# THE LUMINOSITY FUNCTION OF MAIN-SEQUENCE STARS WITHIN 80 PARSECS FROM HIPPARCOS DATA

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# ABSTRACT

The basic data discussed here consist of parallaxes and apparent magnitudes of more than 6000 stars with  $\delta < -26$  degrees from the Hipparcos Catalogue which were selected from the Michigan Spectral Catalogue to have estimated spectroscopic distances within 80 parsecs. The statistical properties of the sample are briefly discussed, and the luminosity function of main sequence stars is determined by the maximum-volume method from nearly 3000 stars with observed parallaxes greater than 12.5 mas.

Key words: space astrometry; stars: luminosity function.

# 1. INTRODUCTION

Up to the present time, studies of the stellar content of the Galaxy in the immediate vicinity of the Sun have been based on various compilations of data for stars selected according to some limit of observed parallax, such as the catalogues of Gliese (1969), Gliese & Jahreiss (1979), Gliese & Jahreiss (1991) and Woolley et al. (1970). While primarily selected according to a limit to trigonometric parallax, all these catalogues also use spectroscopic or photometric data. The formal limit adopted by Gliese was  $\pi > 0.045$  arcsec whereas for the other catalogues it was  $\pi \ge 0.04$  arcsec. Broadly speaking, these catalogues contain data for rather fewer than 4000 stars which are supposed to be within 25 parsecs of the Sun.

To a large extent, the selection of a star to be included in any of these catalogues depended on the criteria adopted for measurement of its trigonometric parallax. A major exception to this being the dwarf K and M stars selected from spectroscopic surveys. The early parallax programmes were based on the apparently bright stars, and subsequently there has been a well-known bias in favour of stars with large proper motion; thus, apart from the late-type dwarfs, stars with small transverse velocities relative to the Sun are under-represented.

Another serious problem in the statistical interpretation of data from these catalogues has been the inhomogeneity, both in system and in accuracy, in the data from different parallax programmes.

The Hipparcos project provides a great opportunity to overcome these difficulties, while, at the same time, extending the volume of space defining the 'solar neighbourhood' which can be accurately surveyed. Ideally one would like to have data for a sample of stars complete within some known radius but clearly this is impractical because of the observational limit set by apparent magnitude. As a first step, we selected a list of stars for observation by Hipparcos, from Volumes 1, 2 & 3 of the Michigan Catalogue of Two-Dimensional Spectral Types for the HD Stars (Houk & Cowley (1975), Houk (1978), Houk (1982)). These first three volumes cover the sky south of  $\delta = -26$  degrees, which represents about 28 per cent of the sphere, and were the only ones published when the proposal was submitted in 1982. Only stars in luminosity classes IV-V and V, with estimated spectroscopic distances less than 80 parsecs, were selected, and are discussed in this paper; the intrinsically brighter stars within this distance limit will automatically be included in the general survey programme of Hipparcos, for which data must await the general release of the Hipparcos Catalogue.

### 2. STATISTICAL PROPERTIES OF THE SAMPLE

A total of 6845 stars were selected according to the stated criteria. Of these, 4400 have observed parallaxes greater than  $\pi = 10$  mas. In Figure 1 the total proper motion (ordinate) is plotted against observed parallax (abscissa) for these 4400 stars. The error ellipses have been calculated from the formal errors and correlation coefficients in the Hipparcos Catalogue. Loci corresponding to transverse speeds of  $10 \,\mathrm{km \, s^{-1}}$ ,  $20 \,\mathrm{km \, s^{-1}}$  and  $200 \,\mathrm{km \, s^{-1}}$  are shown; this latter appears to represent the effective upper limit. There are clearly many stars with small proper motions which would not have been included in classical parallax programmes. 795 stars have annual proper motions less than 50 mas, of which 78 are within 50 parsecs. 91 stars have transverse speeds less than  $5 \text{ km s}^{-1}$  of which 23 are within 50 parsecs.



Figure 1. Parallax (abscissa) versus total proper motion (ordinate) for stars with  $\pi \ge 10$  mas.

The subsequent discussion is restricted to 3905 stars for which  $\sigma_{\pi}/\pi \leq 0.125$ ; 66 per cent of the 495 omitted stars are fainter than  $m_V = 9$ . The formal variances of measured parallaxes are plotted against the  $m_V$  magnitude from the Hipparcos Catalogue, in Figure 2. While there is a clear dependence on magnitude, the global median variance of these stars is  $0.96 \text{ mas}^2$ , and only four stars have variances larger than 10 mas<sup>2</sup>.



Figure 2. Variance of observed parallaxes with  $\sigma_{\pi}/\pi \leq 0.125$ .

### 3. DENSITY DISTRIBUTION

The basic star counts in bins of  $M_V$  and shells at intervals of 10 parsecs radii are given in Table 1. The absolute magnitudes have been calculated from the apparent  $m_V$  given in the Hipparcos Catalogue without any allowance for interstellar absorption.

The last two lines of Table 1 give the distance modulus at the radius of each shell and the corresponding volume within each shell, in units of  $10^3 \text{ pc}^3$ . From the counts in Table 1 the incompleteness due to the apparent magnitude limit of about  $m_V \leq 9$  is evident. Also, there is a clear decrease in stellar density with increasing distance, even for luminous stars that should be bright enough to enter the sample even at 80 pc.

This observation is at first sight puzzling because the scale-height of the stellar density within the Galactic disk,  $\sim 300 \,\mathrm{pc}$ , significantly exceeds 80 pc and, moreover, the centre of our field lies at (l, b) = $(-57^{\circ}, -27^{\circ})$ , far from a Galactic pole. Hence we expect the surveyed population to be spatially homogeneous to a high degree. The radial density profile of even luminous stars within the sample is nevertheless non-uniform because the photometric distance moduli that were used to select the sample were subject to significant errors, even for luminous stars. Hence some stars that really lie well inside 80 pc were excluded from the sample because they were thought to lie at  $d > 80 \,\mathrm{pc}$ . Of course errors in the distance moduli caused other stars that really lie outside 80 pc, to be included in the sample, but these stars were ejected from the sample when the Hipparcos data became available. Hence errors in photometric distance moduli have thinned the underlying distribution towards the outer edge of the volume surveyed even at the brightest absolute magnitudes.

Let Q(r, s) ds be the probability that a star with true distance r is placed in the distance bin (s + ds, s) by being assigned a distance modulus that is in error by  $\Delta = 5 \log(s/r) = (5 \log e)y$ , where  $y \equiv \ln(s/r)$ . We assume that  $\Delta$  is distributed Gaussianly with dispersion  $\sigma$ , so that:

$$Q(r,s) ds = \frac{e^{-\Delta^2/2\sigma^2}}{\sqrt{2\pi\sigma}} d\Delta$$
$$= \frac{\beta}{\sqrt{\pi}} e^{-\beta^2 y^2} dy \qquad (1)$$

where  $\beta \equiv 5 \log e/(\sqrt{2}\sigma)$ . The probability P(r) that a star that has true distance r will be included in the sample is:

$$P(r) = \int_{0}^{s_{\max}} Q(r,s) \, \mathrm{d}s = \frac{\beta}{\sqrt{\pi}} \int_{-\infty}^{\ln(s_{\max}/r)} \mathrm{e}^{-\beta^{2}y^{2}} \, \mathrm{d}y$$
$$= \frac{1}{2} \{ 1 + \mathrm{erf} [\beta \ln(s_{\max}/r)] \}$$
(2)

where  $s_{\text{max}} = 80 \text{ pc}$  is the limiting photometric distance of the sample. The effective volume within distance d can now be calculated as:

$$V(d) \equiv \int_0^d P(r)r^2 \mathrm{d}r \tag{3}$$

## 4. THE LUMINOSITY FUNCTION

The most direct method for estimating the luminosity function near the Sun has been the counting of numbers of stars within particular distance limits. Wielen et al. (1983) applied this method to stars in Gliese's (1969) catalogue, supplemented by more recent data on intrinsically faint stars. Clearly the volume within which star counts can be regarded as complete will depend on luminosity. The maximum

Radius(pc)		10	20	30	40	50	60	70	80	90	100	
$M_V$												Sum
	0	0	0	0	1	0	0	0	0	0	0	1
-2.5	0	0	0	0	0	0	0	0	0	0	0	0
-1.5	0	0	0	0	1	0	0	0	0	0	0	1
-0.5	0	-	- 1	- 1	- 1	- 4	- 3	- 1	- 9	- 9	-	15
0.5	0	1	1	1	1	- -	0	1	2	2	0	41
1.5	0	1	1	0	9	8	8	2	3	ن 	0	41
2.5	1	1	7	4	19	16	24	16	24	27	0	139
2.5	0	6	22	36	59	109	119	101	121	121	3	697
3.5	<b>2</b>	13	40	89	119	184	220	236	197	153	1	1254
4.5	2	19	58	117	166	216	228	186	114	26	2	1134
5.5	5	19	41	91	138	89	64	11	1	0	0	459
6.5	4	29	44	43	12	1	0	0	0	0	0	133
7.5	0	-0 16	7			-	0	0	0	0	0	25
8.5	0	10	1	2	0	0	0	0	0	0	0	20
9.5	2	1	0	0	0	0	0	0	0	U	0	3
	3	0	0	0	0	0	0	0	0	0	0	3
Sum	19	105	991	389	525	627	666	553	462	339	6	3905
m - M	10	0.0	$1.5^{221}$	2.4	3.0	3.5	3.9	4.2	4.5	4.8	$5.0^{-1}$	0000
Vol.	1.2	8.2	23	43	72	107	149	199	256	318		

Table 1. The basic star counts in bins of  $M_V$  and shells at intervals of 10 parsecs radii.

radial distance considered by Wielen et al. was 20 parsecs, for  $M_V \leq 9.5$  .

A more satisfactory approach for analysis of the present sample is to use the maximum-volume method (Schmidt 1968). In order to apply this it is necessary to know the limiting apparent magnitude,  $m_{\rm lim}$ , of the sample. The faintest star in our sample has  $m_V = 10.4$ , and more than 100 are fainter than  $m_V = 9.5$ , but it is not evident that the sample is complete even to this magnitude. The basic source for selection for the Michigan survey is the Henry Draper Catalogue, for which the magnitude limit is not well defined. It is also necessary to adopt some value for  $\sigma$ , the dispersion in errors of distance moduli which is a combination of the dispersion in absolute magnitudes at a given spectral type and of errors in the apparent magnitudes used in calculating the distance moduli.

For each star, we evaluate from Equation 3 the effective volume, V, and the corresponding maximumvolume  $V_{\text{max}}$ , within which it would remain in the sample, according to the adopted selection criteria. Assuming that the sample is complete, and that the space density is uniform within the volume, the luminosity function is given by  $\sum (1/V_{\text{max}})$  summed over all stars for each interval of  $M_V$ . Completeness, and uniformity can be tested from the distribution of the statistic  $V/V_{\text{max}}$  which should be uniform in (0,1).

Figure 3 shows the distribution of  $V/V_{\rm max}$  with  $M_V$ , derived by adopting  $m_{\rm lim} = 9$  and  $\sigma = 0.6$  mag. These values of  $m_{\rm lim}$  and  $\sigma$  were chosen from various trials, to ensure that the median of  $V/V_{\rm max} = 0.5$ over all  $M_V$ . This solution is based on 2428 stars; 74 stars with  $\sigma_{\pi}/\pi > 0.125$  were not included. The resulting luminosity function  $\Phi(M_V)$ , in units of stars per 1000 cubic parsecs, is given in Table 2; the errors associated with each value are calculated from  $\Phi(M_V)/\sqrt{N}$  where N is the total number of stars in each absolute magnitude bin. The main body of the table gives the numbers of stars in each 0.1 bin of  $V/V_{\text{max}}$ . The last column gives the effective distance limit,  $r_{\text{eff}}$ , of the sample at each  $M_V$ ; for the intrinsically bright stars this is the adopted distance limit for the analysis, whereas for  $M_V > 4.5$  it is set by the value of  $m_{\text{lim}}$ .



Figure 3.  $V/V_{\rm max}$  versus absolute magnitude, for the adopted solution.

In Figure 4 we show this luminosity function compared with that of Wielen et al. with their corresponding error bars. Clearly the results from the present sample contribute very little, for  $M_V > 9$ , because of the relatively bright apparent magnitude limit. At brighter values of  $M_V$  our results appear to be very well determined, although generally lower than those of Wielen et al. This may be partly due to

Table 2. The derived Luminosity Function  $\Phi(M_V)$  in units of stars per 1000 cubic parsecs, together with the number of stars in each 0.1 bin of  $V/V_{\text{max}}$ . The last column gives the effective distance limit,  $r_{\text{eff}}$ , of the sample at each  $M_V$ .

$M_V$	$\Phi(M_V)$	$V/V_{ m max}$									$r_{ m eff}$		
_		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
-3.5	$0.002\pm0.002$	0	0	1	0	0	0	0	0	0	0		80.0
-2.5	$0.000\pm0.000$	0	0	0	0	0	0	0	0	0	0		80.0
-1.5	$0.002\pm0.002$	0	0	1	0	0	0	0	0	0	0		80.0
-0.5	$0.023\pm0.007$	1	1	1	2	2	2	0	1	0	1		80.0
0.5	$0.074 \pm 0.012$	5	7	5	4	4	4	3	2	1	0		80.0
1.5	$0.188\pm0.020$	10	10	11	11	5	11	6	12	5	8		80.0
2.5	$0.958\pm0.045$	39	46	36	52	52	54	42	45	41	47		80.0
3.5	$1.908\pm0.063$	86	85	84	97	83	87	102	82	100	98		80.0
4.5	$2.381 \pm 0.092$	69	74	60	74	77	65	61	68	62	59		79.4
5.5	$2.371 \pm 0.175$	23	9	15	16	16	16	20	23	22	23		50.1
6.5	$3.312\pm0.405$	8	8	4	8	5	6	11	7	4	6		31.6
7.5	$1.778\pm0.629$	0	0	0	0	1	1	1	1	2	2		20.0
8.5	$2.002 \pm 1.156$	0	0	1	0	0	0	0	1	1	0		12.6
9.5	$6.857 \pm 4.849$	0	1	0	0	0	0	1	0	0	0		7.9
10.5	$11.653 \pm 11.653$	0	0	0	0	0	0	0	0	1	0		5.0

the fact that our sample includes only stars in luminosity classes IV-V and V, so that giants have been excluded.



Figure 4. Luminosity function for class IV-V & V stars, compared with Wielen et al. (1983).

The results presented here should be regarded as provisional pending further investigation into completeness limit of apparent magnitudes which should be possible from the Tycho data. Furthermore, the magnitude limit of completeness of the Hipparcos Catalogue, depending on galactic latitude, is well defined, so that a definitive evaluation of the bright end of the luminosity function will be possible when the catalogue becomes generally available.

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