THE ABSOLUTE MAGNITUDE OF RR LYRAE STARS

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ABSTRACT

We derive the absolute magnitude of RR Lyrae stars by two independent maximum-likelihood estimations, using Hipparcos proper motion and parallax data, respectively. By applying the statistical parallax method to 99 Halo RR Lyrae stars, we obtain the mean absolute magnitude $< M_V >$ $= 0.69 \pm 0.10$ at <[Fe/H]>=–1.58. Independently of this method, we furthermore derive the M_V – [Fe/H] relation from Hippparcos parallaxes for 125 RR Lyrae stars including negative ones by the method of Ratnatunga & Casertano. We obtain M_V=0.91(±0.88)+0.21(±0.63)[Fe/H]. For 99 Halo RR Lyrae stars, $< M_V > = 0.65 \pm 0.33$ is also derived.

Key words: RR Lyrae stars.

1. INTRODUCTION

The absolute magnitude (M_V) of RR Lyrae stars is an indicator for distance and age in our Galaxy. Especially it is a key parameter to determine the age of Galactic globular clusters. However, the value of M_V defies a consensus and splits into a faint value and a bright value. The faint value of $M_V \sim 0.75$ at the characteristic abundance of halo ([Fe/H]=–1.6), which gives a short distance scale, is derived by e.g., statistical parallax method (Barnes & Hawley 1986, Strugnell et al. 1986, Layden et al. 1996) and Baade-Wesselink analysis (Jones et al. 1992, Liu & Janes 1990). On the other hand, the bright value ~ 0.45, which gives a long distance scale, is derived by e.g., LMC RR Lyraes (Walker 1992) and Sandage's pulsation theory (Sandage 1993). Such a difference yields more than 3 Gyr difference in the derived age of the Galactic globular clusters, and it represents the main uncertainty in the determination of cluster ages.

In general, a linear relation between M_V and [Fe/H] has been assumed. The Baade-Wesselink results give the values of M_V over the range of [Fe/H]= –2.2 to 0, which are fitted by the relation: M_V=1.02+0.16[Fe/H] (Storm et al. 1994). However, the controversial situation holds also for determining the slope of M_V–[Fe/H] relation. Sandage's pulsation theory gives much steeper slope, whereas statistical palallax analyses support very weak (or no) dependence of M_V on [Fe/H]. The slope of M_V–[Fe/H] relation has a strong influence on the inferred age difference in the Galactic globular clusters, which gives the dynamical timescale for halo formation.

In this paper, we determine M_V using two methods, i.e., the statistical parallax method and the statistical treatment of trigonometric parallaxes. The former method is a classical one to derive M_V, whereas the latter method yields a 'first' direct estimation of M_V.

2. M_V DERIVED FROM HIPPARCOS PROPER MOTION

First we perform a maximum-likelihood statistical analysis to determine the absolute magnitude and kinematics of metal-poor halo RR Lyrae stars. Our application is taken from Murray (1983) and Hawley et al. (1986). The data of [Fe/H], radial velocity, apparent magnitude, and reddening for each star are taken from Layden et al. (1996)) and Layden (1994). Combining Hipparcos proper motions with these data for a halo component (<[Fe/H]>= –1.58), we find $< M_V >$=0.69±0.10 together with the solar motion with respect to the galactic center ($U_\odot$,$V_\odot$,$W_\odot$)=(–12±17,–200±11,+1±3)km s^{-1} and the velocity dispersions (σ_U,σ_V,σ_W) =(160±13,104±9,86±7) km s^{-1}. The RR Lyrae stars as a whole rotate with $V_{\rm rot} \simeq 30$ km s^{-1}, assuming $V_{\rm LSR}$=220 km s^{-1} and a solar peculiar motion of 16.5 km s^{-1} in the direction l=53° and b=25°. These results are in good agreement with those obtained by Layden et al. (1996) and Fernley et al. (1997). The results of our analysis compared with those of Layden et al. (1996) are shown in Table 1.

Unfortunately we have only 26 RR Lyrae stars for [Fe/H]>–1, which belong to the thick disk component. The number is too small to apply the maximum-likelihood statistical analysis for these stars. At least 50 stars are needed to obatin the meaningful solutions (Hawley et al. 1986). Therefore we cannot discuss the dependence of M_V on [Fe/H] in the present analysis.

Table 1. Results of statistical parallax analysis.

	$N_{\rm stars}$	<[Fe/H]>	$U_\odot$	$V_\odot$	$W_\odot$	σ_U	σ_V	σ_W	M_V
this work	99	−1.58	−12±17	−200±11	+1±3	160±13	104±9	86±7	0.69±0.10
Layden et al. (1996)	162	−1.61	+9±14	−210±12	-12±8	168±13	102±8	97±7	0.71±0.12

Table 2. Precision of Hipparcos Parallaxes for RR Lyrae stars.

	total	$\sigma_\pi/\pi < 0.1$	$\sigma_\pi/\pi < 0.2$	$\sigma_\pi/\pi < 0.3$	$\sigma_\pi/\pi > 1$	$\pi < 0$
$N_{\rm stars}$	173	0	1	2	65	57

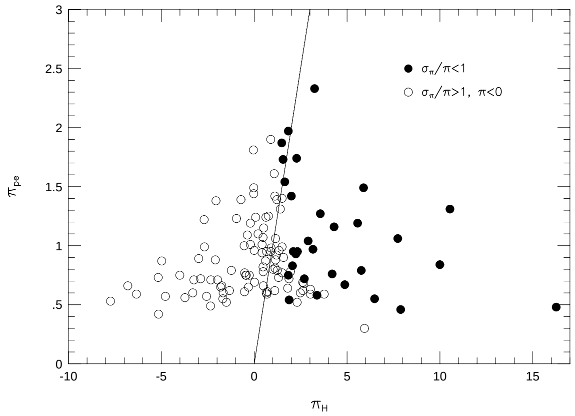

Figure 1. The comparison between Hipparcos parallax $\pi_{\rm H}$ and the photometric parallax $\pi_{\rm pe}$. The straight line indicates the relation $\pi_{\rm H}$=$\pi_{\rm pe}$.

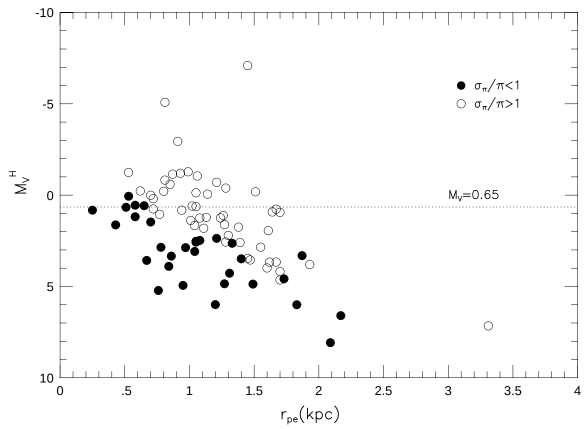

Figure 2. The absolute magnitude $M_V^{\rm H}$ inferred from Hipparcos parallax as a function of the photometric distance.

3. HIPPARCOS PARALLAXES

As shown in Table 2, most of Hipparcos parallaxes of RR Lyrae stars are measured with large errors. Figure 1 shows the comparison between Hipparcos parallax $\pi_{\rm H}$ and the photometric parallax $\pi_{\rm pe}$, which is defined as:

$$1/\pi_{\rm pe} = r_{\rm pe} = 10^{0.2(V-M_V^c-10)} \tag{1}$$

where V is the apparent magnitude corrected for reddening and M_V^c is the calibrated absolute magnitude, which can be written as a function of [Fe/H]:

$$M_V^c = a + b[{\rm Fe/H}] \tag{2}$$

Here a=0.92, b=0.21 are adopted (see the next section). The filled circles represent the stars meaured with relative error σ_π/π smaller than 1. The deviation of Hipparcos parallaxes from the photometric ones is very large, but the mean value of Hipparcos parallaxes including low accuracy and negative parallaxes coincide with the photometric parallaxes.

Figure 2 shows the absolute magnitude $M_V^{\rm H}$ inferred from Hipparcos parallaxes ($M_V^{\rm H} = V + 5 + 5\log\pi_{\rm H}$) of each star with positive parallax as a function of the photometric distance $r_{\rm pe}$. Such a tendency that stars with larger relative errors have brighter luminosities, i.e., have smaller parallaxes, appears clearly when the true parallax is small, compared with error of parallax. Similarly the distant stars have too faint luminosities, i.e., have too large parallaxes, mainly because the true parallax is much smaller than error of parallax.

Only one RR Lyrae star (HIC95497:[Fe/H]=−1.37) is measured with a high accuracy $\sigma_\pi/\pi = 0.13$. Therefore, $M_V^{\rm H}$ of this star derived directly from its parallax may represent an accurate estimate of M_V. However, we should notice that trigonometric parallax measurements are subject to a systematic bias, which yields a tendency for the observed parallaxes to be larger than their actual ones. Owing to this bias, the luminosities derived directly from their parallaxes are underestimated. To compensate for this effect, we should apply the Lutz-Kelker corrections (Lutz & Kelker 1973). Here we use the formulation given by Hanson (1979) to estimate the magnitude of the Lutz-Kelker correction and obtain the corrected value $M_V^H = 0.62$–0.72 for HIC95497, depending on the space distribution of stars.

In the next section, we determine M_V from the parallax data including even negative parallaxes by a statistical treatment.

4. M_V DERIVED FROM HIPPARCOS PARALLAXES

Deriving the absolute magnitude from trigonometric parallaxes, we should make corrections for the biases caused by the intrinsic dispersion of magnitude (the Malmquist bias) and by errors in parallax (Lutz & Kelker 1973). Ratnatunga & Casertano (1991) proposes a maximum-likelihood estimation method to make all of such corrections. This method allows us to use even negative parallaxes. Therefore it is a powerful tool for distant stars which are observed by Hipparcos satellite with very low accuracy like RR Lyrae stars. The detail of the results is described in Tsujimoto et al. (1997).

4.1. The Calibration Model

Owing to the intrinsic magnitude dispersion σ_M, the true absolute magnitude M_V is expected to deviate from the calibrated magnitude M_V^c. We define such a deviation as Δ_V by:

$$M_V = M_V^c - \Delta_V \tag{3}$$

The distribution of Δ_V is represented by the Gaussian distribution function $P_G(\Delta_V, 0, \sigma_M)$, which is defined as:

$$P_G(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\mathrm{pi}}} \exp[-\frac{1}{2}(\frac{x-\mu}{\sigma})^2] \tag{4}$$

In practice, the deviation Δ_V is caused by not only the intrinsic dispersion but also the dispersion due to photometric errors. So we define the effective magnitude dispersion by:

$$\sigma_M^{\mathrm{eff}} = \sqrt{\sigma_M^2 + \sigma_V^2 + b^2\sigma_{[\mathrm{Fe/H}]}^2} \tag{5}$$

We adopt here $\sigma_V = 0.02$ mag and $\sigma_{[\mathrm{Fe/H}]} = 0.15$ dex.

Combining Equation [1] with Equation [3], the true distance r can be written in the form:

$$r = r_{\mathrm{pe}} 10^{0.2\Delta_V} \tag{6}$$

Taking into account the number density of stars $n(r)$, we obtain the probability distribution of the magnitude deviation:

$$p(\Delta_V)d\Delta_V \propto P_G(\Delta_V, 0, \sigma_M^{\mathrm{eff}})n(r)r^2dr \tag{7}$$

where $n(r)r^2dr$ denotes the number of stars per unit solid angle between r and $r+dr$. Using Equation [6], we rewrite Equation [7] in the form:

$$p(\Delta_V) \propto P_G(\Delta_V, 0, \sigma_M^{\mathrm{eff}})n(r)10^{0.6\Delta_V} \tag{8}$$

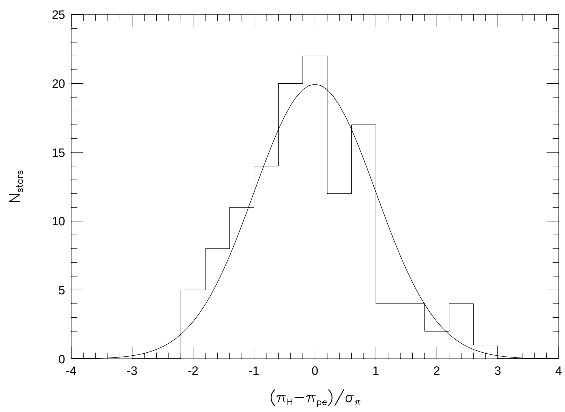

Figure 3. The distribution of the normalized residuals, ($\pi_\mathrm{H} - \pi_\mathrm{pe}$) divided by the error on Hippracos parallax.

The error on the observed parallax π_{obs} is supposed to be Gaussian $P_G(\pi_{\mathrm{obs}}, \pi, \sigma_\pi)$ around the true parallax π. Then the probability distribution of the true parallax $p(\pi|\pi_{\mathrm{obs}})$ for the observed parallax π_{obs} is:

$$p(\pi|\pi_{\mathrm{obs}}) = P_G(\pi_{\mathrm{obs}}, \pi, \sigma_\pi)p(\Delta_V) \tag{9}$$

By integrating $p(\pi|\pi_{\mathrm{obs}})$ over the true parallax and its normalization, we obtain the conditional probability of the observed parallax:

$$p(\pi_{\mathrm{obs}}) = \frac{\int_0^\infty p(\pi|\pi_{\mathrm{obs}})d\pi}{\int_0^\infty \int_{\pi_{\mathrm{lower}}}^{\pi_{\mathrm{upper}}} p(\pi|\pi_{\mathrm{obs}})d\pi_{\mathrm{obs}}d\pi} \tag{10}$$

where $[\pi_{\mathrm{lower}}, \pi_{\mathrm{upper}}]$ is an allowable range in parallax, outside which the star is defined as an outlier. If the parallax data used for the calibration has not been censored, the correspondent range of $[\pi_{\mathrm{lower}}, \pi_{\mathrm{upper}}]$ is $(-\infty, \infty)$.

The log-likelihood $\ln L$ of the aggregate of all the probabilities:

$$\ln L = \Sigma \ln p(\pi_{\mathrm{obs}}) \tag{11}$$

is a function of the model parameters (a,b,σ_M). According to the likelihood principle, the best values of the model parameters are those which $\ln L$ is a maximum.

4.2. Results

We apply the above method to 125 RR Lyrae stars in the metallicity range –2.49<[Fe/H]<0.07. Setting $\pi_{\mathrm{lower}} = -\infty$ and $\pi_{\mathrm{upper}} = \infty$ and assuming $n(r)$=constant, we obtain the following relation;

$$M_V = 0.91(\pm 0.88) + 0.21(\pm 0.63)[\mathrm{Fe/H}] \tag{12}$$

with $\sigma_M = 2.6 \times 10^{-4} \pm 0.29$. If we fix σ_M at the value of 0.1–0.2 (Layden et al. (1996)), the same results are derived. The calibrated relation is in excellent agreement with the one derived from a survey

Table 3. $< M_V >$ *at <[Fe/H]>=–1.58 for the various censorship.*

$[\pi_{\rm lower},\pi_{\rm upper}]$	M_V	$N_{\rm stars}$
$(-\infty,\infty)$	0.65 ± 0.33	99
$(0,\infty)$	0.67 ± 0.37	66
$(1,\infty)$	0.67 ± 0.39	50
$(2,\infty)$	0.87 ± 0.41	30
$(-\infty,5)$	0.59 ± 0.37	89
$(-\infty,2)$	0.90 ± 0.81	69
$(-\infty,0)$	1.27 ± 1.96	32

of the current literature by Chaboyer et al. (1996a): $M_V = 0.98 + 0.20$[Fe/H]. To see the dependence of the results on the stellar density distribution $n(r)$, we calculate the case for $n(r) \propto r^2$ (see Reid 1997) and obtain the same result, by which the insensitivity of the results to the density distribution $n(r)$ is checked. The calibrated relation has a large error; our best estimate is M_V=0.57 at [Fe/H]=–1.6

The distribution of the difference between Hipparcos parallaxes and the photometric parallaxes derived by Equation [12], normalized by the error on Hipparcos parallax, is ploted in Figure 3 for 125 RR Lyrae stars. This distribution is compared with the Gaussian distribution (0,1).

We also determine the mean absolute magnitude for the halo component ([Fe/H]<–1) of Hipparcos RR Lyrae stars, assuming no dependence on [Fe/H] (i.e., the model parameters to be determined are a and σ_M). The solution for 99 RR Lyrae stars (<[Fe/H]> =–1.58) is $< M_V >= 0.65 \pm 0.33$. This method permits the censorship on parallax at our disposal. In principle, the same results should be obtained for any range $[\pi_{\rm lower},\pi_{\rm upper}]$ of observed parallaxes. However, if a censorship on the observed parallax is introduced, the reliability of solutions reduces, in general, because the number of stars used for calibration decreases. The values of M_V at <[Fe/H]>=–1.58 for the various censorship are tabulated in Table 3.

5. CONCLUSIONS

We derive the absolute magnitude of RR Lyrae stars directly from Hipparcos trigonometric parallaxes as well as from statistical parallax analysis based on Hipparcos proper motions. The results obtained are as follows:

- Statistical parallax analysis gives

  $< M_V >$=0.69±0.10 at <[Fe/H]>=–1.58

- Trigonometric parallax gives

  M_V=0.91(±0.88)+0.21(±0.63)[Fe/H]

  for –2.49<[Fe/H]<0.07

  and

  $< M_V >$=0.65±0.33 at <[Fe/H]>=–1.58

Recently, Feast & Catchpole (1997) derive M_V of RR Lyraes using the Cepheid distance to the LMC determined by Hipparcos parallaxes of Cepheid (LMC distance modulus=18.70) and the data of LMC RR Lyrae stars by Walker (1992). They gives $M_V = 0.25$ at [Fe/H]=–1.9. Moreover, Ried (1997) estimates the distance scale to the globular clusters by subdwarf main-sequence fitting, using Hipparcos parallax data, leading to M_V=0.15 at [Fe/H]=–2.1. These bright values of M_V give much younger age for the oldest globular clusters than the previous estimation (Bolte & Hogan 1995, Chaboyer et al. 1996b). However, the statistical parallax analysis and the 'direct' estimation of M_V in the present study support much fainter value than those obtained by them. That means, the age of the oldest globular clusters in our Galaxy still conflicts with the standard cosmological model of a flat matter dominated universe with the Hubble constant estimated from almost all observations. It appears that the age problem remains unresolved.

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