 16.20).



Figure 16.13. Same as Figure 16.11, but for the standard errors in parallax (σ_{π}).



Figure 16.14. Distribution of parallaxes in the NDAC sphere solutions and the Hipparcos Catalogue. The improvement of the successive solutions can be judged from the height and steepness of the distribution, and from the tail of negative parallax values.



Figure 16.15. Distribution of formal standard errors in position (σ_{pos} in Equation 16.20) for the FAST sphere solutions (F12 to F37.3) and for the Hipparcos Catalogue (HIP). The data refer to the approximate mean epoch of each solution as given in Table 16.6.



Figure 16.16. Same as Figure 16.15, but for the standard errors in proper motion (σ_{μ} in Equation 16.20).


Figure 16.17. Same as Figure 16.15, but for the standard errors in parallax (σ_{π}) .



Figure 16.18. Distribution of parallaxes in the FAST sphere solutions and the Hipparcos Catalogue. The improvement of the successive solutions can be judged from the height and steepness of the distribution, and from the tail of negative parallax values.



Figure 16.19. Median standard error in position plotted against the length of the data set included in the solution. A logarithmic scale is used on both axes. The expected improvement as $t^{-1/2}$ is shown by the dashed line. The restricted solutions (no proper motion estimated) are marked 'R'. The FAST iterated 18-months solution (F18.1) is marked 'Iter'.



Figure 16.20. Same as Figure 16.19 but for the standard errors in proper motion. These are expected to improve as $t^{-3/2}$.



Figure 16.21. Same as Figure 16.19 but for the standard errors in parallax.



Figure 16.22. This diagram shows the percentage of negative parallaxes in the various solutions as a function of the length of the data set. Note that the points for the merged catalogues H18, H30 and HIP (filled squares) are significantly below the corresponding points for the FAST and NDAC solutions.

Table 16.8. Orientation and spin differences between the various sphere solutions and preliminary catalogues, compared with the final Hipparcos Catalogue. See Equation 16.21 for the definition of the orientation differences (ε_0 , referred to epoch J1991.25) and spin differences (ω). The last two columns give the mean standard errors of the orientation and spin angles. H37C is the final merged catalogue before it was rotated to the extragalactic reference frame; the orientation and spin parameters in the final row are thus precisely the values adopted for rotating H37C to HIP.

	Orientation (Sol–HIP)		Spi	Spin (Sol–HIP)			Standard errors	
Solution	ε_{0x}	ε_{0y}	ε_{0z}	ω_X	ω_y	ω_Z	$\sigma_{\mathcal{E}}$	σ_{ω}
(Sol)	mas	mas	mas	mas/yr	mas/yr	mas/yr	mas	mas/yr
N0R	+17.123	-30.384	+72.228	-0.754	+0.563	+1.362	0.095	0.079
N12	-40.411	-41.606	+67.784	-1.201	+0.974	+1.239	0.026	0.030
N12R	-39.827	-42.139	+67.815	-0.549	+0.253	+1.302	0.027	0.032
N18	-39.937	-41.468	+67.559	-1.235	+0.952	+1.080	0.008	0.012
N30	-39.823	-41.541	+67.565	-1.425	+1.013	+0.968	0.003	0.005
N37.1	-39.904	-41.593	+67.659	-1.335	+0.803	+1.035	0.003	0.003
N37.5	-39.748	-41.651	+67.619	-1.314	+0.707	+0.978	0.002	0.003
F12	-24.615	-30.000	+78.124	-0.012	+1.683	+1.910	0.062	0.079
F12R	-23.949	-22.528	+69.379	-0.516	+0.531	+1.312	0.027	0.034
F18	-24.174	-27.995	+56.402	-0.462	-0.886	+4.109	0.009	0.013
F18.1	-24.029	-29.734	+58.680	-0.567	-3.251	+6.132	0.007	0.011
F30	-24.192	-27.965	+56.493	-0.524	-0.809	+4.013	0.003	0.004
F37.1	-24.201	-27.554	+56.230	-0.582	-0.468	+3.665	0.003	0.003
F37.3	-24.218	-27.532	+56.190	-0.590	-0.453	+3.661	0.002	0.002
H18	-19.351	-9.315	+22.123	-0.704	+0.035	+0.471	0.033	0.010
H30	-19.108	-8.510	+20.914	-0.707	+0.042	+0.473	0.002	0.003
H37C	-19.1	-8.5	+20.9	-0.73	+0.05	+0.47		

and spin differences (ε_{0x} , ε_{0y} , ε_{0z} , ω_x , ω_y , ω_z) were determined by a robust least-squares method, using the following four observation equations for each star in the basic subset:

$$(\alpha_{\rm S} - \alpha_{\rm H})\cos\delta = -\varepsilon_{0x}\sin\delta\cos\alpha - \varepsilon_{0y}\sin\delta\sin\alpha + \varepsilon_{0z}\cos\delta$$

$$\delta_{\rm S} - \delta_{\rm H} = +\varepsilon_{0x}\sin\alpha - \varepsilon_{0y}\cos\alpha$$

$$(\mu_{\alpha*})_{\rm S} - (\mu_{\alpha*})_{\rm H} = -\omega_x\sin\delta\cos\alpha - \omega_y\sin\delta\sin\alpha + \omega_z\cos\delta$$

$$(\mu_{\delta})_{\rm S} - (\mu_{\delta})_{\rm H} = +\omega_x\sin\alpha - \omega_y\cos\alpha$$

[16.21]

Here, subscripts S and H respectively signify the astrometric parameters in the sphere solution and the Hipparcos Catalogue, always referred to the epoch J1991.25.

Remaining differences in the astrometric parameters were analysed by a variety of methods, with an aim to characterise both the random and the systematic differences between the solutions. The most significant results of this analysis are described hereafter.

Random Differences

For each of the five astrometric parameters, the distributions of the differences between the NDAC and FAST solutions are shown in Figures 16.23–16.27. As a measure of the width of each distribution, a robust 'standard deviation' was computed for each curve; these values are given in Table 16.9. The standard deviations were computed from the

quantiles $x_{\Delta a}(f)$ of the differences, i.e. the values below which a given fraction f of the differences Δa fall. Specifically, the first and fifth sextiles of the distributions were used:

$$\sigma_{\Delta a} = 0.5168 \left[x_{\Delta a} \left(\frac{5}{6} \right) - x_{\Delta a} \left(\frac{1}{6} \right) \right]$$
[16.22]

For a normal distribution this gives the approximate standard deviation. Here, Δa represents the difference in any of the parameters: $\Delta \alpha *$, $\Delta \delta$, $\Delta \pi$, $\Delta \mu_{\alpha *}$, $\Delta \mu_{\delta}$, with the asterisk signifying an implicit $\cos \delta$ factor, and with all differences taken in the sense NDAC minus FAST.

Table 16.10 similarly gives the standard deviations of the differences of each solution with respect to the Hipparcos Catalogue. The overall convergence of the FAST and NDAC solutions towards each other, and towards the final catalogue, is readily seen in these diagrams and tables. Apart from the obvious improvements resulting from the addition of more observations, it is remarkable how the FAST solutions improved by iteration (from F18 to F18.1, and from F37.1 to F37.3; also the NDAC solutions N37.1 to N37.5).

The robust method of Equation 16.22 was introduced because the distributions in Figures 16.23–16.27 are not quite Gaussian: empirically, the far wings tend to decay exponentially, i.e. much slower than for a normal curve. However, even if the errors of each consortium were normal random variables, the distributions in these diagrams could not be expected to be Gaussian, simply because they contain a mixture of populations with different standard deviations. In order to test whether the differences behave normally, they should be scaled by their respective standard errors. Unfortunately the standard errors of the individual differences cannot easily be estimated. As a simple substitute, normalised differences were computed as:

$$\overline{\Delta a} = \frac{a_{\rm N} - a_{\rm F}}{\sqrt{\sigma_{a,\rm N}^2 + \sigma_{a,\rm F}^2}}$$
[16.23]

where *a* stands for any of the five astrometric parameters and σ_a for its standard error from the sphere solution. If the standard errors are correctly estimated and not too unequal, then $\overline{\Delta a}$ should be approximately normal with standard deviation $\sqrt{1-\rho}$, where ρ is the correlation between the NDAC and FAST errors.

The distributions of the normalised differences in parallax are shown in Figure 16.28. The standard deviation of $\overline{\Delta \pi}$ decreases from about 0.78 in the early solutions to 0.63 in the final ones, possibly indicating an increased correlation between the consortia solutions. More significant is perhaps the fact that the curves in Figure 16.28 are much more Gaussian-like than in Figure 16.27. Probability plots of $\overline{\Delta \pi}$ (Figure 16.29) show the deviations from normality much more clearly: for the 30-month solutions and for the final solutions these deviations are remarkably small.

Large-Scale Differences

The Hipparcos mission was designed to make global measurements, directly linking widely separated parts of the sky by means of the basic angle of 58°. It is therefore of great interest to see how well different regions of the sky are connected to the mean reference frame defined by all the regions taken together. Some of the extragalactic link data (in particular the VLBI, MERLIN and HST observations; see Chapter 18) provide an external check on possible large-scale distortions of the Hipparcos reference frame,

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Table 16.9. Standard deviations of the differences in astrometric parameters between the FAST and NDAC sphere solutions, after each solution had been aligned with the Hipparcos Catalogue by application of the orientation and spin differences in Table 16.8. The second and third columns give the number of stars used in each comparison, and the epoch for the comparison of positions. The standard deviations were computed by the robust method of Equation 16.22.

Solutions	No. of	Epoch	Stand	dard de	viations	s (mas, n	nas/yr)
compared	stars		$\Delta lpha *$	$\Delta\delta$	$\Delta \pi$	$\Delta \mu_{\alpha*}$	$\Delta \mu_{\delta}$
N12-F12	39 199	1990.40	3.59	3.19	4.28	11.48	10.30
N12R-F12R	39 199	1990.40	2.09	1.80	2.42	_	_
N18-F18	86 450	1990.70	1.71	1.48	2.09	3.85	3.35
N18-F18.1	86 536	1990.70	1.44	1.26	1.77	3.29	2.86
N30-F30	91616	1991.15	1.14	0.97	1.35	1.59	1.33
N37.1-F37.1	100 702	1991.25	1.00	0.83	1.19	1.27	1.05
N37.5-F37.3	100 894	1991.25	0.97	0.81	1.17	1.19	0.98

Table 16.10. Standard deviations of the differences in astrometric parameters between the successive sphere solutions and the Hipparcos Catalogue (HIP), after each solution had been aligned with the Hipparcos Catalogue by application of the orientation and spin differences in Table 16.8. The second and third columns give the number of stars used in each comparison, and the epoch for the comparison of positions. The standard deviations were computed by the robust method of Equation 16.22.

Solutions	No. of	Epoch	Stan	dard de	viation	s (mas, n	nas/yr)
compared	stars		$\Delta \alpha *$	$\Delta\delta$	$\Delta \pi$	$\Delta \mu_{\alpha*}$	$\Delta \mu_{\delta}$
N0R-HIP	13 887	1990.40	7.57	7.70	9.38	_	_
N12-HIP	43053	1990.40	1.32	1.27	1.95	5.57	5.28
N12R-HIP	75919	1990.40	1.62	1.44	2.43	_	_
N18-HIP	94 210	1990.70	1.26	1.08	1.69	3.58	3.06
N30-HIP	96 881	1991.15	0.89	0.74	1.09	1.43	1.17
N37.1-HIP	100 717	1991.25	0.70	0.59	0.86	0.94	0.79
N37.5-HIP	101 071	1991.25	0.59	0.49	0.73	0.72	0.60
F12-HIP	30 411	1990.40	3.55	3.15	4.33	10.25	10.24
F12R–HIP	44756	1990.40	2.13	1.80	2.77	_	_
F18-HIP	88 922	1990.70	1.50	1.29	1.93	3.87	3.40
F18.1-HIP	89 040	1990.70	1.14	0.99	1.54	3.31	2.86
F30-HIP	95025	1991.15	0.86	0.73	1.01	1.36	1.14
F37.1-HIP	101 222	1991.25	0.57	0.48	0.68	0.70	0.56
F37.3-HIP	101 189	1991.25	0.51	0.43	0.62	0.64	0.51
H18-HIP	96 692	1990.70	1.34	1.25	1.39	3.11	2.66
H30-HIP	100 293	1991.15	0.68	0.57	0.84	1.18	0.97

Solutions	$\Delta \pi_0$	Solutions	$\Delta \pi_0$
compared	mas	compared	mas
N12-F12	+0.461	N30-F30	+0.019
N12R-F12R	+0.142		
N18-F18	-0.005	N37.1-F37.1	+0.015
N18-F18.1	-0.025	N37.5-F37.3	+0.007

Table 16.11. North-south asymmetry of the parallax zero points, when comparing the successive FAST and NDAC sphere solutions. The asymmetry $\Delta \pi_0$ is defined by Equation 16.24.

but only in very few points. Again, a comparison of successive sphere solutions could however give an idea of the consistency of the data on a large scale.

When comparing the 12-month solutions, a significant difference was noted in the FAST and NDAC parallaxes, depending on ecliptic latitude. The difference could be described as a north-south asymmetry: north of the ecliptic the NDAC parallaxes were systematically larger (by about 0.4–0.5 mas) than the FAST ones, while the opposite was true in the southern sky. This could be understood as an effect of the relative scarcity and weakness of data in the ecliptic region, aggravated by the elliptic satellite orbit causing many more Earth occultations than would have been the case in the nominal orbit. When calculating the parallaxes of stars at high ecliptic latitudes, stars in the ecliptic region served as a reference for the parallax zero point (by having a much smaller parallax factor projected on the reference great circles); if they were absent from the sphere solution, the abscissa zero points may have been shifted to produce systematically different parallaxes in the two hemispheres. The problem was predicted to disappear as more observations accumulated, permitting good solutions also of the stars in the ecliptic region. This was indeed the case (Table 16.11). Defining the parallax asymmetry as:

$$\Delta \pi_0 = \frac{1}{2} \left[\left\langle \pi_{\rm N} - \pi_{\rm F} \right\rangle_{\delta > 0} - \left\langle \pi_{\rm N} - \pi_{\rm F} \right\rangle_{\delta < 0} \right]$$
[16.24]

where the angular brackets denote the median value, it was found that the asymmetry decreased to below 0.01 mas in the final solutions.

One rather powerful method of looking for inconsistencies in the system of positions and proper motions is to determine the orientation and spin parameters ϵ_0 and ω from only part of the sky, e.g. separately for different hemispheres. Table 16.12 gives the results for the final sphere solutions (N37.5–F37.3), with the sky divided in eight equal parts according to the given intervals in α and δ . (This analysis was made after both solutions had been globally aligned with the Hipparcos Catalogue, so the mean orientation and spin differences over the whole sky are equal to zero.) The largest deviations amount to 0.08 mas in orientation and 0.13 mas/yr in the spin.

Small-Scale Differences

Figures 16.30–16.34 show the differences in the five astrometric parameters, calculated in the sense N37.5–F37.3 and plotted versus position on the sky in colour coding. The spatial resolution of the maps is 2° . At this resolution the differences are typically about ± 1.5 mas, but as can be seen from the maps, variations are more pronounced in the ecliptic region where the mean number of observations per star is much smaller than at higher positive or negative ecliptic latitudes (Figure 16.35).

Table 16.12. Orientation and spin differences between the final NDAC and FAST solutions as determined from octants of the sky defined by the given limits in α and δ . The all-sky orientation and spin differences have been removed before the regional differences were calculated. See Equation 16.21 for the definition of the orientation differences (ε_0 , referred to epoch J1991.25) and spin differences (ω). The standard errors are typically about 0.012 mas in the orientation components and 0.015 mas/yr in the spin components.

Octant		Orientat	Orientation (N37.5–F37.3)			Spin (N37.5-F37.3)		
conside	ered	ε_{0x}	$\varepsilon_{0,y}$	ε_{0z}	ω_X	ω_y	ω_Z	
α	δ	mas	mas	mas	mas/yr	mas/yr	mas/yr	
0-90°	< 0	-0.003	-0.034	-0.046	-0.003	-0.004	-0.029	
90-180°	< 0	-0.006	+0.004	+0.002	-0.010	+0.001	+0.004	
$180-270^{\circ}$	< 0	-0.011	+0.002	+0.078	-0.031	+0.016	+0.125	
$270 - 360^{\circ}$	< 0	+0.020	-0.027	-0.049	-0.015	-0.007	+0.003	
0–90°	> 0	-0.017	+0.012	-0.024	+0.062	+0.012	+0.070	
90-180°	> 0	+0.017	-0.003	-0.011	+0.039	-0.014	-0.025	
$180-270^{\circ}$	> 0	+0.009	+0.001	+0.016	-0.012	+0.003	+0.004	
$270-360^{\circ}$	> 0	+0.019	+0.020	+0.006	+0.001	+0.060	-0.097	

The angular scale of the differences can be studied more quantitatively by means of the sample correlation function. For any astrometric parameter *a*, the correlation function is defined in terms of the normalised differences Δa as:

$$R(\theta) = \frac{\langle \overline{\Delta a_i} \overline{\Delta a_j} \rangle}{\sqrt{\langle \overline{\Delta a_i}^2 \rangle \langle \overline{\Delta a_j}^2 \rangle}}$$
[16.25]

where the averages are calculated over all pairs of stars (i, j) whose angular separations are in the range $\theta \pm \Delta \theta/2$.

Figure 16.36 shows the sample correlation function for the parallax differences, calculated with a resolution of $\Delta \theta = 0^{\circ}.1$ by considering all $\simeq 5.09 \times 10^{9}$ pairs of 100 890 stars common to F37.3, N37.5 and the basic subset. The first few degrees are also shown in Figure 16.37. At angular separations less than a few degrees the correlation is strongly positive, but decreases to almost negligible values for separations greater than $\simeq 4^{\circ}$. An empirical fit to the first part of the correlation function is given by the function:

$$R(\theta) = R(0) \exp(-0.14\theta - 1.04\theta^2 + 0.41\theta^3 - 0.06\theta^4)$$
[16.26]

where θ is measured in degrees and R(0) = 0.59; this function is shown by the solid curve in Figure 16.37. At greater separations (> 4°) the sample correlations are remarkably small, generally on the ±(0.001 to 0.002) level, while there are more significant negative correlations for $\theta \simeq 180^{\circ}$. Several features of $R(\theta)$ can probably be related to fundamental properties of the great-circle reductions and in particular to the value of the basic angle (58°) and the size of the field of view (0°.9). Note for instance the presence of (small but statistically significant) peaks near $\theta = 58^{\circ}$, $174^{\circ} = 3 \times 58^{\circ}$, and $12^{\circ} = 360^{\circ} - 6 \times 58^{\circ}$.

It is likely that the actual parallax errors in the merged catalogue exhibit a similar spatial correlation, but perhaps with a different scale on the vertical axis. For instance, prelaunch simulations of the astrometric errors resulting from the great-circle reductions and sphere solution gave a mean spatial correlation function with a similar initial decrease as in Figure 16.37, but with R(0) = 0.16 (Lindegren 1988). Given an assumed shape of the correlation function, e.g. according to the above formula, the normalising factor R(0) may in principle be estimated from the dispersion of parallax values in open clusters.

Solutions	Colour	Colour dependent orientation			depende	nt spin
compared	ε'_{0x}	ε'_{0y}	ε'_{0z}	ω'_X	ω'_y	ω'_Z
		mas mag	-1	mas	s yr ⁻¹ mag	g ⁻¹
N12-F12	+0.098	-0.400	+0.230	+2.720	+0.834	-0.705
N12R-F12R	+0.130	-0.045	+0.225	—		_
N18-F18	-0.006	+0.008	+0.027	-0.670	+0.166	+0.116
N18-F18.1	-0.722	+0.259	-0.009	+0.945	-0.291	+0.141
N30-F30	-0.395	+0.089	-0.008	+0.709	-0.216	-0.048
N37.1-F37.1	+0.045	-0.022	-0.038	-0.064	-0.014	-0.008
N37.5-F37.3	+0.011	-0.016	-0.020	-0.094	+0.020	+0.014

Table 16.13. Colour dependence of the orientation and spin differences between the NDAC and FAST solutions (see Equation 16.27).

However, for a detailed examination of astrometric correlations in a small area of the sky it is necessary to consider the elementary observations at abscissa level. This is possible by means of the Hipparcos Intermediate Astrometric Data (Volume 1, Section 2.8) and the empirical abscissa correlation functions discussed in Section 17.8.

Colour and Magnitude Effects

In the early sphere solutions it was found that the orientation and spin differences were colour dependent (and perhaps, to a much smaller degree, magnitude dependent). The effect is most clearly seen if the stars are divided in two equal parts according to their colour index or magnitude, and the rotation parameters ε_0 and ω are determined separately for the two halves. Dividing at V - I = 0.7 gives a mean colour index of $\simeq 0.42$ for the bluer half and $\simeq 1.06$ for the redder half. The differences in the rotation parameters divided by the difference in mean colour index (0.64 mag) give the chromatic rotation parameters in Table 16.13:

$$\boldsymbol{\varepsilon}_{0}^{\prime} = \frac{\Delta \boldsymbol{\varepsilon}_{0}}{\Delta (V - I)}, \qquad \boldsymbol{\omega}^{\prime} = \frac{\Delta \boldsymbol{\omega}}{\Delta (V - I)}$$
[16.27]

There were very significant colour effects up to and including the 30-month solutions; in the 37-month solutions they are suddenly reduced by an order of magnitude. This drastic improvement is almost entirely due to some change in the FAST solutions between F30 and F37.1, as can be shown by a direct comparison of these two solutions. The explanation appears to be related to the FAST modelling of the chromaticity (Section 16.4), but the actual mechanism is not understood.

A similar division according to magnitude reveals much smaller differences. For the final solutions (N37.5–F37.3) the effect is only marginally significant at ~ 0.007 mas mag⁻¹ in orientation and ~ 0.009 mas yr^{-1} mag⁻¹ in the spin. The effect could in fact be merely a reflection of the change in mean colour with magnitude.



Figure 16.23. Distributions of the differences in right ascension ($\Delta \alpha * = \Delta \alpha \cos \delta$) between the NDAC and FAST solutions. In this and subsequent figures, the curves may be identified by means of the labels printed at the level of each peak.



Figure 16.24. Distributions of the differences in declination between the NDAC and FAST solutions.



Figure 16.25. Distributions of the proper motion differences in right ascension ($\Delta \mu_{\alpha*} = \Delta \mu_{\alpha} \cos \delta$) between the NDAC and FAST solutions.



Figure 16.26. Distributions of the proper motion differences in declination between the NDAC and FAST solutions.



Figure 16.27. Distributions of the differences in parallax between the NDAC and FAST solutions.



Figure 16.28. Distributions of the normalised differences in parallax between the NDAC and FAST solutions (Equation 16.23).



Figure 16.29. Distributions of the normalised differences in parallax between the NDAC and FAST solutions (same as in Figure 16.28) shown as normal probability plots. A Gaussian distribution would give a straight line in this plot. The top and bottom curves are plotted against the scales shown on the left axis; other curves are vertically displaced by 2, 4, . . . units for improved visibility. The distributions of N37.5–F37.3 and N30–F30 are very nearly Gaussian, while the other distributions have more extended wings.



Figure 16.30. Map of differences in right ascension between the final NDAC and FAST sphere solutions, $\Delta \alpha * = N37.5 - F37.3$. Mean differences were computed in cells of $2^{\circ} \times 2^{\circ}$.



Figure 16.31. Map of differences in declination between the final NDAC and FAST sphere solutions, $\Delta \delta = N37.5 - F37.3$. Mean differences were computed in cells of $2^{\circ} \times 2^{\circ}$.



Figure 16.32. Map of differences of proper motions in right ascension between the final NDAC and FAST sphere solutions, $\Delta \mu_{\alpha*} = N37.5$ -F37.3. Mean differences were computed in cells of $2^{\circ} \times 2^{\circ}$.



Figure 16.33. Map of differences of proper motions in declination between the final NDAC and FAST sphere solutions, $\Delta \mu_{\delta} = N37.5$ -F37.3. Mean differences were computed in cells of $2^{\circ} \times 2^{\circ}$.



Figure 16.34. Map of differences in parallax between the final NDAC and FAST sphere solutions, $\Delta \pi = N37.5 - F37.3$. Mean differences were computed in cells of $2^{\circ} \times 2^{\circ}$.



Figure 16.35. Map of the mean number of abscissae per star used in the final solutions N37.5 and F37.3. Mean values were computed between the NDAC and FAST numbers; these were then averaged in cells of $2^{\circ} \times 2^{\circ}$.



Figure 16.36. Mean sample correlation coefficient of the normalised parallax difference $(\overline{\Delta \pi})$ as a function of angular separation. The first part of the curve is shown in more detail in the next figure.



Figure 16.37. The points with error bars give the sample correlation coefficients as in the previous figure, but only for small angular separations. The solid curve is the fitted analytical function in Equation 16.26.

Intercomparison by the Method of Infinitely Overlapping Circles

An independent investigation into possible systematic errors remaining in the Hipparcos astrometric results after the final sphere iteration was conducted by B. Bucciarelli and M. Lattanzi, following the prescriptions of a similar study performed on the FAST and NDAC 30-month solutions (Kovalevsky *et al.* 1995). The catalogues compared were the final FAST and NDAC catalogues (F37.3 and N37.5) and the merged catalogue before rotation to the extragalactic system (H37C).

The method of infinitely overlapping circles was used; see Bucciarelli *et al.* (1994), and references therein. Briefly, the method consists of a generalised moving mean algorithm used to find an optimum weighting of the stars in order to evaluate the local systematic differences between two catalogues. For each star (the 'central star'), the catalogue differences are averaged over all stars within a certain radius R of the central star, using the weights:

$$w(r) = \frac{2}{\pi} \left[\arccos(r/R) - (r/R)\sqrt{1 - (r/R)^2} \right], \quad 0 \le r \le R$$
 [16.28]

depending on the angular distances r of the contributing stars from the central star. The central star enters with full weight, since w(0) = 1. By using such a definition of statistical weight one naturally generates continuous systematic differences, while still treating the random part of the individual residuals in a statistically correct way, i.e. the formal expectation of the random part is still zero.

For the present investigation the radius of the small circle was set to $R = 2^{\circ}$, giving an average of about 30 stars per circle. This choice was driven by the requirement to minimise the influence of random errors, while still probing small-scale systematics. This instance is crucial insofar as the random errors of the astrometric parameters are of the same order of magnitude, and even larger, than the systematic effects that are investigated. The method was applied to all five astrometric parameters as a function of position on the celestial sphere. As a representative example of the results, Figure 16.38 shows the average parallax differences as a function of ecliptic latitude and longitude. As expected, the values are small and, when interpreted as residual systematics of one of the two catalogues, they are typically of the order of, or less than, 0.1 mas.

Another powerful way of internally checking the statistical properties of the Hipparcos Catalogue is to compare the empirical distributions of the normalised differences between the NDAC and FAST catalogues with the theoretical distribution. In each astrometric parameter (*a*, e.g. ecliptic longitude) the test statistic is:

$$\epsilon_a = \frac{|a_{\rm N} - (a_{\rm F} + \Delta a_{\rm NF})|}{\sqrt{\sigma_{\rm N}^2 + \sigma_{\rm F}^2 - 2\rho_{\rm NF}\sigma_{\rm N}\sigma_{\rm F}}}$$
[16.29]

where σ_N^2 and σ_F^2 are the variances of the parameter *a* in the two catalogues and $\Delta a_{\rm NF}$ is the catalogue-to-catalogue systematic difference (in the sense NDAC–FAST) derived with the averaging technique of the infinitely overlapping circles; $\rho_{\rm NF}$ is the assumed correlation between the catalogues. The predicted distribution for the test statistic ϵ_a is a folded Gaussian with a mean of $\sqrt{2/\pi} = 0.798$ and a standard deviation of $\sqrt{1-2/\pi} = 0.603$. The actual values of the first two moments of the distributions are in good agreement with the theoretical expectations.



Figure 16.38. Residual systematic differences in parallax between the final FAST and NDAC sphere solutions, plotted as a function of ecliptic longitude (top panel) and latitude (bottom panel). The data were binned in intervals of 4° and 2° , respectively.

Note that the distribution of ϵ_a is degenerate for the case of complete overlap between the two catalogues. However, to obtain the results shown in Figure 16.39 and discussed below, a correlation coefficient $\rho_{\rm NF} = 0.79$ was assumed, i.e. some 15 per cent lower than could be expected from theoretical considerations. Part of the foundation for such a diminishment of the catalogue-wise correlation coefficient is the different processing paths adopted by the two consortia, which differentiate the catalogues more than would be expected from the number of common observations, thereby lifting (in practice) the apparent degeneracy of the problem.

Figure 16.39 shows the empirical distribution functions ϵ_{λ} , ϵ_{β} , $\epsilon_{\mu_{\lambda*}}$, $\epsilon_{\mu_{\beta}}$, and ϵ_{π} and their theoretical counterparts. Mean and standard deviation values of the empirical distributions are reported in Table 16.14. The bottom right diagram in Figure 16.39 was obtained by comparing the FAST catalogue with the merged one—the corresponding comparison for NDAC yields similar results. In this case the correlation coefficient which gave the best estimation of $\langle \epsilon_{\pi} \rangle$ and $\sigma_{\epsilon_{\pi}}$ (0.759 and 0.576 respectively) was $\rho_{\rm FH} = 0.96$, instead of 0.79 found for the NDAC–FAST comparison. This increase in the empirical correlation was expected as the merged catalogue is basically a weighted combination of the consortia catalogues.

In all cases a relatively small number of outliers were found (\leq 3 per cent), which were not taken into account in the calculation of the mean values. The presence of such outliers is usually explained as a discrepancy between the actual differences and the formal errors given in the catalogues.



Figure 16.39. The first three panels (from top left) show the distributions of the test statistic ϵ_a comparing the NDAC and FAST results in each of the five astrometric parameters. The fourth panel (bottom right) shows the statistic ϵ_{π} for the comparison between the FAST and merged catalogues. Dashed curves—theoretical distributions; solid curves—observed distribution in longitude (top) or parallax (bottom); dotted curves—observed distribution in latitude.

In conclusion, this analysis shows that the level of (internal) residual systematics is at the level of, or smaller than, what was expected from pre-launch estimates. Also, the formal errors, as tested by the ϵ_a distributions, appear to have a high degree of consistency with statistical theory.

16.7. Conclusions

The comparison of successive sphere solutions shows very clearly the progress of the data reductions as more and more observations are included, and also the significant improvements obtained by iterating the whole reduction chain (e.g. F18.1 versus F18). The FAST/NDAC comparisons revealed important systematic differences in the early sphere solutions, which were gradually eliminated as calibrations and instrument modelling improved. For the final sphere solutions F37.3 and N37.5, all comparisons indicate that the results behave extremely well, especially in view of the known limitations of the actual mission, such as the sub-optimally sampled ecliptic region. Other tests, for instance of the parallax zero point (Chapter 20), confirm this conclusion. That Hipparcos recovered the total gravitational light deflection (proportional to $(1 + \gamma)/2$) to within 0.4 per cent (NDAC) or 0.2 per cent (FAST) of the value according to General Relativity, corresponding to 0.016 mas or 0.008 mas for the mean observation at right

Statistic	Mean value	Standard Deviation	
	$\langle \epsilon \rangle$	σ_ϵ	
ϵ_{λ}	0.769	0.565	
$\epsilon_{m eta}$	0.762	0.561	
ϵ_{π}	0.759	0.560	
$\epsilon_{\mu_{\lambda}*}$	0.808	0.588	
$\epsilon \mu_{\beta}$	0.798	0.581	
theory	0.798	0.603	

Table 16.14. Sample mean values and standard deviations of the test statistics computed according to Equation 16.29, assuming $\rho = 0.79$.

angles to the solar direction, is also an impressive testimony of its ability to perform accurate global astrometry.

It must be emphasized that the comparison of the consortia solutions cannot prove anything about the quality of the Hipparcos Catalogue. It does however provide considerable insight into the properties and possible shortcomings of the solutions, and hence of the Hipparcos Catalogue. In the end, the quality of the catalogue must be judged from the results of its many applications in astronomy and astrophysics, and from future confrontations with even more accurate measurements.

L. Lindegren, M. Fræschlé, F. Mignard

17. ASTROMETRIC CATALOGUE MERGING

Once the FAST and NDAC final sphere solutions were obtained, the results for each star had to be merged in order to provide the final astrometric parameters together with their associated covariance matrix. Instead of a simple weighted average of the astrometric parameters, this merging was done at the great-circle level. The astrometric parameters of each consortium were first transformed to a common reference frame. Then the FAST and NDAC abscissa residuals with respect to a reference astrometric solution were studied in order to calibrate empirically their correlation coefficient. Finally, for each star, the abscissae and the calibrated covariance matrix between them were used to obtain a merged solution. This merging was applied to the standard solutions of apparently single stars, using five astrometric parameters, but also to accelerated solutions, orbital astrometric binaries and stochastic solutions.

17.1. Introduction

The series of successive sphere solutions described in Chapter 16 ended with two catalogues, F37.3 and N37.5, obtained respectively by the FAST and NDAC Consortia. These catalogues were final in the sense that little improvement was expected from further iterations.

Figures 16.23–16.28 illustrate how the correlation between the FAST and NDAC results increased with the amount of satellite data (from 12 to 37 months of data) and the number of iterations. This can be understood with the following rough model: the random errors on the obtained astrometric parameters consisted of one component attributable mainly to the photon noise of the raw data, and which was therefore common to FAST and NDAC, and another component including the 'modelling errors' proper to each reduction procedure. The second component decreased with the improvement brought by each iteration; it did not vanish however, so that the final correlation between the consortia results was less than one, or about 0.7 on the average.

For this reason, a statistical improvement of the astrometric parameters could be expected from a merging of FAST and NDAC results. This was indeed the case with the merged H18 and H30 Catalogues (Figure 16.22); however these preliminary catalogues were obtained with a simple average of the FAST and NDAC astrometric parameters, with an equal weight for both. Clearly, better results were foreseen using a more adequate weighting, depending both on the standard errors of the FAST and NDAC

parameters and on the correlation between them. The optimal weight would be different for each star, and possibly also different for each of the five parameters. Assuming that it were possible to compute these weights optimally, there would however still remain another problem, namely how to compute the complete covariance matrix of the averaged parameters.

A better scheme was proposed by C.A. Murray. Going back one step in the data reductions, the astrometric parameters of a given star were estimated in each consortium by a least-squares solution in which the abscissae determined by that consortium were regarded as independent 'observations' of the star. If now the FAST and NDAC abscissae were taken together, and considered as correlated observations in a new least-squares solution for the astrometric parameters, this would give not only the optimally combined parameters, but also the correct covariance matrix for these data.

From this general principle, successive merged catalogues were created, with various improvements brought at each step: in order to combine the FAST and NDAC abscissae, the weights of the abscissae had to be revised, and the correlation between the abscissae was empirically calibrated using an unbiased estimator; finally the least-squares procedure which combined the abscissae was adapted for robustness and in order to produce the different types of solutions required (the standard five-parameter solution and the G, O and X type solutions described in Volume 1, Section 2.3). The last catalogue created by this process was called H37C and was used for the link to the extragalactic system (Chapter 18); after rotation to the final Hipparcos reference frame this became the main body of the Hipparcos Catalogue.

17.2. Astrometric Parameters and Abscissa Residuals

The data provided by FAST and NDAC for the catalogue merging consisted of a superset of the final sphere solutions F37.3 and N37.5 described in Chapter 16. This superset contained values of the astrometric parameters $\mathbf{a} = (a_1 \dots a_5)'$ for every star, even when no accepted solution had been found. In addition, the complete set of abscissa residuals Δv_j with respect to the given parameters was provided, together with the partial derivatives $\partial v_j/\partial a_i$ of the computed abscissae with respect to the astrometric parameters. (Recall that the star abscissa v is the angle, as seen from the reference great-circle pole, from the ascending node of the reference great circle on the equator to the satellitocentric coordinate direction of the star at the epoch of the reference great circle; see Figure 11.1.) The standard errors on the abscissae had been computed, as described in Chapter 9, from the formal propagation of the errors through the great-circle reduction procedure, but with empirical corrections derived in connection with the sphere solutions (Chapter 11).

In the following, subscripts $_{\rm F}$ and $_{\rm N}$ will be used to designate data referring to the individual consortia, and subscript $_{\rm H}$ will be used for the corresponding merged quantities.

Table 17.1 shows the number of stars, observations (abscissae), and reference great circles among the initial data given by the consortia, and among the data used for the astrometric merging. For NDAC, the data collected during one orbit defined a single reference great circle. In FAST, this was also generally the case, but a few orbits were split into two great circles. For the merging procedure, a one-to-one correspondence between FAST and NDAC data was needed. Consequently, in the case of split orbits,

Data	Stars	Abscissae	Great circles
FAST	118 160	3668140	2 262
NDAC	111 708	3570643	2 326
Merge	118 300	7227401	2 341

Table 17.1 Statistics of the data used for the astrometric merging.

a single FAST abscissa residual was computed from the weighted average of the two great-circle abscissa residuals, and the partial derivatives were simply averaged. Thus, for the merging, each star had at most one FAST and one NDAC residual on each orbit or reference great circle.

The discrepancy between the total number of stars from FAST and NDAC is due to the fact that the known double stars were not reduced using the main NDAC reduction chain. For this reason, the great-circle abscissae for some of the stars were not available for the merging procedure. The number of great circles used differ because the consortia applied different criteria for accepting a great-circle reduction.

In order to combine the FAST and NDAC results, F37.3 (after transformation to the equatorial frame) and N37.5 had to be rotated first to a common, provisional reference frame approximately aligned with the FK5 Catalogue. This frame was in practice defined by the H30 catalogue, so the orientation and rotation differences F37.3–H30 and N37.5–H30 were used for this purpose. After these corrections of $\mathbf{a}_{\rm F}$ and $\mathbf{a}_{\rm N}$, an initial set of merged parameters was computed as the simple average:

$$\mathbf{a}_{\mathrm{H}} = \frac{1}{2} (\mathbf{a}_{\mathrm{F}} + \mathbf{a}_{\mathrm{N}})$$
 [17.1]

Successive merged solutions replaced the parameters \mathbf{a}_{H} by improved values.

The abscissa residual Δv is the difference between the observed abscissa and the abscissa calculated from a given set of astrometric parameters. The original abscissa residuals provided by the consortia could thus be written:

$$\Delta v_{\rm F}(\mathbf{a}_{\rm F}) = v_{\rm F}^{\rm obs} - v_{\rm F}^{\rm calc}(\mathbf{a}_{\rm F})$$

$$\Delta v_{\rm N}(\mathbf{a}_{\rm N}) = v_{\rm N}^{\rm obs} - v_{\rm N}^{\rm calc}(\mathbf{a}_{\rm N})$$
[17.2]

A direct comparison or averaging of these residuals would not be meaningful, as the reference parameter vectors \mathbf{a}_{F} and \mathbf{a}_{N} were always different. It was therefore necessary to compute new abscissa residuals with respect to the parameters \mathbf{a}_{H} . Around each estimate, a linear expansion was used:

$$v^{\text{calc}}(\mathbf{a} + \Delta \mathbf{a}) = v^{\text{calc}}(\mathbf{a}) + \sum_{i=1}^{5} \frac{\partial v}{\partial a_i} \Delta a_i$$
 [17.3]

resulting in the new abscissa residuals:

$$\Delta v_{\rm F}(\mathbf{a}_{\rm H}) = \Delta v_{\rm F}(\mathbf{a}_{\rm F}) - \sum_{i=1}^{5} \frac{\partial v_{\rm F}}{\partial a_i} (a_{{\rm H}i} - a_{{\rm F}i})$$

$$\Delta v_{\rm N}(\mathbf{a}_{\rm H}) = \Delta v_{\rm N}(\mathbf{a}_{\rm N}) - \sum_{i=1}^{5} \frac{\partial v_{\rm N}}{\partial a_i} (a_{{\rm H}i} - a_{{\rm N}i})$$
[17.4]

For 21 nearby, high-velocity stars, the FAST residuals were also modified to take into account the secular acceleration (Volume 1, Section 1.2.8) which was already included in the NDAC residuals of these stars.

The end result of these transformations was a set of $n_{\rm F}$ residuals $\Delta v_{{\rm F}j}$, with associated formal standard errors $\sigma_{{\rm F}j}$, and a corresponding set of $n_{\rm N}$ residuals $\Delta v_{{\rm N}j}$ with standard errors $\sigma_{{\rm F}j}$, both sets now referring to the same astrometric parameters $\mathbf{a}_{\rm H}$.

As the residuals of a given star came from a previous five-parameter astrometric solution, they had lost five degrees of freedom from the total number of observations, $n = n_{\rm F} + n_{\rm N}$. When computing the unit-weight error, i.e. the sample standard deviation of the normalised residuals $\Delta v_{\rm Fj}/\sigma_{\rm Fj}$, etc., the multiplicative factor $\sqrt{n/(n-5)}$ had to be applied in order to obtain an unbiased estimate of the dispersion of the residuals. For the same reason, a correction 5/n should be added to the sample correlation coefficient between the normalised residuals.

In fact, the exact degree of freedom of statistics computed from the abscissae was not known: other general parameters had been determined by each consortium, such as the great-circle origins, and the reference parameters for the residuals were different from the initial solution obtained by the consortia. For this reason, the statistics used in the merging were, as far as possible, based on the differences of the residuals, which were equal to the difference between observed abscissae, $\Delta v_{Fj} - \Delta v_{Nj} = v_F^{obs} - v_N^{obs}$. This relation holds for all but the first merged solution, where it was only approximate.

17.3. Scaling Corrections of Consortia Formal Errors

Unbiased formal errors were needed both for the computation of the correlation coefficients, described in the next section, and more generally for the weight matrix of the astrometric least-squares solution. The residuals were systematically plotted as functions of all available data: orbit number, position, proper motion, magnitude, colour. For this purpose, only stars considered previously as single stars, with good solutions obtained both by FAST and NDAC, were kept (about 96 000 stars); the observations rejected during the astrometric solutions of the consortia were also rejected for this calibration (but not necessarily in the final merging). Finally, great circles with less than 10 observations common to FAST and NDAC were not used for the calibration (2242 orbits remained).

No systematic trend of the mean residuals was found as a function of astrometric or photometric data. However, the unit-weight errors of the abscissa residuals, $u_F = [Var(\Delta v_F/\sigma_F)]^{1/2}$, $u_N = [Var(\Delta v_N/\sigma_N)]^{1/2}$, computed from the formal standard errors supplied by the consortia, showed some significant variations around unity as functions of time, as may be seen in Figure 17.1.

Small standard errors were also found to have been overestimated (Figure 17.2). For this reason, a scaling of the standard errors had to be introduced. To take into account the variations in Figure 17.2, u_F and u_N were approximated by polynomials of degree four for formal errors below 10 mas, and by constants for the larger standard errors. This calibration obviously induced a magnitude effect, and the unit-weight errors were thereafter calibrated also against magnitude, where linear corrections were sufficient. This procedure was iterated once. A slight colour correction was also applied for stars



Figure 17.1. Evolution of the unit-weight errors (u_F and u_N) of the abscissa residuals, based on the formal abscissa errors supplied by the reduction consortia. Each curve is a running median over 50 orbits ($\simeq 22$ days).



Figure 17.2. Unit-weight errors (u_F and u_N) of abscissa residuals as functions of the formal standard errors.

redder than V-I = 2 mag, namely 1+0.035(V-I-2) for FAST and 1+0.016(V-I-2) for NDAC.

Having fitted the calibration functions $f_F(\sigma_F, Hp, V - I)$ and $f_N(\sigma_N, Hp, V - I)$ to the unit-weight errors, the standard errors could then be corrected through multiplication by these functions. These corrections were smaller than 30 per cent for FAST data (3 per cent on the average), and not more than 20 per cent for NDAC (1 per cent on the average). The corrected unit-weight errors became approximately constant (except for the very small formal errors), with no significant magnitude or colour effects. There was however still a significant variation with time, from one orbit to the next, for which an *ad hoc* correction factor was finally applied.

Astrometric solutions using only FAST or NDAC data were performed in order to verify the usefulness of the corrections which had been done. As may be seen in Figure 17.7, the standard errors on the FAST parallaxes decreased by about 4 per cent on the average. That this was not just a reduction of the formal errors is demonstrated by the fact that the number of negative parallaxes decreased by 2 per cent. For the NDAC data the overall changes brought by the scaling corrections were much smaller.

17.4. Correlation Between Abscissae

The correlation coefficient between FAST and NDAC abscissae varied essentially with magnitude, although there were also some variations with ecliptic longitude and orbit number. It would then have been logical to calibrate the correlation against magnitude. However, this would have created a practical problem for variable stars (where the correlation should then take into account the magnitude at each observation), since epoch photometry was not available at the time of the merging. For this reason it was decided to calibrate the correlation primarily against the abscissa standard errors (which are of course related to the epoch magnitude).

Let ρ be the statistical correlation between the residuals Δv_F and Δv_N , with standard deviations σ_F and σ_N . The variance of the abscissa difference $\Delta v_F - \Delta v_N$ is given by $\sigma_F^2 - 2\rho\sigma_F\sigma_N + \sigma_N^2$. Using the calibrated standard errors described in the previous section, the sample correlation coefficient was computed with the following formula:

$$\rho = \frac{\sigma_{\rm F}^2 + \sigma_{\rm N}^2 - \operatorname{Var}(\Delta v_{\rm F} - \Delta v_{\rm N})}{2\sigma_{\rm F}\sigma_{\rm N}}$$
[17.5]

This was done in bins of (σ_F , σ_N). Assuming that ρ depends only on the standard errors, the residuals in a given bin belonged to the same population and an unbiased estimate of ρ could be obtained, provided that the standard errors were also unbiased. Results are shown in Figure 17.3.

As can be expected from the rough model outlined in Section 17.1, the correlation increases with the standard errors and a relation $\rho \simeq 1 - \text{const}/(\sigma_F \sigma_N)$ was expected. The correlation also decreases with the difference between the variances of the consortia abscissae, producing a rather sharp ridge along the diagonal $\sigma_N = k\sigma_F$, where $k \simeq 1.2$. It should be noted that the number of abscissa pairs on which the calculation of ρ was based drops quickly when going away from the diagonal, causing large statistical fluctuations in these areas of the diagram. The 'ridge' behaviour can also be understood in terms of the model mentioned above, as being due to increased modelling errors

on some abscissae in one or the other consortium. Based on these considerations, the following empirical form was chosen for the calibration of ρ as a function of the standard errors:

$$\rho_0(\sigma_{\rm F},\sigma_{\rm N}) = 1 - \frac{a + \frac{1}{2}|b + k^2 \sigma_{\rm F}^2 - \sigma_{\rm N}^2|}{\sigma_{\rm F} \sigma_{\rm N}} p_1\left(\frac{1}{\sigma_{\rm F} \sigma_{\rm N}}\right)$$
[17.6]

where $a \simeq 9.978 \text{ mas}^2$, $b \simeq -0.160 \text{ mas}^2$ and $k \simeq 1.168$ are constants, and p_1 is the polynomial:

$$p_1(x) = 1 - 16.221x + 141.983x^2 - 663.074x^3 + 1661.218x^4 - 2099.526x^5 + 1047.439x^6$$
[17.7]

The fitted function is shown in Figure 17.4.

In order to verify this calibration, the statistic

$$\delta v = \frac{\Delta v_{\rm F} - \Delta v_{\rm N}}{[\sigma_{\rm F}^2 - 2\rho_0(\sigma_{\rm F}, \sigma_{\rm N})\sigma_{\rm F}\sigma_{\rm N} + \sigma_{\rm N}^2]^{1/2}}$$
[17.8]

was computed for each star and analysed as a function of astrometric or photometric data. The random variable δv should have zero expectation and unit variance if the correlation coefficient is correctly calibrated and if the standard errors σ_F and σ_N are representative of the true variations of the random errors on the abscissae. A magnitude effect was however found, due to the fact that, for bright stars, the abscissa standard errors were only weakly correlated with magnitude. Developing Var(δv) – 1 to first order in ρ allowed the required correction to ρ_0 to be found. This was fit with a cubic polynomial in magnitude.

Finally, there was another variation of the correlation coefficient with time, the correlation being maximum around half-way through the mission and smaller at the beginning and the end of the mission. This effect is probably related to the precision of the proper motions. Using the same method as described above, an additional correction was determined as a cubic polynomial of time.

In summary, the calibration of the correlation coefficient was finally expressed as a function of the standard errors, magnitude and time:

$$\rho(\sigma_{\rm F}, \sigma_{\rm N}, Hp, t) = \rho_0(\sigma_{\rm F}, \sigma_{\rm N}) - 0.1205$$

- 0.02770Hp + 0.010990Hp² - 0.0006509Hp³
- 0.0038t - 0.0314t² + 0.00415t³ [17.9]

where ρ_0 is given by Equation 17.6 and *t* is the time in years from J1991.25; the calibrated correlation was further constrained to the interval $0.2 \le \rho \le 0.99$.

17.5. The Least-Squares Solutions

Together with the standard error of the FAST and NDAC abscissae, the calibrated correlation coefficients provided the necessary information about the covariance matrix V of the observations. The covariances between abscissae from different great circles were neglected. Grouping the observations by pairs, corresponding to the FAST and



Figure 17.3. Sample correlation coefficient ρ between the FAST and NDAC abscissae, calculated in bins of the standard errors σ_F and σ_N .



Figure 17.4. Calibration function ρ_0 (Equation 17.6) fitted to the sample correlation as a function of the standard errors σ_F and σ_N .

NDAC abscissae on each orbit, results in a block-diagonal structure of the covariance matrix:

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_1 & \mathbf{0} & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{V}_2 & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{0} & \mathbf{V}_3 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$
[17.10]

where the submatrix for the *j*th pair is given by:

$$\mathbf{V}_{j} = \begin{pmatrix} \sigma_{Fj}^{2} & \rho_{j} \sigma_{Fj} \sigma_{Nj} \\ \rho_{j} \sigma_{Fj} \sigma_{Nj} & \sigma_{Nj}^{2} \end{pmatrix}$$
[17.11]

For orbits in which only one consortium provided an abscissa \mathbf{V}_j reduces to the 1×1 matrix containing $\sigma_{F_j}^2$ or $\sigma_{N_j}^2$.

Given the partial derivatives of the abscissae with respect to the five astrometric parameters, $\partial v_j / \partial a_i$, the corrections Δa_i to the reference parameters a_i were found by minimising:

$$\chi^{2} = (\Delta \mathbf{v} - \frac{\partial \mathbf{v}}{\partial \mathbf{a}'} \Delta \mathbf{a})' \mathbf{V}^{-1} (\Delta \mathbf{v} - \frac{\partial \mathbf{v}}{\partial \mathbf{a}'} \Delta \mathbf{a})$$
[17.12]

This differs from a standard weighted least-squares solution only in that the weight matrix \mathbf{V}^{-1} is not diagonal, which requires a trivial modification of the normal equations.

The algorithm was also modified for robustness. Initially all observations were kept, but outliers were not unexpected, due for instance to veiling-glare effects. Robust methods using a non-Euclidean metric were tried, but abandoned as the variance of the estimated astrometric parameters increased too much for stars without outliers, which were in the majority. A conventional 3σ rejection was used instead: when the absolute value of a residual exceeded three times its formal error, the observation was excluded and a new solution computed. This process was iterated until the set of outliers remained constant. Pairs of observations were also rejected if their normalised difference (Equation 17.8) was greater than $3\sqrt{2}$.

For the single stars only, an average of 62 observations were used per star, with an outlier rate of 0.5 per cent. The median unit-weight error of the astrometric solutions was about 1.01, and this slight departure from unity explains why the median of the goodness-of-fit statistic F2 (Field H30) is about 0.2. The unit-weight error exhibited no significant variations with astrometric or photometric data.

17.6. Merged Solutions of Non-Single Stars

The standard five-parameter model did not always adequately represent the observations. More complex models were constructed (Volume 1, Section 2.3) and tested on all stars. Seven- and nine-parameter solutions (type 'G') were obtained the same way as in Equation 17.12, using the supplementary partial derivatives:

$$\frac{\partial V}{\partial a_{i+5}} = \frac{1}{2}(t^2 - 0.81)\frac{\partial V}{\partial a_i}$$
[17.13]

$$\frac{\partial V}{\partial a_{i+7}} = \frac{1}{6}(t^2 - 1.69)\frac{\partial V}{\partial a_{i+3}}$$
[17.14]

for i = 1, 2, where *t* is the time in years from J1991.25. The constants 0.81 yr² and 1.69 yr² were chosen to make the quadratic and cubic terms approximately orthogonal

to the position and proper motion terms. Orbital solutions (type 'O') with up to twelve parameters were similarly obtained, using a special routine to calculate the partial derivatives of the tangential coordinates ξ and η (Volume 1, Section 2.3.4) with respect to the seven orbital elements **o**, from which:

$$\frac{\partial V}{\partial \mathbf{o}'} = \frac{\partial V}{\partial a_1} \frac{\partial \xi}{\partial \mathbf{o}'} + \frac{\partial V}{\partial a_2} \frac{\partial \eta}{\partial \mathbf{o}'}$$
[17.15]

Several trials were made before defining the criteria for selection of a model for a given star. The adopted criteria are described in Volume 1, Section 2.3.1.

Although good results were obtained for almost all stars, there existed some stars where none of the above-mentioned models was adequate, thus leading to a high outlier rejection rate or a bad goodness-of-fit. This could be due for instance to unrecognised duplicity or orbital motion. Following a suggestion by R. Wielen, a stochastic model was assumed for these stars. Superposed on the uniform motion of the centre of mass, unmodelled photocentric displacements were assumed to behave in a stochastic manner, with a standard deviation given by the 'cosmic error' ϵ . The cosmic error was determined by adding a variance ϵ^2 to each of the individual abscissa variances, the constraint being that the unit-weight variance $u^2 = \chi^2/(n-5)$ of the five-parameter astrometric solution must be equal to unity. A normal rate of possible outliers (≤ 2 per star) was however still allowed. The cosmic error was added quadratically to the abscissa standard errors, and the modified standard errors ($\sigma_F^2 + \epsilon^2$)^{1/2}, ($\sigma_N^2 + \epsilon^2$)^{1/2} were used to compute the correlation coefficient between the FAST and NDAC abscissae according to Equation 17.9. The calculated correlation coefficient therefore increased in the presence of a cosmic error.

For each star, the adopted procedure was the following iteration:

- 1. a normal astrometric solution, as described above, was performed. If $u^2 \neq 1$, the maximum allowed number of outliers (rejected abscissae) was set to 2;
- 2. if no cosmic error was indicated $(u^2 \le 1)$, the maximum number of outliers was decreased to 1, then 0. Evidently, the standard five-parameter model was adequate for the majority of the stars, in which case the cosmic error was set to $\epsilon = 0$;
- 3. otherwise, ϵ was computed from the non-rejected observations. Assuming Gaussian residuals, $(n-5)u^2$ follows a χ^2_{n-5} distribution, where $n = n_{\rm F} + n_{\rm N}$ is the total number of observations, and the standard error on ϵ is then approximately given by:

$$\sigma_{\epsilon} = \left[\frac{2}{n-5}\right]^{1/2} \left[2\epsilon \frac{\partial(u^2)}{\partial(\epsilon^2)}\right]^{-1}$$
[17.16]

In order to avoid unrealistically small standard errors on some stars, a lower limit of $\epsilon/\sqrt{2(n-5)}$ was introduced on σ_{ϵ} ;

4. the covariance matrix **V** was computed for a new iteration.

This procedure was applied to the whole catalogue. A sample of about 94 000 single stars was first defined, i.e. stars not recognised as double or multiple, with a goodness-of-fit statistic F2 < 3, and with at most two rejected abscissae. From these stars, it was empirically found that the criterion $\epsilon > 5\sigma_{\epsilon}$ corresponded to a significance level similar

to the usual 3σ two-sided test on a Gaussian distribution. This criterion was therefore used to decide whether a stochastic solution was significant.

One of the possible causes for the presence of a cosmic error is unrecognised duplicity. Known double stars should therefore also produce significant cosmic error if given a stochastic solution instead of the proper solution with two or more components. In order to test this hypothesis, stochastic solutions were computed for all Hipparcos stars resolved as double and thus contained in Part C of the Double and Multiple Systems Annex. The cosmic errors were plotted against the effective magnitude difference D (Equation 13.40) of the doubles. This quantity, equal to the real magnitude difference ΔHp when the separation exceeds 0.32 arcsec but increasing for small separations, is a measure of the difficulty by which the secondary may be detected in the Hipparcos detector signal. A strong correlation was found between the cosmic error and the effective magnitude difference (Figure 17.5).

Other stars which received a stochastic solution may be astrometric binaries with a period of less than a few years, and the cosmic error could then be taken as an order-of-magnitude estimate of the semi-major axis of the orbit of the photocentre (a_0) . Significant stochastic solutions were obtained for some of the objects finally retained as orbital astrometric binaries in the Hipparcos Catalogue. In Figure 17.6 the cosmic errors for these objects are compared with the semi-major axes as given in Field DO4 of the Double and Multiple Systems Annex. As expected, there is a rough proportionality between the two quantities, with $a_0 \simeq 2.4\epsilon$ at least for small a_0 .

A grossly erroneous starting position for the astrometric adjustment usually resulted in a number of grid-step errors in the abscissa residuals, which were then more or less uniformly distributed between -s/2 and +s/2, where s = 1.2074 arcsec is the grid step. These cases show up among the stochastic solutions with a cosmic error of the order of $s/\sqrt{12} \simeq 300$ mas (Figure 17.5 contains a number of such solutions). A similar effect tends to occur for non-detections (very weak signal, e.g. due to erroneous pointing of the instantaneous field of view). After elimination of all systems solved by other models, a number of stars with large cosmic errors ($\epsilon > 100$ mas) still remained and had to be examined more closely. Some of them were indeed found to be affected by grid-step errors, and good five-parameter solutions could be obtained by changing the reference positions and the abscissae by multiples of the grid step. Remaining cases with a cosmic error exceeding 100 mas were however rejected as being most probably invalid astrometric solutions.

17.7. Comparison with a Weighted Mean

An alternative to the adopted merging procedure described above would be to use, for each astrometric parameter *a* of a given star, a weighted mean of the FAST and NDAC parameters:

$$a_{\rm W} = w a_{\rm F} + (1 - w) a_{\rm N}$$
 [17.17]

This method (with a constant w = 0.5) was used for the construction of the provisional catalogues H18 and H30, but a slightly better result would be obtained by optimising the weight *w* for each star, or even for each parameter. It is of interest to compare the precision of this simple method to the more elaborate merging adopted for the final catalogue.



Figure 17.5. Cosmic error ϵ (mas) versus the effective magnitude difference D (Equation 13.40) for systems retained as resolved double stars in the final catalogue (Part C of the Double and Multiple Systems Annex). Stochastic solutions with cosmic errors above some 200 mas are generally due to grid-step errors, which were not eliminated in this sample.



Figure 17.6. Cosmic error (mas) versus the semi-major axis of the barycentric orbit of the photocentre. The 95 systems shown in this diagram were retained as orbital astrometric binaries in the final catalogue (Part O of the Double and Multiple Systems Annex), but stochastic solutions were also computed for this comparison.



Figure 17.7. Median precision of parallaxes of single stars versus magnitude Hp for various astrometric solutions: (a) initial FAST solution; (b) initial NDAC solution; (c) FAST solution with re-weighted abscissa standard errors; (d) NDAC solution with re-weighted abscissa standard errors; (e) merged solution adopted for the final Hipparcos Catalogue; (f) a weighted mean of the FAST and NDAC parallaxes.

The value *w* which minimises the variance σ_W^2 of a_W is:

$$w = \frac{1 - \rho q}{1 - 2\rho q + q^2}$$
[17.18]

where $q = \sigma_F / \sigma_N$ is the known ratio of the standard errors and ρ the (initially) unknown correlation coefficient between a_F and a_N . The variance of the weighted mean parameter is given by:

$$\sigma_{\rm W}^2 = \frac{1 - \rho^2}{1 - 2\rho q + q^2} \,\sigma_{\rm F}^2 \tag{17.19}$$

The correlation coefficient may be estimated the same way as in Equation 17.8 under the assumption that ρ and q are constant for a given magnitude; the result is:

$$\rho \simeq \left\langle \frac{1+q^2}{2q} \right\rangle \left[1 - \operatorname{Var}(\overline{\Delta a}) \right]$$
[17.20]

where $\overline{\Delta a} = (a_{\rm F} - a_{\rm N})/(\sigma_{\rm F}^2 + \sigma_{\rm N}^2)^{1/2}$ are the normalised differences introduced in Equation 16.23. This procedure allowed ρ to be computed in bins of magnitude. Each of the two factors of the product in Equation 17.20 was then calibrated as a polynomial in the *Hp* magnitude. Introducing this $\rho(Hp)$ into Equations 17.17–17.19 gives the weighted astrometric parameter and associated standard error for a star of magnitude *Hp*.

Applying the above method to the parallaxes of single stars gives the median value of σ_W as drawn in Figure 17.7. In terms of the astrometric standard errors, the adopted merging procedure appears roughly equivalent to the present method using a simple weighted mean of the astrometric parameters. However, the more elaborate merging of the abscissa data allows the covariances of the merged astrometric parameters also to be computed in a rigorous manner; this is not possible in the simpler method.



Figure 17.8. The auto and cross-correlation functions $\rho_{FF}(\theta)$, $\rho_{FN}(\theta)$ and $\rho_{NN}(\theta)$ of the abscissa residuals in orbit number 1001, plotted against the angular separation of the stars (θ). A bin size of 0°.35 in θ was used to compute the correlations, which were then smoothed with a Gaussian kernel of standard deviation $\simeq 0$ °.43.



Figure 17.9. Same as Figure 17.8 but for orbit 2000, illustrating the typical variation of the correlation functions from one orbit to another.


Figure 17.10. The first bin of the auto and cross-correlation functions $\rho_{FF}(\theta)$, $\rho_{FN}(\theta)$ and $\rho_{NN}(\theta)$, i.e. for separations $\theta < 0^{\circ}.35$, plotted against the orbit number. Each curve is a running median over 50 orbits ($\simeq 22$ days).

17.8. Correlations Between Different Stars on the Same Great Circle

Already in the early stages of the preparations of the Hipparcos data reductions it was recognised that the observation mode, and especially the great-circle reductions, would generate cyclic correlation patterns among the stars observed on the same great circle (Høyer *et al.* 1981). More detailed predictions were included in the pre-launch documentation (Perryman *et al.* 1989 Volume III, Chapter 23). A significant positive correlation (of a few tenths) was expected between the abscissae v_i and v_j of stars for which $|v_i - v_j|$ was less than a few degrees or close to a small multiple of the basic angle, modulo 2π .

The abscissa residuals from the merging allowed the sample correlation coefficients to be computed among pairs of the normalised residuals on the same great circle $(\Delta v_{Fi}/\sigma_{Fi}, \Delta v_{Fj}/\sigma_{Fj}, \Delta v_{Ni}/\sigma_{Ni} \text{ and } \Delta v_{Nj}/\sigma_{Nj} \text{ for } i \neq j)$. This was done as functions of the angular separation θ between the stars (see Equation 16.25). Note that the actual angular separation of the stars was used, rather than the abscissa difference. Two autocorrelation functions [$\rho_{FF}(\theta)$ and $\rho_{NN}(\theta)$] and one cross-correlation function [$\rho_{FN}(\theta)$] were calculated, using a bin size of $180^{\circ}/512 \simeq 0.35$. The results for two fairly typical great circles are shown in Figures 17.8–17.9.

Although most correlation functions show the expected peaks at multiples of the basic angle $(0^{\circ}, 58^{\circ}, 116^{\circ}, 174^{\circ}, 128^{\circ}, ...)$, the amplitude of the peaks varies considerably between different great circles, and also between the consortia results on the same



Figure 17.11. The average auto and cross-correlation functions $\rho_{FF}(\theta)$, $\rho_{FN}(\theta)$ and $\rho_{NN}(\theta)$ obtained by considering all 2341 available great circles.



Figure 17.12. Same as Figure 17.11 but on an enlarged scale showing the correlations for small angular separations.

great circle. The variation of the correlations for small separations (the first bin, i.e. $\theta < 0.35$) is shown in Figure 17.10. The mean correlation functions for the whole mission are shown in Figures 17.11–17.12. The auto-correlations for the FAST data are slightly smaller than the NDAC ones, and the cross-correlation is much smaller than the autocorrelations.

It is interesting to see the influence of the abscissa correlations on the final astrometric parameters. As seen in Figures 16.36–16.37, the small-scale correlations ($\theta \leq 2^{\circ}$) remain whereas the other correlation peaks at multiples of 58° are very strongly damped. The reason for this is that any given pair, separated by 58° on the sky, was very rarely observed on the same reference great circle. The peak at 174° is relatively less damped because stars at diametrically opposite points on the sphere were more likely to be observed on the same great circle.

The auto- and cross-correlation functions can be used, together with the Hipparcos Intermediate Astrometric Data (Volume 1, Section 2.8), to estimate the full covariance matrix of the abscissae for any group of stars, such as in a cluster. This may be useful for evaluating the correlations between the astrometric parameters of the different stars in a more rigorous manner than using the average correlation discussed in Section 16.6.

17.9. Conclusions

By using the FAST and NDAC star abscissae as a basis for the merging, rather than the astrometric solutions of each consortium, it was possible to find an optimum solution for all five (or more) astrometric parameters, including estimates of the standard errors and correlations of the merged astrometric parameters. The method required however very careful calibration of the standard errors of the abscissae and the correlation of abscissa errors between the consortia, for which special techniques were developed. The overall correlation was about 0.7, allowing a significant improvement in precision by the merging, as can be seen in Figure 17.7. The merging of the abscissae also made it relatively simple to choose between the several different models of star motion, from the standard five-parameter model to the twelve-parameter model for orbital binaries.

The final merging resulted in a catalogue called H37C, in which the positions and proper motions were still expressed in the provisional *ad hoc* reference frame defined by earlier mergings. The main part of the Hipparcos Catalogue was created from H37C by applying the rigid-body rotation derived from comparing H37C with the extragalactic reference frame, as described in Chapter 18. Detailed statistics of the Hipparcos Catalogue are given in Volume 1, Part 3.

An important by-product of the merging process was the generation of the complete set of FAST and NDAC abscissa residuals, expressed on the same reference frame as the Hipparcos Catalogue and including the calibrated standard errors and correlations; for future investigations this data set is made available on ASCII CD-ROM, in the form of the Hipparcos Intermediate Astrometric Data described in Volume 1, Section 2.8.

18. THE LINK TO AN EXTRAGALACTIC SYSTEM

The positions and proper motions in the Hipparcos and Tycho Catalogues refer to the International Celestial Reference System, ICRS. This means that the coordinate axes of the catalogues have been aligned with the reference frame determined through VLBI radio observations of extragalactic sources, and remain fixed with respect to that reference frame. Since extragalactic sources (with the exception of 3C273) were not directly accessible for observation by Hipparcos, it was necessary to use other observational techniques to link the Hipparcos reference frame to the extragalactic frame. Since the Hipparcos and Tycho Catalogues are the first large-scale realisations of the ICRS in the optical domain, great importance was attached to the task of achieving this link to the best possible accuracy. This chapter describes the different techniques that were employed, the work done by the several groups contributing to the link, and how their results were combined in order to derive the reference frame finally adopted for the catalogues.

18.1. Motivation for the Link

Hipparcos was able to measure the angles between objects on its observing list very accurately. From these angles, and their variation in time, the positions and proper motions of the stars could be calculated in a single coordinate system covering the whole sky, and their absolute trigonometric parallaxes were obtained at the same time. However, because the angles between stars are invariant with respect to a rigid rotation of the coordinate axes, there was a basic indeterminacy in the instantaneous orientation of the axes that could not be removed from an analysis of the angular measurements alone. Given the kinematical constraint that stars in general have uniform space motions (as incorporated in the modelling of the observations; see Volume 1, Section 1.2.8), it can be shown that the intrinsic indeterminacy of the Hipparcos reference system has six degrees of freedom (Betti & Sansò 1983), corresponding to the inertial spin of the system and its orientation at a given epoch. More precisely, there is a sixdimensional manifold of solutions for the positions and proper motions, each solution being equally consistent with the observations and differing from the others by a uniform rotation. From this manifold of possible reference frames a single one had to be selected for the published catalogues. The selected reference frame should correspond to the International Celestial Reference System (ICRS), as discussed in Section 18.2.

The merging of the final FAST and NDAC sphere solutions, described in Chapter 17, resulted in a catalogue called H37C, the precise axes of which were in an unknown state

of uniform rotation with respect to the desired extragalactic frame. It was the purpose of the link observations to determine this state by all available means, and then apply the corresponding corrections to the positions and proper motions in H37C in order to produce the Hipparcos Catalogue.

A direct determination of the relation between H37C and the extragalactic system would have been possible if the Hipparcos observing programme had included a sufficient number of extragalactic sources, some of which with accurately known radio positions. However, due to the rather bright limiting magnitude of Hipparcos, no such objects were included. (The programme did include the brightest quasar, 3C273 (HIP 60936), and some 45 stars in the Magellanic Clouds; however, the quasar was still too faint to contribute significantly to the link, and the Magellanic Clouds are expected to have proper motions of a few milliarcsec per year, and therefore cannot be used as reference directions. See Section 18.8 for a discussion of the observations of these objects.) Consequently, indirect methods had to be used to bridge the gap between the optically bright objects on the observing list and the extragalactic sources observed either at radio wavelengths or at much fainter optical magnitudes. These methods and their results are described in subsequent sections. Since the actual orientation and spin of H37C found by these methods are only of historical interest (they are given in Table 16.8), while the deviations of the various methods from the adopted mean result are of considerable interest for judging the quality of the link, all results in this chapter are given relative the adopted mean result. Thus, the results of the individual link methods are presented as if the Hipparcos Catalogue (and not H37C) had been compared with the extragalactic frame.

18.2. Reference System for the Hipparcos Catalogue

The choice of a reference system for the Hipparcos Catalogue was initially not an obvious one. The traditional definition of the fundamental celestial directions in terms of the mean equator and equinox was based on dynamical principles and its practical implementation required observations both of solar system objects (to determine the ecliptic) and of the Earth's rotation axis (to determine the equator), as well as a dynamical theory for the inertial variations of these directions. However, it was clear from the outset that a kinematical definition of a non-rotating frame (i.e. with respect to distant galaxies) was much preferred for Hipparcos, because it would be both easier to implement and more accurate than a dynamical (inertial) system. This choice eliminated three degrees of freedom, but still left the orientation of the system unspecified. The situation was clarified in 1991, when the IAU adopted a resolution (Bergeron 1992) stating that the next celestial reference system should be based upon positions of extragalactic radio sources, but that it will come into effect only when there is a realisation of the system in the optical domain. It was then understood that this realisation should be the Hipparcos Catalogue, given its expected high precision and extension to more than a hundred thousand stars.

Since 1988, the International Earth Rotation Service (IERS) has implemented and maintained an extragalactic reference frame containing an increasing number of extragalactic radio sources observed by several VLBI networks throughout the world (Arias *et al.* 1995). At the request of the IAU working group on reference frames, IERS finalised this iterative process and provided a definitive list of objects and coordinates in October 1995 (Ma *et al.* 1997). This list is the International Celestial Reference Frame (ICRF) of 610 sources (IERS 1996). The axes of this catalogue are to remain fixed with respect to the quasars, and constitute the International Celestial Reference System (ICRS). As a result of the present link, all the coordinates published in the Hipparcos and Tycho Catalogues refer to the ICRS. It is expected that in 1997 the IAU will approve ICRS as the new reference system replacing the FK5 system. The Hipparcos and Tycho Catalogues are its first realisation for optical astronomy.

18.3. Link Equations

The analytical tools for comparing two reference frames related by a uniform rigid-body rotation were derived in Lindegren & Kovalevsky (1995) and are summarised here to the extent that they are directly applicable to the various link observations.

The extragalactic reference frame (ICRF) is represented by the triad of unit vectors, $\mathcal{E} = [\mathbf{x}_E \ \mathbf{y}_E \ \mathbf{z}_E]$. Similarly the Hipparcos reference frame is represented by the triad $\mathcal{H} = [\mathbf{x}_H \ \mathbf{y}_H \ \mathbf{z}_H]$. Following the principle of coordinate transformations in Section 1.5.3 of Volume 1, the arbitrary direction **u** is written:

$$\mathbf{u} = \mathcal{E} \quad \cos \delta_{\mathrm{E}} \cos \alpha_{\mathrm{E}} \qquad \cos \delta_{\mathrm{H}} \cos \alpha_{\mathrm{H}}$$
$$\mathbf{u} = \mathcal{E} \quad \cos \delta_{\mathrm{E}} \sin \alpha_{\mathrm{E}} \qquad = \mathcal{H} \quad \cos \delta_{\mathrm{H}} \sin \alpha_{\mathrm{H}}$$
$$\sin \delta_{\mathrm{E}} \qquad \qquad \sin \delta_{\mathrm{H}}$$
$$[18.1]$$

where (α_E, δ_E) and (α_H, δ_H) are the celestial coordinates of **u** in the two frames. The column matrices in Equation 18.1 containing the direction cosines can also be written \mathcal{E}' **u** and \mathcal{H}' **u**, respectively. They are related through the matrix equation:

$$\mathcal{E}'\mathbf{u} = (\mathcal{E}'\mathcal{H})\mathcal{H}'\mathbf{u}$$
[18.2]

where $\mathcal{E}'\mathcal{H}$ is an orthogonal 3×3 matrix whose elements consist of the scalar products $\mathbf{x}'_E \mathbf{x}_H$, etc.

The relation between \mathcal{E} and \mathcal{H} can be represented by the time dependent vector $\boldsymbol{\epsilon}(T)$ such that a triad initially aligned with \mathcal{H} will become aligned with \mathcal{E} after rotation through the angle $\varepsilon = |\boldsymbol{\varepsilon}|$ about the unit vector $\mathbf{e} = \boldsymbol{\varepsilon}\varepsilon^{-1}$. In the small-angle approximation (neglecting terms of order ε^2) the frames are related by:

$$\mathcal{E} \simeq \mathcal{H} + \boldsymbol{\varepsilon} \times \mathcal{H}$$
 [18.3]

and the transformation matrix in Equation 18.2 becomes:

$$\mathcal{E}'\mathcal{H} \simeq \mathbf{I} + (\boldsymbol{\varepsilon} \times \mathcal{H})'\mathcal{H} = \begin{array}{cc} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{array}$$
[18.4]

Here, **I** is the 3×3 identity matrix and ε_x , ε_y , ε_z are the components of ε in either reference frame. (The actual orientation errors occurring in the link equations were less than 0.1 arcsec, so the small-angle approximation was always adequate. The rigorous expressions for arbitrarily large rotations are given in Lindegren & Kovalevsky 1995.) It can be noted that $\mathcal{E}'\varepsilon = \mathcal{H}'\varepsilon$ strictly holds, so that the components of the rotation vector are the same in the two frames.

If \mathcal{H} is rotating with constant angular velocity $\boldsymbol{\omega}$ relative to \mathcal{E} , then its time dependent orientation error can be written (again in the small-angle approximation):

$$\boldsymbol{\varepsilon}(T) = \boldsymbol{\varepsilon}_0 + (T - T_0)\boldsymbol{\omega}$$
 [18.5]

where ε_0 is the orientation error at the reference epoch $T_0 = J1991.25$. The link observations aim at the estimation of the six components of ε_0 and ω in the \mathcal{E} or \mathcal{H} frame, and the link equations express the observations in terms of these six unknowns, or a subset of them.

Positional Observations

Observations linking the positions of objects in the two frames provide information on the orientation difference ϵ at the (mean) epoch of observation, $T = T_0 + t$. Three kinds of positional link observations are considered here: (1) observation of the position of a Hipparcos star in the extragalactic frame; (2) observation of the position of an extragalactic object in the Hipparcos frame; and (3) measurement of the angular separation between two objects, one of which is known in the extragalactic frame, the other in the Hipparcos frame. Observations undertaken for each of these cases can be summarized as follows:

(1) Radio interferometric observations of a radio star allow its barycentric position $(\alpha_{\rm E}, \delta_{\rm E})$ in the extragalactic frame at the (mean) epoch *T* of the radio observations to be determined. (It can be assumed that the observations are corrected to the barycentre, using either the Hipparcos parallax or a parallax determined from the radio observations themselves.) Let $(\alpha_{\rm H}, \delta_{\rm H})$ be the barycentric position of this star at the same epoch *T*, as calculated from the position and proper motion data in the Hipparcos Catalogue. The two sets of celestial coordinates are related through Equations 18.1, 18.2 and 18.4. Neglecting terms of order $(\alpha_{\rm H} - \alpha_{\rm E})\varepsilon$ and $(\delta_{\rm H} - \delta_{\rm E})\varepsilon$ gives the following equations of condition:

$$\frac{-\sin\delta\cos\alpha - \sin\delta\sin\alpha \cos\delta}{\sin\alpha - \cos\alpha 0} \epsilon(T) = \frac{(\alpha_{\rm H} - \alpha_{\rm E})\cos\delta}{\delta_{\rm H} - \delta_{\rm E}}$$
[18.6]

The combination of several such observations spread over a number of years allows determination of ϵ_0 and ω separately by substituting Equation 18.5 in the left-hand side.

(2) Relative astrometric observations by means of photographic or CCD techniques, using Hipparcos stars as reference points, allow the position of an extragalactic object in the Hipparcos frame, (α_H, δ_H) to be determined. If its position (α_E, δ_E) in the extragalactic frame is also known, the same equations of condition result as in the previous case, Equation 18.6, where $T = T_0 + t$ is the epoch of the relative astrometric observation.

(3) Observations with the Hubble Space Telescope Fine Guidance Sensors allow the angular separation ϕ of two objects, e.g. between an extragalactic object (at \mathbf{u}_1) and a Hipparcos star (at \mathbf{u}_2) to be measured. The coordinates of the extragalactic object are known in the extragalactic frame ($\mathcal{E}'\mathbf{u}_1$), while those of the Hipparcos star are known in the Hipparcos frame ($\mathcal{H}'\mathbf{u}_2$). This type of observation differs from the previous two in that none of the objects is accurately known in both frames. The following implicit form of link equation is readily obtained by means of Equations 18.2 and 18.4:

$$2\sin\frac{\phi}{2} = |\mathbf{u}_{1} - \mathbf{u}_{2}| = |\mathcal{E}'\mathbf{u}_{1} - \mathcal{E}'\mathbf{u}_{2}| = |\mathcal{E}'\mathbf{u}_{1} - (\mathcal{E}'\mathcal{H})\mathcal{H}'\mathbf{u}_{2}|$$
$$= \frac{\cos\delta_{E1}\cos\alpha_{E1}}{\cos\delta_{E1}\sin\alpha_{E1}} - \frac{1}{-\varepsilon_{z}} \frac{\varepsilon_{z}}{1} - \frac{-\varepsilon_{y}}{\varepsilon_{x}} \frac{\cos\delta_{H2}\cos\alpha_{H2}}{\cos\delta_{H2}\sin\alpha_{H2}}$$
[18.7]
$$\sin\delta_{E1} - \frac{\varepsilon_{y}}{\varepsilon_{y}} - \varepsilon_{x} - \frac{1}{\varepsilon_{x}} \frac{\sin\delta_{H2}}{\sin\delta_{H2}}$$
[18.7]

An explicit form can be obtained by linearisation.

Proper Motions

Let $(\mu_{\alpha*H}, \mu_{\delta H})$ and $(\mu_{\alpha*E}, \mu_{\delta E})$ be the proper motion components of one and the same object, expressed in the Hipparcos and extragalactic frames. Their differences give directly an observation equation for the spin difference:

$$\frac{-\sin\delta\cos\alpha - \sin\delta\sin\alpha \cos\delta}{\sin\alpha - \cos\alpha 0} \omega = \frac{\mu_{\alpha*H} - \mu_{\alpha*E}}{\mu_{\delta H} - \mu_{\delta E}}$$
[18.8]

Stellar proper motions in the Hipparcos frame are known from the space observations. The proper motions of some of these stars were also known in the extragalactic frame, either from VLBI observations (in the case of radio stars), or from photographic surveys determining 'absolute' stellar proper motions with respect to background galaxies. A third kind of observations leading to the same form of link equation is the measurement of the apparent proper motions of sufficiently distant extragalactic objects in the Hipparcos reference frame; in this case $\mu_{\alpha * E} = \mu_{\delta E} = 0$ is assumed.

Use of Earth Orientation Parameters

The Earth Orientation Parameters (EOP) are a set of time dependent angles describing the orientation of the Earth's spin axis and the phase of the spin about the axis. The spin axis orientation is given in the terrestrial system by the two components of the polar motion (denoted *x*, *y*), and in the celestial system by the offsets in obliquity ($\Delta \epsilon$) and longitude ($\Delta \psi \sin \epsilon$) of the nutation angles from a conventional model of precession and nutation. The instantaneous phase of the spin is given by universal time (UT1), and the corresponding Earth orientation parameters set is taken to be its offset from the international atomic time scale, UT1–TAI. Since 1980 these angles are derived with sub-milliarcsec accuracy from VLBI observations relative to extragalactic radio sources in the IERS reference system. The celestial orientation of the Earth, given by $\Delta \epsilon$, $\Delta \psi \sin \epsilon$ and UT1–TAI, are thus accurately known in the extragalactic reference system.

But the Earth orientation parameters can also be derived from latitude and time observations obtained by optical instruments, typically zenith tubes and astrolabes. Indeed, this was the standard method before the advent of radio interferometry. In this case the celestial orientation of the Earth is determined with respect to the optical reference system of the stars used in the observations. Clearly a comparison of the Earth orientation parameters as derived by VLBI and by the traditional optical means provides an indirect link between the two reference systems. In terms of the orientation vector $\boldsymbol{\varepsilon}$ at the epoch of observation, the link equations are:

$$\varepsilon_{x} = -(\Delta \epsilon_{\rm H} - \Delta \epsilon_{\rm E})$$

$$\varepsilon_{y} = (\Delta \psi_{\rm H} - \Delta \psi_{\rm E}) \cos \epsilon \qquad [18.9]$$

$$\varepsilon_{z} + \Delta \lambda = 15\,041\,(\rm{UT1}_{\rm H} - \rm{UT1}_{\rm E})$$

where $\epsilon \simeq 23$ °.44 is the obliquity of the ecliptic and $\Delta\lambda$ is the longitude difference between the two realisations of the terrestrial system. The numerical factor 15 041 converts seconds of Universal Time to mas. Unfortunately $\Delta\lambda$ is essentially unknown, at the accuracy level of interest here, and the Earth orientation parameters method can therefore only be used to determine the *x* and *y* components of the link. The timedependent part of Equation 18.9 gives the observation equations for ω . Assuming that $\Delta\lambda$ is constant, it should be possible to obtain all three components of $\boldsymbol{\omega}$ from these equations; however, the actual results indicate that there is also a drift in $\Delta\lambda$, so that only ω_x and ω_y can be determined.

18.4. Results of the Different Link Programmes

In the following subsections, the various programmes used for the determination of the extragalactic link are described individually. The participants of each group are listed in Section 18.9. The link programmes are presented in the following order:

- 1. radio and optical techniques providing high-precision positional links to a small number of Hipparcos stars. These define the orientation parameters ε very accurately, but contribute less to the determination of the spin (ω) due to the small number of objects and the relatively short time span of the observations;
- 2. use of proper motion surveys where the motions of large numbers of stars are measured relative to galaxies. These only contribute to the determination of ω ;
- 3. special photographic link programmes;
- 4. use of Earth orientation parameters.

The numerical results of the individual link solutions, expressed as residuals with respect to the adopted global solutions, are given in Tables 18.3–18.4. Further details on the individual solutions are found in Kovalevsky *et al.* (1997) and in separate papers prepared by the different groups.

VLBI Observations

Multi-epoch VLBI observations were conducted between 1984 and 1994 to determine the positions, proper motions and parallaxes of 12 radio-emitting stars. Their positions on the sky are shown in Figure 18.1. The observations were conducted on the US VLBI Network, NASA Deep Space Network, NRAO Very Large Baseline Array (VLBA) and European VLBI Network. The data processing and analysis is described in Lestrade *et al.* (1995). All the VLBI observations for each star were phase-referenced to an angularly nearby extragalactic radio source on the ICRF list. The resulting uncertainties in the astrometric parameters of the radio stars are presented in Table 18.1. The parallax results are discussed in Section 20.3.

The six components of ε_0 and ω were simultaneously solved by a least-squares fit as described in Lestrade *et al.* (1995), using weights based on the combined VLBI and Hipparcos *a priori* measurement uncertainties. Two objects were however downweighted by increasing their positional uncertainties by a factor of three: for HIP 12469 (LSI 61°303) because of its jet structure on a 10 mas scale, and for HIP 19762 (HD 283447) because of its known duplicity on a scale $\simeq 0.1$ arcsec (Ghez *et al.* 1993) which is difficult for Hipparcos. No modifications were made on the *a priori* proper motion uncertainties. After this adjustment of the weights, the unit-weight residual of the solution was close to unity.

Tests were done by splitting the 12 stars in various subsets and calculating independent solutions for each subset. This showed that the fit is quite robust: for instance, the



Figure 18.1. Sky distribution of radio stars used for the link by VLBI (crosses) and MERLIN (circles). Equatorial projection with α increasing from – 180° to + 180° right-to-left.



Figure 18.2. Sky distribution of Lick NPM1 fields (light grey) and Yale/San Juan SPM fields (dark grey) used in the link solutions. Equatorial projection with α increasing from – 180° to + 180° right-to-left.

Hipparcos		Standard errors		
number (HIP)	Star name	Pos. (mas)	P.M. (mas/yr)	Par. (mas)
12469	LSI 61°303 = V615 Cas	3.0	0.30	0.62
14576	Algol	0.61	0.18	0.59
16042	UX Ari	2.1	0.20	0.39
16846	HR 1099	0.48	0.31	0.47
19762	HD 283447	3.0	0.28	0.25
23106	HD 32918	1.5	1.00	0.80
66257	HR 5110	1.28	0.16	0.45
79607	σ^2 CrB	0.29	0.05	0.10
98298	Cyg X1 = V1357 Cyg	1.50	0.14	0.30
103144	HD 199178	1.95	0.43	0.33
109303	AR Lac	0.94	0.19	0.37
112997	IM Peg	1.42	0.47	0.68

Table 18.1. Uncertainties of the absolute positions (at epoch J1991.25), proper motions and trigonometric parallaxes of the 12 link stars as determined by VLBI observations.

differences between the fits of two subsets of six stars each were within the combined uncertainties, i.e. less than 1 mas for the orientation components and less than 0.6 mas/yr for the spin components.

Observations with MERLIN

MERLIN is a real-time radio-linked radio interferometer array with a maximum baseline of 217 km, giving a resolution of approximately 40 mas at 5 GHz. See Thomasson (1986) for a general description of MERLIN. As in the VLBI observations described above, the positions of weak radio stars were obtained by using ICRF sources as phase calibrators. Typically, the star-calibrator separation was 5° and the cycle time was 5 to 10 min.

A total of 13 radio stars were observed between 1992 and 1995, four of which are common with the VLBI set: HIP 12469 (LSI 61°303), HIP 14576 (Algol), HIP 16879 (HD 22403), HIP 19431 (HD 26337), HIP 53425 (DM UMa), HIP 65915 (FK Com), HIP 66257 (HR 5110), HIP 79607 (σ^2 CrB), HIP 85852 (29 Dra), HIP 91009 (BY Dra), HIP 108644 (FF Aqr), HIP 116584 (λ And), and HIP 117915 (II Peg). The positions of individual stars, relative to the ICRF sources, are estimated to have individual errors of approximately 4 mas. Two of the stars (HIP 85852 and HIP 79607) were not included in the link solution because of problems related to the duplicity of these objects. The distribution of the retained stars on the sky is shown in Figure 18.1.

The Hipparcos proper motions and parallaxes were used to reduce the MERLIN geocentric positions to the barycentre and mean epoch of Hipparcos, i.e. J1991.25. For the triple star Algol, a further correction from the radio emitting close pair AB to the centre of mass of the AB–C system was applied, using the orbital elements and mass ratios by Pan *et al.* (1993). Compared with that reference, however, the position angle of the line of nodes had to be rotated by 180° to obtain agreement with the MERLIN data. The solution for ϵ (at the mean epoch J1994.0) gives standard errors of 2.2 to 2.6 mas in the components. Compared with the VLBI solution which is virtually independent, there is a close agreement which lends confidence to the stability of the link which could have been distorted by significant offsets between the optical and radio emission of some of the binary stars.

Observations with the VLA

As part of the link programme, observations were also carried out with the Very Large Array (VLA) operated by the National Radio Astronomy Observatory. For this observing programme the procedures outlined in Florkowski *et al.* (1985) were followed. Between March 1982 and August 1995, radio emitting stars were observed differentially with respect to unresolved extragalactic radio sources. Cleaned maps of the sky near the stellar radio emission were created using the Astronomical Image Processing System (AIPS). The position of the star relative to the absolute phase centre of the map was obtained by fitting a two-dimensional Gaussian function to the stellar emission. The stellar positions (in the extragalactic reference frame) were moved to the epoch J1991.25 by means of the Hipparcos proper motions and parallaxes. The differences between the radio and Hipparcos positions were then used to solve for the orientation vector $\boldsymbol{\epsilon}$ at the mean epoch of observation (around 1986). The standard error in each component of the vector was about 5 mas.

Optical Positions of Compact Sources

The Hamburg/USNO reference frame programme has been described in Johnston *et al.* (1995), Ma *et al.* (1990), and Zacharias & de Vegt (1995), and the reader is referred to these publications for details. The programme is aimed at the determination of precise optical and radio positions of about 400 to 500 selected compact radio sources which display optical counterparts, mostly QSOs and BL Lac's, within a visual magnitude range of 12 to 21 mag. Optical positions in the Hipparcos reference frame were obtained via a system of secondary reference stars in the magnitude range 12 to 14 mag. The procedure thus required two steps: first the establishment of the secondary reference positions by means of astrograph plates, and then the observation of the radio sources with respect to the secondary frame by means of larger telescopes.

The secondary frame was established using wide field ($\simeq 5^{\circ}$) astrographs on both hemispheres. For each field, four plates centred on the source position were taken and measured on the CCD-camera based Hamburg astrometric measuring machine. The measurements included all Hipparcos stars in the whole plate field (typically 50 to 100 stars), and secondary reference stars selected from the Hubble Space Telescope Guide Star Catalog in the central 1° field. Formally, the Hipparcos reference frame could be transferred locally to each radio source field with a precision better than 10 mas.

Optical source positions were then obtained using plates from Schmidt telescopes and the prime focus of large telescopes. Plate or CCD solutions were obtained using the secondary reference star catalogue. The precision of the optical source positions based on several plates and/or CCD frames is better than 30 mas in each case. The programme therefore provides optical positions of the extragalactic reference frame sources in the Hipparcos frame. The link solution used here was based on the CCD frames of 78 globally selected sources at mean epoch J1988.5 and gives a formal error of about 5 mas in each component of the orientation vector at that epoch. The full programme will

eventually determine the orientation parameters on the 1 mas accuracy level, based on all 400 sources.

Observations with the Hubble Space Telescope

The Fine Guidance Sensors (FGSs) of the Hubble Space Telescope (HST) have been used to measure the angular separation of Hipparcos stars from extragalactic objects. Within the instrumental frame the FGSs measure relative positions of targets to a precision of a few milliarcsec (Benedict *et al.* 1992). However, since the absolute orientation of the FGS frames is not accurately known, the relative positions in the FGS field of view cannot be transformed to differences in α and δ on the sky. The angular separation of two objects, being independent of the FGS orientation, is therefore the most accurate datum available for the link work.

78 separations of 46 Hipparcos stars next to 34 extragalactic objects were measured from April 1993 through December 1995. GaussFit, a non-linear least-squares package (Jefferys *et al.* 1988), was used to determine the orientation and spin parameters from these data. The Hipparcos proper motion and parallax values were used to calculate the topocentric directions of the stars at the times of observation. The analysis required the application of a time-dependent field distortion calibration and the inclusion of a time-dependent scale factor among the fitted parameters. The major sources of error are the HST/FGS data (estimated at 3 to 4 mas for a single separation measurement) and the propagation of the Hipparcos proper motion errors to the epochs of the HST observations.

Use of the Lick Proper Motion Program

The published Part 1 of the Lick Observatory Northern Proper Motion Program (NPM), also known as NPM1 (Klemola *et al.* 1987, 1993, 1994), contains 149 000 stars from 899 NPM fields north of $\delta = -23^{\circ}$ for which the proper motions have been determined relative to background galaxies. The mean number of galaxies per field is 80. The typical precision of the NPM1 absolute proper motions is 5 mas/yr.

In total 13 455 stars are common to the NPM1 catalogue and the Hipparcos Catalogue. Preliminary comparisons of Hipparcos proper motions with the NPM1 indicated a linear magnitude equation of about 1 mas yr^{-1} mag⁻¹ in the NPM1 data essentially down to the magnitude limit of the Hipparcos Catalogue. The magnitude equation is coordinate-independent, although in declination it shows a different slope for stars north and south of $\delta = -2^{\circ}5$. It should be noted that due to the lack of measurable multiple grating images in the same exposure, it is impossible to eliminate the magnitude equation internally. Furthermore, there are no absolute proper motions available which could readily be used to correct the magnitude equation, the Hipparcos data cannot be used to correct the magnitude equation, the Hipparcos data cannot be used to correct the magnitude equation.

Two different groups have independently analysed the Hipparcos–NPM1 differences in an attempt to contribute to the extragalactic link of the Hipparcos Catalogue. Their conclusions are separately reported below.

The Yale Analysis: With regard to the magnitude equation described above, affecting the bright NPM1 stars, various solutions to the problem were tried, including the

Magnitude	Number	Spin components		
range	of stars	ω_X	ω_y	ω_Z
(no limit)	9236	-0.70	-0.27	-2.14
$10.5 < m_B < 11.5$	2616	-0.76	+0.17	-0.85
$10.9 < m_B$	2220	-0.72	+0.02	+0.10
$11.9 < m_B$	510	-0.25	-0.12	+0.84
10.0 < Hp < 12.2	2535	-0.81	+0.11	-0.25
10.6 < Hp < 12.2	1135	-0.85	+0.16	+0.60
Formal errors:	1135	0.25	0.20	0.20

Table 18.2. Results of the Heidelberg solutions for the components of the spin vector $\boldsymbol{\omega}$ (in mas/yr) using the Lick NPM1 proper motions. The first line gives the solution without magnitude limits for the selection of stars; subsequent lines give the results for stars in certain intervals of the Lick (m_B) or Hipparcos (Hp) magnitude. The last line gives the formal standard errors for the solution with 1135 stars.

use of additional parameters in the link equations corresponding to linear magnitude equations in each coordinate. Unambiguous disentangling of the magnitude effect from the spin parameters turns out to be very difficult and perhaps impossible, due to the strong correlations among these parameters. A careful inspection of how the magnitude equation affects the spin components indicates the significance of the star distribution over the sky. One possibility of minimising the effect of a coordinate-independent magnitude equation is to seek a well-balanced distribution of the stars. In other words, for the spin components ω_x and ω_y the stars must be distributed such that the sums of the corresponding geometrical weighting factors in Equation 18.8 are close to zero. In practice, the distribution of the NPM1 stars used in the solution was balanced by introducing a fictitious 44°-width 'zone of avoidance' perpendicular to the galactic plane. In addition, the stars with $\delta < -2.5$ (only in the declination solution) and $m_B < 10$ mag were deleted from the sample in order to reduce further the effect of the magnitude equation. Four different solutions were computed from images of different colours (blue, visual) and components (α , δ).

The Heidelberg Analysis: In order to investigate a possible magnitude equation in the Lick data, spin solutions were computed for stellar samples of different brightness (Table 18.2). It turned out that the dependence on magnitude is relatively unimportant for ω_x and ω_y . (The case of $m_B > 11.9$ mag was not considered representative, because of the small number of stars in that sample.) ω_z , on the other hand, shows a strong dependence on magnitude. Moreover, there seems to be no asymptotic behaviour when going to fainter stars. The conclusion is that ω_z cannot be reliably determined from the Lick proper motions. These findings are confirmed by Hanson (1996, private communication) who reported on small systematic errors in the Lick proper motions in right ascension.

Catalogue of Faint Stars (KSZ)

A general catalogue of absolute proper motions of stars with respect to galaxies was compiled by Rybka & Yatsenko (1996), using data from 185 sky areas produced in Kiev, Moscow, Pulkovo, Shanghai and Tashkent. The catalogue includes 977 Hipparcos stars in the magnitude range 4 to 13 mag.

Proper motion differences were analysed according to Equation 18.8. The residuals in each coordinate were analysed as functions of magnitude, colour and position on the sky. No significant dependency on these variables was found: the residuals represent random errors only. However, different results for ω were obtained when the whole interval of stellar magnitudes was used and when only bright (≤ 9.0 mag) or faint (> 9.0 mag) stars were used. Since the stellar data in KSZ were obtained relative to faint galaxies it was assumed that the solution using only the fainter stars is the more reliable one for the link. For that solution, 415 stars were kept from 154 areas of the sky, yielding standard errors of about 0.8 mas/yr for the spin components.

The Yale/San Juan Southern Proper Motion Program

The Yale/San Juan Southern Proper Motion program (SPM) is an extension of the Lick Observatory Northern Proper Motion program to the sky south of $\delta = -17^{\circ}$. A brief description of the observational material can be found in van Altena *et al.* (1990) and Platais *et al.* (1995). In total 63 SPM fields, containing about 4100 Hipparcos stars, were measured and reduced for the Hipparcos link (Figure 18.2). The mean number of reference galaxies per field is 250 on blue plates and 190 on visual plates, yielding a mean uncertainty in the correction to absolute proper motions of 1.0 mas/yr for each field. Since the Hipparcos stars are represented by several images per star (ten in the most favourable case), the single proper motion precision in each colour (blue or visual) can be as good as 2 to 3 mas/yr. If this error were composed entirely by the random measurement and modelling errors, the precision of each spin component with the given number of the Hipparcos stars in hand could be in the range of 0.1 to 0.2 mas/yr. However, the link solutions indicate a somewhat larger scatter in the spin components when compared to this precision estimate. This may very well be due to a small systematic error remaining after the correction for the magnitude equation.

A preliminary study of the systematic errors in the SPM plates (Platais *et al.* 1995) clearly showed the presence of a significant magnitude equation in the SPM coordinates. The bulk of the magnitude equation in coordinates and, presumably, in proper motions was removed using the grating-image offset technique formulated by Jefferys (1962) and modified by the present group. This technique has inherent limitations set by the small number of stars at the bright end, and by the fact that the magnitude equation may have a complicated form, too difficult to model adequately. In addition, the magnitude equation in the SPM plates is stronger and more complex in declination than in right ascension. It was therefore believed that the link solution using only the proper motions in right ascension was less affected by systematic errors related to the magnitude effect.

The Bonn Link Solution

The Bonn link solution uses series of photographic plates characterised by very large epoch differences, typically 70 years and up to 100 years (Brosche *et al.* 1991). Each series contains a compact extragalactic source and several Hipparcos stars, from which the (apparent) proper motion of the extragalactic object in the Hipparcos frame can be derived. The plates were predominantly taken with the f = 5 m double refractor of the Sternwarte Bonn. For some fields the relative proper motions were calibrated using a large number of stars and galaxies on Schmidt plates and Lick astrographic plates.

The link solution used 88 Hipparcos stars in 13 fields distributed over the northern celestial hemisphere. The median uncertainty for each field was 1.3 mas/yr. No significant correlations of the residuals from this solution with magnitude, colour, spherical coordinates or relative position within a field were found.

The Potsdam Link Solution

The Potsdam programme (Dick *et al.* 1987) is based on measurements of plates (using MAMA and APM) taken with the Tautenburg Schmidt telescope (134/200/400 cm). Proper motions of 360 Hipparcos stars were derived in 24 fields (each of about 10 square degrees) well distributed over the northern sky. From 200 to 2000 galaxies per field were used to link the proper motions to the extragalactic reference system. With at least two plate pairs per field and epoch differences of 20 to 40 years, an internal precision of 3 to 5 mas/yr was achieved for the proper motions of Hipparcos stars (Kharchenko *et al.* 1994). Due to the large number of galaxies in each field the formal zero point error is less than 1 mas/yr.

Previous investigations showed that systematic, magnitude-dependent errors could affect the proper motions of bright stars measured on Tautenburg plates (Scholz & Kharchenko 1994, Kharchenko & Schilbach 1995). A significant magnitude equation was indeed found by comparison with the bright Hipparcos stars. To minimise the effect, only 256 Hipparcos stars fainter than $m_B = 9.0$ mag were used for the link, yielding formal errors of 0.5 mas/yr on the components of $\boldsymbol{\omega}$. The rms residual in the proper motions of stars was 6.9 mas/yr.

Use of Earth Orientation Parameters

VLBI determines the five Earth orientation parameters $\Delta \epsilon$, $\Delta \psi \sin \epsilon$, UT1–TAI, *x* and *y*, in the extragalactic frame, at roughly 5-day intervals. The same parameters, referred to the celestial optical system tied to the stars of the Galaxy, can be determined by optical astrometry following the algorithms outlined in Vondrák (1991, 1996) and Vondrák *et al.* (1992, 1995). Using the preliminary Hipparcos Catalogue, all the latitude and UT observations made with 46 instruments at 29 different observatories all over the world were recalculated into that reference frame. About 3.6 million observations were used to derive Earth orientation parameters at 5-day intervals between 1899.7 and 1992.0. However, only the last twelve years in common with the VLBI observations were actually used for the link. Moreover, only the first two components of the orientation vector $\boldsymbol{\epsilon}$ were determined, for the reasons explained in Section 18.3.

Summary of Numerical Results

The results of the individual link solutions are summarised in Tables 18.3–18.4 and presented graphically in Figures 18.3–18.4. The galactic x and y components of the solutions are also shown in Figures 18.5–18.6. As previously explained, the results given in these tables and figures are the residuals of the individual solutions with respect to the adopted solution found by the synthesis described in Section 18.7. For the orientation components the residuals refer to the approximate mean epoch of observation of the link observations in question given in the last column of Table 18.3.



Figure 18.3. Projections onto the xy and xz planes of the individual solutions for the orientation vector $\boldsymbol{\varepsilon}$ (residuals with respect to the adopted vector at the mean epoch of each solution). The components are expressed in mas. The solutions are labelled as in Table 18.3. See also Figure 18.5 for the projections onto the galactic xy plane.



Figure 18.4. Projections onto the xy and xz planes of the individual solutions for the spin vector $\boldsymbol{\omega}$ (actually residuals with respect to the adopted vector). The components are expressed in mas/yr. The solutions are labelled as in Table 18.4. See also Figure 18.6 for the projections onto the galactic xy plane.

Table 18.3. Results of the individual link solutions for the orientation vector $\boldsymbol{\epsilon}$, expressed as residuals with respect to the adopted solution. The second column contains the abbreviations used to identify the solutions in Figure 18.3. The (formal) standard errors supplied with the individual solutions are given in parentheses. The last column gives the approximate mean epoch of the link observations used to determine the orientation.

Method	Label	Orientation components (mas)			Epoch
		ε_X	ε_y	ε_Z	1900+
VLBI	VI	-0.10 (0.47)	+0.08 (0.49)	+0.16 (0.50)	91.3
MERLIN	ME	+1.41 (2.60)	-0.64 (2.20)	+0.51 (2.40)	94.0
VLA	VA	+4.27 (4.70)	-3.75 (5.30)	-5.76 (5.20)	86.3
Hamburg/USNO	HP	+3.38 (5.00)	-0.06 (4.90)	-9.20 (4.70)	88.5
HST/FGS	ST	-6.10 (2.16)	-3.25 (1.49)	+5.42 (2.14)	94.3
EOP	EO	+2.33 (0.88)	+7.80 (0.90)		85.0

Table 18.4. Results of the individual link solutions for the spin vector $\boldsymbol{\omega}$, expressed as residuals with respect to the adopted solution. The second column contains the abbreviations used to identify the solutions in Figure 18.4. The (formal) standard errors supplied with the individual solutions are given in parentheses. The components marked with an asterisk (*) were not used in the synthesis.

Method	Label	Spin components (mas/yr)		
		ω_X	ω_y	ω_Z
VLBI	VI	-0.16 (0.30)	-0.17 (0.26)	-0.33 (0.30)
HST/FGS	ST	-1.60 (2.87)	-1.92 (1.54)	+2.26(3.42)
NPM (Heidelberg)	LH	-0.77 (0.40)	+0.15 (0.40)	+0.23 (*)
NPM (Yale)	LY	+0.09 (0.18)	-0.20 (0.18)	+1.46 (*)
KSZ Kiev	ΚZ	-0.27 (0.80)	+0.15 (0.60)	-1.07 (0.80)
SPM (blue, α)	YBA	+0.23 (0.13)	+0.50 (0.20)	0.00 (0.08)
SPM (blue, δ)	YBD	+0.07 (0.15)	+0.58 (0.08)	
SPM (visual, α)	YVA	+0.44 (0.12)	+0.71 (0.18)	-0.30 (0.07)
SPM (visual, δ)	YVD	+0.30 (0.12)	+0.76 (0.06)	
Bonn plates	BP	+0.93 (0.34)	-0.32 (0.25)	+0.17 (0.33)
Potsdam plates	PP	+0.22 (0.52)	+0.43 (0.50)	+0.13 (0.48)
EOP	EO	-0.93 (0.28)	-0.32 (0.28)	

18.5. Discussion of the Individual Solutions

Before describing the methods and results of the synthesis, some remarks should be made on the data provided by the various link techniques.

Radio Techniques

The relative precisions provided by the three interferometric techniques in the determination of the orientation (ε) are consistent with their baseline lengths and are therefore considered as realistic. The precision of the determination of the spin (ω) depends both on the basic uncertainty of the observations and on the time span. This gives a major advantage to the VLBI observations both for the orientation and spin. An attempt was made to determine ω with MERLIN observations, but the result was not retained, the time span being notably insufficient. A major concern with the radio techniques was the small number of link objects and the consequently rather high sensitivity of the solution to possible offsets of the radio and optical centres of emission. It was therefore very important to support these techniques by independent optical links to other extragalactic sources.

Optical Determination of the Orientation

The full Hamburg/USNO programme comprises some 400 extragalactic sources, but only about 20 per cent were completed at the time of the link. The Hubble Space Telescope observations started very late due to the well known problems with the telescope. In both cases, the uncertainties are large, but the methods are promising and would have given better results with more data. Nevertheless, the results obtained are in acceptable agreement with the radio techniques and support the adopted link within the uncertainties of the optical techniques, namely a few milliarcsec.

Photographic Catalogues Referred to Galaxies

These techniques are much more sensitive to magnitude-dependent errors than the preceding two optical methods. Most of the stars measured in these surveys are faint and comparable in magnitude with the reference galaxies. However, for the link one had to choose only the brightest of the survey stars, for which the effect is likely to be much larger. The magnitude dependence is not necessarily linear at this end of the survey population and the link results depend strongly upon either the model applied or the magnitude cut-off adopted. This is illustrated by the discussions of the NPM, SPM and KSZ programmes in the preceding section. No result can be considered as being unbiased in this respect, although the SPM solutions have an advantage in that the magnitude equation could be calibrated internally by the grating image technique.

The formal errors given by the authors of these methods are small because of the large number of stars, and cannot be considered as realistic. Additional biases related to the magnitude, and perhaps to other factors less well studied, certainly exist. This has justified a significant down-weighting of the results provided. The difference between the results obtained at Yale and Heidelberg in their analyses of the NPM1 stars also justifies this policy.

Special Photographic Link Programmes

Relying on archival plates for the first-epoch measurements, these programmes are also prone to magnitude equation, with little possibility of controlling or studying the effect. Because of this, their formal errors are probably underestimated. Strangely enough, there seems to have been little gain in having a very long time span. Possibly the magnitude-dependent errors are larger or more difficult to model in the old plates, offsetting the advantage of the long time baseline.

Earth Orientation Parameters

In a sense this method is less direct than the others, as it depends on an intermediate (terrestrial) reference frame, whose relations in the optical and radio domain may not be completely understood. Apart from the problem with the *z* components discussed in the preceding section, an additional uncertainty arises from the fact that the mean epoch of the observations is 1985 and that all were performed before the Hipparcos mean epoch. In the synthesis method where only ε_0 was estimated, this led to a considerable down-weighting of the data. However, even when the strong correlation between ε_0 and ω was taken into account, the formal errors had to be substantially increased to make sense in relation to other data.

18.6. Synthesis of the Link Solutions: General Methods

The synthesis of the individual link solutions was made independently by L. Lindegren and J. Kovalevsky, using methods (referred to as Method A and Method B below) which differ not only in implementation but also in the detailed treatment of the orientation and spin components and in the weighting of the individual solutions.

Method A

This method is described in detail in Lindegren & Kovalevsky (1995) and was strictly followed. As shown in that paper, it is possible to cast the results of each individual link solution (*j*) in the form of an information array $[\mathbf{N}_j \mathbf{h}_j]$ representing the normal equations $\mathbf{N}_j \mathbf{s} = \mathbf{h}_j$ for the six-dimensional state vector $\mathbf{s} = [\varepsilon_{0x} \varepsilon_{0y} \varepsilon_{0y} \omega_x \omega_y \omega_y]'$. The symmetric 6×6 matrix \mathbf{N}_j has full rank only for the techniques providing an estimate of all six components of the state vector, which is then given by the solution $\mathbf{s}_j = \mathbf{N}_j^{-1} \mathbf{h}_j$ together with its formal covariance matrix \mathbf{N}_j^{-1} . Other link solutions providing only partial information on the state vector can still be expressed as an information array $[\mathbf{N}_j \mathbf{h}_j]$, but \mathbf{N}_j is then singular, with $r_j = \operatorname{rank}(\mathbf{N}_j) < 6$ being the number of state vector components determined. In setting up the information arrays, the full set of correlations between the orientation and spin components were important for a uniform treatment of the different mean observation epochs shown in Table 18.3.

The individual link solutions $j = 1 \dots J$ were given a *posteriori* weight factors $0 < w_j \le 1$ according to a semi-automatic procedure described below. This corresponds to a set of multiplicative factors $w_j^{-1/2} \ge 1$ on the formal standard errors provided by the link groups. The weighted synthesis solution is then given by:

$$\mathbf{s} = \begin{array}{ccc} & & & \\ J & & & \\ \mathbf{s} = & & W_j \mathbf{N}_j & & W_j \mathbf{h}_j \\ & & & j=1 \end{array}$$
[18.10]

where the inverse matrix also provides the estimated covariance of **s**.

The main problem is to assign the weight factors w_j . This must be done in such a way that the distances of the individual link solutions from **s** are compatible with the (reweighted) standard errors, taking into account the correlations among the components of the individual solutions \mathbf{s}_j and the synthesised solution. This is complicated by the fact that only few of the link techniques provide an estimate of the full state vector. In practice a goodness-of-fit of each solution was computed from the re-weighted residual vector of the normal equations:

$$\mathbf{d}_j = w_j (\mathbf{h}_j - \mathbf{N}_j \mathbf{s})$$
[18.11]

Assuming that $w_j N_j$ is the correct information matrix for estimate *j*, the expected covariance of **d**_j is then given by:

$$\mathbf{D}_{j} = w_{j}\mathbf{N}_{j} - w_{j}\mathbf{N}_{j} \qquad \begin{matrix} J & & ^{-1} \\ & & W_{j}\mathbf{N}_{j} & & W_{j}\mathbf{N}_{j} \end{matrix}$$
[18.12]

which has the rank r_i . The goodness-of-fit statistic for solution *j* was computed as:

$$q_j = \mathbf{d}_J' \mathbf{D}_j^+ \mathbf{d}_j \tag{18.13}$$

where \mathbf{D}_{j}^{+} is the generalised inverse. q_{j} is expected to follow the χ^{2} distribution with r_{j} degrees of freedom. The global statistic $Q = \int_{j} q_{j}$ should similarly follow the χ^{2} distribution with $R = \int_{j} r_{j}$ degrees of freedom.

The procedure for determining the individual weights w_j was roughly as follows. Starting from some *a priori* set of weights, the synthesised solution **s** was computed according to Equation 18.10, and hence the statistics q_j according to Equations 18.11–18.13. Typically this gave too high a value of Q due to unrealistically small standard errors in some of the individual solutions. The most discrepant solution was identified by comparing the normalised statistics q_j/r_j , the weight of that solution was halved, and a new synthesised solution was computed with revised q_j and Q. This process was iterated until $Q \simeq R$ and all $q_j \simeq r_j$, at which point the synthesised solution **s** was accepted and assigned the covariance given by the inverse matrix in Equation 18.10.

Method B

This method is based upon the fundamental assumption that the errors obtained by every task are Gaussian. This means that the probability density function is given in its most general form for *n* variables by:

$$f(x_1, x_2, ..., x_n) = (2\pi)^{-n/2} |\mathbf{V}|^{-1/2} \exp \left[-\frac{1}{2} \int_{i=1}^n \int_{k=1}^n \mathbf{V}^{-1} (x_i - \overline{x}_i)(x_k - \overline{x}_k)\right]$$
(18.14]

where **V** is the variance-covariance matrix of the variables and \overline{x}_i their mean values.

This probability density function can be computed from the data provided by each individual link task (j), namely the estimated variables with their standard errors and the correlation matrix. Now, the joint probability density function of J Gaussian distributions is the product of the probability density functions of these distributions:

$$\varphi = \int_{j=1}^{J} f_j(x_1, x_2, \dots, x_n)$$
 [18.15]

which is easily computed, and has exactly the same form as Equation 18.14, since the quantities in the exponential add. One can therefore compute back the variancecovariance matrix corresponding to φ and derive the standard errors and the correlations of the merged solution. This approach explicitly assumes Gaussian error distributions and it would not be correct to apply it to other distributions. However, in the particular case to which it is applied, no indication of a non-Gaussian behaviour was given by the individual link solutions which all made the same assumption.

Although formulated in a probabilistic framework, this method is in principle equivalent to a weighted least-squares method and should yield similar results as Method A. However, the practical implementations differ, and Method B was also applied separately to the spin and orientation components, which may add some insight into the properties of the individual solutions. As in Method A, a main problem is to adjust the relative weights of the contributing solutions.

18.7. Synthesis of the Link Solutions: Results

Individual Link Data

The results obtained by various individual solutions were given in three different forms depending on the type of solution:

- if the technique only allowed the determination of the orientation of the Hipparcos frame relative to the extragalactic frame, then the vector $\epsilon(T)$ was given for the mean epoch *T* of the link observations (MERLIN, VLA, Hamburg/USNO);
- if the technique only allowed the determination of the spin of the Hipparcos frame relative to the extragalactic frame, then only the vector $\boldsymbol{\omega}$ was given (NPM, KSZ, SPM, Bonn, Potsdam);
- if the technique allowed the determination of both the orientation and spin of the Hipparcos frame relative to the extragalactic frame, then the vectors ε_0 and ω were given, with the former referring to the fixed epoch T_0 = J1991.25 (VLBI, HST/FGS, EOP).

In each case the standard errors and the associated covariance matrix were also supplied. These data (except the correlations) are summarised in Tables 18.3–18.4 in the form of the residuals with respect to the finally adopted solution ε_0 , ω . For the orientation components in Table 18.3 the given data are the residuals at the mean epoch of observation, i.e. $\varepsilon(T) - \varepsilon(T)$, where *T* is the datum in the last column.

Results of Method A

The Heidelberg and Yale analyses of the NPM1 do not represent independent determinations. They were therefore averaged prior to the synthesis, and the more pessimistic standard errors from the Heidelberg analysis were adopted for the average. Similarly the blue and visual SPM solutions were averaged, but in this case giving more weight to the blue solutions. Thus, effectively 12 different solutions were considered in this synthesis.

In a first attempt the original standard errors supplied by the different groups were retained; i.e., $w_j = 1$ was adopted for all j. This gave a value of $\simeq 324$ for the χ^2 variable Q, with R = 44 degrees of freedom. This shows that the individual solutions are quite incompatible if the given standard errors are taken at face value. Consequently it was necessary to reduce the weights of at least some solutions. It is interesting to note, however, that even this initial weighting gave a solution that was within 1.5 mas in orientation and 0.2 mas/yr in spin from the final one.

The semi-automatic procedure described in the previous section was used for the downweighting. It is not obvious that this process converges to a unique result. Indeed, slightly different weights were obtained if the starting point was taken at some *a priori* judgement of the relative weights. However, the various synthesised solutions resulting from such experiments rarely differed by more than 0.1 mas and 0.1 mas/yr, and the final result was not very sensitive to additional changes in the weights. Independent of the starting point, it was found that the solutions from the SPM programme and the Earth orientation parameters had to be severely down-weighted, and the Bonn and HST/FGS solutions slightly down-weighted, but otherwise the given standard errors were roughly consistent with the overall solution. The final goodness-of-fit was Q = 47.9 with R = 44degrees of freedom; the resulting standard errors of the orientation and spin parameters were multiplied by the unit-weight error, $(Q/R)^{1/2} = 1.043$, to take into account the remaining excess in Q.

The results of Method A are summarised by the following rotation parameters (with standard errors in parentheses) referred to the epoch J1991.25:

$$\varepsilon_{0x} = +0.01 \ (0.46) \text{ mas}$$

 $\varepsilon_{0y} = -0.20 \ (0.47) \text{ mas}$
 $\varepsilon_{0z} = +0.12 \ (0.49) \text{ mas}$
 $\omega_x = +0.06 \ (0.16) \text{ mas/yr}$
 $\omega_y = -0.05 \ (0.15) \text{ mas/yr}$
 $\omega_z = +0.00 \ (0.14) \text{ mas/yr}$
[18.16]

The correlation matrix for the solution was:

$$\mathbf{R}_{A} = \begin{bmatrix} 1 & +0.25 & +0.05 & -0.04 & -0.01 & +0.00 \\ +0.25 & 1 & -0.13 & +0.01 & -0.11 & +0.01 \\ +0.05 & -0.13 & 1 & +0.01 & +0.01 & -0.06 \\ -0.04 & +0.01 & +0.01 & 1 & +0.05 & -0.16 \\ -0.01 & -0.11 & +0.01 & +0.05 & 1 & -0.16 \\ +0.00 & +0.01 & -0.06 & -0.16 & -0.16 & 1 \end{bmatrix}$$
[18.17]

The relative contributions of the various link techniques to the synthesised solution can be estimated from the diagonal elements of the re-weighted information matrices $w_j \mathbf{N}_j$. It turns out that for the orientation parameters, the VLBI observations dominate strongly. For the determination of the spin, the contributions are more evenly spread among the several techniques, but with the SPM programme and the VLBI observations together contributing about half of the total weight. It should be noted that in this method the MERLIN, VLA, Hamburg/USNO and HST/FGS links also contribute to the determination of the spin components by virtue of the spread in their mean observational epochs.

Results of Method B

For this method the down-weighting was essentially based upon the considerations given in Section 18.5, moderated by the examination of how the modifications of the weights affected the goodness-of-fit of the synthesised solution and the individual residuals. To begin with, the four solutions obtained from the Yale SPM programme were reduced into a single one by taking a weighted mean. The HST/FGS results for $\boldsymbol{\omega}$ were not used because of their large uncertainties.

In a first approximation, an unweighted solution for ϵ_0 and another solution for ω were computed neglecting the correlations between these quantities. This is justified because they are close to zero in the case of the most accurate method (VLBI), but less justified for the Earth orientation parameters method in which the correlations are about 0.89; in this case the uncertainties are also much larger.

In further iterations weights were modified progressively in order to reduce the largest residuals and the overall goodness-of-fit, as measured by the χ^2 statistic. No systematic procedure was used to modify the weights, but rather a successive approximation technique with steps of 0.2 in the weights.

Then, a global solution taking as unknowns all the six parameters of ϵ_0 and ω was made, starting with the weights obtained in the preceding solutions. This did not change significantly the results, as can be seen from the following summary of the different solutions.

B1. Solution for ε_0 only: The weighted rms residual was 0.8 mas. The solution vector for the epoch J1991.25 was (standard errors in parentheses):

$$\varepsilon_{0x} = 0.00 \ (0.51) \text{ mas}$$

 $\varepsilon_{0y} = -0.04 \ (0.51) \text{ mas}$ B1 [18.18]
 $\varepsilon_{0z} = +0.16 \ (0.53) \text{ mas}$

with correlation matrix:

B2. Solution for ω only: The weighted rms residual was 0.3 mas/yr. The solution vector was:

$$\omega_x = -0.01 \ (0.13) \ \text{mas/yr}$$

 $\omega_y = +0.08 \ (0.13) \ \text{mas/yr}$ B2 [18.20]
 $\omega_z = -0.05 \ (0.18) \ \text{mas/yr}$

with correlation matrix:

$$\mathbf{R}_{\rm B2} = \begin{array}{cccc} 1 & +0.04 & -0.11 \\ +0.04 & 1 & -0.14 \\ -0.11 & -0.14 & 1 \end{array}$$
[18.21]

B3. Solution for both ϵ_0 and ω : the weighted rms residual was 1.2 mas for the orientation components and 0.4 mas/yr for the spin components. The solution vectors were:

$$\varepsilon_{0x} = +0.16 (0.41) \text{ mas}$$

 $\varepsilon_{0y} = +0.18 (0.43) \text{ mas}$
 $\varepsilon_{0z} = -0.06 (0.46) \text{ mas}$
 $\omega_x = -0.06 (0.08) \text{ mas/yr}$
 $\omega_y = +0.05 (0.09) \text{ mas/yr}$
 $\omega_z = 0.00 (0.14) \text{ mas/yr}$
[18.22]

with correlation matrix:

The correlations obtained in solutions B1 and B2 agree rather well with those obtained from Method A, while the correlations in B3 deviate somewhat. However, the correlations are in all cases small or only moderately large, showing that the combination of link solutions gives a well-conditioned determination of all the parameters at the central epoch of the Hipparcos Catalogue.

Final Results

After a comparison of these results and their discussion, it appeared that a mean value of the two methods should be considered as the final solution for the link. The adopted orientation and rotation vectors for the provisional catalogue H37C, given in Table 16.8, were derived from a combination of the solutions A and B3. In the conventions of the present chapter, where all results are given as residuals with respect to the adopted solution, this corresponds to all parameters equal to zero.

The standard errors of the parameters were estimated to be 0.6 mas in each of the components of ε_0 , and 0.25 mas/yr in the components of ω . These numbers were obtained by a conservative rounding of the formal errors resulting from the synthesis, taking into account also the spread of values obtained in the different synthesis solutions and the uncertainty in the relative weights of the different link techniques.

18.8. Verification and Conclusions

A completely independent and accurate verification of the extragalactic link is not possible at present, as practically all available means were already employed in the link. With one important exception, the checks that are at hand can at best demonstrate that the adopted link is not inconsistent with independent data. The exception is the use of stellar kinematics, which is in principle very powerful, but depends on a very simplified statistical description of the Galaxy. This section summarises the independent checks made after the construction of the final Hipparcos Reference Frame.

3C273 (HIP 60936)

The only quasar included in the Hipparcos observing programme was 3C273. Its median magnitude during the mission was $Hp \simeq 12.8$ mag. In addition to its faintness, the position near the ecliptic and equator was quite unfavourable for observation. As a consequence, the standard errors in the five astrometric parameters were in the range 4 to 6 mas or mas/yr. The measured parallax, $\pi = 3.59 \pm 6.07$ mas, is consistent with the assumed cosmological distance (the latter implying a parallax of the order of 10^{-9} arcsec). The position and proper motion measured for this object by Hipparcos were not used in the link, and therefore constitute a completely independent check. The proper motion components measured by Hipparcos, $\mu_{\alpha*} = -11.01 \pm 6.74$ mas/yr and $\mu_{\delta} = +4.38 \pm 4.28$ mas/yr (with a correlation coefficient of -0.49) are hardly significant: the probability of having errors as large as these is 0.19 in the normal case. It is possible that variable source structure could contribute to the measured proper motion at the level of 0.5 mas/yr.

The offset of the Hipparcos position from the ICRF position of the radio source 3C273B (= ICRF J122906.70+020308.6) is $\Delta \alpha * = +9.61 \pm 7.14$ mas, $\Delta \delta = -2.12 \pm 5.44$ mas, where the standard errors are the quadratically combined standard errors of the positions in the two catalogues. The difference from zero offset is not statistically significant. The rather strong negative correlations between the position and proper motion components ($\rho_{\alpha *}^{\mu_{\alpha *}} = -0.68$, $\rho_{\delta}^{\mu_{\delta}} = -0.62$), in combination with the standard errors, show that the effective epoch of observation was close to J1991.85 (see Equation 1.2.10 of Volume 1). At that epoch the offset of the Hipparcos result from the radio position was only $\Delta \alpha * = +3.00\pm 5.41$ mas, $\Delta \delta = +0.51\pm 4.63$ mas. This strengthens the conclusion that the proper motion derived from the Hipparcos data is mainly due to noise in the observations, rather than a real motion of the photocentre due to variability.

Magellanic Clouds

Standard models of the motions of the Magellanic Clouds assume that they lead the Magellanic Stream, a narrow band of neutral hydrogen extending some 100° away from the clouds, which then defines the orbit and direction of motion of the clouds. The models predict proper motions of about 1.5 to 2 mas/yr (see Westerlund 1995 for a review). The mean proper motions of the Clouds as derived from the Hipparcos data are consistent in direction and magnitude, to within about 0.4 mas/yr, with e.g. the numerical model by Gardiner *et al.* (1994).

Stellar Kinematics

Galactic kinematics can be used to define an inertial frame based on a statistical model of stellar motions. In the simplest form, the main assumption is that the peculiar motions of the stars are, in a statistical sense, symmetric with respect to the galactic plane. The velocity components *along* the galactic plane are highly systematic and cannot be used to define an inertial frame based on simple kinematical considerations. Consequently only the component of the proper motion in galactic latitude, μ_b , is useful for this purpose.

Let **v** and **v**_{\odot} be the peculiar velocities of the star and the Sun, respectively, and **u** the unit vector from the Sun towards the star. If the Hipparcos frame is rotating with angular velocity $\boldsymbol{\omega}$, then the observed proper motion vector of the star is given by:

$$\boldsymbol{\mu} = (\mathbf{U} - \mathbf{u}\mathbf{u}')(\mathbf{v} - \mathbf{v}_{\odot})\pi/A - \boldsymbol{\omega} \times \mathbf{u} + \boldsymbol{\xi}$$
[18.24]

where **U** is the unit tensor, π is the parallax, *A* the astronomical unit, and $\boldsymbol{\xi}$ represents the error of observation. With $\mathbf{p}_{\rm G}$ and $\mathbf{q}_{\rm G}$ denoting the unit vectors respectively in the directions of increasing galactic longitude and latitude (Volume 1, Equation 1.5.15), then $\mu_b = \mathbf{q}'_{\rm G} \boldsymbol{\mu}$ or:

$$\mu_b = -\mathbf{p}'_{\rm G}\boldsymbol{\omega} - \mathbf{q}'_{\rm G}\mathbf{v}_{\odot}\pi/A + v_b\pi/A + \xi_b \qquad [18.25]$$

where $v_b = \mathbf{q}'_G \mathbf{v}$ is the latitude component of the star's peculiar velocity, and $\xi_b = \mathbf{q}'_G \boldsymbol{\xi}$ is the observational error in galactic latitude.

Under the assumption that v_b and ξ_b are independent centred random variables with approximately known standard deviations, Equation 18.25 can be used in a least-squares determination of $\boldsymbol{\omega}$ and \mathbf{v}_{\odot} from the observed values of μ_b and π . Noting that $\mathcal{G}'\mathbf{p}_G = (-\sin l \cos l 0)'$ and $\mathcal{G}'\mathbf{q}_G = (-\sin b \cos l - \sin b \sin l \cos b)'$, where $\mathcal{G} = [\mathbf{x}_G \mathbf{y}_G \mathbf{z}_G]$ is the galactic triad, it is seen that only the first two components of $\mathcal{G}'\boldsymbol{\omega}$ may be determined in such a solution, while $\mathbf{z}'_G \boldsymbol{\omega}$ obviously cannot be estimated from the proper motions in latitude; in contrast, the complete vector of the solar peculiar velocity can be determined.

The Hipparcos proper motions and parallaxes for practically all the 'single' stars were used in a robust least-squares solution based on Equation 18.25, assuming a standard deviation of 25 km s⁻¹ for v_b . The results for the first two galactic components of $\boldsymbol{\omega}$ were:

$$\omega_1 \equiv \mathbf{x}'_G \boldsymbol{\omega} = -0.15 \pm 0.04 \text{ mas/yr}$$

$$\omega_2 \equiv \mathbf{y}'_G \boldsymbol{\omega} = -0.09 \pm 0.05 \text{ mas/yr}$$
[18.26]

Using different selections of stars depending on distance, galactic latitude or colour gives results within ± 0.2 mas/yr of the values above. This result is consistent with the adopted link in ω and its estimated uncertainty of 0.25 mas/yr in each coordinate.

Graphical Summary

Figures 18.5–18.6 illustrate the verification results from 3C273 and stellar kinematics in the galactic *x* and *y* coordinates, together with the results of the various link solutions from Tables 18.3–18.4. Considering the spread of the individual solutions and their formal standard errors (shown by error circles or bands), the results from 3C273 and stellar kinematics are fully consistent with the link solutions and with the adopted mean result (represented by the origin of each diagram) to within its stated uncertainty.



Figure 18.5. Summary of the results for the orientation vector $\boldsymbol{\varepsilon}$, expressed in the galactic x and y coordinates $(\varepsilon_1 = \mathbf{x}'_G \boldsymbol{\varepsilon} \text{ and } \varepsilon_2 = \mathbf{y}'_G \boldsymbol{\varepsilon})$. The individual results from Table 18.3 are shown as error circles with a radius of one standard deviation. In the case of the Earth orientation parameters, only two of the equatorial components of $\boldsymbol{\varepsilon}$ were determined, defining a band in the $(\varepsilon_1, \varepsilon_2)$ plane corresponding to the $\pm 1\sigma$ uncertainty. The verification result from the position of 3C273 at its mean epoch of observation is also shown as a band corresponding to its $\pm 1\sigma$ uncertainty.



Figure 18.6. Summary of the results for the spin vector $\boldsymbol{\omega}$, expressed in the galactic x and y coordinates ($\omega_1 = \mathbf{x}'_G \boldsymbol{\omega}$ and $\omega_2 = \mathbf{y}'_G \boldsymbol{\omega}$). The individual results from Table 18.4 are shown as error circles with a radius of one standard deviation. In cases where only two equatorial components of $\boldsymbol{\omega}$ were determined, the corresponding uncertainty bands (of width $\pm 1\sigma$) are shown. The verification result from galactic kinematics, Equation 18.26, is shown by the error circle labelled 'G'.

Conclusions

The procedure for determining the Hipparcos Reference Frame strictly followed the IAU intentions for the new conventional celestial reference system, namely that it should be non-rotating with respect to distant matter and that the fundamental directions are set by the precise coordinates of extragalactic radio sources. As a matter of principle, the procedure was not allowed to be influenced by considering the relationship to the dynamical reference frame of the solar system or to the kinematical frame defined by motions in our Galaxy.

Such considerations could nevertheless be applied *a posteriori* as a check of the Hipparcos Reference Frame. For instance, observations of solar system objects in the Hipparcos frame, together with a dynamical theory of the planetary motions, will determine the direction of the total angular momentum of the solar system, which is expected to remain fixed in the extragalactic frame to very high accuracy. This check must however await a detailed (re-)analysis of the solar system observations, both from Hipparcos and from the ground.

The various checks described in this section are all consistent with the stated accuracy of the extragalactic link, namely 0.6 mas in the orientation and 0.25 mas/yr in the spin, although no significant test of the orientation was obtained by these methods. The strongest test of the spin is provided by the galactic kinematics, supporting the conclusion that the Hipparcos frame is inertial to within a few tenths of a milliarcsec per year.

18.9. Organisation of the Work

The importance of linking the Hipparcos Catalogue to the extragalactic system was stressed already in the planning of the observing programme. Extensive preparations were made by the INCA Consortium to initiate and collect relevant ground-based observations of radio stars and stars in the fields of compact extragalactic radio sources, and to ensure that suitable link stars were included on the observing list (Argue 1989, 1991; Jahreiß *et al.* 1992). A special working group for the determination of the extragalactic link was appointed by the Hipparcos Science Team in 1993. It contained representatives of all the groups participating in the link observations and was coordinated by J. Kovalevsky and L. Lindegren, who were also responsible for the synthesis of the different link determinations.

The members of the various groups contributing to the determination of the link are listed hereafter.

VLBI: J.F. Lestrade led this group in close collaboration with R.A. Preston, D.L. Jones (JPL) and R.B. Phillips (Haystack) for the northern stars, and J. Reynolds, D. Jauncey (CSIRO) and J.C. Guirado (JPL) for the southern hemisphere.

MERLIN: This group comprised S.T. Garrington and R.J. Davis (NRAL, Jodrell Bank), L.V. Morrison and R.W. Argyle (RGO), and A.N. Argue (IoA, Cambridge).

VLA: This task was organised by K.J. Johnston (USNO); the computation of the link was done by D.R. Florkowski (USNO).

Hamburg/USNO: This link was realised by C. de Vegt (Hamburg) and N. Zacharias (USNO).

HST/FGS: Based on observations with the NASA/ESA Hubble Space Telescope, this work was carried out by many people, but the data collection and analysis was the result of continued efforts of P.D. Hemenway, E.P. Bozyan, R.L. Duncombe, A. Lalich, B. MacArthur, E. Nelan and the Hubble Space Telescope Team.

Lick (NPM): The analysis of the NPM1 data for the Hipparcos link was made at Yale Observatory by I. Platais, T.M. Girard, and V. Kozhurina-Platais, and at the Astronomisches Rechen-Institut, Heidelberg, by S. Röser.

Catalogue of Faint Stars (KSZ): This link was realised at Kiev Observatory by N.V. Kharchenko, V.S. Kislyuk, S.P. Rybka and A.I. Yatsenko.

Yale/San Juan (SPM): This work was shared by I. Platais, T.M. Girard, V. Kozhurina-Platais; H.T. MacGillivray and D.J. Yentis furnished positions and magnitudes from the COSMOS/UKST data base of the southern sky and W.F. van Altena provided many useful suggestions.

Bonn: This work was shared by H.-J. Tucholke, P. Brosche, M. Geffert, M. Hiesgen (Münster), A. Klemola (Lick), M. Odenkirchen and J. Schmoll.

Potsdam: This link was realised by E. Schilbach, S. Hirte and R.-D. Scholz.

Earth Orientation Parameters: This task was performed by J. Vondrák, I. Pešek and C. Ron.

J. Kovalevsky, L. Lindegren, M.A.C. Perryman

19. COMPARISONS WITH GROUND-BASED ASTROMETRY

The Hipparcos Catalogue constitutes the best materialization of the optical reference frame with a precision of the order of 1 mas in position and 1 mas/yr in proper motion. It will supersede several astrometric catalogues currently in use such as the FK5 and the PPM. In this chapter an analysis of the FK5 and PPM positions and proper motions against Hipparcos allows the global rotations existing between the Hipparcos(ICRS) and the FK5 and PPM systems to be derived. Beyond the global rotation, systematic and regional effects as large as 100 mas, functions of the right ascension and declination, are found and described. Analyses of new reductions of astrolabe and meridian observations using Hipparcos astrometry are included as well as an assessment of the astrometric measurements carried out with the Mark III interferometer.

19.1. Introduction

The Hipparcos Catalogue is referred to the International Celestial Reference System (ICRS) and constitutes the optical counterpart of the inertial system materialised by a set of radio sources observed by the VLBI technique. Its internal precision is typically below 1 mas in right ascension and declination for all the bright stars and about 1 mas/yr for the components of the proper motion. In addition it is expected to be free of regional errors at the level of 0.1–0.2 mas, a level much lower than any existing global catalogue. This provides the opportunity of using Hipparcos astrometry as a virtually error-free reference to determine the true errors of other catalogues at the Hipparcos epoch and to devise rules to correct for their systematic errors.

Up until now, the FK5 provided the basic stellar reference frame as adopted by the IAU in 1976. It was considered to be the best realisation of a stationary system through the accurate coordinates and proper motion of 1535 bright stars (Fricke *et al.* 1988). Although the ICRS is nominally consistent with the FK5 for the mean equinox and equator of the standard epoch J2000, there is still a global rotation between the two reference systems due to the uncertainty of the stellar positions in the FK5.

Beyond this rotation, regional systematic differences between the FK5 stars (and hence PPM) and the Hipparcos positions do exist up to 100 mas. In this chapter these systematic differences are evaluated and characterized as a function of the right ascension and declination. Due to the global nature of the construction of the Hipparcos Catalogue and its intrinsic accuracy (on a global scale better than 0.2 mas) there is no doubt that

these regional differences must be attributed to the FK5 (and hence PPM) and not to Hipparcos. Eventually the knowledge of these differences might allow old astrometric observations tied to the FK5 frame to be reprocessed.

The PPM Catalogue constitutes the most up to date (to be superseded by the availability of the Tycho Catalogue) compilation Catalogue for the position and proper motion of reference stars. Although it was nominally constructed in the same frame as the FK5, it is desirable to determine its main systematic differences with respect to Hipparcos independently and to assess the external error in position at the epoch J1991.25 and, more important, the true quality of its proper motions.

There is an important difference between the comparison of the FK5 and the PPM to the Hipparcos results. In the former situation, all the FK5 stars are in the Hipparcos solution, so there is no risk of bias due to a truncated distribution of the Catalogue. In the case of the PPM, there is only one quarter of the PPM stars (see Section 19.3) used in the comparison (because the others are not in the Hipparcos Catalogue). Therefore the sky distribution of this sample reflects only the selection made for the Hipparcos programme and deviates from the PPM as a whole.

19.2. Comparison with the FK5 Catalogue

The FK5 Stars

The compilation of the FK5 represented a major effort by the Astronomisches Rechen-Institut to provide a realization of a dynamical frame from the analysis of more than 300 individual catalogues primarily observed with meridian circles. It represents a revision of the FK4 and results from the determination of systematic and individual corrections to the mean positions and proper motions of the FK4, the elimination of the error in the FK4 equinox, and the introduction of the IAU (1976) system of astronomical constants. Its content of 1535 bright stars (the FK5 extension to 3522 stars is not considered here because of its lower accuracy) has an expected accuracy of 0.03 arcsec at the mean epoch of the catalogue: 1955 in right ascension and 1944 in declination. The mean error quoted for the proper motion is 0.6 mas/yr for the northern hemisphere and 1.0 mas/yr for the southern. By propagating the FK5 positions directly to the Hipparcos epoch J1991.25, this leads to an expected error in the right ascension and declination of 40 to 60 mas according to the hemisphere. About 95 per cent of the stars of the FK5 have a Hipparcos magnitude in the range 2 to 7 mag, i.e. brighter than the average Hipparcos star, thus their Hipparcos accuracies are better than average.

Method of Analysis

All the 1535 stars of the FK5 have been observed successfully by Hipparcos and their positions are known at epoch J1991.25 with an accuracy typically 0.4 ± 0.1 mas in declination and 0.6 ± 0.2 mas in right ascension (Figure 19.1), the larger scatter in the latter case being the result of the strong dependence of the Hipparcos accuracy in right ascension with the declination. The corresponding figures for the proper motions are 0.7 ± 0.2 mas/yr and 0.55 ± 0.15 mas/yr with the same kind of dependence on the coordinates as the positions.



Figure 19.1. Distribution in the Hipparcos Catalogue of the formal errors in right ascension and declination of the 1535 FK5 stars.



Figure 19.2. Formal error of proper motions of the 1535 FK5 stars in the Hipparcos Catalogue.

Data Filtering

A number of the FK5 stars have been found either to be double (97 cases) or to present a non-uniform motion (95 cases), indicating that some may actually be astrometric binaries. As a consequence the Hipparcos proper motion constructed on a timebase shorter than the orbital period might be biased. For another 78 entries the Hipparcos solution has been constructed by adding to the standard astrometric parameters one or several orbital elements as supplementary unknowns. Therefore the astrometry refers in this case to the centre of mass, which may differ from the photocentre used in the FK5; these stars were not considered reliable enough for the comparison. Finally there were 22 solutions with residuals significantly larger than the measurement error and another 10 with an apparent motion of the photocentre ascribed to the variability of one of the components of a binary star. All these stars have been excluded from the analysis, resulting in 1233 reliable solutions remaining. Most of the 302 discarded stars in fact have a good Hipparcos solution, but because of their multiplicity they may exhibit systematic differences with the FK5 positions arising from a physical origin while for the remaining 1233 single stars the differences could be accounted for as zonal errors.

Global rotation

The Hipparcos Catalogue was referred to the ICRS after the final astrometric solution had been rotated as explained in Chapter 18. Nominally the ICRS was to maintain the continuity with the previous dynamical reference system realized by the FK5 Catalogue. However due to its limited accuracy the alignment of the ICRS pole and origin of right

Orientation (mas)	Spin (mas/yr)		
$\epsilon_{\textit{X}} = -18.8 \pm 2.3$	$\omega_X = -0.10 \pm 0.10$		
$\epsilon_y = -12.3 \pm 2.3$	$\omega_y = +0.43 \pm 0.10$		
$\epsilon_{Z} = +16.8 \pm 2.3$	$\omega_Z=+0.88\pm0.10$		

Table 19.1. Global orientation and spin differences between the Hipparcos and FK5 Catalogues.

ascension with the corresponding pole and origin of right ascension of the FK5 could not be achieved with consistency better than 20 mas for the pole and 80 mas for the origin of the right ascension. The final Hipparcos solution, ICRS(Hipparcos) and the optical reference frame defined by the FK5, J2000(FK5), differ by a pure rotation and numerous zonal differences of various wavelengths.

Both the rotation and zonal effects can be analysed globally by means of the decomposition of the vectors fields $[(\alpha_F - \alpha_H) \cos \delta, \delta_F - \delta_H]$ and $[(\mu_{\alpha*})_F - (\mu_{\alpha*})_H, (\mu_{\delta})_F - (\mu_{\delta})_H]$ on a set of orthogonal vectorial harmonics. The first degree of these harmonics represents the pure rotation while the harmonics of higher degree account for the zonal differences at decreasing wavelengths with increasing degree. The global rotation and spin differences, together with their uncertainty, are given in Table 19.1. These values were used to rotate the positions and proper motions of the 1233 selected stars to the ICRS at epoch J1991.25. The remaining differences are analysed below.

Epoch transformation

The stellar positions and proper motions in the FK5 are given for the epoch J2000 in the FK5 system, while the Hipparcos Catalogue being an observation catalogue is referred to an epoch close to the average observation time, namely $T_0 = J1991.25$ (TT). All the FK5 positions have been propagated from J2000 to the epoch T_0 using the FK5 proper motions in a straightforward manner. No attempt has been made to estimate the errors in the FK5 coordinates at the epoch T_0 for each star, using in the following discussion an overall and global estimate of the errors. In the following, these positions (rotated and propagated to T_0) are denoted by α_F , δ_F while the Hipparcos positions are labelled α_H , δ_H .

Results of the Comparison

For each of the 1233 comparison stars the positional differences:

$$\Delta \alpha \cos \delta = (\alpha_{\rm F} - \alpha_{\rm H}) \cos \delta_{\rm H}$$
[19.1]

$$\Delta \delta = (\delta_{\rm F} - \delta_{\rm H})$$
[19.2]

have been determined and analysed from a statistical point of view. A similar approach was taken for the proper motions. Results are shown in a series of diagrams as a function of the right ascension and declination: in Figures 19.3–19.4 for the positions, and Figures 19.5–19.6 for the components of the proper motion. A fit has been made through the data points using a robust fitting technique with a moving window of 100


Figure 19.3. Difference in right ascension between the FK5 and the Hipparcos Catalogue at epoch J1991.25. The solid line results from a robust smoothing of the data. Differences are in the sense FK5 – Hipparcos.



Figure 19.4. Difference in declination between the FK5 and the Hipparcos Catalogue at epoch J1991.25. The solid line results from a robust smoothing of the data. Differences are in the sense FK5 – Hipparcos.

data points. If the Hipparcos formal errors are correct, virtually all the scatter in the plots must originate from the FK5 positions.

There are several notable features in the plots of the positional differences between Hipparcos and the FK5:

- 1. the ICRS and FK5 equators are about 60 mas apart, leading to a systematic effect in declination between the two catalogues of the same magnitude. This effect is clearly seen in Figure 19.4 (in the left plot) with the average of $\Delta \delta \simeq -60$ mas;
- 2. both coordinates show significant regional differences as large as ± 100 mas, an amplitude which is definitely larger than the expected accuracy of the FK5 at the Hipparcos epoch. Recent observations with meridian instruments have confirmed this effect and support the claim that these are local distortions in the FK5 rather than regional errors of Hipparcos;
- 3 both the north and south polar regions exhibit larger discrepancies and scatters than the regions at intermediate declinations;
- 4. the scatter in each of these diagrams is a good and robust measure of the FK5 external error at epoch J1991.25. From this analysis:

$$\sigma_{\alpha*} \sim \sigma_{\delta} \sim 80$$
 to 100 mas

and:

$$\sigma_{\mu_{\alpha*}} \sim \sigma_{\mu_{\delta}} \sim 2.0$$
 to 2.5 mas/yr

for the global inaccuracy, combining the random component (about 55 mas in both coordinates), and the contribution of the regional errors (which amounts to about 60 mas). The random error in declination is larger in the southern hemisphere (~ 70 mas) than in the northern (~ 50 mas). For the proper motion the random component is 1.7 mas/yr and the contribution of the zonal distortion to the standard deviation is 1.5 mas/yr, with no clear distinction with the sign of the declination.

These figures are markedly larger than the expected error at epoch J1991.25 and than the quoted uncertainty for the proper motion, even if only the random components are considered. One might have expected that locally, over a small field, the proper motion components would have been consistent below 1 mas/yr, which is definitely not the case. However the size of the fields used in this analysis are not very small (200 square degrees) as a result of the small number of stars, and the distortion on a very small scale cannot be separated from the truly random errors.

Using this uncertainty of 2 mas/yr for the proper motion the propagation from the mean observation epoch to J1991.25 yields precisely the observed uncertainty in the position as deduced from the comparison with the Hipparcos position. This consistency indicates that the standard errors found for the position and proper motion are broadly correct and that the external accuracy, including zonal errors, of the FK5 is not as good as has been believed. This discrepancy has already been pointed out by Morrison *et al.* (1990) from their meridian observations;

5. the regional errors in proper motion behave in a similar manner to the positional errors (as a function of declination). For example, the overall shapes of the curves representing $\Delta \alpha \cos \delta$ and $\Delta \mu_{\alpha} \cos \delta$ as a function of the declination are rather similar. The same is true for the declination and the corresponding proper motion. Since the FK5 positions are propagated from the mean observation epoch of the FK5 to J1991.25, i.e. over about 50 years, a local error of 2 to 3 mas/yr in proper motion gives rise to a local distortion in the position of about 100 to 150 mas at the same latitude. Thus, the wavy pattern in the positional differences with the declination might be simply a consequence of the zonal error in proper motion. No similar correlation can be drawn from the analysis as a function of right ascension.

Further Investigations

The complete characterisation of the regional distortions of the FK5 needs to be investigated more deeply, in particular in order to classify the various errors according to their characteristic scale over the sphere. As noted previously a decomposition on the vectorial harmonics on the sphere has been used to determine the global rotation and spin and the associated uncertainties. In addition, it appears clearly that most of the power spectrum lies in the harmonics of degree 1 and 2, both in their spheroidal and toroidal form (Mignard & Morando 1989). A complete investigation will allow easy-to-use formulae that will be sufficient to describe the main zonal errors of the FK5 frame to be derived.



Figure 19.5. Difference in proper motion in right ascension between the FK5 and the Hipparcos Catalogue at epoch *J1991.25.* The solid line results from a robust smoothing of the data. Differences are in the sense FK5 – Hipparcos.



Figure 19.6. Difference in proper motion in declination between the FK5 and the Hipparcos Catalogue at epoch J1991.25. The solid line results from a robust smoothing of the data. Differences are in the sense FK5 – Hipparcos.

19.3. Comparison with the PPM Catalogue

The basic catalogue for position and proper motion was until the early 1990's the SAO Catalogue, published in 1966. The SAO was intended for satellite-tracking purposes and contains a compilation of positions and proper motions for 258 997 stars reduced to a common system, nominally the B1950(FK4) coordinate system. At epoch 1990 the typical errors in position and proper motion were respectively 1 arcsec and 1.5 arcsec/century. The PPM Catalogue (Röser & Bastian 1991; Bastian & Röser 1993) was designed to provide a more accurate net of reference stars on the J2000(FK5) system based on multiple positional epochs rather than the usual two in the SAO.

The PPM Stars

The PPM north gives the J2000 positions and proper motions for 181 731 stars north of a declination $-2^{\circ}.5$ and brighter than 10.5 mag, although a small sample of fainter stars is included. The published mean error of positions at epoch J1990 and proper motions are respectively 0.27 arcsec and 0.43 arcsec/century. The PPM south covers

the rest of the celestial sphere and comprises 197 179 stars with an astrometric precision of 0.11 arcsec for the positions at J1990 and 0.30 arcsec/century for the proper motions up to a magnitude of 10.5 mag, with few fainter stars. Both catalogues are constructed to represent as closely as possible the reference frame defined by the FK5.

There are 108 046 Hipparcos stars in the two PPM Catalogues, 54 801 in PPM north and 53 245 in PPM south respectively. Most of the faint stars of the Hipparcos programme, Hp > 10.5 mag are missing in the PPM and so are not included in this analysis. Unlike the FK5, the comparison sample is now, but for the faintest stars, the same as the whole Hipparcos Catalogue but only 25 per cent of the PPM content. Thus the error distribution (on the Hipparcos side) as a function of the magnitude and of the position on the sky is the same as the Hipparcos distribution, of the order of 1 mas and 1 mas/yr for the position and proper motion, virtually error-free compared to the PPM accuracy.

The Global Rotation

As noted previously, the PPM is nominally in the FK5 system, which implies that the global rotation between the Hipparcos(ICRS) and the PPM should be given by the rotational parameters of Table 19.1. An analysis of the systematic differences:

$$\Delta \alpha \cos \delta = (\alpha_{\rm P} - \alpha_{\rm H}) \cos \delta_{\rm H}$$
 [19.3]

$$\Delta \delta = (\delta_{\rm P} - \delta_{\rm H})$$
[19.4]

by decomposition of a set of orthogonal vectorial harmonics gives the global rotation as the component of first degree in this representation. A similar decomposition for the differences in proper motions yields the spin components. Results are shown in Table 19.2 and are relatively consistent with the rotation of the FK5, at least for the components ϵ_x and ϵ_z , and slightly outside the probable error for ϵ_y . This confirms that, as far as positions are concerned, the PPM Catalogue is globally aligned with the FK5 system, which was not obvious to achieve owing to the very small density of FK5 stars. For the rotation rate, the differences are more significant, i.e. above the 3σ level.

Despite the much larger number of stars in the PPM comparison compared to the FK5, the global rotation is defined with exactly the same accuracy, indicating again that there is probably no single global rotation valid for all categories of stars. The technique of harmonic analysis is naturally global and does not permit a rotation to be defined separately for the north and the south. Therefore, both the values and the formal standard errors of the global rotation and spin parameters given in Table 19.2, should not be taken too literally. Since the zonal deviations reach a few tenths of an arcsec, the concept of a 'PPM system' is loosely defined at the level of a few mas. If, instead of the Hipparcos Catalogue, the Tycho Catalogue had been used for the comparison (the latter being strictly on the Hipparcos system), a different result might have been obtained due to the very different distribution of the stars on the sky. The comparison sample would have been the entire PPM with its clear-cut concentration towards the galactic plane.

The number of stars allows the global rotation to be studied according to different star groupings. As an illustration, an analysis of the rotation per interval of magnitude is shown in Figure 19.7. There is a clear trend in ϵ_z indicating a variable position of the origin in right ascension according to the star brightness. There is also a small trend in ϵ_x but this is less prominent. The standard error for each data point is of the order of

Table 19.2. Global orientation and spin differences between the Hipparcos and PPM Catalogues.

Orientation (mas)	Spin (mas/yr)
$\epsilon_{X} = -22.5 \pm 2.4$	$\omega_X = -0.66 \pm 0.07$
$\epsilon_y = -7.0 \pm 2.4$	$\omega_y = +0.84 \pm 0.07$
$\epsilon_{Z} = +16.8 \pm 2.4$	$\omega_Z=+0.14\pm0.07$



Figure 19.7. Global rotation of the PPM with respect to Hipparcos for each class of magnitude.

4 mas for the most populated classes in the magnitude range 7.5 to 9.5 mag, well below the level of variation of the rotation components.

Regional Differences

The differences between PPM and Hipparcos have been determined at J1991.25 by computing PPM positions at this epoch with the PPM proper motions. The global rotation and spin have been removed from the differences in Equations 19.3–19.4 and the remaining residuals have been analysed as a function of the right ascension and declination in the same way as for the FK5. Results are shown in Figures 19.8–19.10 for the positions and in Figures 19.11–19.13 for the proper motions.

The following features are noted:

1. not surprisingly the overall patterns are the same as for the differences between the FK5 and Hipparcos, in particular in the combination of the north and south catalogue. There are however noticeable differences between the two PPM Catalogues. For example, in Figure 19.8 there is a shift of about 50 mas between the PPM north and south in the systematic difference $\Delta \alpha \cos \delta$ between the PPM and Hipparcos. Otherwise the curves are rather alike. In Figure 19.9, one sees again the effect observed with the FK5 regarding the position of the FK5 equator compared to the ICRS. The departure of the order of -50 mas in the north and -80 mas for



Figure 19.8. Median difference in right ascension between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the right ascension. PPM north on the left and PPM south on the right.



Figure 19.9. Median difference in declination between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the right ascension. PPM north on the left and PPM south on the right.



Figure 19.10. Median difference in right ascension and declination between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the declination. Difference in right ascension on the left and in declination on the right.



Figure 19.11. Median difference in proper motion in right ascension between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the right ascension. PPM north on the left and PPM south on the right.



Figure 19.12. Median difference in proper motion in declination between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the right ascension. PPM north on the left and PPM south on the right.



Figure 19.13. Median difference in proper motion in right ascension and declination between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the declination. Difference in right ascension on the left and in declination on the right.

	Positior	ns (mas)	Proper Motions (mas/yr)		
	North	South	North	South	
Random	215	125	4.5	3.7	
Regional	85	105	2.8	2.6	
Global	230	165	5.3	4.5	

Table 19.3. External accuracy (1σ) of the PPM for the random, regional and global components. The figures apply to both right ascension ($\sigma_{\alpha} \cos \delta$) and declination (σ_{δ}), and similarly for the proper motion components.

the south. The difference as a function of the declination is comparable to that observed in the difference FK5–Hipparcos;

- 2. in the analysis as a function of declination, the error bars in positions are significantly smaller in the south than in the north (this is not obvious with the resolution of the plots, but these are typically 2.5 mas for the south and 3.5 mas for the north for the error of the median for each class of about 5000 stars). The effect is less pronounced, although visible, for the proper motion (respectively 0.07 mas/yr and 0.08 mas/yr). This is in agreement with the *a priori* better quality of the PPM south compared to the north;
- 3. on a more quantitative basis the random scatter about the median can be used to assess the true precision of the PPM Catalogues. The standard error on the median multiplied by the square root of the population of the class provides a robust estimate of the scatter in the core of the distribution, excluding automatically the outliers. From the analysis of both the right ascension and declination as a function of the right ascension one gets 235 mas for the north and 145 mas for the south. For the analysis as a function of declination the figures are respectively 220 mas and 155 mas. For the north this is slightly better than the quoted precision of 270 mas, but for the south the above figures are somewhat larger than the 110 mas given in the presentation of the PPM. The results of an attempt to discriminate between random and regional errors are given in Table 19.3 for the difference in position and proper motion between the PPM and Hipparcos;
- 4. for the proper motion, the figures lead to an external accuracy of 5 mas/yr and 4.5 mas/yr for the north and south instead of 4.0 and 3.0 mas/yr for the standard error given in the presentation of the PPM. These numbers are about ten times larger than the spin rate corrections and the uncertainty on this rotation does not affect the above conclusion. The propagation over 50 years with these figures yields too large an error in the position at epoch J1991.25 in comparison with the departure of the PPM positions with respect to Hipparcos at this epoch.

At first sight, this discrepancy appears somewhat surprising, since various uses of the PPM have indicated that the precision of the relative proper motions over small fields (such as star clusters, the Magellanic Clouds and the Cygnus superbubble region) agrees much more closely with the stated errors. Thus the discrepancy must be due to large regional errors on the scale of 5° to 10° , which could not be taken into account in the present investigation. That regional errors of the required size could indeed be present is indicated by detailed comparison of the PPM system of positions with a preliminary Hipparcos Catalogue by Lindegren *et al.* 1995 (more specifically Figure 7 of that paper).

A study of the dispersion of the differences of the proper motions PPM–Hipparcos in cells of about 10 square degrees, yields a dispersion of 4.5 mas/yr for the north and 3.7 mas/yr for the south. This scatter is a good indicator of the level of the random errors, provided that there is no large systematic effect of typical size less than a few degrees. On the other hand the inter-cell scatter represents the contribution of the systematic differences over wavelengths larger than few degrees, and amounts to 2.7 mas/yr, which eventually leads to the 4.5 to 5.3 mas/yr for the external accuracy of the PPM;

- 5. the medians of the systematic differences in position at epoch J1991.25 and proper motion are very significant, often larger than five times the standard errors. As in the case of the FK5, the most conspicuous feature is the systematic shift between the ICRS and FK5 equator of about 50 mas to 80 mas. There are large zonal errors (~ 100 mas) in both coordinates;
- 6. the relationship between the differences in proper motions and in positions is more striking than in the case of the FK5. A comparison between Figures 19.8–19.10 and Figures 19.11–19.13 is instructive in this respect. The corresponding plots in each set are very similar in shape with roughly a scale factor between them, linked to the time span between the mean epoch of the PPM (1931) and J1991.25. Unlike the FK5 this remark applies fully to both the analysis as a function of right ascension and declination. Thus the zonal errors seen in position very likely are the consequence of the unsatisfactory knowledge of the proper motions, a situation which will not be improved until the re-reduction of the Astrographic Catalogue in the Hipparcos system and its combination with the Tycho data.

19.4. Comparison with the Mark III Interferometer Results

The Mark III interferometer was set up by Shao *et al.* (1990) at Mount Wilson Observatory in the late 1970s, and became operational for astrometric observations in September 1986. Observations were discontinued in 1993 with the development of a new instrument located at Mount Palomar Observatory. It was designed for observations in amplitude and phase mode, the latter mode allowing global astrometry to be conducted by careful laser monitoring of the delay lines. The astrometric measurement carried out in 1988 over 12 FK5 stars yielded an average 1 σ error for fifty observations of the order of 10 mas in right ascension and 6 mas in declination (Shao *et al.* 1990). However the conclusion of this first run was that an extended series of measurements was needed to ascertain the true accuracy that the instrument could achieve in absolute astrometry.

Repeated measurements over four years have been obtained by Hummel *et al.* (1994) for 11 stars at four different epochs between 1988.6 and 1992.7. Measurements at two different wavelengths were used to correct for the refractive index fluctuations in the atmosphere. Although the number of stars was too small to allow an absolute determination of the declination, the analysis of the offsets with respect to FK5 led to an accuracy of 13 mas in declination and 23 mas in right ascension. Using Hipparcos data, which for these bright stars are better than 1 mas and 1 mas/yr in position and proper motions, one can see whether these estimates are realistic or not, and assess the real potential of the interferometers for the measurements of stellar positions.

HIP	FK5	Name	Нр	α (deg)	δ (deg)
3031	19	ϵ And	4.3	9.63	+29.31
4436	33	μ And	3.9	14.18	+38.49
7607	52	-	3.8	24.49	+48.62
8903	66	β Ari	2.7	28.65	+20.80
10670	79	γ Tri	4.0	34.32	+33.84
101076	1534	-	4.1	307.34	+30.37
106481	1568	ρ Cyg	4.1	323.49	+45.59
109410	835	π^2 Peg	4.4	332.49	+33.17
111169	848	α Lac	3.8	337.82	+50.28
112748	862	μ Peg	3.7	342.50	+24.60
116805	1619	κ And	4.1	355.10	+44.33

Table 19.4. List of stars with astrometric solution obtained with the Mark III optical interferometer.

Because of a difference in the reference system, which is difficult to resolve with a small unevenly distributed sample of stars, the comparison is done directly on the arc-lengths between stars of the Mark III programme and the corresponding arcs computed from the Hipparcos astrometric solution. This method has the advantage of being insensitive to a global rotation but also has the drawback of the lack of independence of the set of arc-lengths, which precludes a rigorous statistical analysis. In addition it is impossible to scale separately the accuracy in declination and right ascension.

The list of the stars with an interferometric astrometric solution is given in Table 19.4, which contains also other relevant parameters. Among the 11 stars, HIP 8903 = FK5 66= β Arietis was solved by Hipparcos with a significant orbital motion, and was eventually excluded from the comparison. Each of the ten remaining stars was observed at four different epochs by Mark III. For each of these observations the Hipparcos positions were computed using the Hipparcos astrometric parameters. The number of arcs was then $4 \times 10 \times 9/2 = 180$, for only $4 \times 10 \times 2 = 80$ independent measurements.

The distribution of the differences is shown in Figure 19.14 for the 180 arcs. They are ordered as follows: the arcs of HIP 3031 with HIP 4436 for the four epochs, then the four arcs of HIP 3031 with HIP 7607, and so on until the set of arcs is exhausted with the four arcs of HIP 112748 with HIP 116805.

The median of the differences is -1.2 mas, and the scatter measured by the standard deviation 32 mas. The latter is in agreement with the results on nine stars by Lindegren *et al.* (1995) in a comparison made against Hipparcos preliminary results using, for each star, a normal positions derived from the four individual observations. The median is not significantly different from zero over this small sample. The significance of the dispersion must be evaluated with reference to the uncertainty of the positions of each pair of stars, given that the Hipparcos error can be neglected.



Figure 19.14. Distribution of the differences of the arc-lengths between the Mark III observations and the corresponding arcs from the Hipparcos Catalogue.

The propagation of the errors on the positions at each epoch to the arc-lengths was estimated by a Monte-Carlo procedure, using the mean coordinates of the ten stars to determine the reference lengths of the 45 basic arcs and then adding a Gaussian noise component on each coordinate with a standard error of 13 mas in declination and 23 mas in right ascension (σ_{α} and not $\sigma_{\alpha} \cos \delta$). From 500 such experiments it appears that the distribution of the standard error (1 σ) on the arc-lengths was between 19 mas and 30 mas with a well defined average of 24 mas, somewhat smaller than the quadratic propagation of the error on the positions of the two end-points of an arc. This can be understood as follows : for two stars more or less on the same celestial meridian, the arc-length is insensitive to an error in right ascension which displaces the two stars in a direction perpendicular to the arc. A similar reasoning holds for the error in declination for two stars with similar declinations.

Thus the observed scatter of 32 mas in the arc-lengths is significantly larger than the expected value (i.e. $\simeq 24$ mas) determined with the reported accuracy of $\sigma_{\alpha} = 23$ mas and $\sigma_{\delta} = 13$ mas, indicating that the astrometric accuracy is overestimated by about 30 per cent. One cannot conclude with certainty as to whether the right ascension or the declination should be incriminated; probably both.

19.5. Astrometric Reductions of Schmidt Plates

A preliminary study was made to test the ultimate accuracy attainable in reducing plates, using intermediate Hipparcos and Tycho 30-month solutions (Robichon *et al.* 1995). To minimise the effect of the errors on proper motions, especially for Tycho data, plates with epochs close to the Hipparcos mid-mission epoch were chosen for this test. This study has not yet been repeated using final Hipparcos and Tycho Catalogues but its results are nonetheless meaningful since the difference between the 30-month and the final solutions are much smaller than the typical error in the plate reduction.

The Observational Material: Hipparcos and Tycho 30-month Solution

The intermediate Hipparcos astrometric catalogue constructed from the first 30 months of data obtained from the satellite is described in Kovalevsky *et al.* (1995), Lindegren *et al.* (1995) and in Chapter 16 of this volume. The intermediate Tycho Catalogue version used below is T30d, the first iteration containing both proper motions and parallaxes. The properties of this version are described in Høg *et al.* (1995), Høg (1995), and in Chapters 11 and 16 of Volume 4.

The Observational Material: Schmidt Plates

The plates used in this study were taken with the CERGA and ESO Schmidt telescopes, as part of a programme on eight open clusters observed by Hipparcos. Plates were obtained in U, B, V, R, I, with a long (1–2 hours) and a short (15–30 min) exposure for each colour.

Six short-exposure plates have been used to test the ultimate accuracy attainable on positional determination from Schmidt plate measurements, and two long-exposure for comparison. Astrometric reductions were made using successively PPM data (PPM North: Röser & Bastian 1993; PPM South: Bastian & Röser 1993), and preliminary Hipparcos and Tycho data. Each field, of about 25 square degrees, contains approximately 350 PPM stars (ranging from 250 to 400), 85 Hipparcos stars (from 70 to 200), and 1000 Tycho stars (from 550 to 3000).

Mean errors of PPM positions and proper motions are of the order of 300 mas and 4 mas/yr respectively in the north (i.e. fields 1, 2, 3, 8) and 150 mas and 3 mas/yr respectively in the south (i.e. fields 4 to 7), to be compared with 1.5 mas and 1.5 mas/yr for preliminary Hipparcos results, and with 30 mas and 30 mas/yr for preliminary Tycho results. The plates were scanned with the Machine Automatique à Mesurer pour l'Astronomie (MAMA) of the Centre d'Analyse des Images in Paris.

The Results

The results obtained show that the ultimate accuracy expected from a single Schmidt plate is better than 0.10 arcsec (0.06–0.07 arcsec) for stars brighter than 11 mag, to be compared with 0.15–0.37 arcsec obtained using the PPM Catalogue. The model is more reliable when the reduction is carried out using the numerous, though less accurate data from Tycho than the more accurate, but less numerous data from Hipparcos. A decrease of the errors on the positions of the reference stars below 0.01 arcsec does not seem to improve the results. The modelling errors are of the order of 0.05 arcsec. An rms error of about 0.04 arcsec for the centring of the images is consistent with our results. This study emphasizes the importance of obtaining good proper motions for Tycho Catalogue stars, using Tycho data in conjunction with first epoch astrometric data such as the Astrographic Catalogue, in order to take advantage of these accurate data for plates taken at epochs very different from the Hipparcos mid-epoch.



Figure 19.15. Residual in the astrolabe catalogue after five years of observations for the right ascension of 68 FK5 stars. On the left, all the reduction has been done with the FK5 Catalogue coordinates and on the right with Hipparcos.



Figure 19.16. Residual in the astrolabe catalogue after five years of observations for the declination of 48 FK5 stars. On the left, all the reduction has been done with the FK5 Catalogue coordinates and on the right with Hipparcos.

19.6. Analysis of Recent Meridian Circle Observations

The impact of the Hipparcos and Tycho Catalogues on the meridian circle observations has been known for years. A critical examination of the residuals obtained with respect to the FK5 has been made by Réquième *et al.* (1993) with the very first positions based on the Hipparcos observations, without improvement of the proper motions. Subsequently an analysis of recent meridian circle observations, using successively the FK5 and Hipparcos as reference has been done by Réquième *et al.* (1995) with the Hipparcos 18-month solution. A remarkable improvement in the post-fit residuals of the least-squares adjustment was obtained for observations made by the Bordeaux and La Palma automatic circles. The difference between the two fits confirmed the existence of systematic errors in the FK5 and revealed small instrumental errors of 30 mas in right ascension and 50 mas in declination that were hidden by the larger noise brought about by the FK5 regional errors.

19.7. Analysis of Recent Astrolabe Observations

Astrolabe observations have been performed at OCA/CERGA on a regular basis for the last twenty years. During the last decade they were done on an impersonal photoelectric and highly automated instrument. In addition to determining the daily orientation of the local vertical, the yearly analysis of the residuals permits corrections to the star catalogue to be derived.

The processing of the same observations performed over five years centred on the Hipparcos epoch, from J1988.5 to J1993.5, has been done by using successively the FK5 coordinates and proper motions and the Hipparcos data for the same stars. The results expressed as corrections to apply to the star positions given in the astrolabe catalogue are shown in Figure 19.15 and Figure 19.16 respectively for the right ascension and declination.

The comparison between the left and right plots in each figure confirms the real improvement of the Hipparcos reference frame compared to FK5 and gives for the first time the true level of accuracy of the observations carried out with a photoelectric astrolabe. The remaining scatter in the right plots is a combination of the instrument limitation and the photon noise and should not be interpreted as errors in the Hipparcos positions.

Prior to this comparison it was difficult to decide from the residuals on the FK5 stars on the respective contribution of the instrument and that due to the FK5 uncertainties. The difference between the internal error obtained during the processing, of the order of 12 mas in right ascension and 14 mas in declination (Vigouroux *et al.* 1995), and the true external error which is in fact closer respectively to 20 mas and 28 mas is evident.

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20. VERIFICATION OF PARALLAXES

Hipparcos parallaxes will play a major role in the astrophysical applications of the Hipparcos results and in this respect their accuracy is more important than their precision, at least for investigations of a statistical nature. In this chapter, the systematic errors of the Hipparcos astrometric parameters, including the parallaxes, are evaluated by examining the possible sources of bias arising in the data reduction process. Then, the external errors of the parallaxes are further studied on the basis of individual or statistical comparisons to ground-based distances. The validity of the Hipparcos standard errors are also investigated.

20.1. Introduction

The determination of distances for a large number of stars was probably the most eagerly awaited product of the Hipparcos mission and was indeed the key element that led eventually to the decision to design a dedicated space experiment. Distances are the foundation on which virtually all stellar and galactic astronomy rests, and the future development of astronomical research in these areas will rely to a large extent on the Hipparcos parallaxes. It was then of the utmost importance to validate the results, to certify the standard errors and to assess the magnitude and the kind of systematic errors that may be present in the data.

In practice this validation is not easily achieved. It is commonplace with the Hipparcos data to state that the results have so good an internal accuracy that there is no sample of ground-based data which would allow the pattern of the external errors to be assessed, at least statistically. This is particularly true for the parallaxes because of the relative paucity of ground-based measurements matching the Hipparcos precision and accuracy. As a consequence the comparison to external data is based on a carefully selected sample of stars whose distance is statistically well known, even though this is not necessarily true for individual objects.

20.2. Assessment of Possible Errors

The Hipparcos trigonometric parallaxes are essentially absolute, which is not the case of those obtained with ground-based programmes. In principle, given the way the Hipparcos observations were performed and the data reduced, no systematic errors above 0.1–0.2 mas are expected in the Hipparcos parallaxes. However, the possibility of a zero-point shift cannot be ruled out, for example if there were periodic variations of the basic angle of the instrument beam-combining mirror (Lindegren *et al.* 1992).

Systematic errors of the order of, or smaller than, 0.1 mas may be evident only with samples of several hundred error-free parallaxes, e.g. typically a set of stars known to be farther than few kiloparsecs or cluster members of known distance. The Magellanic Clouds fall short in fulfilling this criterion, because there are less than 50 such stars in the Hipparcos programme which, in addition, are predominantly faint stars. One has then to resort to galactic clusters.

Photometric calibrations $(uvby\beta)$ are also used in order to get estimates of the interstellar extinction and to derive visual absolute magnitudes. With these data and a simple galactic model it is possible to compute an unbiased estimate of the global zero-point of the parallaxes of distant stars along with its unit-weight error.

The absence of a significant zero-point error on parallaxes would probably imply the same absence on the other parameters, as the parallax does not play a special role in the astrometric reduction. It is also possible to have a general view of the systematic errors on all the astrometric parameters, using the residuals from astrometric reduction. For this reason, the Hipparcos data are systematically studied as a function of the astrometric and photometric data of the stars: positions, parallaxes, proper motions, apparent magnitudes and colours.

Regarding random errors, the standard errors of the Hipparcos parallaxes vary mostly with magnitude, and also with ecliptic latitude as a result of the scanning law of the satellite. Internal tests by Lindegren (1995) and external tests by Arenou *et al.* (1995) on the 30-month solution reached the conclusion that the standard errors on parallaxes were good estimates of true external errors. However, in the H30 catalogue, the astrometric parameters were obtained with a straight average of FAST and NDAC data, and their assigned standard error was the quadratic average of FAST and NDAC standard errors; unlike the final merged solution, these averages did not take into account the correlation between Consortia data. It was thus necessary to study the random errors in the final Catalogue. Given their large range (from 0.5 to 5 mas at the faint end), the standard errors themselves are not evaluated directly but the unit-weight error is studied instead.

20.3. Comparison with Ground-Based Data

In this section, Hipparcos parallaxes are compared to various samples of ground-based parallaxes. Ground-based measurements are generally affected by atmospheric or mechanical effects and suffer from lack of homogeneity. Thus, while the ground-based data could not be used to assess the external precision of the Hipparcos parallaxes, Hipparcos data could be used to determine the systematic errors present in ground-based measurements down to the mas level.

In the following comparisons, robust estimates have been used to secure results insensitive to outliers. The estimates rely heavily on the median of the distributions instead of the average as location parameter, and on the half-width between the 15.85th and 84.15th percentile as an unbiased estimate of the standard deviation.



Figure 20.1. Comparison between Hipparcos and USNO parallaxes.

USNO Parallaxes

The US Naval Observatory has been conducting a systematic photographic programme for trigonometric parallaxes since 1964 with the 61-inch telescope at Flagstaff. The latest list has brought the programme to 1013 stars and over the years the typical parallax precision for a completed series, has evolved from ± 4 mas to ± 2 mas. This programme is now discontinued and superseded by the parallaxes determined by the CCD initiated in 1983. Results from that programme demonstrated that relative parallaxes with formal mean errors in the 0.5 to 1.2 mas range are readily achieved if suitable reference star frames are available (Monet *et al.* 1992).

For the present comparison to the Hipparcos parallaxes, a set of $n_{\pi} = 88$ stars (Harrington & Dahn 1980, Harrington *et al.* 1993) has been used. The median quoted formal precision for these stars is $\simeq 2.5$ mas. Differences between Hipparcos and USNO results are plotted in Figure 20.1 which shows that very good agreement is found, with no obvious outliers. The median of the differences between these ground-based parallaxes and their Hipparcos counterparts is 0.2 ± 0.35 mas, typically of the order of $\sigma n_{\pi}^{-1/2}$, suggesting the absence of bias and of systematic differences between the two techniques. The distribution of normalized differences computed as:

$$\frac{\pi_{\rm USNO} - \pi_{\rm H}}{\sqrt{\sigma_{\rm USNO}^2 + \sigma_{\rm H}^2}}$$
[20.1]

has a standard deviation of 0.96, a good indication that the formal errors are probably realistic.

HIP	Name	Нр	α	δ	π_{H}	$\sigma_{\rm H}$	π_{VLBI}	σ_{VLBI}
		mag	deg	deg	mas	mas	mas	mas
12469	LSI61303	10.7	40.1	61.2	5.65	2.28	-0.66	0.62
14576	Algol	2.1	47.0	41.0	35.14	0.90	32.51	0.59
16042	UX Ari	6.5	51.7	28.7	19.91	1.25	19.89	0.39
16846	HR 1099	5.8	54.2	0.6	34.52	0.87	33.88	0.47
19762	HD 283447	10.9	63.6	28.2	9.88	2.71	6.93	0.25
23106	HD 32918	8.1	74.6	-75.3	3.43	0.61	4.02	0.80
66257	HR 5110	4.9	203.7	37.2	22.46	0.62	22.21	0.45
79607	$\sigma^2 CrB$	5.2	243.7	33.9	46.11	0.98	43.93	0.10
98298	Cyg X1	8.8	299.6	35.2	0.58	1.01	0.73	0.30
103144	HD 199178	7.2	313.5	44.4	10.68	0.73	8.59	0.33
109303	AR Lac	6.1	332.2	45.7	23.79	0.59	23.97	0.37
112997	IM Peg	5.9	343.3	16.8	10.33	0.76	11.29	0.68

Table 20.1. List of radio stars observed in the VLBI programme.

VLBI Parallaxes

The systems of positions and proper motions resulting from the analysis of the Hipparcos data have very high internal consistency, meaning that the angular separation between two stars is known with millisecond accuracy, but without any connection to any predefined reference system. In order to link the Hipparcos reference system to the ICRS, several link programmes were undertaken (Lindegren and Kovalevsky 1995, Chapter 18 of this Volume) and used to rotate the provisional Hipparcos solution to the ICRS. Al-though this link has no influence on the parallaxes, it happens that the extragalactic link programme based on the VLBI observations of radio stars carried out by Lestrade *et al.* (1995), yielded positions, proper motions and parallaxes of 12 optically bright radio-emitting stars to the outstanding precision of 0.2–1 mas, the only instance where individual ground-based parallaxes are of better quality than Hipparcos.

The 12 VLBI stars are listed in Table 20.1 with the parallaxes measured by Hipparcos and by radio-interferometry (Lestrade *et al.* 1997). The comparison illustrated by the plot of Figure 20.2 shows that good agreement is found between the two sets of measurements. Among the VLBI stars, three are detected and solved as double stars (HIP 16042, 16846, 79607), one astrometric binary (HIP 14546 = Algol) the solution of which refers to the barycentre after correction of the circle abscissae for the orbital motion, and one variable double (HIP 19762), with a poor solution. Given the accuracy of the VLBI data, and the fact that as far as Hipparcos is concerned, these stars are representative of the difficulties encountered in the processing, the comparison looks very favourable for the Hipparcos determination, although the small number of objects precludes from too general a conclusion being drawn.



Figure 20.2. VLBI versus Hipparcos parallaxes (mas). Two stars were down-weighted for the extragalactic link, due respectively to their jet structure or duplicity, and five stars are in the Hipparcos Double and Multiple Systems Annex.

Yale Parallaxes

The Yale University Observatory published in 1995 a completely revised and enlarged edition of the General Catalogue of Trigonometric Stellar Parallaxes, containing 15 994 parallaxes for 8112 stars published before the end of 1995 and obtained at various places. (GCTP, van Altena *et al.* 1995). The mode of the parallax accuracy for the \simeq 1700 newly added stars of 4 mas is considerably better than in the previous editions (about 16 mas). The relative parallaxes which constitute the basic data, are corrected to absolute parallaxes using corrections that are based on an improved model of the Galaxy. Altogether the median formal errors of the GCTP parallaxes is about 10.5 mas. An attempt is made by the authors to determine the accidental and systematic errors of the parallaxes.

Compared to the small samples studied in the previous sections, the General Catalogue of Trigonometric Stellar Parallaxes provides a sample of 4292 stars suitable for the comparisons with the Hipparcos single stars. A more in-depth cross-identification process could probably have yielded more stars, however the sample has been considered large enough for our comparison purpose, considering the extra effort needed to get a comprehensive intersection of the two catalogues.

A straight comparison between GCTP and Hipparcos parallaxes gives a median difference $\pi_{GCTP} - \pi_H = 1.8 \pm 0.2$ mas, which differs significantly from zero. This bias comes partly from distant stars: the difference amounts to 2.6 ± 0.3 mas for stars farther away than 50 parsecs whereas it is only 0.5 ± 0.4 mas for stars nearer than 20 parsecs, i.e. hardly significant. It could originate from the transformations applied to correct to the

absolute parallaxes using a model of the Galaxy, although this statement needs to be substantiated.

However, the main source of bias comes from zonal errors, as may be seen in Figure 20.3. Systematic errors, up to 7 mas at declination $\delta = -30^{\circ}$, and to a smaller extent in right ascension, are found. If the comparison is restricted to the northern hemisphere, the median difference between GCTP and Hipparcos parallaxes is reduced to 1.2 ± 0.3 mas for stars farther than 50 parsecs. The difference between the two hemispheres is striking, and comes as no surprise given the number of observatories and variety of instruments involved in the compilation made by van Altena *et al.* (1995). Moreover, variations with magnitude cannot be ruled out: a bias is also possibly present at the bright and faint ends.

Apart from the systematic errors reported above, no indisputable outliers were found (the largest deviation is of 4.7σ). The width of the normalised differences (see Equation 20.1) is 1.04 ± 0.01 , indicating that their is no global scale defect in the formal errors of the General Catalogue of Trigonometric Stellar Parallaxes.

20.4. Systematic Errors of the Hipparcos Astrometric Parameters

The search of a zero-point error, or of more complex systematic effects, on the five astrometric parameters is not straightforward since their observed values cannot be compared to their unknown true values. It is however possible to test for neglected terms in the position, by reprocessing the final adjustment of the great-circle abscissae to the astrometric parameters, with an improved model including either a constant term or by extending the five-parameter model of star motion which was adopted for the majority of the Hipparcos stars, including systematically acceleration components in right ascension and declination. These terms, being physically spurious, should average out to zero. If the observed averages happen not to be significantly different from zero, one could conclude that the astrometric parameters are also free of significant systematic errors of global nature.

During the data processing, every star has been tested for the significance of the acceleration terms. When the test was negative, the usual five parameter model was taken as the baseline. Now, if all the double stars and the suspected astrometric binaries are excluded, and all the other stars are processed with the extended model, the average value of the components of the acceleration should be zero. Any departure from this would be an indication that small systematic effects could pervade the astrometric solution. One must add that there are only a handful of nearby stars with perspective acceleration larger than 0.1 mas and they do not affect the overall statistics.

A dedicated run of the astrometric processing was set, with either a six-parameter model (a constant term *c* was also computed) or a seven-parameter (including the acceleration components $g_{\alpha*}$ and g_{δ}). Only stars never flagged as double, were considered. This amounts to $\simeq 92\,000$ stars for the six-parameter solution, with an *a priori* exclusion of outliers, and $\simeq 95\,000$ stars for the seven-parameter solution. On average, the formal errors on the offset *c*, and the acceleration components $g_{\alpha*}$ and g_{δ} were respectively about 0.6 mas, and 3.1 and 2.4 mas/yr². In both models, the unit-weight error of these terms were found to be 1.07, suggesting that the standard errors of the Hipparcos astrometric parameters might be slightly underestimated.



Figure 20.3. Distribution of the parallax differences between the General Catalogue of Trigonometric Stellar Parallaxes and the Hipparcos Catalogue.

The medians of the three terms are plotted in Figure 20.4 as a function of magnitude and colour, and as a function of the five Hipparcos astrometric parameters. Significant variations larger than 0.1 mas are clearly visible. Although this limit may appear very small, it is about one quarter of the best standard errors of the parallaxes (0.42 mas) in the Hipparcos Catalogue. Possible departures from zero of the plotted data should however be appreciated with their formal errors in mind, at a 2σ level for instance. The quoted error bars depend both on standard errors (which increase with magnitude) and on the number of stars in each bin.

The main results are as follows:

- 1. for the brightest stars a significant offset is found: the median value of *c* for the $\simeq 1000$ stars brighter than Hp = 5 mag is 0.11 ± 0.01 mas;
- 2. the chromaticity effect played an important role in the Hipparcos data reduction; a clear trend may be seen, especially concerning redder stars. For the \simeq 900 stars



Figure 20.4. Variation of a constant term and of the acceleration components, obtained respectively with a six- and seven-parameter astrometric model, as a function of photometric and Hipparcos astrometric data. For clarity, only c error bars are indicated; the errors on $g_{\alpha*}$ and g_{δ} are about 5 and 4 times larger respectively. Within their error bars, these terms are expected to be around 0 if the astrometric parameters are free from systematic errors.

with V - I > 2.5 mag, one finds a median *c* of 0.24 ± 0.04 mas, significantly larger than 0.1 mas. The acceleration components exhibit the same trend. Significant peaks around V - I = 0.6 mag and V - I = 1.8 mag are also found;

- 3. no significant effect is found as a function of position;
- 4. for parallaxes, no conclusion may be drawn from the small parallaxes or from the negative tail, since in this case the parallax value represents merely the observation error, which is obviously correlated with the observation errors on *c*, $g_{\alpha*}$ and g_{δ} ; however, for larger parallaxes, the *c* term remains constant and significantly positive;
- 5. variations of accelerations with high proper motions, noticeable in particular for $\mu_{\alpha*} < -200$ mas/yr, are possibly due to the expected correlation between *g* and μ .

Although systematic errors greater than 0.2 mas may occur for the reddest stars, it must be stressed that this analysis was done by adding one or two unknowns in the astrometric reduction. In the case of the baseline model with five astrometric parameters, these errors are probably distributed among the five unknowns. Apparently, parallax and proper motions are more sensitive to this effect than coordinates.

In any case, the number of stars affected by a possible systematic error above 0.1 mas remains very small. As seen in Figure 20.4, the bulk of the Hipparcos stars ($Hp \sim 9$ mag, $\pi_{\rm H} \sim 3$ mas, low proper motion) correspond to values of *c*, $g_{\alpha*}$ and g_{δ} which are completely negligible on the average.

20.5. The Zero-Point and Unit-Weight Error of the Parallaxes

It was shown in the previous section that the astrometric parameters may have small, but significant, systematic errors. The purpose of this section is to assess the magnitude of the zero-point *z* of the Hipparcos parallaxes. Simultaneously, the standard errors of the parallaxes are also studied by means of the determination of the unit-weight error $k = \langle \sigma_{\text{ext}} / \sigma_{\text{H}} \rangle$, i.e. the ratio of the external to the internal errors. If both parallaxes and standard errors are unbiased, the expected values are $z \simeq 0$ and $k \simeq 1$.

Magellanic Cloud Stars

Magellanic Clouds stars were included in the Hipparcos programme in order to determine the proper motion of the Small Magellanic Cloud and the Large Magellanic Cloud. They are distant enough, with parallaxes of $\simeq 0.02$ and 0.015 mas, that they can be used to search for a systematic bias in the Hipparcos parallaxes. Out of the 46 Hipparcos stars lying in the Magellanic Clouds which were regularly observed during the mission, 8 have been solved with a poor parallax accuracy. They have been detected as non single stars and placed in the Double and Multiple Systems Annex. Three of these stars belong to the category of the stochastic solutions, since it was impossible to reconcile the final residuals with the *a priori* abscissa errors.

Using the 38 remaining single stars, the average weighted parallax is $z_{\rm M} = -0.1\pm0.23$ mas. However, due to the correlation between great-circle abscissae, the precision on the mean parallax of a group of *n* adjacent stars is about $\sigma_{\pi} n^{-0.35}$ instead of the expected $\sigma_{\pi} n^{-1/2}$ (Lindegren 1989). This has not been taken into account in the quoted error bar of the average parallax. The unit-weight error is $k_{\rm M} = 1.04 \pm 0.12$. This analysis on a very limited and peculiar sample (the stars in the Magellanic Clouds are predominantly faint) leads to the conclusion that the zero-point in the parallax determination is not larger than 0.4 mas, too high an upper bound to qualify the Hipparcos distances.

Open Cluster Stars

Open star clusters are the most recognisable stellar systems and are easily observable even with a small telescope. Astronomers have long appreciated their use in the understanding of stellar evolution as well as their link with the physics and dynamics of the Galaxy. To date, there are just over 1200 known open clusters, nearly all within 2000 parsecs.

Since the members of a star cluster form a more or less bound system, they are essentially all at the same distance. This property, associated with the assumption of a common origin, has made it possible to measure the distance of an open cluster with some confidence. The distances of galactic open clusters are believed to be known with a relative error of the order of ten per cent. Using distant clusters (> 200 parsecs) and assigning to each member of a particular cluster, the distance of this cluster, allows an absolute error on their parallax to be obtained to better than 0.5 mas.

These estimates provide a reliable basis for a comparison with the Hipparcos parallaxes, provided that all test stars are true members of the corresponding clusters. To assess the cluster membership, the average proper motion of the cluster was computed with all the candidates stars. Then all the stars with a proper motion component relative to the average, five times greater than its standard error, were rejected.

Using the BDA cluster data base (Mermilliod 1992), and the distance moduli quoted by Lyngå (1987), parallaxes were available for 391 stars, after exclusion of non-members. The median difference between the Hipparcos and cluster parallaxes was found to be $z_{\rm C} = 0.04 \pm 0.06$ mas, thus not significantly different from zero, and the unit-weight error is $k_{\rm C} = 1.06 \pm 0.07$. This is a much more significant result than that obtained with the Magellanic clouds, although the contribution of the uncertainty of the distance of the clusters to the error of the median would require a more refined appraisal.

Estimation Using Photometric Data

After trigonometric and moving cluster parallaxes, calibrated intrinsic luminosities provide the most widely used and reliable distance estimators for individual stars. Many $uvby\beta$ calibrations were used in order to obtain an estimate of the photometric distance modulus for all available stars. The major part of the Hertzsprung-Russell diagram was covered: dwarfs B to M2, supergiants B to G5, population II F stars; red giants are of course missing. A programme was built to automatically choose the calibration which must be applied, and from these calibrations, estimates of intrinsic (corrected for the reddening) photometric indices, B-V colour excess, interstellar extinction A_V , absolute magnitude, effective temperature, gravity and metallicity were obtained. Photometric errors were propagated through the different steps so that formal errors on the stellar parameters were also estimated. Eventually the absolute magnitude, the extinction, and the apparent magnitude were used to determine the distance modulus $t = V - M_V - A_V$.

The $uvby\beta$ input data came from the Hauck & Mermilliod (1990, 1996) Catalogue in an updated version. In order to minimize the error on the distance modulus based

on photometric data, only the most distant stars must be kept since a relative error in parallax translates directly into an absolute error in the distance modulus. For this reason, the sample was restricted to stars with a distance modulus 8.5 < t < 14.5. In addition, stars known to have a variability > 0.2 mag, having a joint photometry associated to binaries or those with $\sigma_t > 0.35$ were not included in the sample. After all these filters were applied 467 stars remained.

The truncation in distance moduli combined with the random measurement errors caused the sample average parallax to be biased. In order to take this bias into account and limit its adverse effect, a specific statistical method was applied by Arenou *et al.* (1995) and is now briefly summarised.

The conditional probability density function that the Hipparcos parallax of a star is $\pi_{\rm H}$, given its observed distance modulus *t*, its galactic latitude *b*, the Hipparcos zero-point error (*z*) and the unit-weight error (*k*), is:

$$f(\pi_{\rm H}|t, b, z, k) = \frac{\int_{0}^{+\infty} p_1(\pi_{\rm H}|\pi, k, z) p_2(t|\pi) p_3(b|\pi) p_4(\pi) \,\mathrm{d}\pi}{\int_{-\infty}^{+\infty} \int_{0}^{+\infty} p_1(\pi_{\rm H}|\pi, k, z) p_2(t|\pi) p_3(b|\pi) p_4(\pi) \,\mathrm{d}\pi \,\mathrm{d}\pi_{\rm H}}$$
[20.2]

where the conditional probability distributions p_1 to p_4 are determined in Arenou *et al.* (1995). In this equation the unknown parameters are the zero-point and the unitweight errors; they can be estimated from the observed parallaxes and distance moduli. The estimator of (k, z) is found numerically from the maximum of log-likelihood function $\mathcal{L} = \sum \ln f(\pi_{H_i} | t_i, b_i, z, k)$ of the *n*-sample. The method also checks the quality of the fit to the model, filters out the outliers and gives the standard errors of the unknowns.

The distribution of the errors on Hipparcos parallax was shown to be approximately Gaussian by Arenou *et al.* (1995). Thus p_1 is a Gaussian of expectation $\pi + z$ and standard deviation $k\sigma_H$. A possible censorship on π_H was taken into account, although no truncation was actually applied to Hipparcos parallaxes. The moduli *t* were assumed Gaussian around the true value $-5 \log \pi - 5$ and the truncation on *t* was also explicitly taken into account. For the joint distribution of the galactic latitude and parallax, $p(b, \pi) = p_3(b|\pi)p_4(\pi)$, the distribution perpendicular to the galactic plane was assumed exponential with a mean scale height of 100 pc. However this assumption is not critical for the sample investigated here.

Applying this method to the available sample of n = 467 stars, the zero-point found was $z_{\rm P} = -0.05 \pm 0.05$ mas, thus not statistically different from 0, the unit-weight error being $k_{\rm P} = 1.04 \pm 0.04$. The uncertainty of the median is in good agreement with $1/\sqrt{n}$ mas. No outlier was found in the sample.



Figure 20.5. Zero-point and unit-weight of Hipparcos parallaxes, from external comparisons using distant stars.



Figure 20.6. Variation of the parallax zero-point versus V - I colour, using cluster and photometric data of distant stars. There is one data point for every decile of each population.

20.6. Conclusions

Results obtained with the external comparisons are summarized Figure 20.5. The global zero-point error of Hipparcos parallaxes can be safely assumed to be smaller than 0.1 mas. Another important conclusion is that the standard errors of the parallaxes have probably not been underestimated by more than 10 per cent.

These results have been derived from distant stars only, so that one may ask whether they are representative of the whole Hipparcos Catalogue. This is probably indeed the case. Firstly, the absolute value of the distance played no specific role in the Hipparcos data processing, and it is difficult to imagine a systematic effect on the parallax which would be function of the parallax itself. Also, no bias was found in the comparisons to the USNO or VLBI parallaxes despite the fact that they cover a large range of parallaxes.

The chromaticity effect exhibited in the previous section may also be studied with the distant stars. Although no red star was available for this comparison, Figure 20.6 shows that variations of the zero-point with colour of about some tenths of mas cannot be excluded even for blue stars. It is however difficult to assess whether these variations are really in the Hipparcos data or due to ground-based data used for the comparison purpose.

Eventually the Hertzsprung-Russell diagram constructed with the Hipparcos provisional data (Hipparcos parallaxes, colour indices and magnitudes of the 30-month solution) has provided an important confirmation of the quality of the parallaxes and the photometry through the overall consistency of the diagram for a wide range of stars and distances (Perryman *et al.* 1995). This is particularly meaningful for the parallaxes whose uncertainty would broaden the main sequence with the standard error of the absolute magnitude $\simeq 2.1 \sigma_{\pi}/\pi$. As discussed by Perryman *et al.*, the observed width of the main sequence is likely to be attributable to intrinsic dispersion of physical origin rather than to some random or systematic effect of the parallaxes.

F. Arenou, F. Mignard, J. Palasi

21. VALIDATION OF PHOTOMETRIC RESULTS

The Hipparcos photometric solution was one of the highlights of the mission and proved to be very successful. As was done for other parameters determined from the Hipparcos observations, the aim of this chapter is to provide an evaluation of the quality of the results from the study of the internal consistency and, when this was possible, from direct comparisons with ground-based data of comparable quality. Various aspects are taken up successively: the reliability and the precision of the calibration and in particular the error of the zeropoint, the meaning of the internal errors published in the Catalogue with their relation to the true errors, the stability of the photometric system over the three years of instrument ageing and the validity of the period determination of newly identified periodic variables. Comparisons with three ground-based photometric systems show both the overall reliability of the data, as well as some of the features encountered when combining Hipparcos and groundbased photometry.

21.1. Introduction

Chapter 14 of this Volume presents the methods and algorithms that were developed and implemented to compute the magnitudes of all the Hipparcos programme stars. The magnitude estimates were obtained at every field transit, providing an average of 110 observations per star. This proved to be a rather complex task: starting from the recorded photon counts as indicators of the stellar intensity and relying on a set of photometric standard stars, defined within the assumed photometric system, instrument parameters were derived that determined the transformation of the observed intensities to magnitudes in this photometric system. At various stages of the processing the photometric system and the associated values of the photometric standard stars were re-adjusted and followed by a complete re-reduction of all photometric data available at that time (see also Section 14.5). An internal assessment of the quality of the final product was highly desirable and could be based on the consistency of all the determinations, the demonstration of the stability of the calibration from the fluctuation of the individuals measurements of known (or assumed) constant stars. From this analysis and the knowledge of the various noise contributions considered, our expectation was that the internal accuracy at the grid-crossing level was about 0.012 mag for a single measurement of a star of Hp = 8 mag.

Comparisons with ground-based systems may tell us more about the ground-based system than about Hipparcos, but a good agreement between well calibrated independent systems does provide a clear indication of the level of reliability of the Hipparcos photometry.

21.2. Evaluation of the Calibrations

As explained in Chapter 14, both FAST and NDAC used an instrument model to represent the sensitivity of the Hipparcos detection chain as a function of the position of the stellar image on the grid and of the colour of the star. The basic term of this calibration is the zero-point coefficient, representing the response of the detector at the centre of the grid for a star of colour V - I = 0.65 mag. Its variation with time mirrors the slow decline of the overall photometric response of the instrument, due mainly to the transmission loss in the glass of the optical elements of the zero-point is rather arbitrary and has no deep physical meaning, the scatter of the zero-point about the mean ageing curve and the way the zero-points of the preceding and following field behave are very relevant to assess the ultimate accuracy of the calibration and as a consequence, that of the photometric solution.

Figure 21.1 shows the variations of the zero-point over the mission, expressed in magnitude. The origin of the scale was chosen to be close to the theoretical mid-mission, where the Hipparcos photometric scale was defined through the predicted magnitudes of the set of photometric standards. The plot gives the history of the zero-point for the dc-scale with the preceding field of view on the left scale and the following field of view on the right. The points on these curves are computed as $-2.5 \log X_1$ where X_1 is the FAST zero-point defined in Table 14.5. The ageing has been analysed in Chapter 14 and will not be reconsidered here. What is more important in the context of the validation of the photometric solution, is the fact that the same details appear in the two curves at the same time, although the calibrations for the two fields of view were fully independent and used different observations obtained within the same orbit. Several small incidents are visible on both curves and should be considered significant. See for example the small loss of sensitivity of about 0.02 mag at day \simeq 600, or the isolated increase of the same amplitude at day 934. The complex fine structures around days \simeq 700 are very similar in the two field of views. The same details can be observed from the equivalent NDAC calibration values (see Figure 14.4). The occurrence of these synchronous changes with the same amplitude makes us confident that these effects were real and not artifacts, and that they were properly accounted for by the calibrations that were performed twice a day during the mission.

Figure 21.2 shows the evolution of the difference between the zero-points of the preceding and following field of view. The observed difference is close to zero. This is not a chance effect, as most of the optical and detector chain is identical for the two fields of view. The precise value of the difference is, however, of little or no meaning. On the other hand the fluctuations of the distribution are much more relevant. The long term fluctuations are real and indicate small sensitivity variations that were not perfectly identical in the two fields of view. Similar variations between the two fields of view have been observed for a wide range of parameters (see, for example, Chapters 8 and 10). The overall variation of sensitivity has been of order 0.5 mag over the mission (Figure 21.1). However the evolution of the difference between the two fields, up to the



Figure 21.1. Time variation of the zero-point of the photometric calibration for the dc-magnitude scale. The left scale refers to the preceding field of view (FOV) and the right scale to the following. The two scales in the plot have been shifted by 0.2 mag for better visibility; however the absolute value of the two sensitivities are very similar and should be very close at about day 700.



Figure 21.2. Zero-point differences between the photometric calibrations of the preceding and following fields of view during the mission. The plot refers to the dc-photometric scale.

start of the gyro problems at day 1250, remained within ± 0.01 mag and was perfectly monitored by the calibration. During the sun-pointing mode and at the end of the mission the two fields started behaving differently. This could be contributed to changes in the temperature of the spacecraft, and the effects thereof on the optical system.

Also noted were a few discontinuities revealing quick changes in the instrument or in the calibrations as at day 388, with an increase of 0.003 mag in the sensitivity of the preceding field or the event of day 755 of 0.006 mag in the other direction and which in fact was not transient. The first event was caused by the implementation of the calibrated grid rotation in the observations, causing a general improvement in the pointing of the instantaneous field of view. The latter event was associated with a failure



Figure 21.3. Time variation of the zero-point of the photometric calibration for the ac-magnitude scale. The left scale refers to the preceding field of view (FOV) and the right scale to the following field of view. The two scales have been shifted by 0.2 mag for better visibility; however the absolute value of the two sensitivities are very similar.

of the Thermal Control Electronics (TCE1), and a switch to the redundant system (TCE2) (see Table 2.1). These small changes in sensitivity were all properly restored by the photometric calibration which guaranteed the consistence of the photometric system to within a few millimagnitudes.

On a shorter time scale, the width of the curve is of the order of 0.003 mag and variations are of a random nature. This provides information on the amplitude of the statistical fluctuations of the calibration from one orbit to the next. It provides an indication for the ultimate quality of a calibration, which is about 0.002 mag in each field of view. This means that the individual Hipparcos photometric measurements are not defined in an absolute sense with an accuracy better than 0.002 mag. The internal consistency of the photometric system is, however, much better defined. For a constant bright star, the accuracy and stability of the instrument modelling, the zero-points and the chromatic parameters, were the limiting factors in the error budget of the accumulated photometry. For fainter stars, the photon statistics was the primary contributor to the errors.

A similar investigation was performed for the ac-scale, and led to the plots shown in Figure 21.3 for the evolution of the photometric response at the centre of the field for a star of colour index V - I = 0.65 mag and in Figure 21.4 for the difference between the two fields of view. The overall ageing is the same as for the dc-scale with a total decrease of the sensitivity of about 0.45 mag over the mission. The curves are however more structured over short timescale than for the dc-scale.

This becomes more clear by looking at the differences between the two fields of view in Figure 21.4. The long filaments going downward extending over one month at the beginning of the mission and two months later on, are associated to the slow change of focus, followed by the abrupt refocusing of the instrument (see also Figure 14.3 and Chapter 2 of this Volume, and Chapter 10 of Volume 2). As explained in Chapter 14, the ac-photometric scale is based on the amplitude of the modulation IM_1 , where M_1 is the modulation coefficient of the first harmonic. (The exact formula is slightly



Figure 21.4. Difference between the zero-points of the photometric calibration of the preceding and following fields of view during the mission. The plot refers to the ac-photometric scale. The thread-like features are due to slow changes in focus, the discontinuities are the result of refocusing.

different for FAST, but this is of no importance here.) The zero-point of the ac-scale was therefore directly affected by the change in the modulation coefficient M_1 , and thus by the refocusing, which produces a change in modulation and thus in the apparent sensitivity. The refocusing is global and does not have the same effect in the preceding and following field of view. From Figure 21.3 it is evident that the average sensitivity of the preceding field of view, as far as it can be measured by the zero-point, gets larger just after a refocusing took place whereas it decreases in the following field. This differential behaviour shows up amplified in Figure 21.4 as a significant difference in the zero-points according to the field of view.

21.3. Distribution of the Unit-Weight Variance

The Overall Distribution

The aim of this section is to investigate the reported standard errors of the photometric transits in order to assess whether they are representative of the true errors. The best way would be to study the true error of the Hipparcos photometry with respect to ground-based photometric measurements of comparable or better quality, and more or less obtained at the same time. Unfortunately few such measurements are available, and when they are, there remains the problem of transforming from the ground-based photometric system to Hipparcos, or *vice versa*, with an accuracy of few millimagnitudes.

The approach adopted here instead was to study the statistical distribution of the errors. Basically the scatter about the mean or the median of the 40 to 380 individual magnitudes of a constant star, should be related to the standard error given for the individual transits. If the standard errors are too optimistic the scatter appears too large and one can conclude that there is a lack of consistency. This is equivalent to studying the χ^2 distribution of the deviation of the individual measurements with respect to the median.



Figure 21.5. Distribution of the reduced unit-weight variance of Hp in the Wilson-Hilfert normal approximation.

For the *i*th star and the magnitudes Hp_{ij} obtained at the times t_j with the standard deviations σ_{ij} and a median of Hp_i , one has:

$$\chi_i^2 = \frac{n_i}{\sum_{j=1}^{n_i} \frac{(Hp_{ij} - Hp_i)^2}{\sigma_{ij}^2}}$$
[21.1]

which follows a χ^2 distribution with $v_i = n_i - 1$ degrees of freedom. The Wilson-Hilfert cube root transformation has been used (Kendall & Stuart 1977) in the analysis because it gives a remarkably useful normal approximation to the χ^2 distribution as:

$$\mathcal{Z} = \frac{9\nu}{2} \frac{\chi^2}{\nu} \frac{\chi^2}{\nu} + \frac{2}{9\nu} - 1$$
 [21.2]

which follows asymptotically a normal law with zero mean and unit variance.

For constant stars with Gaussian errors, the distribution of the $\mathcal{Z}(\chi_i^2)$ should be close to a standardized normal. When the star is variable, the systematic deviations from the mean are larger than the deviation expected from the purely random noise, since the total variance of the time series is the combination of the random component and that brought about by the true light variations. As explained in Volume 1, Section 1.3, Appendix 2, the χ^2 has been one of the basic variability indicators. Thus it is not possible to eliminate the variable stars to determine whether the χ^2 distribution of the remaining stars is adequately distributed. It should be the case by construction.

What has been done is the following: the negative range of \mathcal{Z} predominantly comprises constant stars, even though just about the same number are in the positive region, but indistinguishable from the population of the true variables which are there as well. The argument is supported by the distribution of the reduced χ^2 in its Wilson-Hilfert version, shown in Figure 21.5. Roughly, from $-5 < \mathcal{Z} < 5$ there is a somewhat symmetrical distribution centred around $\mathcal{Z} = 0$, as expected.

The fact that the maximum of the distribution is about $\mathcal{Z} = 0$, is equivalent to saying that the unit-weight variance of the constant stars does not depart from unity on average. As a consequence for the bulk of the catalogue the standard errors of the accepted photometric transits are representative of the scatter observed between the individual transits and may be considered as realistic. Later in this section the particular case of the bright end will be considered separately.

Focussing on the positive range of the distribution, the population of the classes with $\mathcal{Z} > 2$ are systematically larger than their negative counterpart, indicating the presence of variable stars of small amplitude or with short duty cycle like the eclipsing binaries. The extension of the distribution to larger \mathcal{Z} values is entirely populated by variable stars. Altogether there are 41 per cent of the stars with $\mathcal{Z} < 0$, 48 per cent with $0 < \mathcal{Z} < 5$ and 11 per cent with $\mathcal{Z} > 5$. So there must be from this distribution 7 per cent of weak variables and 11 per cent of perfectly detectable variables, i.e. about 13 000 such cases. This is the population of stars flagged U, M or P in Field H52 of the Hipparcos Catalogue.

Consider now the χ^2 distribution of the 41 per cent of the stars which show less scatter about the mean than expected. This amounts approximately to the 46 000 entries with the flag C in Field H52; the tests applied were not strictly the χ^2 . Ideally, for a homogeneous population, the negative region of Figure 21.5, should be similar to the negative region of a standard normal law of zero mean and unit standard deviation. To see that better, a plot of the distribution and its mirror image with respect to $\mathcal{Z} = 0$ has been drawn (Figure 21.6).

By construction it is perfectly centred and symmetric, but its width is related to the scatter of the χ^2 of the constant stars. The standard deviation of this distribution is very close to 1.5 and a normal distribution of zero mean and $\sigma = 1.5$ is also plotted (the solid line in Figure 21.6). Other estimates of the standard deviation of the distribution can be obtained by restricting to the distribution shown in Figure 21.5 by using the quantiles < 0.5. With the quantiles 0.2, 0.3, 0.4 one gets for the standard deviation of the assumed normal distribution 1.53, 1.52 and 1.5 respectively. The mean of the distribution for Z < 0 is under the normal assumption $\sigma \sqrt{2/\pi}$. The observed mean of Z with Z < 0 is 1.19, which yields for the estimate of $\sigma \simeq 1.49$. Finally the same study has been made by partitioning the stars according to their brightness using four classes of similar size with Hp < 7.5 mag, 7.5 mag < Hp < 8.5 mag, 8.5 mag < Hp < 9.5 mag and Hp > 9.5 mag which happen to be centred on Hp = 7, 8, 9 and 10 mag. The respective standard deviations are 1.56, 1.60, 1.52, 1.48.

This phenomenon cannot be ascribed simply to a mere scale factor in the standard errors, since this would also change the mean dramatically and make the mode of the χ^2 distribution much larger than unity, even for a small underestimation of the standard error (and much smaller than unity for a small overestimation). It seems to indicate rather that the population is not a homogeneous one, even after scaling by the standard deviation, and there are some categories of stars where the standard deviation is too small compared to the actual scatter and other groups where just the opposite conclusion applies. On the average the distribution remains well centred, but its negative wing gets wider due to the population with too large standard errors which displaces the whole distribution to the left. A more detailed investigation of this effect goes beyond the objectives of this volume, but is worth doing in the future.

Bright Stars

For the bright stars an investigation has been conducted to determine the χ^2 distribution within each class of magnitude. In the Hipparcos Catalogue there are 1473 stars brighter than Hp = 5 mag consisting of 1264 single stars and 209 double and multiple stars. Obviously the recognition of the variability becomes much more sensitive for these stars and the χ^2 distributions show extended positive wings with significant population. The



Figure 21.6. Symmetrised distribution of the unit-weight variance of Hp.

Table 21.1. The unit-weight variance for the bright stars as a function of *Hp*. The second line gives the average of the reduced χ^2 for the constant stars within a class of magnitude. The factor κ represents the coefficient by which the standard deviations should be multiplied to make the χ^2 centred at one.

Нр	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
χ^2	4	3.4	2.8	2.2	1.9	1.5	1.3	1.1	0.99
κ	2	1.85	1.67	1.48	1.38	1.25	1.14	1.04	1.00

number of stars in each class is too small to apply the symmetrised distribution used above to a sub-population assumed to comprise only constant stars.

Instead, stars with $Hp_{min} - Hp_{max} < 0.02$ mag have been considered as constant and the median of their χ^2 distribution has been computed as a function of the magnitude. It departs markedly from one as shown in Table 21.1. Interpreting this offset as an underestimation of the standard errors, one can determine the factor κ by which one should multiply the standard errors to scale down the unit weight variance.

21.4. Analysis of the Periods of Variable Stars

Period searches were undertaken for stars detected as variable from the variance analysis. Several methods were applied to determine reliable periods, usually using a Fourier analysis as a first step. However severe limitations were placed on the efficiency of any method as a result of the poor and irregular time coverage of the observations. The character of the scanning law made the period recognition particularly difficult in the range 5 to 100 days. A description of some of the methods actually employed is given in Section 1.3 of Volume 1.

For many variable stars, a ground-based determination of the period is available and can be used to assess the quality of the Hipparcos determination based only on the observations made during the mission. There are a total of 2541 periods that were successfully computed from the Hipparcos data, of which 1272 have a ground-based


Figure 21.7. Distribution of the periods of periodic variable stars computed from the Hipparcos data for which a ground-based determination is known. The abscissa gives the logarithm of the period expressed in days.

counterpart and 1269 can be considered as new determinations, as far as the search in the literature can be considered as comprehensive, a goal almost impossible to fulfill. These figures should be regarded as a good order of magnitude for each category.

The period distributions for the two groups are shown in Figure 21.7 and Figure 21.8. The two distributions are broadly similar but differ in the details. They cover the same range of periods from about one hour to 1000 days. The upper limit is smaller for Hipparcos newly determined periods, with an upper bound of about 500 days, i.e. half the mission duration. The ground-based periods covers roughly three regions centred at 5 hours, 4 days and 250 days with a significant depletion at $P \simeq 40$ days. As for the distribution of the new determinations many periods appear in the range 0.6 days to 5 days, with a regular decrease in the detection of larger periods. The decrease in number of periods just above 5 days should be ascribed to the observation window which makes the period analysis difficult in this range. The depletion at about 40 days is less prominent than in the case of the periods with ground-based measurements, but this could also be due to the window function: periods in that range could quite easily have been wrongly assigned.

To judge the validity of the period determination relying only on the Hipparcos data, it is useful to compare the 1272 determinations which have an equivalent from the ground, where it should be realized that there is considerable variation in the reliability of ground-based periods, as well as that some variable star periods are changing with time. A plot showing the ground-based period against the Hipparcos value is shown in Figure 21.9. It is found that $|\Delta P|/P$ is less than 1 per cent for 917 stars out of the 1272 examined, less than 10 per cent for 1150 and larger for the remaining \simeq 100. As a consequence most of the data points in Figure 21.9 lie along the first diagonal. The small straight spikes at periods 0.1, 0.2 and 0.3 days result from the rounding of periods in the published data.

In a logarithmic plot, all the data points well outside the first diagonal correspond to very large disagreement between the Hipparcos and the ground-based period. However as a rule, there is either a close agreement or a strong disagreement between Hipparcos and the ground-based data. As the period range of the new variable stars measured by Hipparcos is not very different from the periods measured in the set of known variable stars, we may infer that the new periods have the same level of reliability.



Figure 21.8. Distribution of the periods of periodic variable stars computed from the Hipparcos data with no known ground-based determination. The abscissa gives the logarithm of the period expressed in days.



Figure 21.9. Comparison of the periods of variable stars found in the literature to the determinations based only on the Hipparcos observations.

21.5. Stability of the Photometric System

Selection of Constant Stars

The ageing of the instrument was significant during the mission, reaching 0.5 mag for a star with V - I = 0.65 mag as it was shown in Section 21.2. The effect was colour dependent, being more pronounced for the early type stars than for the reddest (see Chapter 14, Figure 14.4). Over the 37 months of observation, the calibration process intended to reduce all the observations onto a unique and constant photometric system, close to the actual sensitivity curve of Hipparcos at mid-mission. One way to check

-		5 6	,			0				
HIP	HD	Sp.	V – I mag	<i>N</i> ₁	<i>N</i> ₂	Var.	<i>Hp</i> mag	σ _{Hp} mag	σ _{transit} mag	
11698	15596	G5 III	0.970	57	55	С	6.3760	0.0011	0.0062	
13473	18149	F5 V	0.505	150	148	Х	6.0318	0.0006	0.0064	
34622	54810	K0 III	1.026	88	86	Х	5.0792	0.0007	0.0049	
37447	61935	K0 III	1.007	153	145	С	4.0986	0.0004	0.0039	
41307	71155	A0 V	-0.024	75	71	С	3.8987	0.0003	0.0033	
49712	88206	B3 IV	-0.100	149	126	Х	4.8151	0.0006	0.0047	
58345	103932	K4 V	1.219	92	86	Х	7.0986	0.0009	0.0078	
59750	106516	F5 V	0.526	143	133	Х	6.2143	0.0007	0.0062	
69722	124780	F0 V	0.335	82	82	Х	6.6416	0.0007	0.0067	
75181	136352	G2 V	0.715	80	76	С	5.7821	0.0007	0.0056	

Table 21.2. List and basic data on the ten constant stars that have been used in this work to determine the stability of the Hipparcos photometric system. N_1 = number of photometric transits, N_2 = number of accepted transits, Var. = Variability flag of the Hipparcos Catalogue.

whether this goal was achieved is to investigate the 'light curve' of known constant stars. If the system remained well defined and constant during the mission, the scatter of the individual measurements about the mean should be compatible with the standard errors computed at the transit level and no trend should appear in the data between the beginning and the end of the mission (however, note that a trend could still appear if the wrong colour index had been applied for a star in its data reductions; see also Section 14.5).

The computation of the unit-weight variance has been the baseline to identify about 46 000 stars as constant, i.e. not detected as variable with the Hipparcos data. It would have been a vicious circle to select few constant stars from this sample to investigate the stability of the photometric system. Instead, a list of eleven photometric standards of various spectral types and classes has been kindly provided by E. Chapelier (private communication). One of the stars was detected and solved as double by Hipparcos and was not included in the analysis. The remaining ten stars should be photometrically constant at the level of 0.01 mag.

The list of selected stars is given in Table 21.2 with the spectra and colour of each star and the summary data of the Hipparcos observations. The number of accepted transits corresponds to the photometric transits considered as particularly reliable, with no anomaly detected. The median value of Hp and its standard error are based only on the accepted transits and are the same as the corresponding numbers given in the Catalogue. The last column gives the average of the standard errors of the individual transits for each star computed from the transit data in the Hipparcos Epoch Photometry Annex. The ratio between the numbers of the last two columns is always of the order of $\sqrt{N_2}$ as expected. The variability index is particularly interesting in the present context: only four of these stars were labelled constant and none was discovered as variable. Being 'constant' is an attribute which is not the opposite of being variable.

The individual observations are plotted in Figure 21.10 and Figure 21.11 as a function of the observation number. The data points appear in chronological order, but the scale is not linear with the time. However for every star there are observations performed during the first months and the last months of the mission. The error bars are given by the standard error of the individual transits published in the Hipparcos Epoch Photometric Annex. The full scale of the *Y*-axis is the same for all the plots with a range of 0.1 mag. From a visual inspection the distribution of the individual observations look very satisfactory at first glance. The stability of stars like HIP 41307 and HIP 75181 appears particularly outstanding.

Application of a χ^2 Test

For each star χ^2/ν and its \mathcal{Z} approximation have been computed with Equations 21.1 and 21.2. Results shown in Table 21.2 indicate, as expected, no evidence of variability. However the values are large enough for HIP 37747 and HIP 49712 to exclude them from the category of constant stars, until further examination. For example, for HIP 37747, it appears that from the data points plotted in Figure 21.10, the large χ^2 value is based on a few measures that deviate from the median by several standard deviations and also because of a larger scatter at the end of the mission. A different test, able to locate outliers would probably conclude that this star is sufficiently stable to be classified as constant.

Study of the Range

The χ^2 is an all purpose test and is not always sensitive to all the possible defects of the photometric reduction. The columns H_{\min} and H_{\max} in Table 21.3 give respectively the magnitude for the observed maximum and minimum brightness, computed as the 5th and the 95th percentiles of the magnitudes measured at every field transit. The values have been recomputed from the transits data and have one more decimal figure than in the Catalogue. The range is the difference between these two numbers and under the assumption that the star is constant and the error Gaussian, the expectation of the range is 3.29 standard deviations. The observed and expected values are given in Table 21.3 as Δ_1 and Δ_2 . They are relatively consistent with each other; the most significant departure being again HIP 37747 and to a lesser degree HIP 58345 and HIP 59750. For the other stars the agreement is very satisfactory.

With the results from the χ^2 tests, the following conclusions can be drawn:

- (1) the estimate of the standard deviation at the level of the individual observations is compatible with the scatter of the observations, indicating that the errors given in the catalogue are probably realistic, except as noted earlier, for the very bright stars;
- (2) for constant stars there is no large tail in the distribution of errors. Otherwise the test based on the range would have failed. This test is very sensitive to small departures from the expected distribution.

Testing for the Presence of Trends

The third study carried out on this set of constant stars aimed at detecting possible systematic trends with the time, a possibility arising as a result of the instrument chromatic ageing (see Section 14.5). A linear model $Hp(t) = \alpha + \beta(t-t_0)$ has been fitted to the data.



Figure 21.10. Light curves based on the Hipparcos data for a sample of known constant stars. The observations are ordered according to the observation number in chronological order.



Figure 21.11. Light curves based on the Hipparcos data for a sample of known constant stars. The observations are ordered according to the observation number in chronological order.

HIP	χ^2	Z	H _{max}	Hmin	Δ_1	Δ_2	Trend
			mag	mag	mag	mag	10 ⁻⁶ mag/day
11698	0.66	-1.95	6.367	6.386	0.019	0.020	$+1.6\pm2.6$
13473	1.07	+0.59	6.020	6.044	0.024	0.021	-1.7 ± 2.0
34622	1.11	+0.70	5.071	5.091	0.019	0.017	$+2.6\pm1.9$
37447	1.42	+3.16	4.090	4.108	0.018	0.013	$+4.4\pm1.4$
41307	0.85	-0.89	3.894	3.906	0.012	0.011	-0.8 ± 1.7
49712	1.51	+3.49	4.806	4.826	0.019	0.016	$+2.2\pm1.6$
58345	1.08	+0.55	7.090	7.120	0.031	0.026	$+9.5\pm2.9$
59750	1.12	+0.99	6.204	6.229	0.025	0.020	-6.7 ± 2.1
69722	0.78	-1.44	6.634	6.653	0.020	0.022	-7.8 ± 3.8
75181	0.87	-0.78	5.772	5.791	0.018	0.018	-1.9 ± 2.2

Table 21.3. Summary results of the checks performed on the constant stars listed in Table 21.2. $\Delta_1 = H_{\min} - H_{\max}$, $\Delta_2 =$ its expected value with σ_{transit} of Table 21.2.

The fitted gradient of the trend β is given together with its error in the last two columns of Table 21.3. The trend is just above the 3σ level for the same three stars for which the range was found anomalous. A trend between 4×10^{-6} to 9×10^{-6} mag per day is equivalent to a total systematic effect in the range 5 to 10 millimag over the mission. For the other stars the trend is totally negligible which confirms at the same time that these stars are not long period variables and that the Hipparcos photometric system does not change with the time. The fact that few stars exhibit a trend does not invalidate this conclusion: as was stated above, trends can be the result of errors in the colours used in the data reductions, and can occasionally be related to actual astrophysical phenomena. As long as the average star does not show a significant trend, it can be concluded that the photometric system is stable.

21.6. Comparison with the Walraven Photometric System

The Walraven photometric system (Lub & Pel 1977) is amongst the best internally calibrated systems. A complete recalibration of all available data has been undertaken by Lub & Pel, based upon a consistent programme of standard star measurements made between 1980 and 1985. This programme resulted in the complete redefinition of the *VBLUW* system of standard stars as it was available on the Dutch telescope at La Silla between 1979 and 1991. The system available prior to 1979 in South Africa, though closely related to the La Silla one, has to be treated separately and will not be further discussed here. Based upon the reductions which Lub provided for all available data, Pel produced a list of in total 1972 'calibration' stars of high quality observations, and



Figure 21.12. Comparison of Walraven photometric V-band measurements with Hipparcos median Hp magnitudes. Above: the direct comparison of the data (the V_W is expressed in log I), and the fitted curve. Below: the residuals left after subtracting the fitted curve.

spanning a large range of intrinsic properties, as a preparation to a full rediscussion of the merits of the *VBLUW* photometric system.

Pel (1991) applied this basic catalogue to an intercomparison between the four photometric systems with most data available in the southern hemisphere. Whereas the comparison with the Cousins' *V* magnitude and (B - V) colour, and similarly with the Strömgren *y* and (b - y) versus Walraven *V* and V - B showed no clear systematic effects as a function of magnitude, colour, right ascension and declination, problems were encountered in the comparison with the Geneva *V* and (B2 - V1) (see also Section 21.7).

A comparison between two photometric systems will show that either one of the systems is having problems (without distinguishing which system), or that both systems are reliable at the level of detection. The chance of both systems being affected by exactly the same discrepancies is too small to be considered seriously, in particular when backgrounds of these systems are very different. Accordingly it was considered to be of great interest to compare the same high-quality catalogue with the Hipparcos photometry. Of the 1972 stars, 1720 were identified as Hipparcos stars (only the 1948 stars with HD numbers were checked), and of these 944 had a χ^2 based probability of being constant higher than 20 per cent.

In the comparison only 843 stars of luminosity classes III to V were used, after verification that there were no systematic effects observed between stars of these classes. Stars



Figure 21.13. The differences between Hp and transformed V_W magnitudes as a function of declination (top) and right ascension (bottom), showing no signs of dependence of residuals with position on the sky.

detected as possibly variable had been excluded. Comparisons were made with respect to median Hp magnitudes in the Hipparcos system. In the Walraven system, all data are represented as $\log(I)$ values, which requires a multiplication by -2.5 to obtain the equivalent in magnitudes.

Figure 21.12 shows the comparison between the Walraven V and the Hp magnitudes, as a function of the Walraven (V - B) index. As the system was originally intended for the study of early type stars, to which the sensitivity to a three-dimensional classification for the F and G stars was added later, the majority of comparison points are found among the B, A, F and G type stars. To every point an error was assigned, based on the error on the median as given in the Hipparcos catalogue, and the error on the mean for the Walraven data (the quoted standard deviation divided by the square root of the number of observations). The errors were dominated by the contribution from the Walraven system. The data were subsequently fitted with a cubic spline function, using 6 knots. The unit-weight standard deviation after this fit was 1.7, which may reflect the length of time covered by the Walraven data (> 10 years). There is also an indication that some of the remaining dispersion is still related to astrophysical properties: a comparison between the (B - U) colour index and the remaining residuals showed some (difficult to represent) systematic behaviour. The remaining noise is at a level of 0.003 mag, which can be considered as very good for both the Hipparcos and the Walraven system. Systematic differences are well below the 0.001 mag level.

Figure 21.13 shows the residuals as a function of right ascension and declination. In some photometric systems there are small but significant calibration inconsistencies as a function of position on the sky, due to the complicated process of establishing a fully reliable calibration sequence in the presence of seasons and a fixed location. The

B-V	Corr	B-V	Corr	B-V	Corr	B-V	Corr
-0.50	-0.0083	0.20	+0.0037	0.70	+0.0041	1.30	-0.0072
-0.40	-0.0068	0.30	-0.0007	0.80	+0.0032	1.35	-0.0086
-0.30	-0.0053	0.35	-0.0024	0.90	+0.0030	1.40	-0.0088
-0.20	-0.0034	0.40	-0.0018	1.00	+0.0020	1.50	-0.0070
-0.10	-0.0015	0.50	+0.0013	1.05	-0.0010	1.60	-0.0023
0.00	+0.0003	0.60	+0.0049	1.10	-0.0022	1.70	+0.0028
0.10	+0.0022	0.65	+0.0053	1.20	-0.0045	1.80	+0.0080

Table 21.4. Corrections needed to transform the *V* magnitudes in the main catalogue (Field H5) to Johnson V_J magnitudes for luminosity class III to V and spectral type O to G5.

Walraven system appears to be free of systematic effects down to a level of at least 0.001 mag, and the same has to be true for the Hipparcos system (see also Pel 1991).

This comparison is, due to its limited range in colour, only providing information on the system quality for relatively blue stars (B - V < 1.3), and it requires other systems to make similar comparisons for redder stars. However, the very good level of agreement shows that the Hipparcos system can be considered as internally consistent and very reliable for at least all but the most extreme-colour stars.

21.7. Additional Comparisons with Ground-Based Systems

The Magnitude Scale

The comparison of Hp magnitudes to ground-based data required a reduction of Hp to Johnson V_J magnitude through a single relation for stars earlier than G5 stars or with (B - V) < 0.9, and relations distinct for giants and dwarfs of later type. The colour corrections are expressed as function of (V - I) and lead to the V magnitude as given in Field H5 in the main catalogue (see Volume 1, Sections 1.3 and 2.1). The set of standard stars was obtained by reducing all data from the major photometric system to Johnson V, (B - V) and Hp. The comparison of V, as provided in Field H5, with genuine UBV data shows small chromatic residuals (Table 21.4). They are to be added to the V given in the Hipparcos catalogue to obtain the Johnson V for O to G5 stars and later type red giants. Similar tables must be used to reproduce V magnitudes from different photometric systems.

Once corrected for chromatic residuals, V magnitudes from Hipparcos may be checked against ground-based data. The mean difference $V_J - V(Hp)$ is plotted as function of the visual V_J magnitude in Figure 21.14, retaining stars in common with Geneva photometry and *UBV* photometry respectively, and excluding M giants and variable stars. The common feature is a small but significant non-linearity with a mean slope of -0.0017 mag/mag in the range V = 3-9 mag. The bump around V = 4.7 mag is present both with *UBV* and Geneva data. This global behaviour is not understood and may be due to ground-based data as well as to Hipparcos instrumentation. The occurrence of similar departures for two distinct photometric systems is an indication that space magnitude scale may be slightly distorted. Uncertainty on the intensity transfer function would affect only the brightest stars.



Figure 21.14. The mean errors V(ground)-V(Hp) as a function of the apparent V magnitude from Geneva photometry (filled symbols) and from UBV photometry (open symbols).

By construction the Hipparcos Hp system is tied to the UBV system by the relation: $Hp = V_J$ for B - V = 0.000 mag. With the above mentioned chromatic corrections the mean difference $\langle V_J(UBV) - V(Hp) \rangle$ is only -0.00006 mag for 15 386 non-variable comparison stars.

The Brightest Stars

The Hp magnitudes of the brightest stars show some departures from the expected magnitude computed from ground-based Johnson V_J or Geneva V(Gen), and colour correction (Hp - V) derived from $(B - V)_J$ or $[B - V]_G$. The departure Hp(obs) – Hp(true) reaches 0.005 mag at V = 3.5 mag, and 0.025 mag at $V \simeq 2$ mag. A few stars around V = 0 mag show departures up to 0.04 mag. Hp magnitudes are slightly underestimated in the range 0 to 4 mag and overestimated if brighter than $V \simeq 0$ mag. This non-linearity effect is attributed to the satellite detection chain calibration and not to the ground-based photometry since the departures between observed and predicted Hp magnitudes are the same for two independent data sets, UBV and Geneva, obtained with different detectors.

The brightest stars are often used as spectro-photometric standards, like HIP 91262, α Lyr. The estimated *Hp* for the three brightest stars are given in Table 21.5 together with data from Geneva and *UBV* photometry. The other bright stars, with V > 0 mag, have observed *Hp* coherent with the prediction from ground-based photometry. For Sirius (HIP 32349), the photon flux from Hipparcos is underestimated by about 44 per cent. The reason for this discrepancy was the failure of the analogue mode for the image dissector tube (see Section 5.1), which forced the observations of Sirius to be made in

Name	HIP	Нр	σ_{Hp}	<i>Hp</i> (Gen)	Hp(UBV)
Canopus	30438	-0.5536	0.0066	-0.678	-0.652
Sirius	32349	-1.0876	0.0024	-1.422	-1.453
Vega	91262	0.0868	0.0021	0.048	0.030

Table 21.5. Magnitudes for the brightest stars measured by Hipparcos, compared with measurements in the Geneva and the *UBV* systems.

photon counting mode, even though it was evident that counts would be seriously affected by saturation.

The mean *V* magnitude for the primary standard Vega, HIP 91262, in *UBV* photometry is $V = 0.033 \pm 0.003$ mag with a range from 0.01 to 0.06 mag and few discrepant values up to 0.16 mag. Geneva $V = 0.028 \pm 0.004$ mag is consistent with the magnitudes as measured in the *UBV* and *WBVR* systems: 0.028 mag. A mean value $V_J = 0.030$ mag is adopted to tie the Hipparcos magnitude scale to the spectrophotometric data. With B - V = -0.001 mag, $Hp = 0.030 \pm 0.001$ mag for Vega. Table 21.5 provides a summary of the relevant photometric data for the brightest stars.

Photometric Precision

With the exception of a possible, small, systematic magnitude drift, the Hipparcos photometry shows the highest accuracy ever achieved in stellar photometry. At the time of the Hipparcos Input Catalogue compilation only 49 per cent of these stars had photoelectric magnitudes and colours. The remaining stars had photographic or visual magnitudes. The cumulative distributions of standard errors on V magnitude for non-photoelectric, photoelectric and Hipparcos Hp magnitudes are shown in Figure 21.15. The transition from classical photoelectric photometry to the Hipparcos era represents a jump in accuracy similar to that from photographic to photoelectric, i.e. about one order of magnitude.

Magnitude Homogeneity

The intercomparison of stars over great circles of various orientations ensured the homogeneity of Hp magnitudes, free from systematic errors as a function, for example, of the star coordinates. The comparison with ground-based data reveals errors on classical photometry rather than on Hp magnitudes. For example the distribution of V residuals with respect to the declination, Figure 21.16, shows very small systematic errors as a function of declination, but an increase of quality for UBV data when obtained from southern sites, i.e. for $\delta < 10^\circ$. With a mean error on V(Hp) of 0.0032 mag, the external error on V_J appears to be about 0.026 mag.

The distribution of residuals over the whole sky is displayed in Figure 21.17 for V from UBV system. No large scale systematic errors, of the order of 1 per cent are noticeable although local departures are present. The comparison with Geneva magnitudes, Figure 21.18, shows little scatter locally over the sky but a modulation as a function of right ascension in the southern hemisphere with a peak to peak amplitude of 0.008 mag. The homogeneity of the Hipparcos photometric reference system is high enough to allow a complete re-evaluation of the star magnitudes as measured in the various photometric systems.



Figure 21.15. The cumulative distribution of standard error on Hp magnitude (left curve), on photoelectric V magnitude (central curve) and on photographic or related visual magnitude (right-curve). The vertical jumps corresponds to assumed errors for single source data.



Figure 21.16. The distribution of the residuals $V_J - V(Hp)$ for stars in common with UBV photometry and V - I < 1.8, as a function of declination.



Figure 21.17. The residuals $V_J - V(Hp)$ as a function of equatorial coordinates.



Figure 21.18. The residuals $V_G - V(Hp)$ as a function of equatorial coordinates.



Figure 21.19. The residuals V(Gen)-V(Hp) for stars with declination below 10° (bottom) and above 10° (top), as a function of right ascension.

21.8. Conclusions

Unlike the situation that prevailed with the relative astrometry and photometry of the double stars, or to a lesser extent with the absolute astrometry of single stars, there was no real possibility of assessing the external quality of the photometric treatment from a comparison on a star-by-star basis, to ground-based data of comparable quality, for lack of material and the difficulty in making the transformation of the photometric system with the required accuracy. Therefore the various methods of validation attempted in this chapter had to rely mostly on the internal consistency rather than on an analysis of differences between Hipparcos photometry and independent external measurements. This was, however, possible on less quantitative aspects, such as the constancy of a limited set of bright stars or the comparison of the period of variables concluded from the Hipparcos data to their ground-based counterpart.

The comparisons do not reveal major shortcomings in the Hipparcos evaluation of the magnitudes and the estimate of the internal errors, except maybe for the very bright

stars (Hp < 3 mag) where the errors at the transit levels are probably a bit optimistic on the average. In this range as well, the Hp values may suffer systematic effects and should be used with care. The period determination appears of remarkable quality despite the relatively short time base for the long period variables, and the unfavourable observation window in many cases. Finally there are several pieces of information not evaluated here, but which constitute a major asset of the Hipparcos Photometric Catalogue: the alternate photometric scale ac, the indication for every transit of the complementary field of view and the identification of possible perturbing objects from the Guide Star Catalog. The value of this information should not be underrated in the astrophysical applications.

F. Mignard, M. Grenon, F. van Leeuwen

22. ANALYSIS OF DOUBLE STAR RESULTS

A solution for the relative astrometry and photometry of double stars has been obtained from the Hipparcos observations, strictly based on the methods introduced in Chapter 13 of this volume. Although the Hipparcos processing provides an indication of the internal error, a better evaluation of the true external error can only be obtained by comparison with ground-based observations of comparable accuracy. We discuss in this chapter two such comparisons: the first analyses the results of the relative astrometry with respect to the best ground-based observations by speckle interferometry, for about 1000 stars common to Hipparcos and to the CHARA programme; the second investigates the photometric solution in relation to the CCD photometric observations carried out at La Palma over a sample of similar size common to both programmes.

22.1. Introduction

The details of the methods implemented by FAST and NDAC to determine the astrometry and photometry of double and multiple stars are given in Chapter 13 of this volume, along with the main properties of the solution. The precision for the relative astrometry (separation ρ and position angle θ) was shown to be mainly dependent on the magnitude difference but not very sensitive to the separation, at least for separations less than 15 arcsec. A fit of the median of the standard error of the separation yields the following useful formula for the precision as a function of the magnitude difference Δm :

$$\log \sigma_o \simeq \max(0.75, \ 0.5 + 0.3\Delta m)$$
 [22.1]

where the standard error σ_{ϱ} is expressed in milliarcsec (Figure 22.1). For the subset of easy double stars, with $\varrho \gtrsim 0.2$ arcsec and $\Delta m \lesssim 2$ mag, the separation could be obtained with a precision better than 10 mas and in many instances than 5 mas. For relative photometry, Hipparcos provides the best homogeneous and full-sky coverage for a sample of 12 000 systems with a precision of a few 0.01 mag. The precision of the magnitude difference is, again, primarily dependent upon the magnitude difference itself and to a lesser extent on the separation. For separations larger than 0.3 arcsec a smooth representation of the median of the standard error is given by:

$$\log \sigma_{\Delta m} \simeq -1.7 + 0.25 \Delta m \qquad [22.2]$$



Figure 22.1. Average standard errors of the relative astrometric (left scale) and photometric (right scale) solution of the binary stars as a function of the magnitude difference.

(see Figure 22.1). For smaller separations the typical standard error in Δm grows sharply with $1/\rho$ and the magnitude difference and separation become strongly correlated.

It is difficult to assess to what extent these internal errors are representative of the true errors, despite the effort of the data analysis groups to provide as realistic an evaluation of this error as possible. In addition, systematic errors both in astrometry and photometry are possible and very likely to exist especially at the two extremes: small separations ($\rho < 0.15$ arcsec) and large magnitude differences ($\Delta m > 3$ mag). The analyses presented in this chapter attempt to provide a more objective assessment of these errors through the results of a comparison of the Hipparcos results with the best ground-based data to date, the speckle astrometric measurements and the CCD photometric observations.

22.2. Relative Astrometry

Ground-Based Material

The only sizable set of observations of relative astrometry of multiple systems matching the quality of the Hipparcos data is provided by the speckle observations and occultation timings compiled in the various versions of the CHARA Catalogue. The following work is based on Version 3, available on the World Wide Web (Hartkopf *et al.* 1996). Preliminary comparisons were carried out on a small sample during the Hipparcos data reduction (Mignard *et al.* 1995) and based on a previous version of the CHARA Catalogue or on data published in the Astronomical Journal, and led to the evidence of a small bias in separation between Hipparcos and the ground-based observations for separations above 0.6 arcsec. All these observations are included in the present analysis.

To be more precise, the third CHARA Catalogue includes all measures of binary and multiple star systems obtained by modern high-resolution techniques (speckle interferometry, photoelectric occultation timings) as well as negative examination for duplicity, as of December 1995. For each observation reported, there is an indication of the observer and the method employed. Each system is identified by one or several of the following identifiers: ADS, HR, HD, SAO, WDS. The observation records give the date of observation, position angle, separation (or upper limit when no detection was possible), and, in some cases, an indication of the error. When available, the magnitude of each component is also given, but these data were not used for the present analysis because they are too scarce and have a relatively low accuracy compared to the CCD observations made at La Palma (Section 22.3).

Cross-Identifications

The Hipparcos identification number is obviously not among the various identifiers found in the CHARA Catalogue and it must be searched for by cross-identification. Unfortunately, none of the above identifiers is alone sufficient to find all the Hipparcos stars which are in the CHARA Catalogue. In principle the WDS identifier, available for every entry, should allow the correct identification of all the systems to be found by using properly truncated J2000 coordinates or the CCDM number when the system is known to be double in the Hipparcos Catalogue. While easy to implement, this method is not very reliable, yielding too many wrong identifications simply because of differences of one or two units in the last digit of rounded right ascension or declination between the WDS identifier and the Hipparcos truncated positions.

As a consequence systems clearly in the Hipparcos and CHARA Catalogues went unnoticed (i.e. were not recognised as Hipparcos objects) or were given a false Hipparcos identifier, only because the coordinates of two Hipparcos entries were a few minutes apart. (An example of such a situation arises with HIP 162 and HIP 171, which are different components of a wide system.) This method of identification was not used in a systematic way but only as a last resort after all the other identifiers had been exhausted. The other identifiers like HD or SAO do not suffer from this drawback, but unfortunately do not cover all the Hipparcos entries. Eventually all these possibilities were used in sequence and the final files were merged into a single one without redundant identifications.

There are 6280 entries in the CHARA Catalogue representing about 22 000 individual observations. Five thousand of these entries have a counterpart in the Hipparcos catalogue and 2100 are associated with at least one positive and reliable observation of separation and position angle. The number of systems with a double star solution in the Hipparcos catalogue is obviously smaller, of the order of 1700. Accurate numbers are given in the last column of Table 22.1. The 400 remaining systems are mainly close binaries and therefore not detected as non-single by Hipparcos.

Few systems were solved with more than two components from the Hipparcos observations (there are 249 entries related to triple and quadruple systems). On the other hand the CHARA compilation provides in several instances the observations of individual components for systems with three or more components, in which one particular pair may be associated with a Hipparcos double star solution. Because of the difficulty in making a safe and automatic identification of these components, the CHARA systems with more than two components potentially resolvable by Hipparcos were not considered in the comparison. This sample was in any case too small to affect the conclusions of this investigation.

Categories (see text)	1	2a	2b	3a	3b	Total
Entries in common	906	192	866	104	38	2106
With Hipparcos double star solution	765	159	637	95	24	1680
$\Delta \varrho < -50 \text{ mas}$	18	3	35	3	1	60
$\Delta \varrho > +50 \text{ mas}$	37	4	62	5	6	114
Systems used in the analysis	710	152	540	87	17	1506

Table 22.1. Content of the selected comparison data set between Hipparcos and the CHARA Catalogue.

Astrometric Parameters

To be significant the comparison between the speckle measurements (ρ_S , θ_S) and the Hipparcos observations (ρ_H , θ_H) must be based on nearly contemporaneous observations. As most of the systems considered in CHARA have separations less than one arcsec, and many below 0.3 arcsec, the orbital motion could be large enough to prevent a meaningful comparison at the level of one milliarcsec in the case when the Hipparcos and CHARA epochs differ by more than a few months. Several observing missions collected speckle data at the end of 1990 or in 1991, quite close to the Hipparcos Catalogue epoch of J1991.25, making the epoch difference negligible. In addition not every star in the CHARA sample exhibits a significant annual orbital motion, so that observations carried out a few months before or after the Hipparcos epoch are nonetheless useful in such a comparison. Finally when two CHARA observations bracket the Hipparcos epoch, an interpolated position could be computed at the Hipparcos epoch, provided the orbital motion was not too large over the bracketed interval.

The comparison data set was eventually separated into five categories according to the reliability of the estimation of the separation at the Hipparcos epoch:

- (1) systems with at least two CHARA observations bracketing the Hipparcos epoch. Two kinds of linear interpolations were tried: the first in Cartesian coordinates $X(t) = \rho_S \sin \theta_S$ and $Y(t) = \rho_S \cos \theta_S$, from which the separation and position angle were then computed for the Hipparcos Catalogue epoch $T_0 = J1991.25$; the second method interpolated directly the polar coordinates. It is obvious that for a nearly circular motion the latter is preferable while for a nearly linear motion with a large excursion in position angle the interpolation in rectangular coordinates gives better results. For objects showing a motion of few degrees in position angle the two methods are equivalent at the milliarcsec level. It was found that the statistical analysis did not depend very much on the interpolation method, although for a small number of systems the cartesian and polar interpolation in Cartesian coordinates;
- (2) systems for which the last CHARA observation was made earlier than T_0 , with the following two sub-cases:
 - (2a) the last observation was made after $T_0 0.5$ yr;
 - (2b) the last observation was made before $T_0 0.5$ yr;



Figure 22.2. Difference in the apparent separation of double stars observed by Hipparcos and speckle interferometry. A smooth curve has been fitted to the data points to show the systematic differences. The difference is defined in the sense Hipparcos minus speckle.

- (3) systems for which the first CHARA observation was made later than T_0 , with the following two sub-cases:
 - (3a) the first observation was made before $T_0 + 0.5$ yr;
 - (3b) the first observation was made after $T_0 + 0.5$ yr.

In case 2 the last observation found in the CHARA Catalogue was retained for the comparisons, and in case 3 the first observation in the CHARA Catalogue was used. Attempts to extrapolate from a polynomial fit over the last (case 2) or first (case 3) two or three observations were rapidly abandoned as being too difficult to handle safely.

For all the observations the possible 180° ambiguity between the speckle observations and the Hipparcos solutions was removed by adding 180° to the position angle of the speckle data (θ_S) whenever $\cos(\theta_S - \theta_H) < -0.85$. It turns out that among the 949 'good' systems of categories 1, 2a and 3a appearing in the last line of Table 22.1, the differences in position angle left no room for ambiguity as to when a 180° shift had to be applied. The actual distribution of $\Delta \theta = \theta_S - \theta_H$ had a core of 771 systems with $-20^\circ < \Delta \theta < 20^\circ$ and then two distinct small populations at ±180° with respectively 95 and 83 systems. These are indeed rather small numbers considering the difficulty of removing the 180° ambiguity in the speckle observations.

The content of each category is shown in Table 22.1. In some cases the components considered in the Hipparcos solution were not the same as the two components given in CHARA. For the subsequent statistics all the systems for which the difference in the separation supplied by Hipparcos and CHARA was larger than 50 mas were removed, the difference being considered as too large to belong to the statistical distribution. This indicates an incorrect identification of the system or of the components of a multiple system, or was the consequence of an invalid interpolation of the CHARA data to the Hipparcos epoch. In Table 22.1 the occurrence of more cases of $\Delta \rho$ in the positive wing follows from the fact that the difference was taken in the sense $\rho_{\rm H} - \rho_{\rm S}$ and that for multiple hierarchical systems CHARA usually refers to the close pair and Hipparcos to the distant component. Such systems obviously had to be removed from the comparison sample.



Figure 22.3. Distribution of the differences in separation between the Hipparcos observations and the speckle measurements interpolated at the Hipparcos epoch when possible (see text). The difference is defined in the sense Hipparcos minus speckle.



Figure 22.4. Reduced distribution of the difference in separation between the Hipparcos observations and the speckle measurements. The solid line is the normal distribution of zero mean and unit variance. The difference is defined in the sense Hipparcos minus speckle.

Results and Analysis of the Comparisons

For each of the above categories various analyses were carried out. Table 22.2 summarises the results for the differences $\Delta \rho = \rho_{\rm H} - \rho_{\rm S}$ for each category. As expected the best results are obtained in the first group, when an interpolated separation was computed at T_0 . The scatter in $\Delta \rho$ measured by the standard deviation is 8.8 mas, close to the typical error of the Hipparcos measurement of 6–7 mas for this sample of bright stars with small magnitude difference (Figure 22.1). The slight bias is hardly significant for a population of 700 objects. The scatter is slightly larger in categories 2a and 3a, with the selected observations within six months of the Hipparcos epoch and is much larger for the other two populations. In this last two cases the filtering at $|\Delta \rho| < 50$ mas makes the scatter somewhat too optimistic, since differences may fall in the range 50 to 100 mas.

						_
Categories (see text)	1	2a	2b	3a	3b	
Number of systems considered	710	152	540	87	17	
Mean of $\Delta \varrho$ in mas	+0.5	+0.8	+1.6	+1.1	-1.9	
Median of $\Delta \varrho$ in mas	0	+2	+2	+1	-4	
Standard deviation of $\Delta \varrho$ in mas	8.8	9.1	14.7	11.7	13.8	

Table 22.2. Summary statistics of the comparison of the separations between Hipparcos and speckle interferometry.

The plot in Figure 22.2 shows the data points as a function of $\rho = (\rho_{\rm H} + \rho_{\rm S})/2$ and the running median as a solid line. There are three regimes in this plot. At the smallest separations (< 0.2 arcsec, close to the Hipparcos detection limit), there is a systematic difference of 3–4 mas but also more scatter in the data than for the larger separations. For separations between 0.2 and 0.6 arcsec, there is no noticeable systematic difference. For still larger separations, there is again an increasing systematic difference, reaching a maximum of about 3–4 mas at $\rho \simeq 1$ arcsec. If this latter effect is real, its origin in either the Hipparcos or the speckle data is still unknown, and its full understanding requires further investigation.

Another presentation of the residuals is shown in the histograms of Figure 22.3 and Figure 22.4. The first diagram represents the distribution of the differences in separation in mas, while the second histogram gives the reduced distribution, determined by computing for each star the scaled difference as:

$$\frac{\varrho_{\rm H} - \varrho_{\rm S}}{\sqrt{\sigma_{\rm H}^2 + \sigma_{\rm S}^2}}$$
[22.3]

with $\sigma_S = 3$ mas. For a normal distribution of the errors with the above variances, the scaled difference should follow a normal law with zero mean and unit standard deviation, shown by the solid line in Figure 22.4. The standard deviation of the observed scaled difference is however 1.15, slightly larger than expected and primarily due to the populated tails rather than the distribution between -2 and +2. If the standard errors in the speckle observation are accepted to be less than 5 mas, including the uncertainty induced by the interpolation at T_0 , this may indicate that the quoted Hipparcos errors are too small by about 15 per cent, at least to account for the wings of the distribution.

The comparison in position angle shows that there is no systematic orientation difference larger than $0^{\circ}.05-0^{\circ}.1$. The scatter diagram in Figure 22.5 shows $\rho\Delta\theta$ as a function of the separation with the median smoothed out in the solid line. A systematic difference in orientation would show up as a trend in $\rho\Delta\theta$ such that a difference of 1 mas for $\rho = 1$ arcsec would correspond to $0^{\circ}.05$ in orientation. The other features in this diagram less than 1 mas are not significant.

The summary statistics of $\rho\Delta\theta$ are given in Table 22.3 for each of the categories. The number of systems in each category is smaller than the corresponding numbers in Table 22.2, because an additional filtering has been applied whenever $|\rho\Delta\theta| > 50$ mas, to be consistent with the analysis of separations. As expected the same behaviour as in Table 22.2 is observed, with the smallest scatter in the first category linked to the interpolated positions. However the standard deviations are larger than in the case of $\Delta\rho$, which could be explained by the fact that the apparent orbits are more circular than



Figure 22.5. Difference in the position angle measured by $\rho \Delta \theta$ in mas for the double stars observed by Hipparcos and speckle interferometry. A smooth curve has been fitted to the data point to show the systematic differences. The difference is defined in the sense Hipparcos minus speckle.

Table 22.3. Summary statistics of the comparison of the position angles between Hipparcos and speckle interferometry.

Categories (see text)	1	2a	2b	3a	3b
Number of systems considered	695	152	507	86	16
Mean of $\rho\Delta\theta$ in mas	-1.1	+1.1	+1.7	+0.1	-0.6
Median of $\rho\Delta\theta$ in mas	-0.5	+1.2	+2.1	+0.1	+1.8
Standard deviation of $\rho\Delta\theta$ in mas	10.4	11.8	14.4	10.5	17.2

elongated ellipses and an error of a few months between the Hipparcos and CHARA shows up primarily in $\rho\Delta\theta$ rather than in $\Delta\rho$.

Figure 22.6 shows the difference of relative position of the secondary with respect to the primary computed as:

$$\Delta X = (\varrho_{\rm H} \sin \theta_{\rm H} - \varrho_{\rm S} \sin \theta_{\rm S})$$
[22.4]

$$\Delta Y = (\varrho_{\rm H} \cos \theta_{\rm H} - \varrho_{\rm S} \cos \theta_{\rm S})$$
[22.5]

There is no preferred orientation in this plot and the standard deviations in each direction are nearly identical, respectively 10.2 and 10.4 mas.

22.3. Relative Photometry

Through the history of the double star observations little attention has been given to provide magnitude differences of high quality, at least in comparison with the efforts made to get reliable astrometry. Lately CCD observations have dramatically changed the situation with the possibility of processing digitised images with good photometric calibrations. A large amount of such data has been made available by A.N. Argue *et al.* (1992) from observations carried out at La Palma in 1986–1987.



Figure 22.6. Relative position on the tangent plane on the sky between the position of the secondary with respect to the primary obtained by Hipparcos and the speckle interferometry. The difference is defined in the sense Hipparcos minus speckle.

More than 2300 systems were observed with separations typically larger than 0.7 arcsec and usually in the range 1 to 5 arcsec. Among them, there are 1360 systems of the Hipparcos programme. These were all detected as non-single from the Hipparcos observations and solved for the astrometry and photometry with solutions of higher quality than the average, primarily because there are no close binaries in the sample. The magnitude difference range covers the whole Hipparcos range up to $\Delta m = 4$ mag and the reported accuracy is typically between 0.01 and 0.02 mag, smaller by a factor two to four than the Hipparcos standard error for this sample as seen in Figure 22.1.

The photometric system used in the La Palma observations is different from Hipparcos and the comparison of the magnitude differences cannot be done directly. Argue and his colleagues provide component magnitudes in the V and R wavebands of the Landolt photometric system based on the Johnson *UBV* and Cousins' *RI* systems. Writing the link between the *BV* and *Hp* bands as the function:

$$Hp - V = f[(B - V)]$$
 [22.6]

the magnitude difference between the components *A* and *B* is given by:

$$\Delta m_{\rm LP} = \Delta m_V + f [(B - V)_B] - f [(B - V)_A]$$
[22.7]

where Δm_{LP} is the La Palma magnitude difference in the *Hp* band. For main-sequence stars there is a correspondence between B - V and V - R, which in principle allows the transformation of the V - R of each component as measured by Argue *et al.* into B - V. Only systems with $|\Delta m_V - \Delta m_R| < 0.4$ mag have been selected in order to ensure that Equation 22.7 gave a good approximation to the Hipparcos system. Finally the risk of possible misidentification was limited by excluding from the analysis all the systems with differences in separation between Hipparcos and La Palma larger than 0.3 arcsec. In the end, the comparison sample reduced to 958 systems.

The main results of the comparison are plotted in the two diagrams of Figure 22.7 with $\Delta m_{LP} - \Delta m_{H}$ in ordinate and, in abscissa, the magnitude difference (upper panel)



Figure 22.7. Comparison of the magnitude difference between the components of double stars observed by Hipparcos and by ground-based CCD at La Palma, shown as a function of the magnitude difference (upper diagram) and of the separation (lower diagram).

and separation (lower panel). The average of $\delta(\Delta m) = -0.002$ mag and the standard deviation is 0.13 mag. Up to $\Delta m \simeq 3$ mag there are no systematic differences between the Hipparcos and ground-based measurements. Neither is the separation a factor affecting the difference, at least for $\rho > 1$ arcsec. For smaller separations the scatter is larger and is more likely due to the CCD observations which are less reliable in this range.

Regarding the distribution of the reduced differences:

$$\frac{\Delta m_{\rm LP} - \Delta m_{\rm H}}{\sqrt{\sigma_{\rm LP}^2 + \sigma_{\rm H}^2}}$$
[22.8]

plotted in Figure 22.8, it is quite different from a normal law of zero mean and unit variance. There is a small systematic zero effect of -0.2 mag in reduced values, or -0.01 mag in unscaled values, which is acceptable. The scatter of the distribution however is much larger than expected if the quoted standard deviations are real estimates of the random errors. A normal curve with standard deviation 1.4 provides a good fit to the central part of the observed distribution, but does not account for the tails. Clearly the reduced distribution is not Gaussian and exhibits extended wings. The effect introduced by the difference between the two photometric systems is probably



Figure 22.8. Histogram of the reduced difference of the relative photometry between Hipparcos and the CCD observations of La Palma. The dotted line is a normal distribution of zero mean and standard deviation of 1.4, which best fits the core of the data.

non-negligible at the level of few 0.01 mag and contributes also to increase the random scatter in a way hard to quantify. Another natural source of increased scatter is stellar variability. While the Hipparcos data are averages over a three-year period, the La Palma measures are much more liable to instantaneous deviations in magnitude.

22.4. Conclusions

The above comparisons have been restricted to the relative astrometry and photometry, for which ground-based data of comparable quality exist. The comparisons confirm the excellent overall quality of the Hipparcos results in the astrometry of double stars. The comparison of absolute astrometry was not possible at the same level, because of the lack of an independent sample matching the Hipparcos quality. However there is no real difference between the absolute astrometry of single stars and that of double and multiple stars, except that the latter are not as accurate. The confidence in the astrometry of single stars applies equally well to the double and multiple stars. In particular there are no reasons to suspect that the quoted standard errors are underestimated by more than 10 to 20 per cent.

For the photometry the situation is not so clear and illustrates the loss of accuracy in disentangling the complex signal of a multiple system into that of its components. While the photometry of the single stars (Chapters 14 and 21 of this Volume) is precise and accurate and limited primarily by the photon noise for star fainter than 8 mag, no such feat was achieved for the relative photometry of double and multiple systems. However, it was not possible to assess exactly what kind of systematic effects are to be expected and whether the overall underestimation of the standard errors applies equally to all the stars.

F. Mignard, C. Martin

23. FUTURE PROSPECTS

23.1. The Merits of a Scanning Astrometric Mission

A concept for a future space astrometry mission based on an extrapolation of the principles adopted by Hipparcos has recently been formulated (Lindegren & Perryman 1996), and has been recommended for further study within ESA's long-term scientific programme. A small interferometer, with a baseline of about 2.5 m, and equipped with CCD detectors, should be capable of measuring the astrometric parameters of every object down to 15 mag or fainter (some 50 million or more), with an accuracy of some 10 microarcsec at 15 mag or some 2–3 microarcsec at about 10 mag. Instrumental optimisation could lead to the measurement of a significant proportion of objects down to 20 mag, with an improved accuracy of about 2–3 microarcsec at 15 mag.

Scientifically, the attractions of such a mission are very broad. Distances of objects throughout the Galaxy would be measured (with a 10 per cent accuracy at distances of the galactic centre), and space velocities would be acquired with an accuracy of around 1 km/sec even at 20 kpc. In addition to the detailed motions and properties of individual stars and stellar groups throughout the Galaxy, metric terms would be directly measurable (with a precision in the PPN parameter γ of the order of 1 part in 10⁶), and planetary companions of a few Jupiter masses would be observable out to a few hundred parsecs. The appeal of such large-scale, high-accuracy astrometric measurements, and the technological prospects of conducting them within the next one or two decades, provokes the question of the extent to which the Hipparcos experiences can be carried forward to space astrometry in the future.

In general terms, the measurements conducted by such a continuously scanning satellite can be shown to be almost optimally efficient, with each photon acquired during a scan contributing to the precision of the resulting astrometric parameters. Although every object down to the limiting magnitude of the Hipparcos instrument could not be observed, and significant inefficiencies resulted from the sequential mode of operation of the detector, a future mission would most probably be able to observe the objects passing across the field of view simultaneously, with every star above the corresponding signal-to-noise threshold ultimately contained within the final catalogue. The small conceptual appeal of being able to devote more observing time to a particular object of high scientific interest by means of a payload which can 'stop and stare' at a given region of sky appears to be completely outweighed by the very high accuracy that is achievable in any case on such a large number of objects. One of the scientific targets of a future astrometric mission, indeed, will be the large-scale dynamical motions of stars, associations, clusters, and galactic spiral arms, that can only be tackled by access to the distances and motions of large samples of stars.

The scanning satellite concept also leads directly to the construction of a global reference frame into which each object is placed in an absolute sense. One of the great merits of Hipparcos is that it generates a reference frame within which parallaxes and proper motions are rigidly defined. A future astrometric mission, reaching to 15 mag or fainter, would circumvent one of the problems faced by the Hipparcos mission in linking the resulting reference frame to an inertial system, through the direct observation of extragalactic objects. The wide separation of two separate viewing directions would be preserved, since it leads to the determination of absolute trigonometric parallaxes, and thereby circumvents the problem which has plagued ground-based parallax determinations, namely the transformation of relative parallaxes to absolute distances. The successful implementation of these concepts has been convincingly demonstrated by the Hipparcos mission.

The continuously scanning satellite approach leads to two further important attributes of the resulting data. The first of these is the wealth of photometric information that is acquired by an instrument which continuously scans the celestial sphere in a reasonably uniform manner. The calibrated photometric results from Hipparcos surpass in quantity, quality and uniformity the corresponding ground-based results acquired over many decades. The application of the photometric data to the study of stellar variability, and the direct astrophysical value of high-accuracy magnitudes and colours, is already evident from the Hipparcos results.

The other feature of the global astrometric data which is such an important pointer for the future is the capability of determining the astrometric parameters of double and multiple systems. Although posing a considerable and continual challenge to the instrument design, the data acquisition, the data analysis, and the final catalogue production, the wealth of information contained in the Hipparcos results provides an insight into the importance of double and multiple systems within the context of a future catalogue of 50 million objects with microarcsec accuracy. At this level, the complexity of the systems already evident in the Hipparcos Double and Multiple Systems Annex will be compounded, and a powerful observational system which samples the stellar images and their photocentric motions semi-continuously will reveal much about star formation, the initial and subsequent mass functions, *n*-body interactions, and many other details of stellar structure and evolution. The scanning satellite concept is important in that a semi-continuous sampling of the double or multiple star geometry is possible, and is again directly placed within the overall reference frame of the global catalogue.

Finally, in all of these considerations, it should be stressed that both for Hipparcos, and for an advanced mission based on similar concepts, the number of distinct astrometric observations per star is very much larger than the number of variables characterising the stellar motion. In this sense the overall instrument is self-calibrating, and the resulting astrometric parameters are determined along with estimates of their standard errors and correlations. This provides the possibilities of an accurate and unambiguous calibration of the instrumental geometry, and standard errors of the astrometric parameters which are expected to be a realistic indication of the true errors. For the rigorous scientific exploitation of the astrometric data such confidence in the error estimates is crucial.

23.2. The Space Astrometry Problem Revisited

Looking back on the many years of planning and execution of the data reductions for Hipparcos, it is easy to find instances where a somewhat different approach to the analysis of the satellite data might have been advantageous. In several cases more direct and accurate methods would certainly have been adopted, given the availability of present-day computing facilities. This experience must be taken into account in any future space astrometry project. In this context it is perhaps of some interest to reconsider the space astrometry problem in very general terms.

Stellar astrometric observations from space aim at the determination of a finite set of parameters describing the barycentric motion of each star. These parameters may be summarised in a vector of unknowns, **a**. The observations consist of instantaneous measurements of the centroids of stellar images on the detector, expressed in detector coordinates, such as slits or pixels, denoted *G* and *H*. Each observation, *k*, is therefore characterised by the time t_k , a measurement vector $\mathbf{g}_k = (G_k, H_k)'$, and associated statistics.

Very generally, the space astrometry problem can be formulated as the minimisation problem:

$$\min_{\mathbf{a},\mathbf{n}} \left\| \mathbf{g}^{\text{obs}} - \mathbf{g}^{\text{calc}}(\mathbf{a},\mathbf{n}) \right\|_{M}$$
[23.1]

where \mathbf{g}^{obs} is the vector of all measurements and \mathbf{g}^{calc} the vector of detector coordinates calculated from the astrometric parameters. The norm is calculated in a metric Mdefined by the statistics of the data, which in the general, non-linear case need not be Gaussian. In this equation \mathbf{n} is a vector of parameters which are of no direct interest to the astronomical problem at hand, but which are nevertheless required for a physically realistic modelling of the data and therefore have to be estimated along with the astrometric parameters. The practical formulation of the problem is mainly related to the specification of the 'nuisance parameters' \mathbf{n} , which naturally depends on the type of mission considered. Subsequently a continuously scanning satellite, such as Hipparcos or GAIA, will be assumed.

The modelling of the observables \mathbf{g} is done by three successive transformations: (1) from astrometric parameters to the celestial directions of the star at the instants of observation, using an astrometric model; (2) from celestial to instrumental frame directions using an attitude model; and (3) from instrumental directions to detector coordinates using an instrument model.

Astrometric Model

In the simplest case, as applied to most of the Hipparcos stars, the modelling of the satellitocentric direction to star *i* at time t_k depends on just five parameters intrinsic to the star, the so-called five astrometric parameters: α_i , δ_i , π_i , $\mu_{\alpha*i}$, and $\mu_{\delta i}$, referred to a given epoch and being defined with respect to the solar system barycentre. More generally, the stellar astrometric parameters could include, for instance, the orbital parameters of binary stars.

One of the important insights gained from the Hipparcos mission concerned the impact of stellar duplicity on high-accuracy astrometry, and the sometimes astonishing complications brought about by this well-known, but easily forgotten, phenomenon of the common stars. For the present discussion it is assumed that the vector \mathbf{a}_i includes whatever parameters are needed to represent the motion to the required accuracy.

Calculation of the observable (proper) direction of the star at an arbitrary instant requires a set of auxiliary data **e** which are regarded as known, i.e. not subject to improvement from the observations. Most importantly this set includes the barycentric ephemeris of the satellite. The transformation to proper direction, largely covered in Chapter 12, is written symbolically:

$$\mathbf{u}_{ik} = \mathbf{u}(\mathbf{a}_i | t_k, \mathbf{e})$$
 [23.2]

Note that the auxiliary data \mathbf{e} are not part of the nuisance parameters. Hence they are placed, with time, to the right of the bar in Equation 23.2, indicating that they are 'given'.

Continuous Attitude Model

The attitude specifies the instantaneous orientation of the instrument axes in the same celestial reference frame as used for the astrometric parameters. The instrument axes are defined by means of the celestial projections of certain reference points on the detector. Clearly the attitude angles enter as unknowns in the general problem. There are two rather different ways in which they can be handled: as discrete or continuous variables.

In the discrete case there is an independent set of (three) attitude angles for every instant t_k . Given that each observation provides two coordinates, a prerequisite for this model is that at least two observations are made at each instant. In principle the attitude parameters can be eliminated 'on the spot', leaving a set of equations representing the instantaneous relative measurements, e.g. in the form of the angular separations of stellar images expressed in detector coordinates. A pointing space observatory is the most obvious example where the discrete attitude model applies.

The continuous attitude model is only applicable to a scanning satellite. It describes the attitude in the form of continuous functions of time, using a reduced set of parameters **c**. These could be, for instance, the spline coefficients for the three attitude angles with respect to an analytical reference model. Provided that the actual attitude motion is sufficiently smooth, this model has a significant advantage over the discrete model, owing to the smaller number of parameters, or degrees of freedom, that have to be estimated. The optimum dimension of **c** is a compromise between the measurement-induced error and the modelling error. Considering the relatively short dynamical memory of the satellite it is reasonable to use an independent set of attitude parameters, **c**_{*i*}, for each time interval T_i of several hours.

Angular coordinates on the sky, measured with respect to the projected axes of the instrument, are called 'field angles' and denoted (η, ζ) . Given the proper direction to a star and the attitude parameters, the field angles of the object at the time $t_k \in T_j$ can be written:

$$\mathbf{f}_{ik} = \mathbf{f}(\mathbf{u}_{ik}, \mathbf{c}_j | t_k)$$
 [23.3]

where \mathbf{f}_{ik} is the vector of field angles.

Instrument Model

The final transformation is from field angles **f** to detector coordinates **g**, i.e. G, H; this is the field-to-detector transformation:

$$\mathbf{g}_{ik} = \mathbf{g}(\mathbf{f}_{ik}, \mathbf{d}_j | t_k)$$
 [23.4]

It depends on the instrument parameter vector \mathbf{d}_j describing the scale, detector orientation, optical and mechanical distortions, etc. The set of parameters \mathbf{d}_j is also assumed to be defined on the interval \mathcal{T}_j but may contain a subset which is constant over much longer times, e.g. for the medium- or small-scale distortion.

The practical formulation of the field-to-detector transformation is rather dependent on the hardware of the optics and detector system. The scan field mosaic of the Hipparcos main grid naturally led to a model with two components (Chapter 10): one fixed, medium-scale distortion pattern representing the physical deformations of the scan fields, and a variable, large-scale polynomial component capable of absorbing all kinds of optical distortion, chromaticity, etc.

Model Synthesis

The overall transformation can be written:

$$\mathbf{g}_{ik} = \mathbf{g}(\mathbf{f}(\mathbf{u}(\mathbf{a}_i|t_k, \mathbf{e}), \mathbf{c}_j|t_k), \mathbf{d}_j|t_k)$$

$$\equiv \mathbf{h}(\mathbf{a}_i, \mathbf{c}_j, \mathbf{d}_j|t_k, \mathbf{e})$$
[23.5]

The general minimisation problem thus becomes:

$$\min_{\mathbf{a},\mathbf{c},\mathbf{d}} \|\mathbf{g}^{\text{obs}} - \mathbf{h}(\mathbf{a},\mathbf{c},\mathbf{d}|t,\mathbf{e})\|_M$$
[23.6]

where the indices *i*, *j* and *k* have been dropped since the norm is to be computed over the whole range of the indices.

The observations are invariant with respect to a uniform, rigid rotation S of the celestial coordinate system. The rigorous formulation must therefore be such that:

$$\mathbf{h}(\mathcal{S}\mathbf{a}, \mathcal{S}\mathbf{c}, \mathcal{S}\mathbf{d} | t, \mathcal{S}\mathbf{e}) = \mathbf{h}(\mathbf{a}, \mathbf{c}, \mathbf{d} | t, \mathbf{e})$$
[23.7]

for any such transformation S of the parameter vectors. Only the instrument description, which does not involve celestial coordinates, can be assumed to be independent of this transformation: S**d** = **d**.

In the Hipparcos reductions this invariance was most strikingly demonstrated by the different choices of celestial reference frame—ecliptic versus equatorial—by the two consortia. On a more subtle scale it was manifested in the small global orientation and spin differences found after transformation to equatorial coordinates (Chapter 16). The discussion of the rank-deficiency problem in Chapter 11 showed that this invariance was not an obvious property of the data reduction problem in its usual formulation based on the so-called 'three-step' method (Chapter 4). One conclusion for the future is that the invariance with respect to uniform rotations should be carefully considered and built into the equations from the very start, resulting in minimally constrained solutions for the reference frame.

Method of Solution: Direct Approach

In Equations 23.3 and 23.4 the unknowns \mathbf{c}_j and \mathbf{d}_j were both taken to be defined over the interval \mathcal{T}_j of several hours. While the two parameter sets represent very different physical models, they are thus equivalent from a data processing point of view and may be considered together as parts of the 'local' vector of nuisance parameters, \mathbf{n}_j . However, as was remarked before, \mathbf{d}_j may contain a part which is common to a longer interval, or even the whole mission. These 'global' nuisance parameters may be separated out as the vector $\boldsymbol{\gamma}$, and \mathbf{n} may be redefined to contain only the 'local' nuisance parameters. Equation 23.5 is then recast as:

$$\mathbf{g}_{ik} = \mathbf{h}(\mathbf{a}_i, \mathbf{n}_j, \boldsymbol{\gamma} | t_k, \mathbf{e})$$
 [23.8]

and the general minimisation problem becomes:

$$\min_{\mathbf{a},\mathbf{n},\boldsymbol{\gamma}} \|\mathbf{g}^{\text{obs}} - \mathbf{h}(\mathbf{a},\mathbf{n},\boldsymbol{\gamma}|t,\mathbf{e})\|_M$$
[23.9]

It can be noted that this form, after linearisation, has the same general structure as the least-squares problems encountered in the great-circle reductions (Equation 9.5) and the sphere solution (Equation 11.23), and could in principle be handled by the same direct method as was used in those problems. That is, after sorting the data either chronologically (by the *i* index) or systematically (by the *i* index), the corresponding unknowns (\mathbf{n}_i or \mathbf{a}_i) may be eliminated, resulting in a rather dense system of normal equations for the remaining parameters. For Hipparcos the dimensions of **a** and **n** were, respectively, about 370 000 (the astrometric parameters for the primary reference stars, see Table 11.1) and ~ 2 000 000 (the number of spline coefficients and free instrument parameters in the FAST great-circle reductions). If the \mathbf{n}_i are successively eliminated, the direct solution of the remaining system requires of the order of $n^3/3 \sim 10^{16}$ floatingpoint operations, and the administration of $n^2/2 \sim 6 \times 10^{10}$ double-precision reals (\simeq 500 Gigabyte): a non-trivial task even for supercomputers and parallel processing. It was such considerations that lead to the idea of the 'three-step' decomposition proposed in 1976. However, the practicality of that method was gained at the expense of approximations which should now be avoided.

Global Iterative Solution

Apart from the 'three-step' method, the only alternative to the direct solution proposed to this date seems to be an iterative solution. The basic idea dates back at least to 1977, when Prof. Pierre Lacroute advocated the use of intermittent guiding of the satellite and the use of 'dynamical smoothing' in the quiet intervals. In his introductory talk at the 'Colloquium on European Satellite Astrometry', held in Padova in June 1978, the idea was formulated the following way:

... it is possible to represent the attitude motion during the periods of free motion by using the coordinates of the stars and all their transit times. With the help of mechanical laws the computed attitudes should be very accurate and by using them along with the transit times we could obtain better evaluations of the coordinates.

To iterate this procedure is an obvious possibility. The resulting method, which may be referred to as the 'global iterative solution', was subsequently proposed and studied by a group at the Istituto di Topografia, Fotogrammetria e Geofisica, Milano (Betti, Sansò *et al.*, in Perryman *et al.* 1989 Volume III, Chapter 28) and further discussed by Lattanzi *et al.* (1990). In the present framework it can be described as follows.

Let \mathbf{i}_i be the vector of all observations $\mathbf{g}_{jk}^{\text{obs}}$ of a particular star *i*, and similarly let \mathbf{j}_j be the vector of all observations made in the time interval \mathcal{T}_j . With *I* and *J* denoting the number of stars and time intervals, respectively, $(\mathbf{i}_1, \mathbf{i}_2, ..., \mathbf{i}_I)$ and $(\mathbf{j}_1, \mathbf{j}_2, ..., \mathbf{j}_J)$ are thus different partitions of the total observation vector \mathbf{g}^{obs} . In practice they could be obtained by sorting the observations according to star index or time, respectively, although this may not be necessary depending on the administration of the equations.

If, for a moment, the astrometric parameters **a** and the global parameters γ are regarded as known, or rather as 'given', it is a simple matter to solve, for each time interval *j*, the minimisation problem:

$$\min_{\mathbf{n}_{i}} \|\mathbf{g}_{ik} - \mathbf{h}(\mathbf{a}_{i}, \mathbf{n}_{j}, \boldsymbol{\gamma} | t_{k}, \mathbf{e})\|_{M}$$
[23.10]

involving only the observations \mathbf{j}_j and resulting in a linearised system of equations with $\dim(\mathbf{n}_j)$ unknowns, i.e. typically a few hundred. The solution to this problem may be formally written as the function $\hat{\mathbf{n}}_j(\mathbf{j}_j|\mathbf{e}, \mathbf{a}, \boldsymbol{\gamma})$. This problem is somewhat analogous to the attitude reconstruction problem discussed in Chapter 7.

Conversely, by regarding γ and the local parameters $\mathbf{n} = (\mathbf{n}_1, \mathbf{n}_2, ..., \mathbf{n}_J)$ as given, the astrometric parameters of each star are obtained by solving the problem:

$$\min_{\mathbf{a}_i} \|\mathbf{g}_{ik} - \mathbf{h}(\mathbf{a}_i, \mathbf{n}_j, \boldsymbol{\gamma} | t_k, \mathbf{e}) \|_M$$
 [23.11]

involving only the observations \mathbf{i}_i and resulting in a linearised system of equations with dim (\mathbf{a}_i) unknowns (typically 5). The solution to this problem, analogous to the astrometric parameter determination discussed in Chapter 11, may be written as the function $\hat{\mathbf{a}}_i(\mathbf{i}_i | \mathbf{e}, \mathbf{n}, \boldsymbol{\gamma})$.

Finally, if both the local and astrometric parameters are regarded as given, the global parameters may be obtained as the solution to the problem:

$$\min_{\boldsymbol{\gamma}} \|\mathbf{g}_{ik} - \mathbf{h}(\mathbf{a}_i, \mathbf{n}_j, \boldsymbol{\gamma} | t_k, \mathbf{e}) \|_M$$
 [23.12]

and denoted $\hat{\gamma}(\mathbf{g}^{\text{obs}}|\mathbf{e}, \mathbf{a}, \mathbf{n})$. This problem involves all the observations, but still results in a relatively small system of equations with dim(γ) unknowns.

The global iterative solution is a straightforward sequential application of the above (partial) solutions. The optimal sequence of the three estimators $\hat{\mathbf{n}}_j$, $\hat{\mathbf{a}}_i$, $\hat{\boldsymbol{\gamma}}$ is not obvious, but the following order seems intuitively natural:

 $\langle \mathbf{0} \rangle$

$$\mathbf{a}^{(0)} = \text{initial catalogue} \mathbf{\gamma}^{(0)} = \mathbf{0} \mathbf{n}^{(m)}_{j} = \hat{\mathbf{n}}_{j}(\mathbf{j}_{j}|\mathbf{e}, \mathbf{a}^{(m-1)}, \mathbf{\gamma}^{(m-1)}), \quad j = 1, 2, ..., J \mathbf{a}^{(m)}_{i} = \hat{\mathbf{a}}_{i}(\mathbf{i}_{i}|\mathbf{e}, \mathbf{n}^{(m)}, \mathbf{\gamma}^{(m-1)}), \quad i = 1, 2, ..., I \mathbf{\gamma}^{(m)} = \hat{\mathbf{\gamma}}(\mathbf{g}^{\text{obs}}|\mathbf{e}, \mathbf{a}^{(m)}, \mathbf{n}^{(m)})$$

$$[23.13]$$

If the iterations converge, the end result is evidently equivalent to a direct solution of the global minimisation problem, Equation 23.9.

Concerning the convergence properties, it can be noted that the linearised form of the procedure, written in the form of normal equations, is equivalent to the Gauss-Seidel iteration method for the solution of the linear system of equations. It is well known that this method converges for any symmetric and positive definite matrix. Due to the (theoretical) rank deficiency of the problem, this condition is in principle not satisfied.

However, it can be argued that the particular degeneracy due to the undefined reference frame is of no practical consequence for the iterative solution, since each of the partial minimisation problems (Equations 23.10–23.12) do not suffer from this degeneracy. The tentative conclusion is therefore that the method does converge, namely to the particular solution closest, in some sense, to the initial estimate $\mathbf{a}^{(0)}$, $\mathbf{n}^{(0)}$, $\boldsymbol{\gamma}^{(0)}$.

Intuitively, the global iterative solution is expected to converge as a consequence of the geometrical structure of the problem, namely that in a given interval \mathcal{T}_j many different stars contribute to the determination of \mathbf{n}_j , while, conversely, many different intervals contribute to the determination of a given star. Thus, an initial error in the coordinate of one star gives only a much smaller error in the attitude parameters of the affected intervals, and these errors in turn are diffused, in the next iteration, to a large number of stars, and, in rather few iterations, to the whole set of stars. It can be noted that this diffusion is strengthened by the superposition of the two fields of view in Hipparcos, by the incommensurability of the basic angle to 360°, and by the diversity of scan directions across any point on the sky; i.e. by the very properties that make the Hipparcos reference frame internally 'stiff'. It is a very likely hypothesis that the convergence properties are closely linked with the stiffness of the resulting reference frame: a well-designed space astrometry project should ensure good convergence of the global iterations.

A simplified version of the global iterated solution, using 2000 stars, was in fact implemented by Sansò *et al.* (in Perryman *et al.* 1989 Volume III, Chapter 28), and was found to converge in only two iterations. The block iteration method used for the FAST sphere solution (Equations 11.27–11.29) follows the same general numerical principle (although the detailed equations are different), demonstrating its feasibility for a similar problem with ~ 370 000 unknowns.

The global iterative solution thus appears to be a both practically feasible and intuitively natural method for solving the general space astrometry problem. One possible disadvantage of the method is that it seems to be difficult to estimate reliably the uncertainties of the astrometric parameters. The curvature matrix associated with the restricted problem in Equation 23.11 gives only a lower bound to the covariance matrix of **a**, by neglecting the uncertainties in **n** and γ . This aspect of the global iterative solution requires additional study.

23.3. An Attempted Global Iterative Solution

The 'three-step method' on which both the FAST and NDAC data reductions were based introduced the star abscissae as an intermediate quantity in order to allow a direct, but approximate, solution of the general space astrometry problem. The nature of this approximation was discussed in Sections 11.3 and 11.7. A particular concern was that it might introduce a distortion of the resulting system of positions and proper motions. As shown in the previous section, the approximation could be eliminated by adopting instead the 'global iterative solution'. It was also remarked that some of the key procedures necessary for the global iteration were in fact very similar to procedures already implemented in the data reductions: for Equation 23.10, the attitude reconstruction or, more precisely, the attitude smoothing included in the great-circle reductions; for Equation 23.11, the determination of astrometric parameters.
In 1993, when the end of the main astrometric reductions in NDAC appeared to be within sight, it was therefore natural to start thinking of a possible alternative treatment of the grid coordinates, eliminating the artificial division into the great-circle reductions and sphere solution. A first plan was drafted by L. Lindegren in July 1993, and most of the software was written by C.S. Petersen at Copenhagen University Observatory between March and September 1994. However, because of other commitments and the more urgent requirements of the final iteration of the nominal reductions in NDAC, it was not until April 1995 that a first successful solution was made.

The input to the Copenhagen global iterative solution consisted of two major data sets:

- the attitude files (~ 1.6 Gigabyte), containing the results of the last iteration of the NDAC attitude determination;
- the grid coordinate files (~ 2.8 Gigabyte), containing the phase determinations of all the programme stars observed in each observational frame.

These data sets were essentially the output from the first stage of the data processing (Part A in Section 4.1) performed at the Royal Greenwich Observatory, but with the along-scan attitude component updated from the great-circle reductions. Additionally, three data bases were used:

- the star catalogue from one of the last NDAC sphere solutions (N37.1);
- the instrument parameters determined in the last run of great-circle reductions;
- the mean residual maps (Section 10.3).

The output consisted of the updated star catalogue including the 5×6 normal equations system for each of the $\simeq 118\,000$ programme stars.

In order to make the best use of existing procedures and minimise the need for additional software development, the following simplifications were introduced, in comparison with Equation 23.13:

- no global parameters (γ) were included;
- the local parameters n_j included only the spline coefficients for the corrections to the along-scan attitude angle (Ω), with the knot sequences taken without changes from the last great-circle reduction;
- the instrument parameters were not updated.

Each iteration consisted of three main procedures run in sequence:

- (1) initialisation of the normal equations for all the stars;
- (2) a loop through the attitude intervals T_j to determine the spline coefficients \mathbf{n}_j and, using the residuals of each such fit, update the normal equations for the corresponding stars;
- (3) solution of the normal equations for one star at a time.

The initial catalogue, $\mathbf{a}^{(0)}$, was taken from the NDAC sphere solution N37.1. Only a single iteration was made (m = 1), and took about 18 hours on a Sparc-10 workstation. Nearly all the time was spent on procedure (2) above, the other two procedures being a matter of few minutes only.

Table 23.1. Standard deviations of the differences in astrometric parameters between the global solution NG1 and four other catalogues, after elimination of orientation and spin differences: the Hipparcos Catalogue (HIP), the final FAST and NDAC sphere solutions (F37.3 and N37.5), and the NDAC sphere solution N37.1 used as starting point for the global solution. The positions were compared at the epoch J1991.25. The standard deviations were computed by the robust method of Equation 16.22. The second column gives the number of stars used in each comparison. The last column gives the geometrical mean, D, of the five standard deviations in each comparison, as a somewhat arbitrary measure of the global 'distance' between the catalogues. Differences among the comparison catalogues are given in the lower part of the table (see Tables 16.9–16.10).

Solutions	No. of	Stand	lard dev	viations	(mas, n	nas/yr)	
compared	stars	$\Delta lpha *$	$\Delta\delta$	$\Delta \pi$	$\Delta \mu_{\alpha} *$	$\Delta \mu_{\delta}$	D
NG1-HIP	101 093	0.77	0.66	0.90	1.08	0.94	0.858
NG1-F37.3	101 036	1.03	0.87	1.18	1.37	1.17	1.111
NG1-N37.5	100 919	0.72	0.61	0.84	1.01	0.88	0.800
NG1-N37.1	100 713	0.73	0.62	0.85	1.05	0.93	0.822
F37.3-HIP	101 189	0.51	0.43	0.62	0.64	0.51	0.536
N37.5-HIP	101 071	0.59	0.49	0.73	0.72	0.60	0.619
N37.5-F37.3	100894	0.97	0.81	1.17	1.19	0.98	1.014
N37.5-N37.1	100 589	0.51	0.43	0.62	0.71	0.60	0.566

Results

The first iteration of the NDAC global iterative solution, here denoted NG1, resulted in a star catalogue with 117616 entries. However, in the following only the 101093 entries in common with the basic subset defined in Section 16.2 will be considered, thus avoiding the major complications due to stellar duplicity.

The results of NG1 were compared with the final Hipparcos Catalogue (HIP) and the last FAST and NDAC sphere solutions (F37.3 and N37.5) according to the general principles described in Section 16.6. Additional comparisons were made with N37.1, the catalogue used as a starting approximation, in order to obtain the mean updates produced by the global solution, and between the various comparison catalogues in order to see the typical differences arising in the nominal Hipparcos processing.

First, the global orientation and spin differences of NG1 with respect to HIP were determined. The results were:

$$\boldsymbol{\varepsilon}_{0} = \begin{pmatrix} -39.910 \\ -41.592 \\ +67.666 \end{pmatrix} \text{ mas [J1991.25], } \boldsymbol{\omega} = \begin{pmatrix} -1.389 \\ +0.832 \\ +1.069 \end{pmatrix} \text{ mas/yr}$$
[23.14]

These values are extremely close to the corresponding values for the sphere solution used as starting point for the global iteration, N37.1 (see Table 16.8), showing that the iteration did not introduce any significant change in the global reference frame. After elimination of the orientation and spin differences, the differences in each of the five astrometric parameters were calculated with respect to the comparison catalogues. The standard deviations of the differences, estimated according to Equation 16.22, are shown in Table 23.1.

can be taken as a global measure of the 'distance' between any two solutions. In this sense, the global solution is 'nearest' to the final NDAC sphere solution (D = 0.800), which is not surprising, as they used the same basic input data. In this connection it is worth noting that the distance to the initial catalogue, N37.1, is slightly greater (D = 0.822). Clearly the global solution is rather different from both N37.1 and N37.5, although less different from these than the FAST and NDAC sphere solutions from each other. Interestingly, the global solution, while moving away from N37.1 and N37.5, does not seem to approach the FAST solution (or HIP), but rather behaves to some extent as independent of the FAST and NDAC sphere solutions. This was confirmed by the properties of the parallax distribution (see below).

The median offset in parallax was $\langle \pi_{\rm NG1} - \pi_{\rm HIP} \rangle = -0.012 \pm 0.003$ mas. The offset was found to be slightly dependent on colour, with a mean coefficient of $\pm 0.06 \pm$ 0.01 mas per magnitude of V - I. The hemisphere asymmetry, defined in analogy with Equation 16.24, was $\Delta \pi_0 = -0.028 \pm 0.006$ mas. The width of the parallax distribution indicated that NG1 was slightly more precise than N37.5, while the fraction of negative parallaxes lead to the contrary conclusion. Both criteria showed that a weighted mean of NG1 and N37.5, with about equal weight to the two solutions, would provide a significant improvement of the parallaxes (by $\simeq 8$ per cent in the median standard error). Even with respect to the final Hipparcos Catalogue, the global solution would contribute significant information, reducing the median standard error in parallax by a few per cent. This supports the previous conclusion that the modelling errors in NG1 are rather different from those in the sphere solutions.

Large-scale differences between NG1 and HIP, apart from the global offset in orientation and spin, were investigated by computing the rotational offsets in eight different areas of the sky (see Table 16.12). In position (ϵ_0) the absolutely largest difference was 0.080 mas, while in proper motion (ω) it was 0.175 mas/yr. These values are somewhat larger than the FAST-NDAC differences reported in Table 16.12, but not alarmingly large and probably related to the chromatic effects described below.

By far the most serious systematic effects revealed by the various comparisons are related to the colours of the stars. The slight chromatic offset of the parallaxes was already noted. Chromatic effects are however much more drastic in the positions and, especially, the proper motions. They show up, for instance, as very large chromatic rotation parameters, defined as in Equation 16.27:

$$\boldsymbol{\varepsilon}_{0}^{\prime} = \begin{pmatrix} -0.444 \\ +0.098 \\ +0.688 \end{pmatrix} \text{ mas mag}^{-1} \text{ [J1991.25]}, \qquad \boldsymbol{\omega}^{\prime} = \begin{pmatrix} -1.586 \\ +0.723 \\ +0.911 \end{pmatrix} \text{ mas yr}^{-1} \text{ mag}^{-1} \text{ [23.15]}$$

This is very likely caused by inadequate modelling of instrument chromaticity, in particular the 'constant chromaticity' term c_{00} not included among the instrument parameters (see Equation 10.9 and Figure 16.5). Since this would have to be included among the global parameters γ , it was not taken into account in NG1. The effect has both the magnitude and the strong time dependence needed to explain the strong influence on the proper motion system. Some of the earlier comparisons of the NDAC and FAST sphere solutions showed similar colour-dependent differences (Table 16.13), which disappeared only in the final solutions after careful modelling of the chromaticity.

Although very robust methods were used in the comparisons, the error statistics of NG1 are somewhat degraded by the rather unclean appearance of the solution in comparison with either sphere solution or the Hipparcos Catalogue. This is manifested, for instance, in the number of parallax values below -10 mas, which is 80 in NG1, but only 9, 16, and 12 in F37.3, N37.5, and HIP (as always, only the intersections of these catalogues with NG1 and the basic subset were considered).

Finally it should be remarked that the standard errors in NG1, computed from the curvature matrix of Equation 23.11, were typically underestimated by a factor 0.5 to 0.8 compared with N37.5. This was the case even though the average unit-weight variance of the residuals was close to one. The discrepancy highlights the problem, referred to in the previous section, of finding a practical method to compute reliable covariance matrices for the global iterative solution.

Conclusions

Although the Copenhagen experiment provided only a single iteration step, the practical feasibility of the global iterative solution was clearly demonstrated. Moreover, it resulted in a solution which was not inferior to the standard sphere solution in terms of overall precision, but rather different in terms of the detailed (modelisation) errors. Given more time and work, it is probable that the major shortcomings of NG1—in particular the chromatic errors and the occurrence of outliers—could have been eliminated, resulting in a solution somewhat better than the standard NDAC sphere solution. Furthermore, rigorous inclusion of the instrument and global parameters among the unknowns, as well as fine-tuning of the attitude smoothing and more iterations, would surely result in additional improvements.

An obvious extension of the method would be to merge the FAST and NDAC data already at the grid-coordinate level and perform a global iterative solution on the merged data. However, the degree of improvement in the end results remains uncertain.

While the global iterative solution thus appeared to be a very promising alternative approach to the Hipparcos data reductions, the amount of additional work required was likely to be substantial, and it had to be abandoned as the baseline NDAC contribution to the Hipparcos Catalogue. Further study of the method should nevertheless be encouraged, especially in view of future space astrometry missions.

23.4. The Challenges for the Future

A future space astrometry mission will clearly rest very heavily on the Hipparcos experiences. Certain issues, such as the basic conceptual problems faced by the first global scanning space astrometry experiment—the derivation of absolute trigonometric parallaxes, the determination of the astrometric parameters of complex double and multiple systems, and so forth—have been convincingly demonstrated.

An experiment aiming for the cataloguing of the astrometric parameters of tens of millions of stars will certainly face numerous problems associated with the treatment of such a large quantity of data related to a very large number of stars. Not only will the Hipparcos experience help in preparing such reductions, but developments in

computational power and object-oriented data bases mean that the complexities related only to the data volume will certainly not rise in proportion to the number of objects.

Apart from the instrumental challenges of designing, launching and operating a satellite with the requisite optical and geometric stability, the challenges to be faced in proceeding from milliarcsec to microarcsec astrometry will most likely be of comparable complexity as those involved in the progress to milliarcsec positional accuracy. Metric and light travel time effects will compound the complexities of the astrometric model, and its formulation and practical solution. And the conceptual definition of a reference system in which differential galactic rotation becomes a significant observable effect may demand a more complex representation of the space motion of each object observed.

What appears beyond doubt is that the principles of the Hipparcos space astrometry mission can be carried over to the realms of a microarcsec astrometry experiment, the successful completion of which would characterise to an even more significant degree the structure and evolution of stars, and our Galaxy, in a manner completely impossible using any other methods.

L. Lindegren & M.A.C. Perryman

APPENDIX A GLOSSARY

Some of the terms included in the Glossary of Volume 1 are repeated here, where considered appropriate. Certain additional terms more relevant for the published catalogues may also be found in the Glossary of Volume 1.

- Abscissa: the angular coordinate of a star measured from an arbitrary origin on the reference great circle to the normal projection of the star on that circle. The perpendicular coordinate is known as the ordinate of the star. The collection of abscissa measurements were used to derive the astrometric parameters of each star. The individual abscissa measurements are retained as useful intermediate astrometric data on the ASCII CD-ROMs. See also great-circle reduction.
- ac magnitude: the magnitude derived from measurements of the modulation amplitude of the image dissector tube signal.
- Accumulated photometry: the magnitude *Hp* determined for every star as the median of the *Hp* magnitudes derived from the individual transits.
- Accuracy: the uncertainty of a measured quantity, including accidental and systematic errors. The term is often used synonymously with 'external standard error' (cf. precision).
- Active stars: in the FAST Consortium's great-circle reduction software, stars were divided into 'active stars' for which a rigorous least-squares solution was computed (and which, therefore, defined the geometric reference on the circle) and 'passive stars' (which were defined *a priori* or during the processing itself) which were fitted into the reference framework of the active stars. In the NDAC Consortium, the concept of active and passive stars was replaced by the procedure of re-weighting.
- Astrometric binary: a physical stellar system not observed as a visual double because of its small separation and/or large magnitude difference, but evidently non-single because of the detectable non-linear proper motion of the photocentre. A large residual from a model with five astrometric parameters may also indicate that the actual motion may deviate from the assumed rectilinear motion of the centre of mass.
- Astrometric parameter determination: the final step of the 'three-step' method, which allowed the calculation of astrometric parameters for any (single) star from its observed abscissae on (typically) 50 different reference great circles.
- Attitude determination: the name given to the process by which the data from the satellite (the star mapper transits and the gyro data) were used to derive a description of the

three-axis attitude of the viewing directions of the payload at any instant in time. Onboard, this process (referred to as real-time attitude determination) used the brighter star transit information from the star mapper to yield a three-axis attitude accurate to about 1 arcsec rms. On the ground, this was improved to some 0.1 arcsec for the direction of the spin axis, and to a few milliarcsec for the spin phase.

- Attitude smoothing: as part of the iterations performed during the great-circle reductions, knowledge of the along-scan attitude of the satellite was improved by modelling the attitude evolution, between gas jet actuations, by means of splines. The improved attitude resulted in the effective connection of stars not present simultaneously within the combined field of view, in a correspondingly improved 'rigidity' of the great-circle solution, and, thus, in improved precision on the great-circle abscissae of the programme stars.
- Basic angle: the fixed angle, approximately 58°, between the two viewing directions of the Hipparcos telescope. The exact value of the basic angle was determined during commissioning to a precision of about 1 arcsec, sufficient for the piloting of the image dissector tube to the transiting programme stars. During the great-circle reductions, the basic angle was determined, as part of the geometrical transformation parameters, to much better than a milliarcsec. The stability of the basic angle during the calibration period (of one reference great circle, or about 10.7 hours) was ensured by the payload thermal control.
- B_T magnitude: see Tycho magnitudes.
- Beam-combining mirror: the first element of the Hipparcos payload optics, responsible for combining the light from the two fields of view, separated by the 'basic angle'.
- Complementary field of view: during the observation of a programme star in one of the two fields of view, the region of the sky covered by the other field of view is referred to as the complementary field of view.
- Corrections to origins: the angular correction applied to the arbitrary origin of each great circle to bring these origins into a consistent system (see also sphere solution).
- Cramér-Rao limit (or Minimum Variance Bound): in statistical estimation a lower bound to the variance of an unbiased estimator of a parameter. The practical importance of the limit is that it is often much easier to calculate than the actual variance of a given estimator, and is independent of the choice of estimator: it is given by the negative inverse of the expected curvature (or Hessian matrix) of the log-likelihood function. The realism of the Cramér-Rao limit as an estimator of the variance of a given parameter must be investigated e.g. by Monte Carlo simulations.
- dc magnitude: the magnitude derived from measurements of the zero-level of the image dissector tube signal.

Dynamic smoothing: see geometric smoothing, and attitude smoothing.

- Epoch photometry: the determination of the Hp magnitudes (and in the case of the Tycho Catalogue, the B_T and V_T magnitudes) performed at every grid crossing or transit.
- Field angles: spherical coordinates defined in analogy with the 'field coordinates'.
- Field coordinates: direction cosines (w, z) of an object in one of the two fields of view, with respect to the orthogonal unit vectors **w** and **z** (see Appendix B).
- Field of view: one of the regions of sky, 0.9×0.9 in size, visible at any given instant to the Hipparcos payload. The two fields of view (preceding and following), separated by the basic angle of about 58°, were brought to a common focal surface by means of the 'beam-combining' mirror.

- Field-to-grid transformation: the geometrical relationship between the coordinates of an object on the celestial sphere, as described by the 'field angles' or 'field coordinates', and the 'grid coordinates' measured at the focal surface of the telescope. The parameters of the transformation were calibrated during the great-circle reductions, over a time interval corresponding to that of a reference great circle, or about 12 hours.
- Field transit: the transit of a stellar image across the field of view, often referring to the data collected for that star during this time interval.
- First-look analysis: the data analysis set up by the FAST Consortium, at SRON, Utrecht, to allow a first inspection of subsets of the data within a few days of the generation of the data by the satellite.
- Five-parameter model: the basic model describing the modulated image dissector tube signal in terms of a general two-harmonic trigonometric function, with five unknown parameters. The phases determined from the model fitting were used as inputs to the great-circle reductions. The term may also refer to the standard astrometric model, whereby the apparent motion of a (single, unperturbed) star is described by the five astrometric parameters.
- Fully observable star: a concept defined in the context of the star observation strategy (see Volume 2) indicating that a star was within one of the fields of view throughout a given observation frame (of duration $T_4 = 2.133...$ s), i.e. not in the process of entering or leaving the field. Most observed stars fell into this category (see also partially observable star).
- General parameters: the set of instrumental parameters common to all abscissae determined during the sphere solution.
- Geometric smoothing: the process of improving the knowledge of the satellite attitude by including a geometric model of the attitude evolution (as adopted by FAST) in contrast to an attitude model more dependent on a consideration of the dynamical motion of the satellite (referred to as dynamical smoothing).
- Great circle: one revolution of the satellite, roughly corresponding to a great circle projected on the sky, corresponded to a period of approximately 2.1 hours. Data from several great circles, comprising a reference great-circle set, were reduced together as part of the great-circle reductions.
- Great-circle reduction: the first step in the 'three-step' reduction method, whereby phases determined by the image dissector tube data processing were brought together (over about 5 satellite rotations or revolutions, or about 10.7 hours), to derive the along-scan abscissae of the stars, with respect to an adopted 'reference great circle' by the method of least-squares.
- Grid coordinates: orthogonal rectangular coordinates in the tangent plane at the centre of the main modulating grid, expressed in linear units.
- Grid period (or grid step): the period of the main modulating grid. From the great-circle reductions the mean grid period was found to be 1.207 366 arcsec, with extreme values (depending on position in the field of view) of 1.207 348 and 1.207 371 arcsec. Where only an approximate value of the grid period is relevant, the nominal pre-launch value of 1.208 arcsec is frequently used. Where a more accurate value was appropriate (for example, to correct slit errors in the sphere solution), a value of 1.2074 arcsec has been adopted.
- Grid-step ambiguity: the along-scan phase measurements were made modulo one grid period (approximately 1.208 arcsec), so that stars with relatively poor *a priori* knowledge in their positions (or as a consequence of the poor instantaneous knowledge of the satellite attitude) suffered a corresponding uncertainty in the determination of their grid

coordinates. If different measurements differ within a reference great circle, this fact can be recognised in the great-circle processing and duly corrected—the effect is then referred to as a grid-step inconsistency. Once made consistent at the level of the greatcircle reductions, the grid coordinate may still be incorrect by a multiple of the grid step. This problem is referred to as that of grid-step errors. Such errors do not generally affect the validity of the great-circle abscissae derivations: they are recognised and corrected during the sphere solution process, and updated values are used in iterations of the great-circle reductions to improve the attitude knowledge.

- Grid-step error: see also grid-step ambiguity. In the double-star reductions, a grid-step error may occur for any (or several) stars in a system with poorly known *a priori* positions, and especially for new doubles with a large magnitude difference, the separation may be in error by one or more times 1.2 arcsec (due to differences in the scanning geometry, the unit is not exactly that of the nominal grid period).
- Heliotropic angles: angles within the heliotropic reference frame in which one of the reference axes was constantly pointing towards the (nominal) Sun. This reference frame was used in the NDAC Consortium reductions (for defining the instantaneous deviation of the actual satellite attitude from that given by the nominal scanning law, and in defining the reference framework for the great-circle reductions).
- Hipparcos magnitude: the magnitude, designated by *Hp*, sensed by the (broad-band) main detection system of the Hipparcos payload. The payload response was calibrated as a function of wavelength before launch, and photometric calibration was carried out throughout the mission by means of the reductions to an adopted system defined by standard stars.

Hp: see Hipparcos magnitude.

- Housekeeping data: auxiliary data generated by the satellite, in addition to the main mission data, needed for a full exploitation of the satellite information. It included in-flight calibration data, thermal payload measurements, and instrument status.
- ICRS: the International Celestial Reference System, in which the Hipparcos and Tycho Catalogue positions and proper motions are given. This is consistent with the conventional equatorial system for the mean equator and equinox of J2000, previously realised by the FK5 Catalogue (see Section 1.2.2 of Volume 1 for further details).
- Inclined slits: part of the star mapper grid consisting of four \lor -shaped slits, and used for the determination of the transverse coordinate of star images. The apex of the inclined slits is located in the viewing plane. See also vertical slits.
- Instantaneous field of view: the sensitive area of the image dissector tube (behind the modulating grid) of about 38 arcsec diameter. The image dissector tube allowed a rapid change of the mean position of the instantaneous field of view, making it possible to observe several stars in the field of view almost simultaneously.

Instantaneous scanning great circle: see viewing plane.

Intensity transfer function: the description of the relation between the measured photoncounts (in de-compressed form) and a linear intensity scale.

Julian Year: 365.25×86400 s (exactly).

- Large-scale distortion: the component of the field-to-grid transformation (originating from the payload optics) which was calibrated during the great-circle reductions.
- Longitudinal: this prefix usually signifies a quantity measured or counted in the direction of scanning (i.e. perpendicular to the slits of the main grid), as opposed to the transverse quantity (normal to the scan)—e.g. longitudinal field angle.

- Main grid: the main modulating grid of 2688 parallel slits, each of width 3.13 μ m, and separated by 8.2 μ m, or approximately 1.208 arcsec on the sky. The grid, engraved on the spherical surface of a piece of glass matching the telescope's focal plane curvature, was built up from 168 by 46 elements (each containing 16 lines), referred to as 'scan fields'. With the scanning of the telescope, stellar images moved across the focal plane roughly perpendicular to the grid lines, resulting in a very regular modulation of the light observed from behind the grid.
- Main mission/main experiment/main grid: sometimes used to refer to the Hipparcos Catalogue related aspects of the satellite or mission, in contrast to the 'star mapper' or Tycho Catalogue related aspects.
- mas: milliarcsec (0.001 seconds of arc).
- Medium-scale distortion: the component of the field-to-grid transformation (originating from the method by which the modulating grid was fabricated in 46×168 scan fields—see Volume 2) which was calibrated on ground, and used as input to the data reductions as a matrix of calibration points, depending on the location of the star image on the grid at the instant of observation.
- Modulating grid: see main grid.
- Modulation phase: the phase of the first harmonic in the five-parameter model which increased cyclically from 0 to 2π radians as the satellite rotated. The value of the modulation phase at a specific instant was derived by the process known as 'phase extraction'.
- Nominal scanning law: see scanning law.
- Observational frame: the basic time unit of 32/15 s, also referred to as T_4 , used to fit the photon counts to the five-parameter model.
- Off-line tasks: a collective name given to those reduction tasks which strictly did not fall within the main (three-step) reduction chain: the photometric reductions, the double star and minor planet treatment, simulations and instrument modelling, calibrations and first-look activities, and the link to the quasi-inertial reference systems.
- On-ground attitude determination: see attitude determination.
- Optical transfer function: the description of the modulation coefficients and the phase differences between the first and second harmonics in the modulated signal for the main grid, as a function of field of view, position in the field of view, and star colour.
- Orbital period (of the Hipparcos satellite): the interval between perigee passages. In its geostationary transfer orbit, the orbital period of the Hipparcos satellite was approximately 10.7 hours.
- Ordinate: the angular distance of a star from the reference great circle, reckoned positive towards the great-circle pole. See also abscissa.
- Parallax: the Hipparcos and Tycho Catalogues provide the annual parallax, π , from which the coordinate distance is $(\sin \pi)^{-1}$ astronomical units, or with sufficient approximation, π^{-1} parsec if π is expressed in arcsec. The parallax determinations are trigonometric, absolute (in the sense that the parallax determination of a given star is not dependent upon either the parallaxes, or assumptions concerning the parallaxes, of other stars—including stars close by on the sky), and independent of any previous distance determinations. Analyses place a limit on the global parallax zero-point offset of less than 0.1 milliarcsec, and give confidence that the published standard errors are a reliable indication of their true external errors.
- Partially observable star: a concept defined in the context of the star observation strategy (see Volume 2) indicating that a star was in the process of entering or leaving the

field during a given observation frame (of duration $T_4 = 2.133...s$). Bright partially observable stars were included in the star observations in order to improve the attitude determination during the observation frame.

Passive star: see active star.

- Phase extraction: the derivation of phases from analysis of the image dissector tube data, by fitting of the experimental data to the three- or five-parameter signal model.
- Position: the Hipparcos and Tycho Catalogues provide the barycentric coordinate direction, specified as right ascension, α , and declination, δ .
- Precision: the uncertainty of a measured quantity due to accidental errors. The term 'precision' is often used synonymously with 'internal (or formal) standard error' as derived e.g. from a least-squares solution (cf. accuracy).
- Primary grid: the main modulating grid of 2688 parallel slits, separated by $8.2\mu m$, or 1.2074 arcsec on the sky, located at the focal surface of the combined field of view of the telescope, on which the main Hipparcos measurements were based.
- Primary reference star: a star selected to be included in the sphere solution due to its appropriate properties (bright, single, etc.). These stars (which numbered around 40 000) defined the relative origins of the 2000 or so reference great circles generated throughout the mission. The secondary reference stars were subsequently fitted into the resulting reference system.
- Programme star: one of the stars (approximately 120000) contained in the Hipparcos Input Catalogue, and observed by the main detector. The observing programme was defined before launch and remained essentially fixed for the entire mission duration.
- Proper motion: the Hipparcos and Tycho Catalogues provide the rate of change of the barycentric coordinate direction expressed as proper motion components $\mu_{\alpha*} = \mu_{\alpha} \cos \delta$ and μ_{δ} , in angular measure per unit time (milliarcsec per Julian year).
- Real-time attitude determination: see attitude determination.
- Reference great circle: a reference plane chosen to correspond to the mean scanning motion of the satellite during several hours, and signifying also the collection of observations during this time-interval. In practice the maximum duration of observations constituting the reference great circle was limited by the satellite's orbital period, corresponding to about 5 great-circle scans, or about 10.67 hours—typical lengths of the reference great circles were somewhat shorter. Star abscissae were projected onto the reference great circle (through a knowledge of the three-axis attitude of the satellite) and solved for during the great-circle reductions.
- Scan field: elements of the mosaic in which the main grid and star mapper grids were manufactured. The main grid consisted of 46×168 scan fields, and the star mapper grids of 102 scan fields on either side of the main grid (68 for the chevron slits and 34 for the vertical slits). See also medium-scale distortion.
- Scanning law: the three-axis attitude of the satellite, determining where the two fields of view of the satellite were directed, at any instant of time. The nominal scanning law is a deterministic scanning motion which defined the required satellite attitude. By comparing the target and actual attitude on-board, by means of the star mapper transits, corrections to the actual attitude were effected by means of regular (roughly every 400 s) three-axis gas jet actuations, which brought the attitude back to its target one. In this way, deviations between the actual and nominal scanning law were kept to within about 10 arcmin throughout the mission.

Secondary reference star: see primary reference star.

Set solution: an alternative name given to the great-circle reduction process.

- Small-scale distortion: the component of the field-to-grid transformation (originating from the method by which the modulating grid was fabricated—see Volume 2) which was uncalibrated on the ground, and uncorrected in orbit (see also large- and medium-scale distortion). Typically, the small-scale distortion resulted in a negligible degradation on the phase measurements.
- Solar system objects: the 48 minor planets and three natural satellites observed in the Hipparcos programme.
- Sphere solution: the second step of the 'three-step method', which combined the greatcircle data for a number of reference stars and determined the 'great-circle zero-points'. These zero-points defined the interconnection between the reference great-circle reference systems leading to the global Hipparcos reference system.
- Star mapper: the detection chain (including aperiodic vertical and inclined grids, relay optics and detectors) located on each side of the main grid (two were provided for redundancy reasons). The prime purpose of the star mapper was to provide three-axis (hence, the inclined slits) attitude information to the satellite, in real-time, on the basis of the time of transits of some 40 000 bright reference stars distributed over the sphere. It was also used for the Tycho experiment, and included, for this reason, two photometric channels (B_T and V_T), each sampled by their own photomultiplier tube detectors. In contrast to the detector used for the main field of view, the star mapper detectors sampled the entire signal generated simultaneously by star transits over the entire star mapper grid.
- Star mapper grid: the arrangement of four vertical and four inclined grids, arranged aperiodically at one side of the main grid, used for the satellite real-time attitude determination and the Tycho measurements.
- Star observing strategy: the on-board algorithm which determined the cycle of star observations on the main grid, on the basis of the satellite attitude, and the information contained in the programme star file.
- Telemetry format: $32/3 = 10.66 \dots s = 5$ observation frames (= 256 telemetry frames).
- Telemetry frame: 1/256 of a telemetry format (= 25 star mapper samples = 50 image dissector tube samples = 1/24s).
- Three-parameter model: a constrained form of the image dissector tube signal model, involving only three unknown quantities (as compared with the more general five-parameter model) and which was valid for single stars.
- Three-step method: the break-down of the (directly) intractably large Hipparcos reduction problem (to estimate simultaneously more than 600 000 astrometric parameters along with large numbers of additional satellite attitude unknowns and time-dependent geometrical calibration terms) in three partial steps. The first step is the 'great-circle reduction', the second step the 'sphere solution' and the last step the 'astrometric parameter determination'.
- Transverse: a prefix used for quantities measured in a direction normal to the scan (i.e. along the slits of the main grid), as opposed to the longitudinal (along-scan) direction—e.g. transverse field angle.
- Tycho magnitudes (B_T, V_T) : the magnitude system defined by the Tycho instrument, in reasonable correspondence with the usual Johnson *B* and *V* magnitude system. Transformation equations between the various systems are provided.
- Veiling glare: phase perturbations on the measurements of programme stars on the main grid in the presence of nearby bright stars, from either field of view, caused by the profile of the image dissector tube response.

- Vertical slits: part of the star mapper grid consisting of four slits perpendicular to the scanning motion of the satellite, used for the determination of the along-scan attitude angle. See also inclined slits and attitude determination.
- Viewing directions: the two directions in space towards which the telescope pointed at a given time. The directions refer, more precisely, to the centres of the two fields of view (preceding and following): the angles between them is known as the basic angle.
- Viewing plane: the plane containing the two viewing directions. Its intersection with the celestial sphere is known as the viewing great circle or instantaneous scanning great circle.

 V_T : see Tycho magnitudes.

APPENDIX B NOTATION

It was not practicable to employ a completely uniform system of notations throughout this Volume. The reasons for this were partly historical—certain conventions had developed in the Hipparcos literature and documentation and could not easily be disregarded. There were also practical considerations: strict observance of a general system, if at all possible, would often lead to a profusion of suffixes and mathematical accents obscuring the particular relationships relevant in a given context. Nevertheless, an attempt was made to simplify the cross-referencing of the various chapters by using, as far as practicable, similar notations for quantities with a similar meaning. This Appendix lists a number of notations that had a more than 'local' usage, or at least a meaning outside the specific context in which they were introduced.

Throughout this Volume, the prime symbol (') associated with matrices and vectors denotes transposition. In particular, for vectors, it denotes scalar multiplication: thus $\mathbf{a}'\mathbf{b}$ is equivalent to the scalar product of \mathbf{a} and \mathbf{b} . The angular brackets, when applied to a vector, denote normalisation of the vector length: thus $\langle \mathbf{a} \rangle = \mathbf{a} |\mathbf{a}|^{-1}$ is a unit vector in the direction of \mathbf{a} . For scalar quantities the angular brackets denote averaging. The asterisk has a special meaning in entities like $\mu_{\alpha*}$ and $\sigma_{\lambda*}$, where it signifies an implicit cosine factor, i.e. $\mu_{\alpha*} = \mu_{\alpha} \cos \delta$ and $\sigma_{\lambda*} = \sigma_{\lambda} \cos \beta$. Suffixes F, N and H are often employed to distinguish quantities related to FAST, NDAC, and the Hipparcos Catalogue. Similarly p and f may denote the preceding and following fields of view, while obs and calc usually refer to observed (derived from data) and calculated (theoretical or fitted) quantities.

Α	the astronomical unit (Table 12.1)
A_k	in double-star treatment: same as I_k
<i>a</i> ₁ <i>a</i> ₅	(1) the astrometric parameters as components of the vector ${f a}$
	(2) harmonic coefficients of the main detector signal (FAST)
a ^p , a ^f	instrument parameters for field-to-grid transformation in preceding and following
19 19	field (FAST)
а	(1) vector of the (five) astrometric parameters of a star, α , δ , π , $\mu_{\alpha*}$, μ_{δ} , or their
	differential corrections $\Delta \alpha *$, etc.
	(2) vector of signal parameters $a_1 \dots a_5$ (FAST)
α	(1) right ascension
	(2) solar phase angle for solar system object
α	(1) barycentric right ascension at catalogue epoch T_0
0	(2) geocentric right ascension of the reference point for an observation of a solar
	system object
α;	in double star treatment (FAST): spatial frequency of the grid in the direction
J	of increasing right ascension during transit <i>i</i> (cf. $f_{\rm Y}$, $f_{\rm V}$, $f_{\rm D}$)
αp	right ascension of reference great circle nole
B	magnitude in the (Johnson) LIBV photometric system
B - V	colour index in the (Johnson) UBV photometric system also written $(B - V)_{T}$
B _T	Tycho magnitude in the 'blue' spectral hand of the star mapper
b I	ralactic latitude
b. b.	barmonic coefficients of the main detector signal
$b_1 \dots b_5$ $b_p = a_f$	chromatic instrument parameters for field_to_grid transformation in preceding
^v ij ^{, u} ij	and following field (FAST)
h	(1) general harveentric position
b	(1) general barycentric position (2) vector of signal parameters $h_{i} = h_{i}$
h	(2) vector of signal parameters $b_1 \dots b_5$
DE ba	barycentric position of the Sun
ρ	colimite latitude
р e	ecliptic latitude
PR	parameters of the main detector fue parameter model (NIDAC)
$p_1 \dots p_5$	in dauble ster treatment (EAST), metial frequency of the grid in the direction
Ρj	In double star treatment (FAS1): spatial frequency of the grid in the direction of increasing declination during transit i (of $f = f$. f.)
0	or increasing decimation during transit $f(c_1, I_X, I_Y, I_p)$
	vector of the signal parameters $p_1 \dots p_5$ (NDAC)
	(centred) colour index, e.g. $(V - I) = 0.5$
C_j, S_j	coefficients of the out narmonic of abscissa error for reference great circle <i>j</i>
c	speed of light (Table 12.1)
c _j	abscissa zero point correction for felerence great circle <i>J</i>
c_{ij}	in dealed a starter traction of the starter of the
	in double star treatment: a measure of the difficulty of resolving a double star
$a_1 \dots a_4$	relative positions of the four sitts in a star mapper sitt group
a _{ij}	chromatic instrument parameters for field-to-grid transformation (INDAC)
d	vector of instrument parameters in great-circle reduction
8	declination
δ_0	(1) barycentric declination at catalogue epoch T_0
	(2) geocentric declination of the reference point for an observation of a solar
	system object
$\delta_{\rm R}$	declination of reference great circle pole
δG	medium-scale distortion in the G coordinate on the main grid
-	
δg	small-scale distortion in the G coordinate on the main grid
δg ΔG	small-scale distortion in the G coordinate on the main grid (1) (large-scale) distortion in the G coordinate on main grid or star mapper

	(3) in double-star treatment: component separation projected on grid coordinate
ΔH	distortion in the H coordinate on the main grid
ΔHp	(1) magnitude difference in a double star (also written Δm)
•	(2) difference $Hp_{ac} - Hp_{dc}$, indicator of duplicity or an extended object
Δm	magnitude difference in a double star (also written ΔHp)
$\Delta \gamma_0, \Delta \gamma_1$	instrument parameters representing corrections to the nominal basic angle (FAST)
$\Delta \phi$	difference in modulation phase between components of double star
$E^{-\tau}$	residual sum of squares in great-circle reduction
ε	extragalactic reference frame. $[\mathbf{x}_{\mathbf{F}} \mathbf{v}_{\mathbf{F}} \mathbf{z}_{\mathbf{F}}]$
e	noise (error) vector in great-circle reduction
e f	(1) obliquity of ecliptic (Table 12.1)
0	(2) 'cosmic error' in stochastic solution
	(3) <i>a priori</i> correction of relative intensity in photometric calibration (FAST)
	(4) general noise term
60 60 60	aguatorial components of the orientation vector \boldsymbol{s} at the reference exact T_{c}
$\varepsilon_{0x}, \varepsilon_{0y}, \varepsilon_{0z}$	equatorial components of the orientation vector $\mathbf{\epsilon}$ at the reference epoch T_0
e	Linnerson reference from with respect to the extragelectic reference from a
	(1) longitudinal field angle see r. (
η	(1) longitudinai netu angle, see η , ζ
	(2) local rectangular coordinate on sky, $\sim \Delta \delta$
6	(5) general noise term
η, ζ	neid angles along and normal to the nominal scan direction
η_0	reference position for star mapper slit group (FAS1)
F	(1) statistical index indicating modulation of the main detector signal
-	(2) statistical index for great-circle reductions
F ₃₅	statistical index indicating non-single star (FAST)
f	field of view index (±1 for preceding/following field)
$f_X, f_Y f_p$	derivatives of modulation phase with respect to $\alpha *$, δ , π
g	star mapper slit-group index (0, ± 1 for vertical, upper/lower inclined slit groups)
<i>g</i> 0	reference modulation phase on main grid
<i>g</i> ₁ , <i>g</i> ₂	modulation phases on main grid of first and second harmonic
g _{ij}	instrument parameters for field-to-grid transformation (NDAC)
$g_{\alpha*}, g_{\delta}$	acceleration components in α and δ of binary photocentre from orbital motion
g	(1) general geocentric vector
	(2) vector of gyro readings (NDAC)
g 0	geocentric position of Hipparcos
G	grid coordinate on the main grid
GE	geocentric gravitational constant (Table 12.1)
GS	heliocentric gravitational constant (Table 12.1)
Γ	generic global parameter in sphere solution
Γ	vector of global parameters in sphere solution
γ	(1) basic angle of the Hipparcos instrument ($\simeq 58^{\circ} 00' 30''$)
	(2) parametrised post-Newtonian (PPN) parameter of the heliocentric metric
	(3) in double-star treatment: local position angle of scan
Н	hypothesis for statistical testing
Н	coordinate perpendicular to the nominal scan direction
${\cal H}$	Hipparcos reference frame, [$\mathbf{x}_H \mathbf{y}_H \mathbf{z}_H$]
Нр	magnitude in the Hipparcos main detector photometric system
Hpac, Hpdc	<i>Hp</i> magnitudes derived from the modulated (AC) and mean (DC) detector signal
Hpmax	<i>Hp</i> magnitudes of variable star at maximum luminosity
<i>Hp</i> min	<i>Hp</i> magnitudes of variable star at minimum luminosity (note: $Hp_{min} \ge Hp_{max}$)
h_{ii}	instrument parameters for field-to-grid transformation (NDAC)
\mathbf{h}_0	heliocentric position of the Hipparcos satellite

\mathbf{h}_{E}	heliocentric position of the Earth
Ι	magnitude in Cousins' system; see $V - I$
Ib	background count rate (intensity) of the main detector
I _{b,c}	background count rate (intensity) of the star mapper in channel $c = B_T$ or V_T
I_k	expected count (intensity) for sample k of the main detector
$I_{k,c}$	expected count (intensity) for sample k of the star mapper in channel $c = B_T$ or V_T
Is	mean stellar count rate (intensity) of the main detector
Is,c	peak stellar count rate (intensity) of the star mapper in channel $c = B_T$ or V_T
$I_{XX} \dots I_{ZZ}$	elements of the inertia tensor I in body coordinates
i _G	longitudinal scan field index (main grid)
i _H	transverse scan field index (main grid)
I	inertia tensor of the Hipparcos satellite
1	imaginary unit = $\sqrt{-1}$
κ	(1) condition number of least-squares equations
	(2) correction factor for photometric standard errors of bright stars
1	galactic longitude
λ	ecliptic longitude
$\lambda_{\mathbf{R}}$	ecliptic longitude of reference great circle pole
λ_{\odot}	ecliptic longitude of the Sun
M	in double-star treatment: same as M_1
M_{1}, M_{2}	modulation coefficients for the first and second harmonics of the main detector signal
$\mu_{\alpha*}$	proper motion in right ascension, including $\cos \delta$ factor
μ_B	proper motion in ecliptic latitude
μ_{δ}	proper motion in declination
$\mu_{\lambda*}$	proper motion in ecliptic longitude, including $\cos \beta$ factor
N	in double-star treatment: same as M_2
N(0, 1)	normal (Gaussian) distribution with mean value zero and unit variance
N _k	photon counts for sample k of the main detector
$N_{k,c}$	photon counts for sample k of the star mapper in channel $c = B_T$ or V_T
Ν	vector sum of external torques acting on the Hipparcos satellite
\mathcal{N}_{i}	binned photon counts of the main detector
v	in statistical tests: the number of degrees of freedom
ν, ξ, Ω	heliotropic angles describing the scanning law and instrument attitude (NDAC)
0	'orbit number', sequential numbering of perigee passages and data sets
ω	(1) scan velocity of the grid on the sky ($\simeq 168.75$ arcsec/s)
	(2) angular frequency of main detector signal modulation (\simeq 878 rad/s)
$\omega(x)$	influence function for robust estimation
$\omega_X, \omega_Y, \omega_Z$	(1) components of instrument spin vector $\boldsymbol{\omega}$ in body coordinates
	(2) components of reference frame spin vector in equatorial coordinates
w	(1) inertial spin (angular velocity) of instrument
	(2) spin difference between two reference frames, in particular the spin of the
	provisional Hipparcos reference frame with respect to the extragalactic reference frame
Ω	heliotropic spin phase (NDAC); see v, ξ , Ω
Р	period of photometric variability
Р	unit vector towards the abscissa origin on the reference great circle
p_k	grid modulation phase for sample k relative to the mid-frame time
р	unit vector in the local direction $+\alpha$, see [pqr]
[pqr]	normal triad at ${f r}$ relative to the equatorial or ecliptic frame
π	trigonometric parallax
$\Phi(b)$	normalised galactic luminosity profile (FAST)
ϕ	(1) attitude angle; see ψ , θ , ϕ
	(2) modulation phase of first harmonic of main detector signal (also written g_1 and a_3)

ψ	(1) attitude angle; see ψ , θ , ϕ
	(2) modulation phase of second harmonic of main detector signal (also written a_5)
ψ, θ, φ	attitude angles for instrument in reference great-circle frame
$\Psi(\varrho)$	attenuation profile of the instantaneous field of view
q	unit vector in the local direction $+\delta$, see [p q r]
Ô	unit vector towards the point $v = 90^{\circ}$ on the reference great circle
Q _V	in great-circle reductions: variance-covariance matrix of the vector \mathbf{v}
Ψy r	(1) ordinate of a general direction in the reference great-circle frame
1	(2) in double-star treatment: intensity ratio of components
P. Pa	(2) in double star detailert. Intensity ratio of components
<i>I</i> ₁ <i>I</i> ₃	general vector or direction; in particular the harveentric coordinate direction to an
ľ	chieft see [m a m]
D	object, see [$\mathbf{p} \mathbf{q} \mathbf{r}$]
ĸ	ratio of modulation coefficients, M_2/M_1 of N/M
R_q	single-slit response function of the star mapper
R -	reference great circle triad [P Q R]
R	unit vector defining the pole of the reference great circle
\mathbf{R}_i	3×3 matrix describing a rotation around axis <i>i</i>
ρ	(1) general statistical correlation coefficient
2	(2) angular diameter of solar system object
$\rho_{\alpha*}^{\delta}$ (etc.)	correlation coefficients among the astrometric parameters of a star
Q	angular separation of stars, in particular in double stars
S_0	nominal scale of the field-to-grid transformation (= 170749.01 slits/rad)
S_j	see C_j, S_j
\$	grid step ($\simeq 1.2074$ arcsec per slit interval)
S	general satellitocentric vector
σ_X	(estimated) standard error of the estimated value of the generic variable x
t	general time variable, in particular relative to a given reference epoch
T	(1) astronomical time, in particular Terrestrial Time (TT)
	(2) statistical index indicating failure of five-parameter model of main detector signal
T_0	reference epoch of the Hipparcos Catalogue, $T_0 = J1991.25$ (TT)
T_1	main detector sampling interval (1/1200 s)
T_2	main detector repositioning period ($8T_1 = 1/150$ s)
T_3	observing schedule interlacing period ($20T_2 = 2/15$ s)
T_A	observation frame period $(16T_3 = 32/15 = 2.133s)$
τ	star mapper transit time
$\Theta_1 \dots \Theta_3$	Tait-Bryan angles used in real-time attitude determination
θ	(1) position angle of secondary component in double star
	(2) position angle of scan across a solar system object
	(3) attitude angle: see w A &
An Ac	transverse attitude components in preceding and following field of view (NDAC)
•p, • ₁ A	vector for the orientation error of the reference great-circle frame \mathcal{R}
II	magnitude in the (Johnson) LIBV photometric system
$U(\mathbf{v})$	complex visibility of an extended object in the scan direction
$U(\mathbf{x})$	unit weight error $(u^2 - unit weight variance)$
ū	a interval $(a - a)$ which we get valuate (a)
u A	notional direction to an object
u V	natural direction to all object
V	magnitude in the (Johnson) OBV photometric system
V - I	colour index in Cousins system; also written $(V - I)_{\rm C}$
V _R	radiai veiocity
V_T	Tycno magnitude in the visual spectral band of the star mapper
V	abscissa of a general direction in the reference great-circle frame
Vq	local scan velocity across a star mapper slit group

V	abscissa (in \mathcal{R}) of the Sun
V	barycentric velocity of the Hipparcos satellite
W	relative weight of the second harmonic of the main detector signal (FAST)
W, Z	field coordinates along and normal to the nominal scan direction
\bar{W}, \bar{Z}	normalised field coordinates for field-to-grid transformation (NDAC)
w	unit vector in scanning direction
χ^2	general goodness-of-fit statistic
<i>X</i> , <i>Y</i>	relative Cartesian coordinates of secondary component in double star
х, у	field angles (FAST)
\bar{X}, \bar{Y}	normalised field angles for field-to-grid transformation (FAST)
<i>X</i> , <i>Y</i> , <i>Z</i>	body coordinates fixed in Hipparcos satellite, along axes \mathbf{x} , \mathbf{y} , \mathbf{z}
x	in great-circle reductions: vector of unknowns
x , y , z	(1) orthogonal axes of the equatorial reference frame
	(2) orthogonal axes fixed in instrument or satellite
$\textbf{x}_E\textbf{y}_E\textbf{z}_E$	orthogonal axes of the extragalactic reference frame $\ensuremath{\mathcal{E}}$
$\textbf{x}_{H}\textbf{y}_{H}\textbf{z}_{H}$	orthogonal axes of the Hipparcos reference frame $\ensuremath{\mathcal{H}}$
ξ	(1) local rectangular coordinate on sky, ~ $\Delta \alpha \cos \delta$
	(2) revolving angle (~ 43°); see v, ξ , Ω
Y	see X, Y
у	see x, y
У	(1) axis in equatorial or instrument frame; see \mathbf{x} , \mathbf{y} , \mathbf{z}
У	(2) in great-circle reductions: vector of observations
\mathcal{Z}	test statistic for the distribution of epoch photometry
Ζ	transverse field coordinate; see w, z
Z	direction of telescope nominal spin axis; see also \mathbf{x} , \mathbf{y} , \mathbf{z}
ζ	(1) transverse field angle; see η , ζ
	(2) radial velocity divided by distance, $\zeta = V_{\rm R} \pi / A$

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