# A STATISTICAL STUDY OF THE VISUAL DOUBLE STARS IN THE TYCHO CATALOGUE 

J.-L. Halbwachs ${ }^{1}$, S. Piquard $^{1}$, P. Virelizier ${ }^{1}$, J. Cuypers ${ }^{2}$, P. Lampens ${ }^{2}$, E. Oblak ${ }^{3}$<br>${ }^{1}$ Observatoire Astronomique de Strasbourg, 11 rue de l'Université, F-67000 Strasbourg, France<br>${ }^{2}$ Koninklijk Sterrenwacht van Belgie, Ringlaan, 3, B-1180 Brussel, Belgium<br>${ }^{3}$ Observatoire de Besançon, 41 bis, avenue de l'Observatoire, F-25044 Besançon Cedex, France


#### Abstract

The Tycho Catalogue contains reliable $\left(B_{T}, V_{T}\right)$ photometry for double star components with separations above a limit of about 3 arcsec. These data are used to investigate the distribution of mass ratios of main sequence binaries. For that purpose, two other points are treated first: the correction of Tycho photometry for contamination by neighbour stars, and the derivation of the colour-magnitude diagram in the Tycho photometric system.

It appears that the main sequence in the ( $V_{T}$ abs , $B_{T}-V_{T}$ ) diagram derived from the Tycho double stars is nearly similar to that obtain from the calibrations selected by Schmidt-Kaler (1982). The distribution of mass ratios, as derived from the massluminosity relation, has a rough IMF-like shape with an exponent of about -2 .


Key words: visual binaries; HR diagram; mass function.

## 1. INTRODUCTION

Among other data, the Tycho Catalogue provides the $B_{T}$ and $V_{T}$ magnitudes of one million stars. It is nearly complete for stars brighter than $T=10.9 \mathrm{mag}$, where $T$ is the magnitude in the $B_{T}+V_{T}$ channel (in a simple approximation, $T=\left(B_{T}+V_{T}\right) / 2$; the exact transformation is in Grossmann et al. 1995). The minimum separation between two stars in the catalogue is about 1 arcsec, but, in practice, only stars with separations wider than 3 arcsec received reliable photometry in the $B_{T}$ and in the $V_{T}$ channels. This limit is close enough for permitting the selection of a sample of physical double stars with an acceptable contamination by optical pairs.

In the present paper, a sample of binary stars is extracted from the Tycho catalogue, and it is used to investigate two questions. The first one is the derivation of the main sequence (noted 'MS' hereafter) and of the subgiant and giant branch in the colour-magnitude diagram, with a technique dedicated to double stars. This diagram is used to select a sample of binaries with MS components.

Dwarf binaries are usually used for investigating the statistical properties related to the origin of double stars. Among these ones, the most critical is $f_{q}$, the distribution of the mass ratios $q=\mathcal{M}_{2} / \mathcal{M}_{1}$. For visual binaries, $q$ is estimated from the difference of magnitudes between the components, $\Delta m$. The derivation of the distribution of $\Delta m$ of MS binaries was already treated in several papers since Opik 1923 and Öpik 1924. It was usually concluded that, in the range of separations covered by visual double stars, binaries could have been constitued by random pairing of stars with the same age. This applies to binaries with periods at least as long as about ten or one hundred years (Abt and Levy 1976, Duquennoy and Mayor 1991). However, these studies were usually based on small samples, or on crude photographic or visual estimations of $\Delta m$. Apart from a general shape compatible with random pairing, the statistic of visual $\Delta m$ exhibits also a gap around $\Delta m=2 \mathrm{mag}$ (Halbwachs 1983). This corresponds to a mass ratio $q \approx 0.6$. This gap could be due to a bias in visual estimation, but it may also be interpreted as the transition between random pairing (that should produce binaries with wide separations) and an alternative formation process, that would generate binaries with similar components. This hypothesis is supported by the flat $f_{q}$ found by Duquennoy and Mayor from spectroscopic binaries, and confirmed by Halbwachs et al. 1997, although it was obtained from a sample so small that the distribution of the long-period binaries could not be discarded significantly. More recently, Söderhjelm 1997 found from the close binaries separated in the Hipparcos reduction that $f_{q}$ was depending on the separation: the proportion of stars with $q \approx 1$ was larger for the systems closer than 120 AU than for the binaries above this limit. The distribution of $q$ for separations wider than about 100 AU is re-considered hereafter from the Tycho double stars.

The Tycho catalogue was obtained from observations performed with the star mapper of the Hipparcos satellite. The star mapper was constituted by two slit systems in the focal plane common to the two fields of the satellite (Høg et al. 1992). Due to the configuration of this instrument, it may happen that the transit record of a star was contaminated by another star. The effects of the blended transits in the Tycho reduction was limited by the 'parasite check' and the 'non-linear filtering', both described in the Volume 4 of ESA 1997. However, for double stars with
separations closer than the width of the star mapper slit systems, that is 34 arcsec, it is expected that the photometry of each component may be slightly contaminated by the other one. This contamination must be evaluated and corrected before any use of the data.

## 2. THE TYCHO MAGNITUDES COMPARED TO ON-GROUND MEASUREMENTS

Double stars were extracted from the Tycho Catalogue, by comparison of the coordinates. Only the stars with reliable $B_{T}$ and $V_{T}$ magnitudes that were confirmed in the Tycho astrometric reduction (i.e. with the flag by $T 36=$ ' $M$ ' or ' $N$ ', and with astrometric qualities 1 to 8 , see $\mathrm{H} ø \mathrm{~g}$ 1997) were taken into account. Moreover, the stars known as variable from the Hipparcos observations were discarded, as well as the close double stars that were not separated in the Hipparcos astrometric reduction, according to the H10 flag. The pairs were searched until a limit of 34 arcsec, matching the width of the star mapper slit systems. On the other side, on-ground measurements of $V_{J}$ magnitudes were obtained from two different sources: the Hipparcos catalogue, and unpublished CCD observations of double or multiple systems containing at least one Hipparcos star; a validation of the CCD measurements is presented in Oblak et al. 1997. Merging these two sources, a sample of 517 double star components with on-ground $V_{J}$ magnitude was constitued for checking Tycho photometry.

The on-ground $V_{J}$ magnitudes were transformed in $V_{T}$ magnitudes, using the $B_{T}-V_{T}$ colour indices provided by Tycho. The differences between the Tycho $V_{T}$ mag and these on-ground $V_{T}$ are plotted as a function of $\Delta V_{T}$ in Figure 1. Hereafter, $\Delta V_{T}$ is the difference between the Tycho $V_{T}$ magnitude of the companion and that of the star which is considered; it is positive for the primary components, and negative otherwise.

The majority of stars in this figure are gathered along a sequence close to $V_{T \text { Tycho }}=V_{T \text { on-ground }}$, except for small $\Delta V_{T}$. Appart from this general feature, two clusters of deviating points appear, one on each side of the mean sequence. The upper one is constituted with stars for which the Hipparcos catalogue provides on-ground measurements brighter than the magnitudes measured by Tycho. It is then probable that these on-ground measurements correspond in fact to complete systems, but not to separated components. The lower cluster constains CCD measurements that are much fainter than the Tycho magnitudes. A close examination revealed that about half of these CCD measurements were also excessively faint when they were compared to the Hipparcos $H p$ magnitudes. Moreover, when the CCD measurements disagreed with Tycho as well as with Hipparcos magnitudes, the difference $V_{T \text { Tycho }}-V_{T \text { on-ground }}$ was nearly the same for both components, instead of being related to $\Delta V_{T}$. Therefore, for these stars, the large deviations probably do not come from the Tycho measurements. Nevertheless, it seems that the large scatter of the sequence for negative $\Delta V_{T}$ does really exist, although it is much larger than the uncertainties in the Tycho catalogue.


Figure 1. Verification of the magnitudes in the Tycho catalogue for components of double stars with separations closer than the width of the star mapper slit systems. The Tycho $V_{T}$ magnitudes are compared to estimations of $V_{T}$ derived from on-ground observations. Delta $V_{T}$ is the difference of Tycho $V_{T}$ magnitudes of the components, defined as positive for the primary components and negative for the secondary components. The correction adopted is indicated by the thick line.

For $\Delta V_{T} \in[-3,2]$, the following mean correction was derived:

$$
\begin{equation*}
V_{T_{\text {corrected }}}=V_{T_{\text {Tycho }}}+0.075-0.0375 \times \Delta V_{T} \tag{1}
\end{equation*}
$$

This correction was applied to the Tycho magnitudes in the next sections. The same formula was used to correct the $B_{T}$ magnitudes.

## 3. THE COLOR-MAGNITUDE DIAGRAM DERIVED FROM THE TYCHO DOUBLE STARS

An historic method for obtaining the HR diagram is based on open clusters: since the stars belonging to a cluster have the same distance, their apparent magnitudes are equal to their absolute magnitudes, minus the distance modulus of the cluster. Therefore, a composite HR diagram is obtained from several clusters, by shifting vertically the sequences in order to compensate for the different distances. The same principle is applied hereafter, considering the binaries as clusters of two stars.

### 3.1. Preparation of the Sample

Only the double stars having both components with astrometric qualities 1 to 5 in the Tycho catalogue were considered, since the other stars could be themselves close binaries, or even non-stellar objects. The photometric quality of the sample was ensured by discarding the stars with $\sigma_{B_{T}-V_{T}}$ larger than 0.1 mag . The proportion of optical pairs was restricted within acceptable limits by selecting only the double stars with separations closer than 10 arcsec; moreover, the
pairs of stars with proper motions not compatible at the 3. $\sigma$ level on both axes were discarded. Finally, the double stars with components of the same colour were eliminated by the condition $\left|\Delta\left(B_{T}-V_{T}\right)\right|>0.1 \mathrm{mag}$, since they just increased the scatter of the main sequence determined hereafter. A sample of 776 double stars was thus obtained. The double stars with separation between 50 and 60 arcsec were used to estimate the number of optical pairs in the sample; it was found that, among the 776 double stars, about 75 could be optical.

### 3.2. The Main Sequence

The double stars with MS components were selected by assuming that they had secondary components redder than the primary ones; therefore, the condition $\left(B_{T}-V_{T}\right)_{2}>\left(B_{T}-V_{T}\right)_{1}$ was applied. The location of the MS in the ( $V_{T \text { abs }}, B_{T}-V_{T}$ ) plane was derived with an iterative least-square computation (Virelizier and Halbwachs 1997). It appeared however that the determination of the red part of the sequence, with $B_{T}-V_{T}>1$, was based on very few stars and was sensitive to a contamination by optical pairs containing red giants. This was avoided by discarding the double stars with secondary components more than 2.3 mag above an a priori sequence. The MS presented in Figure 2 was thus derived from 451 double stars. It is defined from $B_{T}-V_{T}=-0.15$ until $B_{T}-V_{T}=1.5$.

The interstellar extinction was not taken into account in the determination of the MS, but it may be ignored because the sequence is almost rectilinear. Since both components of a double star are shifted in $B_{T}-V_{T}$ by the same reddening, and in $V_{T}$ by the same extinction, the slope of the sequence is not affected by this effect. The sequence in Figure 2 looks rather similar to that derived from the Hipparcos parallaxes by Perryman et al. 1995, but it is not so broad. However, this narrow aspect does not correspond to reality, but it is just an artefact due to the least-square computation technique. In Figure 3, the MS is compared with that derived from the calibration of Schmidt-Kaler 1982. When the origin of the absolute magnitudes is chosen in order to coincide for $B_{T}-V_{T}=0$, the agreement is rather good.

### 3.3. The Evolved Stars

It was assumed that the double stars with $\left(B_{T}-\right.$ $\left.V_{T}\right)_{1}>\left(B_{T}-V_{T}\right)_{2}$ had dwarf secondary components, but that their primary components were evolved stars. These stars were plotted on the colourmagnitude diagram of Figure 2, by putting the secondary components right on the MS derived above. For clarity, the few subgiant stars at less than 2 mag from the MS are not represented in this figure. Finally, the subgiant and the giant branches were derived from 203 double stars. As the MS, this sequence is in agreement with that derived by Perryman et al. 1995. Contrarely to the previous section, the location of the branch of the subgiant and giant stars may be affected by reddening. The effect of reddening is double: first, the primary components are shifted toward large colour indices, and, moreover, since the secondaries are reddened too, they are shifted toward the


Figure 2. The colour-magnitude diagram of the Tycho double stars. The crosses represent the components used to fit the MS, indicated by the dotted line. The diamonds are the giant and subgiant components. The origin of the absolute magnitudes is arbitrary, since the sequences were derived only from apparent magnitudes and colour indices.
bottom of the MS; therefore, the evolved stars are also shifted toward the faint magnitudes. This effect is different from the extinction, since it just comes from the reddening and from the slope of the MS.

## 4. THE DISTRIBUTIONS OF $\Delta V_{T}$ AND OF MASS RATIOS OF MS DOUBLE STARS

### 4.1. A Bias-Determined Sample of MS Double Stars

As in the previous section, only stars with astrometric qualities 1 to 5 were considered. The stars with errors $\sigma_{V_{T}}$ or $\sigma_{B_{T}}$ larger than 0.15 mag were discarded. The binaries were selected by maintaining the limit of 10 arcsec in separation. A sample of double stars with MS components was obtained by selecting the couples for which the secondary components were closer than 1 mag to the MS when the primary components were put right on. No restriction was applied on $\Delta\left(B_{T}-V_{T}\right)$; even the double stars with $\left(B_{T}-V_{T}\right)_{1}>\left(B_{T}-V_{T}\right)_{2}$ were retained when their secondary components were less than 1 mag away from the MS. 1344 double stars were thus selected. However, this sample was affected by selection effects,


Figure 3. The main sequence of the colour-magnitude diagram of the Tycho double stars, compared to the MS provided by Schmidt-Kaler 1982, converted in the Tycho photometric system. The origin of the absolute magnitude was set in order to get the same $V_{T}$ abs when $B_{T}-V_{T}=0$.
that must be determined before going to the intrinsic properties of binary stars.

The limits of complete selection of the sample were searched, applying the method of Halbwachs 1983. These limits were expressed as function of the $T$ magnitudes, since the stellar detection in the Tycho reduction was based on the photon counts in the $T$ channel. It was found that the sample was complete as long as the following two conditions were satisfied:

$$
\begin{align*}
T_{2} & <10.9  \tag{2}\\
\Delta T & <\rho / 2-0.5 \tag{3}
\end{align*}
$$

where $\rho$ is the separation, expressed in arcsec. The limit of 10.9 mag in Equation 2 in fact corresponds to the completeness limit of the whole Tycho catalogue. The double stars that were initially included within the sample although they did not satisfy to these conditions were discarded. A sample of 912 binaries with biases completely determined was thus selected. Again, the number of optical pairs was estimated from the double stars fulfilling the same conditions, but with separations in the range 50 to 60 arcsec. A proportion of 2 per cent was found.

### 4.2. The Intrinsic Distribution of $\Delta V_{T}$

The intrinsic distribution of $\Delta V_{T}$ was derived with a generalization of the recurrent method of Halbwachs 1983 for computing the distribution of $\Delta m$. The method consists in two steps:

- for each double star, calculate $\Delta V_{T}^{\mathrm{Max}}$, the maximum value of $\Delta V_{T}$ permitted by the conditions in Equations 2 and 3, assuming the magnitude and the colour index of the primary component, the separation between the components, the position of the MS, and also the deviation of the secondary component from the MS;
- the double stars with $\Delta V_{T}^{\mathrm{Max}}$ larger than a given $\Delta V_{T}^{i}$ are used to derive the intrinsic distribution of $\Delta V_{T}$ from 0 to $\Delta V_{T}^{i}$. The distributions obtained for different values of $\Delta V_{T}^{i}$ are combined in a recurrent calculation, starting from the smallest $\Delta V_{T}^{i}$ to the largest one. The consecutive values of $\Delta V_{T}^{i}$ must be close enough for deriving the final intrinsic distribution from all the stars of the sample.

The distribution of $\Delta V_{T}$ presented in Figure 4 was thus obtained. This distribution looks rather flat, and, although a gap appears near $\Delta V_{T}=2 \mathrm{mag}$, it does not look statistically significant.


Figure 4. The distribution of $\Delta V_{T}$ derived from the $T y$ cho double stars with MS components. The thin line represents the histogramme of the binaries in the biasdetermined sample; the thick line is the intrinsic distribution, when the selection effects are taken into account.

### 4.3. The Intrinsic Distribution of the Mass Ratios

The distribution of the mass ratios was derived with a technique similar to that used for the distribution of $\Delta V_{T}$, although it was a bit more complicated. For each double star, the mass ratio was computed from Tycho photometry with the following procedure: the absolute $V_{T}$ magnitude of the primary component was derived from $\left(B_{T}-V_{T}\right)_{1}$, and from the MS
presented in Figures 2 and 3; the absolute magnitude of the secondary component was then obtained from $\Delta V_{T}$; the absolute magnitudes were converted from the $V_{T}$ band to the Johnson $V$ band, taking the colour indices $B_{T}-V_{T}$ into account; the masses were then derived from the absolute $M_{V}$ magnitudes, and from the mass-luminosity relation taken from Kroupa et al. 1993.

The minimum mass ratio permitted by Equations 2 and $3, q_{\min }$ was derived for each double star, in the same way as $V_{T}^{\mathrm{Max}}$ in the previous section. The intrinsic distribution of $q$ was then computed by merging the various distributions $f_{q_{i}}$, that were each obtained from the binaries with $q_{\text {min }} \leq q_{i}$. The result is in Figure 5.


Figure 5. The distribution of the mass ratios derived from the Tycho double stars with MS components. The thin line represents the histogramme of the binaries in the bias-determined sample; the thick line is the intrinsic distribution, when the selection effects are taken into account. The dashed line is the IMF-like function $f_{q}(q) \propto q^{-2.11}$.

The distribution obtained is roughly fitted with the exponential function $f_{q}(q) \propto q^{-2.11}$. In fact, the value of the exponent is very uncertain: it comes essentially from the bins with the smallest mass ratios, that actually contained very few stars. For $q$ larger than 0.6 , the distribution derived looks too flat for a good agreement with an IMF-like distribution. However, it seems hazardous to conclude that this disagreement is significative, due to the errors coming from the estimation of the masses of the stars, and to a possible contamination by optical systems or by evolved stars.

As a conclusion, although the gap around $q \approx 0.6$ does not exist in our sample, it is quite possible that the binaries with $q \approx 1$ are actually in excess. The IMF-like shape of $f_{q}$ that we found and the excess for $q>0.9$ are in good agreement with the distribution derived by Söderhjelm (1997) for separations wider than 120 AU.

## ACKNOWLEDGEMENTS

The derivation of the HR diagram from visual double stars was suggested by C. Jaschek. The participation to the colloquium was supported by national funding from the CNES.

## REFERENCES

Abt H.A., Levy S.G., 1976, ApJS 30, 273
Duquennoy A., Mayor M., 1991, A\&A 248, 485
ESA, 1997, The Hipparcos and Tycho Catalogues, ESA SP-1200
Großmann V., Bässgen G., Evans D.W., Grenon M., Grewing M., Halbwachs J.L., Høg E., Mauder H., Snijders M.A.J., Wagner K., Wicenec A., 1995, A\&A 304, 110
Halbwachs J.-L., 1983, A\&A 128, 399
Halbwachs J.-L., Mayor M., Udry S., 1997, Brown Dwarfs and Extrasolar Planets Workshop, Puerto de la Cruz, Tenerife, March $17-21,1997$, ASP Conference Series (in press)
Høg E., 1997, this volume
Høg E., Bastian U., Egret D., Grewing M., Halbwachs J.L., Wicenec A., Bässgen G., Bernacca P.L., Donati F., Kovalevsky J., van Leeuwen F., Lindegren L., Pedersen H., Perryman M.A.C., Petersen C., Scales D., Snijders M.A.J., Wesselius P.R., 1992, A\&A 258, 177

Kroupa P., Tout C.A., Gilmore G., 1993, MNRAS, 262, 545
Oblak E., Cuypers J., Lampens P., Seggewiss W., Chareton M., Duval D., 1997, this volume
Öpik E.J., 1923, Publications de l'Observatoire Astronomique de l'Université de Tartu, vol. XXV, N. 5

Öpik E.J., 1924, Publications de l'Observatoire Astronomique de l'Université de Tartu, vol. XXV, N. 6

Perryman M.A.C., Lindegren L., Kovalevsky J., Turon C., Høg E., Grenon M., Schrijver H., Bernacca P.L., Crézé M., Donati F., Evans D.W., Falin J.L., Froeschlé M., Gómez A., Grewing M., van Leuuwen F., van der Marel H., Mignard F., Murray C.A., Penston M.J., Petersen C., Le Poole R.S., Walter H., 1995, A\&A 304,69

Schmidt-Kaler T., 1982, Numerical Data and Functional Relationships in Science and Technology, new series, Group IV vol.2b, Springer-Verlag
Söderhjelm S, 1997, International Workshop Visual Double Stars: Formation, Dynamics and Evolutionary Tracks, Santiago de Compostela, July 29 - August 1, 1996 (in press)

Virelizier P., Halbwachs J.-L., 1997, International Workshop Visual Double Stars: Formation, Dynamics and Evolutionary Tracks, Santiago de Compostela, July 29 - August 1, 1996 (in press)

